A

COMPLETE SYSTEM

OF

ASTRONOMY;

BY THE

REV. S. VINCE, A.M. F.R.S.

PLUMIAN PROFESSOR of ASTRONOMY and EXPERIMENTAL PHILOSOPHY in the UNIVERSITY of CAMBRIDGE.

VOL. I.

CAMBRIDGE,

Printed by J. BURGES, Printer to the UNIVERSITY;

And Sold by J. DEIGHTON, and J. NICHOLSON, Cambridge; F. WINGRAVE, P. EMSLEY, and H. GARDNER, in the Strand; B. & J. WHITE, Fleet-Street; W. H. LUNN, Oxford-Street; R. FAULDER, Bond-Street; F. & C. RIVINGTON, and G. & T. WILKIE, St. Paul's Church-yard; T. & J. EGERTON, White-Hall, London; J. FLETCHER, and D. PRINCE & J. COOKE, Oxford.

MDCCXCVII.

BRITISH MUSEUM

TO

THE REVEREND

NEVIL MASKELYNE, D.D. F.R.S.

ASTRONOMER ROYAL,

THIS SYSTEM OF ASTRONOMY IS DEDICATED

BY THE AUTHOR;

AS WELL

FROM MOTIVES OF PRIVATE FRIENDSHIP,

AS

FROM AN EARNEST DESIRE TO PAY A PUBLIC TRIBUTE

OF SINCERE RESPECT,

JUSTLY DUE TO HIM FOR THE IMPROVEMENTS

WHICH HE HAS MADE IN THIS,

AND EVERY OTHER SUBSIDIARY BRANCH OF PHILOSOPHY;

AND

FOR SO ABLY DISCHARGING

THE ARDUOUS DUTIES OF HIS HIGH AND IMPORTANT

STATION,

BY WHICH HE HAS DONE SO MUCH

HONOUR TO HIMSELF

AND

TO HIS COUNTRY.

THIS Volume contains the PRINCIPLES of PLANE ASTRONOMY, in which the Reader will find references made to a Set of Auxiliary Tables at the end; but the matter having increaſed beyond the Author's expectation, theſe Tables would have enlarged the Volume to an unuſual ſize; they will therefore be found at the end of the next Volume. In conſequence of this, the other ſubjects to be treated of will furniſh matter ſufficient for a ſecond Volume, without the Tables of the Motions of the Sun, Moon, Planets and Jupiter's Satellites; theſe Tables will therefore be afterwards publiſhed by themſelves, with an Introduction explaining their Conſtruction and Uſe.

THE CONTENTS OF THE FIRST VOLUME.

CHAPTER I.

PAGE.

Definitions 1

CHAP. II.

On the Doctrine of the Sphere 9

CHAP. III.

On the Right Aſcenſion, Declination, Latitude and Longitude of the Heavenly Bodies 30

CHAP. IV.

On the Equation of Time 42

CHAP. V.

On the Length of the Year, Preceſſion of the Equinoxes, and Obliquity of the Ecliptic 50

CHAP.

CHAP. VI.

PAGE.
On Parallax 60

CHAP. VII.

On Refraction 76

CHAP. VIII.

On the Syſtem of the World 93

CHAP. IX.

On KEPLER'S *Diſcoveries* 98

CHAP. X.

On the Motion of a Body in an Ellipſe about the Focus 104

CHAP. XI.

On the Oppoſitions and Conjunctions of the Planets 116

CHAP. XII.

On the Mean Motions of the Planets 119

CHAP. XIII.

On the greateſt Equation, Excentricity, and Place of the Aphelia of the Orbits of the Planets 130

CHAP. XIV.

On the Motion of the Aphelia of the Orbits of the Planets 150

CHAP.

CHAP. XV.

PAGE.

On the Nodes and Inclinations of the Orbits of the Planets to the Ecliptic 155

CHAP. XVI.

On the Georgian Planet 166

CHAP. XVII.

On the Apparent Motions and Phaſes of the Planets 177

CHAP. XVIII.

On the Moon's Motion from Obſervation, and it's Phænomena . . . 185

CHAP. XIX.

On the Rotation of the Sun, Moon, and Planets 218

CHAP. XX.

On the Satellites 237

CHAP. XXI.

On the Ring of Saturn 301

CHAP. XXII.

On the Aberration of Light 311

CHAP. XXIII.

On the Projection for the Conſtruction of Solar Eclipſes 339

CHAP.

CHAP. XXIV.

PAGE.

On Eclipſes of the Sun and Moon, and Occultations of the Fixed Stars by the Moon 345

CHAP. XXV.

On the Tranſits of Mercury and Venus over the Sun's Diſc 393

CHAP. XXVI.

On Comets 424

CHAP. XXVII.

On the Fixed Stars 491

CHAP. XXVIII.

On the Longitude 515

CHAP. XXIX.

On the Uſe of the Globes 561

CHAP. XXX.

On the Diviſion of Time 571

A COMPLETE

A COMPLETE SYSTEM OF ASTRONOMY.

CHAPTER I.

DEFINITIONS.

Art. 1. ASTRONOMY is that branch of natural philoſophy which treats of the heavenly bodies. The determination of their magnitudes, diſtances and the orbits which they deſcribe, is called *plane* or *pure* Aſtronomy; and the inveſtigation of the cauſes of their motions is called *phyſical* Aſtronomy. The *former* is determined from obſervations on their apparent magnitudes and motions; and the *latter* from analogy, by applying thoſe principles and laws of motion by which bodies on and near the earth are governed, to the other bodies in the ſyſtem. The principles of plane Aſtronomy come firſt in order to be treated of; and in this, we ſhall begin with the explanation of ſuch terms as are the fundamental principles of the ſcience.

2. A *great* circle of a ſphere is that whoſe plane paſſes through its center; and a *ſmall* circle is that whoſe plane does not paſs through its center.

3. A diameter of a ſphere perpendicular to any great circle is called the *axis* of that circle; and the extremities of the diameter are called its *poles*.

4. Hence the pole of a great circle is 90° from every point of it upon the ſurface of the ſphere; but as the axis is perpendicular to the circle when it

is perpendicular to any two radii, a point on the ſurface of a ſphere 90° diſtant from any two points of a great circle will be the pole.

5. All angular diſtances on the ſurface of a ſphere, to an eye at the center, are meaſured by the arcs of *great* circles; for they being arcs to equal radii will be as the angles.

6. Hence, all triangles, formed upon the ſurface of a ſphere, for the ſolution of ſpherical problems, muſt be formed by the arcs of *great* circles.

7. All great circles muſt biſect each other; for paſſing through the center of the ſphere their common ſection muſt be a diameter, which biſects all circles.

8. *Secondaries* to a great circle are great circles which paſs through its poles.

9. Hence ſecondaries muſt be perpendicular to their great circle; for if one line be perpendicular to a plane, any plane paſſing through that line will alſo be perpendicular to it; therefore as the axis of the great circle is perpendicular to it, and is the common diameter to all the ſecondaries, they muſt all be perpendicular to the great circle. Hence alſo every ſecondary, biſecting its great circle (7)*, muſt biſect every ſmall circle parallel to it; for the plane of the ſecondary paſſes through, not only the center of the great circle, but alſo of the ſmall circles parallel to it.

10. Hence a great circle paſſing through the poles of two great circles, muſt be perpendicular to each; and, vice versâ, a great circle perpendicular to two other great circles muſt paſs through their poles.

11. If an eye be in the plane of a circle it appears a ſtraight line; hence in the repreſentation of the ſurface of a ſphere upon a plane, thoſe circles whoſe planes paſs through the eye are repreſented by ſtraight lines.

12. The angle formed by the circumferences of two great circles on the ſurface of a ſphere is equal to the angle formed by the planes of thoſe circles; and is meaſured by the arc of a great circle intercepted between them, deſcribed about the interſection of the circles as a pole.

FIG. 1. For let C be the center of the ſphere, PQE, PRE two great circles; then as the circumferences of theſe circles at P are perpendicular to the common interſection PCE, the angle at P between them is equal to the angle between the planes, by Euc. B. XI. Def. 6. Now draw CQ, CR perpendicular to PCE; then the angle QCR is alſo the angle between the planes, and therefore equal to the angle at P formed by the two circles; and the angle QCR is meaſured by the arc QR of a great circle, which arc has (4) for its pole the point P, becauſe PQ, PR are each 90°.

13. If at the interſection of two great circles as a Pole, a great circle be deſcribed, and alſo a ſmall circle parallel to it, the arcs of the great and ſmall circles

* Figures included thus in a Parentheſis refer to the articles.

cles intercepted between the two great circles contain the ſame number of degrees.

For draw *AB*, *AD* perpendicular to *PCE*, then as *AB*, *AD* are parallel to *CQ*, *CR*, the plane *BAD* is parallel to the plane *QCR*, and therefore the ſmall circle *BD* of which *A* is the center, is parallel to the great circle *QR*; and as each angle *BAD*, *QCR*, meaſures the inclination of the planes, they muſt be equal, and conſequently the arcs *BD*, *QR* contain the ſame number of degrees. Hence the arc of ſuch a ſmall circle meaſures the angle at the pole between the two great circles. Alſo *QR* : *BD* :: *QC* : *BA* :: radius : cos. *BQ*. Hence *QR* is the greateſt diſtance between the two circles.

14. The *axis* of the earth is that diameter about which it performs its diurnal motion; and the extremities of this diameter are called its *poles*.

15. The *terreſtrial equator* is a great circle of the earth perpendicular to its axis. Hence the axis and poles of the earth are the axis and poles of its equator. That half of the earth which lies on the ſide of the equator which we inhabit is called the *northern hemiſphere*, and the other the *ſouthern*; and the poles are reſpectively called the *north* and *ſouth* poles.

16. The *latitude* of a place on the earth's ſurface is its angular diſtance from the equator, meaſured upon a ſecondary to it. Theſe ſecondaries to the equator are called *meridians*.

17. The *longitude* of a place on the earth's ſurface is an arc of the equator intercepted between the meridian paſſing through the place, and another, called the firſt meridian, paſſing through that place from which you begin to meaſure.

18. If the plane of the *terreſtrial* equator be produced to the ſphere of the fixed ſtars, it marks out a circle called the *celeſtial* equator; and if the axis of the earth be produced in like manner, the points in the Heavens to which it is produced are called *poles*, being the poles of the celeſtial equator. The ſtar neareſt to each pole is called the *pole* ſtar.

19. Secondaries to the celeſtial equator are called *circles of declination*; of theſe, 24 which divide the equator into equal parts, each containing 15°, are called *hour* circles.

20. Small circles parallel to the celeſtial equator are called *parallels of declination*.

21. The *ſenſible* horizon is that circle in the heavens whoſe plane touches the earth at the ſpectator. The *rational* horizon is a great circle in the heavens, paſſing through the earth's center, parallel to the ſenſible horizon.

22. *Almacanter* is a ſmall circle parallel to the horizon.

23. If the radius of the earth to the place where the ſpectator ſtands be produced both ways to the heavens, that point vertical to him is called the *zenith*, and the oppoſite point the *nadir*. Hence the *zenith* and *nadir* are (3) the poles

of the *rational* horizon; for the radius produced being perpendicular to the ſenſible, muſt alſo be perpendicular to the rational horizon.

24. Secondaries to the horizon are called *vertical* circles, becauſe they are (9) perpendicular to the horizon; on theſe circles therefore the altitude of an heavenly body is meaſured.

25. A Secondary common to the celeſtial equator and the horizon of any place, and which therefore (10) paſſes through the poles of each, is the *celeſtial* meridian of that place. Hence the plane of the celeſtial meridian of any place coincides with the plane of the terreſtrial meridian of the ſame place.

26. That direction which paſſes through the north pole is called *north*, and the oppoſite direction is called *ſouth*. Hence the meridian muſt cut the horizon in the *north* and *ſouth* points.

27. Hence the meridian of any place divides the heavens into two hemiſpheres lying to the eaſt and weſt; that lying to the eaſt is called the *eaſtern* hemiſphere, and the other lying to the weſt is called the *weſtern* hemiſphere.

28. The vertical circle which cuts the meridian of any place at right angles is called the *prime* vertical; and the points where it cuts the horizon are called the *eaſt* and *weſt* points. Hence the eaſt and weſt points are 90° diſtant from the north and ſouth. Theſe four are called the *cardinal* points.

29. The *azimuth* of an heavenly body is its diſtance on the horizon, when referred to it by a ſecondary, from the north or ſouth points. The *amplitude* is its diſtance from the eaſt or weſt points.

30. The *ecliptic* is that great circle in the heavens which the ſun appears to deſcribe in the courſe of a year.

31. The ecliptic and equator being great circles muſt (7) biſect each other, and their angle of inclination is called the *obliquity of the ecliptic*; alſo the points where they interſect are called the *equinoctial* points. The times when the ſun comes to theſe points are called the *equinoxes*.

32. The ecliptic is divided into 12 equal parts, called *ſigns*; Aries ♈, Taurus ♉, Gemini ♊, Cancer ♋, Leo ♌, Virgo ♍, Libra ♎, Scorpio ♏, Sagittarius ♐, Capricornus ♑, Aqarius ♒, Piſces ♓. The order of theſe is according to the motion of the ſun. The firſt point of aries coincides with one of the equinoctial points, and the firſt point of libra with the other. The firſt ſix ſigns are called *northern*, lying on the *north* ſide of the equator; and the laſt ſix are called *ſouthern*, lying on the *ſouth* ſide. The ſigns ♑, ♒, ♓, ♈, ♉, ♊ are called *aſcending*, the ſun approaching our (or the north) pole whilſt it paſſes through them; and ♋, ♌, ♍, ♎, ♏, ♐ are called *deſcending*, the ſun receding from our pole as it moves through them.

33. The motion of the heavenly bodies which is according to the order of the ſigns, is called *direct*, or *in conſequentia*; and the motion in the contrary direction

rection is called *retrograde*, or *in antecedentia*. The *real* motion of all the planets is according to the order of the ſigns, but their *apparent* motion is ſometimes in an oppoſite direction.

34. The *zodiac* is a ſpace extending 8° on each ſide of the ecliptic, within which the motion of all the planets is contained.

35. The *right aſcenſion* of a body is an arc of the equator intercepted between the firſt point of aries and a declination circle paſſing through the body, meaſured according to the order of the ſigns.

36. The *oblique aſcenſion* is an arc of the equator intercepted between the firſt point of aries and that point of the equator which riſes with any body, meaſured according to the order of the ſigns.

37. The *aſcenſional difference* is the difference between the right and oblique aſcenſion.

38. The *declination* of a body is its angular diſtance from the equator, meaſured upon a ſecondary to it drawn through the body.

39. The *longitude* of a ſtar is an arc of the ecliptic intercepted between the firſt point of aries and a ſecondary to the ecliptic paſſing through the body, meaſured according to the order of the ſigns. If the body be in our ſyſtem, and ſeen from the *ſun*, it is called the *heliocentric* longitude; but if ſeen from the *earth*, it is called the *geocentric* longitude; the body in each caſe being referred perpendicularly to the ecliptic in a plane paſſing through the eye.

40. The *latitude* of ſtar is its angular diſtance from the ecliptic, meaſured upon a ſecondary to it drawn through the body. If the body be in our ſyſtem, its angular diſtance from the ecliptic ſeen from the *earth* is called the *geocentric* latitude; but if ſeen from the *ſun* it is called the *heliocentric* latitude.

41. Hence if ♈ ♎ be the equator, ♈ *C* the ecliptic, ♈ the firſt point of aries, *s* a ſtar, and the great circles *sr*, *sn* be drawn perpendicular to ♈ *C* and ♈ ♎; then ♈ *n* is its right aſcenſion, *sn* its declination, *sr* its latitude and ♈ *r* its longitude. The circle *sr* is called a *circle of latitude*. FIG. 2.

42. The *tropics* are two parallels of declination touching the ecliptic. One, touching it at the beginning of cancer, is called the *tropic of cancer*; and the other, touching it at the beginning of capricorn, is called the *tropic of capricorn*. The two points where the tropics touch the ecliptic are called the *ſolſtitial* points.

43. *Colures* are two ſecondaries to the celeſtial equator, one paſſing through the equinoctial points, called the *equinoctial* colure; and the other paſſing through the ſolſtitial points, called the *ſolſtitial* colure. The times when the ſun comes to the ſolſtitial points are called the *ſolſtices*.

44. The *arctic* and *antarctic* circles are two parallels of declination, the former about the north and the latter about the ſouth pole, the diſtance of which from

from the two poles is equal to the diſtance of the tropics from the equator. Theſe are alſo called *polar* circles.

45. The two tropics and two polar circles, when referred to the earth, divide it into five parts, called *zones*; the two parts within the polar circles are called the *frigid* zones; the two parts between the polar circles and tropics are called the *temperate* zones; and the part between the tropics is called the *torrid* zone.

Small circles in the heavens are referred to the earth, or the contrary, by lines drawn to the earth's center. Thus the ſmall circle *AxBy*, in the heavens, is referred to *LvMw* on the earth. Hence if *AxBy* be the tropic or the polar circle in the heavens, *LvMw* will be the tropic or polar circle on the earth. Theſe circles therefore retain the ſame relative ſituations, that is, the former is as far from the pole in the heavens, as the latter is from the pole of the earth. The planes of theſe correſponding ſmall circles do not coincide; but when they become great circles, then the planes become coincident.

FIG. 3.

46. A body is in *conjunction* with the ſun, when it has the *ſame* longitude; in *oppoſition*, when the difference of their longitudes is 180°; and in *quadratures*, when the difference of their longitudes is 90°. The conjunction is marked thus ☌, the oppoſition thus ☍, and quadratures thus □.

47. *Syzygy* is either conjunction or oppoſition.

48. The *elongation* of a body is its angular diſtance from the ſun when ſeen from the earth.

49. The *diurnal parallax* is the difference between the apparent places of the bodies in our ſyſtem when referred to the fixed ſtars, if ſeen from the center and ſurface of the earth. The *annual* parallax is the difference between the apparent places of a body in the heavens, when ſeen from the oppoſite points of the earth's orbit.

50. The *argument* is a term uſed to denote any quantity by which another required quantity may be found. For example, the argument of that part of the equation of time which ariſes from the unequal angular motion of the earth in its orbit about the ſun is the ſun's anomaly, becauſe that part of the equation depends entirely upon the anomaly; and the latter being given, the former is found from it. The argument of a ſtar's latitude is its diſtance from its node, becauſe upon this the latitude depends.

51. The *nodes* are the points where the orbits of the primary planets cut the ecliptic, and where the orbits of the ſecondaries cut the orbits of their primaries. That node is called *aſcending* where the planet paſſes from the ſouth to the north ſide of the ecliptic; and the other is called the *deſcending* node. The aſcending node is marked thus ☊, and the deſcending node thus ☋. The line which joins the nodes is called the *line of the nodes*.

52. If a perpendicular be drawn from a planet to the ecliptic, the angle

at

at the ſun between two lines, one drawn from it to that point where the perpendicular falls, and another to the earth, is called the angle of *commutation*.

53. The angle of *poſition* is the angle at an heavenly body formed by two great circles, one paſſing through the pole of the equator and the other through the pole of the ecliptic.

54. *Apparent* noon is the time when the ſun comes to the meridian.

55. *True* or *mean* noon is 12 o'clock, by a clock adjuſted to go 24 hours in a mean ſolar day.

56. The *equation of time* is the interval between *true* and *apparent* noon.

57. A ſtar is ſaid to riſe or ſet *coſmically*, when it riſes or ſets at ſun riſing; and when it riſes or ſets at ſun ſetting, it is ſaid to riſe or ſet *achronically*.

58. A ſtar riſes *heliacally*, when, after having been ſo near to the ſun as not to be viſible, it emerges out of the ſun's rays and juſt appears in the morning; and it ſets *heliacally*, when the ſun approaches ſo near to it, that it is about to immerge into the ſun's rays and become inviſible in the evening.

59. *Curtate diſtance* of a planet from the ſun or earth is the diſtance of the ſun or earth from that point of the ecliptic where a perpendicular to it paſſes through the planet.

60. *Aphelion* is that point in the orbit of a planet which is furtheſt from the ſun.

61. *Perihelion* is that point in the orbit of a planet which is neareſt to the ſun.

62. *Apogee* is that point of the earth's orbit which is furtheſt from the ſun, or that point of the moon's orbit which is furtheſt from the earth.

63. *Perigee* is that point of the earth's orbit which is neareſt to the ſun, or that point of the moon's orbit which is neareſt to the earth.

The terms aphelion and perihelion are alſo applied to the earth's orbit.

64. *Apſis* of an orbit is either its aphelion or perihelion, apogee or perigee; and the line which joins the apſides is called *the line of the apſides*.

65. *Anomaly (true)* of a planet is its angular diſtance at any time from its aphelion, or apogee—*(mean)* is its angular diſtance from the ſame point at the ſame time if it had moved uniformly with its mean angular velocity.

66. *Equation of the center* is the difference between the *true* and *mean* anomaly; this is ſometimes called the *proſthaphereſis*.

67. *Nonageſimal degree* of the ecliptic is that point which is higheſt above the horizon.

68. The *mean* place of a body is the place where a body, not moving with an uniformly angular velocity about the central body, *would* have been, if the angular velocity had been uniform. The *true* place of a body is the place where the body actually is at any time.

69. *Equa-*

69. *Equations* are corrections which are applied to the *mean* place of a body in order to get its *true* place.

70. A *digit* is a twelfth part of the diameter of the ſun or moon.

71. Thoſe bodies which revolve about the ſun in orbits very nearly circular are called *planets*, or *primary* planets for the ſake of diſtinction; and thoſe bodies which revolve about the *primary* planets are called *ſecondary* planets, or *ſatellites*.

72. Thoſe bodies which revolve about the ſun in very elliptic orbits are called *comets*. The ſun, planets and comets, comprehend all the bodies in what is called the *ſolar ſyſtem*.

73. All the other heavenly bodies are called *fixed ſtars*, or ſimply *ſtars*.

74. *Conſtellation* is a parcel of ſtars contained within ſome aſſumed figure, as a ram, a dragon, an hercules, *&c.* the whole heaven is thus divided into conſtellations. A diviſion of this kind is neceſſary, in order to direct a perſon to any part of the heavens which we want to point out.

Characters uſed for the Sun, Moon and Planets.

☉ The Sun.	♂ Mars.
☽ The Moon.	♃ Jupiter.
☿ Mercury.	♄ Saturn.
♀ Venus.	♅ Georgian.
⊕ The Earth.	

Characters uſed for the Days of the Week.

☉ Sunday.	♃ Thurſday.
☽ Monday.	♀ Friday.
♂ Tueſday.	♄ Saturday.
☿ Wedneſday.	

CHAP.

CHAP. II.

ON THE DOCTRINE OF THE SPHERE.

Art. 75. A SPECTATOR upon the earth's ſurface conceives himſelf to be placed in the center of a concave ſphere in which all the heavenly bodies are ſituated; and by conſtantly obſerving them, he perceives that by far the greater number never change their relative ſituations, each riſing and ſetting at the ſame interval of time and at the ſame points of the horizon, and are therefore called *fixed* ſtars; but that a few others, called *planets*, together with the *ſun* and *moon*, are conſtantly changing their ſituations, each continually riſing and ſetting at different points of the horizon and at different intervals of time. Now the determination of the times of the riſing and ſetting of all the heavenly bodies; the finding of their poſition at any given time in reſpect to the horizon or meridian, or the time from their poſition; the cauſes of the different lengths of days and nights, and the changes of ſeaſons; the principles of dialling, and the like, conſtitute the *doctrine of the ſphere*. And as the apparent diurnal motion of all the bodies have no reference to any particular ſyſtem or diſpoſition of the planets, but may be ſolved, either by ſuppoſing them actually to perform thoſe motions every day, or by ſuppoſing the earth to revolve about an axis, we will ſuppoſe this latter to be the caſe, the truth of which will afterwards appear.

76. Let *pep'q* repreſent the earth, *O* its center, *b* the place of a ſpectator, *HZRN* the ſphere of the fixed ſtars; and although the fixed ſtars do not lie in the concave ſurface of a ſphere of which the center of the earth is the center, yet, on account of the immenſe diſtance even of the neareſt of them, their relative ſituations from the motion of the earth, and conſequently the place of a body in our ſyſtem referred to them, will not be affected by this ſuppoſition. The plane *abc* touching the earth in the place of the ſpectator is called (21) the *ſenſible* horizon, as it divides the viſible from the inviſible part of the heavens; and a plane *HOR* parallel to *abc*, paſſing through the center of the earth, is called the *rational* horizon; but in reſpect to the ſphere of the fixed ſtars, theſe may be conſidered as coinciding, the angle which the arc *Ha* ſubtends at the earth becoming then inſenſible from the immenſe diſtance of the fixed ſtars. Now if we ſuppoſe the earth to revolve daily about an axis, all the heavenly bodies muſt ſucceſſively riſe and ſet in that time, and appear to deſcribe circles whoſe planes are perpendicular to the earth's axis, and conſequently parallel to FIG. 4.

each other; thus all the ſtars would appear to revolve daily about the earth's axis, as if they were placed in the concave ſurface of a ſphere having the earth in the center. Let therefore pp' be that diameter of the earth about which it muſt revolve in order to give the apparent diurnal motion to the heavenly bodies, then p, p', are called its poles; and if pp' be produced both ways to P, P' in the heavens, theſe points are called (18) the poles in the heaven, and the ſtar neareſt to each of theſe is called the pole ſtar. Now, although the earth, from its motion in its orbit, continually changes its place, yet as the axis always continues parallel to itſelf, the points P, P' will not, from the immenſe diſtance of the fixed ſtars, be ſenſibly altered; we may therefore ſuppoſe theſe to be fixed points*. Produce Oo both ways to Z and N, and Z is the zenith and N the nadir (23). Draw the great circle $PZHNR$, and it will be the celeſtial meridian (25), the plane of which coincides with the terreſtrial meridian pbp' paſſing through the place of the ſpectator. Let eq repreſent a great circle of the earth perpendicular to its axis pp', and it will be the equator (15), and if the plane of this circle be extended to the heavens it marks out a great circle EQ called the celeſtial equator (18). Hence, for the ſame reaſon that we may conſider the points P, P' as fixed, we may conſider the circle EQ as fixed. Now as the latitude of a place on the earth's ſurface is meaſured by the degrees of the arc be (16), it may be meaſured by the arc ZE; hence as the equator, zenith, and poles in the heaven, correſpond to the equator, place of the ſpectator, and poles of the earth, we may leave out the conſideration of the earth in our further enquiries upon this ſubject, and only conſider the equator, zenith and poles in the heavens, and HR the horizon to the ſpectator.

FIG. 5. 77. Let therefore figure the fifth repreſent the poſition of the heavens to Z the zenith of a ſpectator in north latitude, EQ the equator, P, P' its poles, HOR the rational horizon, $PZHP'R$ the meridian, and draw the great circle ZON perpendicular to $ZPRH$, and it is the prime vertical (28); R will be the north point of the horizon and H the ſouth (26), and O will be the eaſt or weſt points (28) according as this figure repreſents the eaſtern or weſtern hemiſphere. Draw alſo a great circle POP' perpendicular to the meridian. Now as each circle HR, EQ, ZN, PP' is perpendicular to the meridian, its pole muſt be in each (8, 9), therefore their common interſection O is the pole of the meridian. Draw alſo the ſmall circles wH, mt, ae, Rv, yx parallel to the equator; and as the great circle POP' biſects EQ in O, it muſt alſo biſect the ſmall circles mt, ae, in r and c; for as $EO = 90°$, therefore (13) tr and ec each $= 90°$; and as $QO = 90°$, mr and ac each $= 90°$; hence $ac = ce$, and $mr = rt$.

78. As all the heavenly bodies, in their diurnal motion, deſcribe either

* This is not accurately true, the earth's axis varying a little from its parallelism, as will be explained in the proper place.

the

the equator, or ſmall circles parallel to the equator, according as the body is in or out of the equator, if we conceive this figure to repreſent the eaſtern hemiſphere, QE, *ae*, *mt*, may repreſent their apparent paths from the meridian under the horizon to the meridian above, and the points *b*, *O*, *s* are the points of the horizon where they riſe. And as *ae*, QE, *mt*, are biſected in *c*, *O*, *r*, *eb* muſt be greater than *ba*, $QO = OE$, and *ts* leſs than *sm*. Hence a body on the *ſame* ſide of the equator with the ſpectator will be longer above the horizon than below, becauſe *eb* is greater than *ba*; a body *in* the equator will be as long above as below, becauſe $QO = OE$; and a body on the *contrary* ſide will be longer below than above, becauſe *ms* is greater than *st*. The bodies deſcribing *ae*, *mt*, riſe at *b* and *s*; and as *O* is the eaſt point of the horizon, and *R* and *H* are the north and ſouth points, a body on the *ſame* ſide of the equator with the ſpectator riſes between the eaſt and the north, and a body on the *contrary* ſide riſes between the eaſt and the ſouth; and a body *in* the equator riſes in the eaſt at *O*. When the bodies come to *d* and *n*, they are in the prime vertical, or in the eaſt; hence a body on the *ſame* ſide of the equator with the ſpectator comes to the eaſt *after* it is riſen, and a body on the *contrary* ſide, *before* it riſes. The body which deſcribes the circle *Rv*, or any circle nearer to *P*, never ſets; and ſuch circles are called circles of *perpetual apparition*; and the ſtars which deſcribe them are called *circumpolar* ſtars. The body which deſcribes the circle *wH*, juſt becomes viſible at *H*, and then it inſtantly deſcends below the horizon; but the bodies which deſcribe the circles nearer to *P'* are never viſible. Such is the apparent diurnal motion of the heavenly bodies when the ſpectator is ſituated any where between the equator and poles; and this is called an *oblique* ſphere, becauſe all the bodies riſe and ſet obliquely to the horizon. As this figure may alſo repreſent the weſtern hemiſphere, the ſame circles *ea*, *tm* will repreſent the motion of the heavenly bodies as they deſcend from the meridian above the horizon to the meridian under. Hence a body is at the greateſt altitude above the horizon, when on the meridian, and at equal altitudes when equidiſtant on each ſide from it, if the body have not changed its declination. This is the foundation of finding the time of paſſing the meridian from equal altitudes of a body on each ſide.

79. If the ſpectator be at the equator, then *E* coincides with *Z*, and conſequently EQ with *ZN*, and therefore *PP'* with *HR*. Hence as the equator EQ is perpendicular to the horizon, the circles *ace*, *mrt*, parallel to EQ, muſt alſo be perpendicular to it; and as theſe circles are always biſected by *PP'*, they muſt now be biſected by *HR*. Hence all the heavenly bodies are as long above the horizon as below, and riſe and ſet at right angles to it, on which account this is called a *right* ſphere. FIG. 6.

80. If the ſpectator be at the pole, then *P* coincides with *Z*, and conſequently FIG. 7.

quently PP' with ZN, and therefore $E\Omega$ with HR. Hence the circles mt, ae, parallel to the equator, are alſo parallel to the horizon; therefore as a body in its diurnal motion deſcribes a circle parallel to the horizon, thoſe fixed bodies in the heavens which are above the horizon muſt always continue above, and thoſe which are below muſt always continue below. Hence none of the bodies by their *diurnal* motion can either riſe or ſet. This is called a *parallel* ſphere, becauſe the diurnal motion of all the bodies is parallel to the horizon. Theſe apparent diurnal motions of the fixed ſtars remain conſtant, that is, each always deſcribes the ſame parallel of declination.

81. The *ecliptic*, or that circle in the heavens which the ſun appears to deſcribe in the courſe of a year, does not coincide with the equator, for during that time it is found to be only twice in the equator; let therefore COL repreFIG. 5. ſent the ecliptic, which being a great circle muſt cut the equator into two equal parts (7). Hence, as this apparent motion of the ſun is nearly uniform, the ſun is nearly as long on one ſide of the equator as on the other. When therefore the ſun is at q on the *ſame* ſide of the equator with the ſpectator, deſcribing the parallel of declination ae by its diurnal motion, the days are longer than the nights, and it riſes at b from the eaſt towards the north; but when it is on the *contrary* ſide, at p, deſcribing mt, the days are ſhorter than the nights, and it riſes at s from the eaſt towards the ſouth, the ſpectator being on the north ſide of the equator; but when the ſun is in the equator, at O, deſcribing ΩE, the days and nights are equal, and it riſes in the eaſt at O*. If ae, mt be equidiſtant from $E\Omega$, then will $be=ms$ and $ab=st$; hence when the ſun is in theſe oppoſite parallels, the length of the day in one is equal to the length of the night in the other; and the *mean* length of a day at every place is 12 hours. Hence at every place, the ſun, in the courſe of a year, is half a year above the horizon FIG. 6. and half a year below †. When the ſpectator is at the equator, tm, ea being biſected by the horizon, the ſun will be always as long above as below the horizon, and conſequently the days and nights will be always 12 hours long. There will however be ſome variety of ſeaſons, as the ſun will recede $23\frac{1}{2}°$ on each ſide from the ſpectator. When the ſpectator is at the equator, the ſun

* The different degrees of heat in ſummer and winter do not altogether ariſe from the different lengths of times which the ſun is above the horizon, but from the different altitudes of the ſun above the horizon; the higher the ſun is above the horizon the greater is the number of rays which fall on any given ſpace, and the greater alſo is the force of the rays. From all theſe circumſtances ariſe the different degrees of heat in ſummer and winter. The increaſe of heat alſo as you approach the equator ariſes from the two latter circumſtances.

† This is not accurately true, becauſe the ſun's motion in the ecliptic is not quite uniform, on which account it is not exactly as long on one ſide of the equator as on the other. If the major axis of the earth's orbit coincided with the line joining the equinoctial points, the times would be equal.

will

will be vertical to him at noon when it is in the equator. And when the ſpectator is any where between the tropics, the ſun will be vertical to him at noon when its declination is equal to the latitude of the place, and of the ſame kind, that is, both north, or both ſouth. When the ſpectator is at the pole, the ſun at p or q is carried by its *diurnal* motion parallel to the horizon; hence it never ſets when it is in that part of the ecliptic which is above the horizon, nor riſes when in that part which is below; conſequently there is half a year day and half a year night. Hence the variety of ſeaſons ariſes from the axis of the earth not being perpendicular to the plane of the ecliptic, for if it were, the ecliptic and equator would coincide, and the ſun would then be always in the equator, and conſequently it would never change its poſition in reſpect to the ſurface of the earth. If $\odot R = EH = 23°.\ 28'$ the ſun's greateſt declination, then on the longeſt day the ſun would deſcribe the parallel Rv, which juſt touching the horizon at R, ſhows that the ſun does not deſcend on that day below the horizon, and therefore that day is 24 hours long. But when the ſun comes to its greateſt declination on the other ſide of $E\odot$, it deſcribes wH and conſequently does not aſcend above the horizon for 24 hours, and therefore that night is 24 hours long. This therefore happens when EH, the complement of EZ the latitude (16), is $23°.\ 28'$, or in latitude $66°.\ 32'$. If EH, the compliment of latitude, be leſs than $23°.\ 28'$, the ſun will be above the horizon in ſummer, and below in winter, for more than 24 hours, and the longer above or below as you approach the pole, where, as before obſerved, it will be 6 months above and as long below the horizon. The orbits of all the planets, and of the moon, are alſo inclined to the equator, and conſequently their motions amongſt the fixed ſtars muſt be in circles inclined to the equator; therefore the ſame appearances will take place in each, in the time they make one revolution in their orbits. All theſe different appearances in the motion of the moon muſt therefore happen every month. It is evident alſo that theſe variations muſt be greater or leſs as the orbits are more or leſs inclined to the equator; hence they muſt be greater in the moon than in the ſun*. This apparent motion of the ſun, and real motion of the moon and planets amongſt the fixed ſtars is from weſt to eaſt, and therefore contrary to their apparent diurnal motion.

FIG. 7.

FIG. 5.

82. Hitherto we have conſidered the motion of the heavenly bodies in the eaſtern hemiſphere; but if this figure repreſent the weſtern hemiſphere, all the reaſoning will equally apply; hence the bodies will be juſt as long in deſcending from the meridian to the horizon as in aſcending from the horizon to

* On account of the continual change of declination of the ſun, moon and planets, their apparent diurnal motions will not be accurately parallel to the equator; in ſuch therefore whoſe declination alters ſenſibly in the courſe of a day, and in caſes where great accuracy is required, we muſt, in our computations, take into conſideration the change of declination.

the

the meridian, the paths deſcribed will be ſimilar, and they will ſet in the ſame ſituation in reſpect to the weſt point of the horizon as they riſe in reſpect to the eaſt; that is, if a body riſe from the eaſt towards the north or ſouth, it will ſet at the ſame diſtance from the weſt towards the north or ſouth.

83. Having thus explained all the apparent diurnal motions of the heavenly bodies, with the cauſe of the variety of ſeaſons, we ſhall proceed in the next place to ſhew the method of determining the poſitions of the different circles, and the ſituation of the bodies in reſpect to the horizon, meridian or any other circles, at any given time, and to find the time from their poſition.

84. The altitude PR of the pole above the horizon is equal to the latitude of the place. For the arc ZE is (16) the meaſure of the latitude; but $PE=ZR$, each being $=90°$, take away ZP which is common to both, and $EZ=PR$*.

85. The latitude of a place may be found by obſerving the greateſt and leaſt altitude of a circumpolar ſtar, and then applying the correction for refraction, and half the ſum ſo corrected will be the altitude of the pole. For if yx be the circle deſcribed by the ſtar, then, as $Px=Py$, PR muſt $=\frac{1}{2}\times\overline{Ry+Rx}$. The latitude may alſo be found thus. Let eOt be the ecliptic, then when the ſun comes to e its declination is the greateſt, and eH is the greateſt meridian altitude; when the ſun comes to the ecliptic at t, let ts be the parallel deſcribed on that day, and then sH is the leaſt meridian altitude; and as $Ee=Es$, $\frac{1}{2}\times\overline{He+Hs}=HE$ the complement of the latitude.

FIG. 8.

* From hence ariſes the method of meaſuring the circumference of the earth; for if a man travel upon a meridian till the height of the pole has altered one degree, he muſt then have travelled one degree; hence by meaſuring that diſtance and multiplying it by 360, we get the circumference of the earth. This was firſt undertaken by our countryman Mr. NORWOOD, who meaſured the diſtance between London and York, and obſerved the different altitudes of the pole at thoſe places. Afterwards the French mathematicians meaſured a degree. CASSINI meaſured one in France. After that, CLAIRAUT, MAUPERTUIS and ſeveral other mathematicians went to Lapland and meaſured a degree, the length of which appears to be 69,2 engliſh miles in the latitude of 45°; for the earth being a ſpheroid, the degrees in different latitudes are different. This will appear when we treat of the figure of the earth. The figure of the earth was very early diſcovered to be ſpherical; firſt, from the ſhadow of the earth upon the moon when it is eclipſed being always circular; and ſecondly, from the apparent ſpherical figure of the ſea, and from obſerving that as ſhips receded from the ſhore, the hulks firſt became inviſible, and then the maſts gradually from the bottom to the top, which was the laſt part that diſappeared; and this could not happen on a plane ſurface. POSSIDONIUS mentions that CLEOMEDES made uſe of a method ſimilar to this in order to determine the magnitude of the earth. He obſerved that at Alexandria the ſtar Canopus paſſed the meridian at the altitude of $7\frac{1}{2}°$, and at Rodes, lying nearly under the ſame meridian, it paſſed the meridian juſt at the horizon. The diſtance between theſe two places was found to be 3752 ſtades, which multiplied by 48 ($7\frac{1}{2}°$ being the 48th. part of 360°) gives 180096 ſtades for the whole circumference. It is uncertain what is the exact length of the ſtade here made uſe of; it alſo appears from the obſervations of CHAZELLES that the diſtance of theſe two places is not $7\frac{1}{2}°$ but only $5\frac{1}{4}°$; no concluſion therefore can be drawn from hence of the accuracy of this meaſurement.

86. Half

86. Half the difference of the ſun's greateſt and leaſt meridian altitudes is equal to the inclination of the ecliptic to the equator. For half *He*—*Hs*, or half *se*, is equal to *Ee* which (12) meaſures the angle *EOe*, the inclination of the ecliptic to the equator. To determine theſe altitudes without a quadrant, a gnomon *AB* has been erected perpendicular to the horizon *AC*, and the length of the ſhadow *AD* has been obſerved on the longeſt and ſhorteſt day, and knowing *AB*, the angle *BDA* is known, which is the ſun's altitude. Sometimes inſtead of erecting a gnomon, a ſmall hole is made in the wall or roof of a building for the ſun to ſhine through, and a plumb line let fall from the hole to the floor, the length of which is meaſured, and which anſwers to a gnomon *AB*; and the length of the ſhadow *AD* from *A* being alſo meaſured, (*A* being the point where the plumb line meets the floor) the angle *BAC* becomes known as before. Dr. LONG made uſe of this method to find the latitude of Pembroke Hall in this Univerſity, from the known declination of the ſun: He made the latitude 52°. 12′. 55″; ſee his Aſtronomy page 513. FIG. 9.

87. The angle which the equator makes with the horizon, or the altitude of that point of the equator which is on the meridian, is equal to the complement of the latitude. For *ZH* is 90°, and therefore *EH* is the complement of *EZ*; and as *OE*=*OH*=90°, *EH* meaſures (12) the angle *EOH*. FIG. 8.

88. Let *abcdxe* be a parallel of declination deſcribed by any heavenly body in the eaſtern hemiſphere, and draw the circles of declination *Pb*, *Pd*, *Px*, and the circles of altitude *Zb*, *Zc*, *Zx*. Now, as has been already explained, when the body comes to *b* it riſes, at *c* it is at the middle point between *a* and *e*, and at *d* it is due eaſt; and let *x* be its place at any other time. Let us firſt ſuppoſe this body to be the ſun, and not to change its declination* in its paſſage from *a* to *e*, and let us ſuppoſe a clock to be adjuſted to go 24 hours in one apparent diurnal revolution of the ſun, or from the time it leaves any meridian till it returns to it again, then the ſun will always approach the meridian which it has left at the rate of 15° in an hour, or any other circle of declination; alſo the angle which the ſun deſcribes about the pole will be at the ſame rate, becauſe (13) any arc *xe*, which the ſun at *x* has to deſcribe before it comes to the meridian, meaſures the angle *xPe*, called the *hour* angle. If therefore we ſuppoſe the clock to ſhow 12 when the ſun is on the meridian at *a* or *e*, it will be 6 o'clock when it is at *c*. And as the ſun deſcribes angles about the pole *P* at the rate of 15° in an hour, the angle between any circle *Px* of declination

* Knowing the longitude and time nearly at any place, the ſun's declination may be found at that time by firſt taking the declination for noon on the given day from the nautical almanac, and then correcting it for the difference of the meridians of the place and Greenwich, and for the hour of the day, by the 6th of the *Requiſite Tables*. Theſe tables were computed to facilitate the computations from the nautical almanac, and were publiſhed by the Board of Longitude.

paſſ-

paſſing through the ſun at x and the meridian PE, converted into time at the rate of 15° for an hour, will give the time from *apparent* noon, or when the ſun comes to the meridian.

89. Given the ſun's declination and latitude of the place, to find the time of its riſing, and azimuth at that time. The ſun riſes at b; and in the triangle bZP, $bZ=90°$, $bP=$co-dec. $PZ=$co-lat. Now when one ſide of a triangle $=90°$, it may be ſolved by the circular parts, taking the angles adjacent to the ſide $=90°$, and the complement of the other three parts for the circular parts. Hence, rad. : cot. bP :: cot. ZP : cos. ZPb, *or* rad. :: *tan. dec.* : *tan. lat.* : *cos. ZPb the hour angle from apparent noon*; which converted into time at the rate of 15° for an hour, and ſubtracted from 12 o'clock, gives the apparent time of riſing. Alſo ſin. ZP : rad. :: cos. bP : ſin. PZb, or, *cos. lat.* : *rad.* :: *ſin. dec.* : *cos. of the azimuth from the north.*

Ex. Given the latitude of Cambridge 52°. 12′. 35″, to find the time of the ſun's riſing on the longeſt day, and azimuth at that time, aſſuming the greateſt declination of the ſun 23°. 28′.

Rad. - - - -	10.0000000
Tan. 23°. 28′ - -	9.6376106
Tan. 52°. 12′. 35″	10.1104699
Cos. 124°. 2′. 47″* -	9.7480805

Convert this into time (Tab. 1.) and it gives 8*h*. 19′. 6″ which ſubtracted from 12 gives 3*h*. 40′. 54″ for the time when the ſun's center is upon the rational horizon on the longeſt day. Alſo

Cos. 52°. 12′. 35″	0.2127004 *A.C.*
Rad. - - -	10.0000000
Sin. 23°. 28′. -	9.6001181
Cos. 49°. 28′. 9″. az. from north	9.8128185

Hence on the longeſt day the ſun riſes 40°. 31′. 51″ from the eaſt.

90. To find the ſun's altitude at 6 o'clock. The ſun is at c at 6 o'clock, and the angle ZPc is a right one; hence, rad. : cos. ZP :: cos. Pc : cos. Zc, or *rad.* : *ſin. lat.* :: *ſin. dec.* : *ſin. of the altitude.*

91. To find the time when the ſun comes to d the prime vertical, and its altitude at that time. In this caſe the angle $dZP=90°$; hence, cos. ZP : rad. :: cos. dP : cos. ZS, or, *ſin. lat.* : *rad.* :: *ſin. dec.* : *ſin. of the altitude.* Alſo rad. : cot. Pd. :: tan. PZ : cos. ZPd, or, *rad.* : *tan. dec.* :: *cot. lat.* : *cos.* ZPd, which converted into time gives the time from apparent noon.

92. Given the latitude of the place, the ſun's declination and altitude, to find

* This log. 9.7480805 is found in the tables to be the log. coſine of 55°. 57′. 13″, but as the angle is manifeſtly greater than 90°, we muſt take its ſupplement. In the ſolution of ſpherical triangles, ambiguous caſes will frequently ariſe, for the determination of which, where the caſe is not evident, the reader is referred to Dr. MASKELYNE's very valuable Introduction to TAYLOR's Logarithms.

its

its azimuth and the hour. Let x be the ſun's place; then by ſpher. trig. ſin. $xP \times$ ſin. ZP : rad.^{2} :: ſin. $\frac{1}{2} \times \overline{Px+PZ+Zx} \times$ ſin. $\frac{1}{2} \times \overline{Px+PZ-Zx}$: cos. $\frac{1}{2}ZPx^2$; hence ZPx is known, which converted into time gives the time from apparent noon. Alſo ſin. $xZ \times$ ſin. PZ : rad.^{2} :: ſin. $\frac{1}{2} \times \overline{Zx+ZP+Px} \times$ ſin. $\frac{1}{2} \times \overline{Zx+ZP-Px}$: cos. $\frac{1}{2}xZP^2$; hence the azimuth xZP from the north is known.

Ex. Given the lat. 34°. 55′ N, ſun's declination 22°. 22′. 57″ N, and true altitude 36°. 59′. 39″, to find the apparent time. Here ZP=55°. 5′, Zx=53°. 0′. 21″, Px=67°. 37′. 3″; hence,

Px=	67°. 37′. 3″	arith. comp. of ſine	0,034019
ZP=	55. 5. 0	arith. comp. of ſine	0,086193
Zx=	53. 0. 21		
Sum	175. 42. 24.		
½ Sum	87. 51. 12	ſine - -	9.999694
Zx=	53. 0. 21		
Dif.	34. 50. 51	ſine - -	9.756932
			2)19.876838
			9.938419

the coſine of 29°. 47′. 44″ half the angle ZPx, ∴ ZPx=59°. 35′. 28″, which reduced into time gives 3h. 58′. 22″ the time from apparent noon.

93. If the error in altitude be given, we may thus find the error in time. Let mn be parallel to the horizon, and nx repreſent the error in altitude; then as the calculation of the time is made upon ſuppoſition that there is no error in the declination, we muſt ſuppoſe the body to be at m inſtead of x, and conſequently the angle mPx, or the arc qr, meaſures the error in time.

Now nx : xm :: *ſin. nmx* : *rad.*

xm : qr :: *cos. rx* : *rad.* (13).

hence nx : qr :: ſin. $nmx \times$ cos. rx : rad.^{2} ∴ $qr = nx \times \frac{\text{rad.}^2}{\text{ſin. } nmx \times \text{cos. } rx}$; but $ZxP = nmx$, nxm being the complement of both; alſo ſin. ZxP, or nmx : ſin. ZP :: ſin. xZP : ſin. xP, or cos. rx, ∴ ſin. $nmx \times$ cos. rx = ſin. $ZP \times$ ſin. xZP; hence, $qr = nx \times \frac{\text{rad.}^2}{\text{ſin. } ZP \times \text{ſin. } xZP} = nx \times \frac{\text{rad.}^2}{\text{cos. lat.} \times \text{ſin. azi.}}$. Hence, the error is leaſt on the prime vertical. All altitudes therefore to deduce the time from, ought to be taken on, or as near to, the prime vertical as poſſible. In lat. 52°. 12′, if the error in alt. at an azi. 44°. 22′ be 1′, then $qr = 1' \times \frac{1^2}{,612 \times ,699}$ = 2′,334 of a degree = 9″,336 in time.

Hence the perpendicular aſcent of a body is quickeſt when it is on the prime vertical, for nx varies as ſin. azi. when qr and the lat. are given.

94. Given

94. Given the latitude of the place and the ſun's declination, to find the time when twilight begins. Twilight is here ſuppoſed to begin when the ſun is 18° below the horizon; hence draw the circle *kyk* parallel to the horizon and 18° below it, and twilight will begin when the ſun comes to *y*, and $Zy = 108°$; hence ſin. $yP \times$ ſin. ZP : rad.^{2} :: ſin. $\frac{1}{2} \times \overline{PZ + Py + 108°} \times$ ſin. $\frac{1}{2} \times \overline{PZ + Py - 108°}$: cos. $\frac{1}{2} yPZ^2$, hence yPZ is known, which converted into time gives the time from *apparent* noon.

95. To find the time when the apparent diurnal motion of a fixed ſtar is perpendicular to the horizon. Let *yx* be the parallel deſcribed by the ſtar; draw the vertical circle *Zh* touching it at *o*, and when the ſtar comes to *o* its motion is perpendicular to the horizon; and as the angle *ZoP* is a right one, we have, rad. : tan. *oP* :: cot. *PZ* : cos. *ZPo*, that is, *rad.* : *cot. dec.* :: *tan. lat.* : *cos. ZPo*, which converted into time (Tab. 1.) gives the time from the ſtar's being on the meridian. Hence the time of the ſtar's coming to the meridian being found by Art. 105. the time required will be known.

FIG. 5.

96. To find the time of the ſhorteſt twilight. Let *ab* be the parallel of the ſun's declination at the time required, draw *cd* indefinitely near and parallel to it, and *TW* a parallel to the horizon 18° below it; then *vPw*, *sPt* meaſure the twilight on each parallel of declination, and when the twilight is ſhorteſt, the increment of the hour angle being = 0, theſe muſt be equal; hence *vPr* = *wPz*, therefore *vr* = *wz*; and as *rs* = *tz*, and the angles *r* and *z* are right ones, *rvs* = *zwt*; but *Pvr* = 90° = *Zvs*, take *Zvr* from both, and *PvZ* = *rvs*; for the ſame reaſon *PwZ* = *zwt*; hence *PvZ* = *PwZ*. Take *ve* = *wZ* = 90°, then as *Pv* = *Pw*, and the angle *Pve* = *PvZ*, therefore *Pe* = *PZ*; let fall the perpendicular *Py* and it will biſect the baſe *eZ*. Then by trig. cos. $Py = \frac{\text{cos. } Pv}{\text{cos. } vy} = \frac{\text{cos. } Pv}{\text{ſin. } ey}$; alſo cos. $Py = \frac{\text{cos. } Pe}{\text{cos. } ey} = \frac{\text{cos. } PZ}{\text{cos. } ey}$; hence $\frac{\text{cos. } Pv}{\text{ſin. } ey} = \frac{\text{cos. } PZ}{\text{cos. } ey}$, ∴ cos. *Pv*, or ſin. *hv*, $= \text{cos. } PZ \times \frac{\text{ſin. } ey}{\text{cos. } ey} = \text{cos. } PZ \times \text{tan. } ey$, hence rad. : cos. *PZ*, or ſin. lat. :: tan. *ey* = 9° : ſin. *hv* the ſun's declination at the time of ſhorteſt twilight. Becauſe *PZ* is always leſs than 90°, and $Zy = 9°$, therefore *Py* is always leſs than 90°, and therefore its coſine is poſitive; alſo *vy* is always greater than 90°, therefore its coſine is negative; hence cos. *Pv* (= cos. *Py* × cos. *vy*) is negative, conſequently *Pv* is greater than 90°, therefore the ſun's declination is *ſouth*. This is M. CAGNOLI's Inveſtigation.

97. To find the length of the ſhorteſt twilight. As *wPZ* = *vPe*, therefore *ZPe* = *vPw* meaſuring the ſhorteſt time. Now ſin. *PZ*, or cos. lat. : rad. :: ſin. *Zy* = 9° : ſin *ZPy*, which doubled gives *ZPe* or *vPw*, which converted into time gives the length of the ſhorteſt twilight.

Ex.

Ex. To find the time of the year at Cambridge, when the twilight is ſhorteſt; and the length of that twilight.

Rad.	10,0000000
Sin. 52°. 12′. 35″	9,8977695
Tan. 9°	9,1997125
Sin. 7°. 11′. 25″ dec.	9,0974820

This declination of the ſun gives the time about March 2, and October 11.

Cof. 52°. 12′. 35″	0,2127004 *A.C.*
Sin 9°	9,1943324
Rad.	10,0000000
Sin. 14°. 47′. 27″	9,4070328

The double of this gives 29°. 34′. 54″, which converted into time gives 1*h*. 58′. 20″ for the duration of the ſhorteſt twilight, it being ſuppoſed to end when the ſun is 18° below the horizon.

98. To find the ſun's declination when it is juſt twilight all night. Here the ſun at *a* muſt be 18° below the horizon; hence 18° + dec. $\mathcal{Q}a = R\mathcal{Q} = EH$ = comp. of lat. of place; hence the ſun's dec. = comp. lat. − 18°; look therefore into the *Nautical Almanac*, and ſee on what days the ſun has this declination, and you have the time required. The ſun's greateſt declination being 23°. 28′, it follows that if the complement of latitude be greater than 41°. 28′, or if the latitude be leſs than 48° 32′, there can never be twilight all night. If the ſun be on the other ſide of the equator, then it's dec. = 18°. − comp. lat. FIG. 5.

99. If the ſpectator be between *E* and *L*, and the ſun's declination *Ee* be greater than *EZ*, then the ſun comes to the meridian at *e* to the *north* of it's zenith; and if we draw the ſecondary *Zqm* touching the parallel *ae* of declination deſcribed by the ſun, then *Rm* is the greateſt azimuth from the north which the ſun has that day, the azimuth increaſing till the ſun comes to *q*, and then decreaſing again, and the ſun has the ſame azimuth twice in the morning. If therefore we draw the ſtraight line *Zv* perpendicular to the horizon, the ſhadow of this line, being always oppoſite to the ſun, would firſt recede from the ſouth point *H* and then approach it again in the morning, and therefore would go backwards upon the horizon. But if we conſider *PP′* as a ſtraight line, or the earth's axis produced, the ſhadow of that line would not go backwards upon that plane, becauſe the ſun always continues to revolve about that line, and therefore it's ſhadow muſt always go forwards; FIG. 11.

whereas the ſun does not revolve about the perpendicular Zv. Hence it appears, that the ſhadow of the ſun upon a dial can never go backwards, becauſe the gnomon of a dial is parallel to PP', and therefore the ſun muſt always revolve about the gnomon. The time when the azimuth is greateſt is found from the right angled triangle PqZ, by ſaying, rad. : tan. qP :: cot. PZ : cos. ZPq, or *rad. : cot. dec. :: tan. lat. : cos. PZq* the hour angle from apparent noon.

100. It has hitherto been ſuppoſed, that it is 12 o'clock when the ſun comes to the meridian, and that the clock goes juſt 24 hours in the interval of the ſun's paſſage from any meridian till it returns to it again. But if a clock be thus adjuſted for one day, it will not continue to ſhow 12 o'clock every day when the ſun comes to the meridian, becauſe the intervals of time from the ſun's leaving any meridian till it returns to it again are not always equal; this difference between the ſun and the clock is called the *Equation of Time*, as will be explained in Chap. IV. Hence, when the clock does not agree with the ſun, any arc xe is not the meaſure of the time from 12 o'clock, but from the time when the ſun comes to the meridian, or from *apparent* noon*.

FIG. 8.

101. In the ſame manner as we find the hour angle for the ſun, we may alſo find it for any fixed ſtar or planet, it's altitude and declination being given; but when the hour angle is thus found, it is neceſſary to know the time when the body is upon the meridian in order to find the time from thence, the hour angle being the diſtance from the meridian; alſo the method of reducing the hour angle into time will be different. For let E be the earth, $rmsn$ the equator, sr a meridian paſſing through a fixed ſtar S reduced to the equator; then as the meridian returns to the ſtar in $23h.\ 56'.\ 4''$ after leaving it (127), we have $360°$: hour angle :: $23h.\ 56'.\ 4''$: time from the meridian. Now let P be a planet, and the meridian mn to paſs through it; then the meridian will return to that poſition again in $23h.\ 56'.\ 4''$; now let Pv or Pv' be the planet's motion in right aſcenſion in one day, according as it's motion is direct or retrograde, and reduce this into time (t) at the rate of $15°$ for an hour, which will be ſufficiently exact for ſo ſmall an arc, then the meridian returns to the planet again after an interval of $23h.\ 56'.\ 4'' \pm t$; hence the meridian, after leaving the planet, approaches it at the rate of that time for $360°$, becauſe when the meridian leaves the

FIG. 12.

* The converſion of the hour angle into time for the ſun at the rate of $15°$ for an hour, by a clock adjuſted to mean ſolar time, is not accurate, becauſe the ſolar days are not all accurately equal to 24 hours, but to $24h. \pm$ the variation (e) of the equation of time for that day, according as the equation is increaſing or decreaſing; hence, to reduce the hour angle to give accurately the time from apparent noon, ſay, $360°$: hour angle ($a°$) :: $24h. \pm e$: time $= \frac{a°}{360°} \times \overline{24h. \pm e}$; for, in this caſe, the meridian, inſtead of returning to the ſun in $24h.$ returns to it in $24h. \pm e$. This quantity e is ſometimes $30''$, and therefore if $a° = 60°$, the correction would be $5''$. A clock is adjuſted to *mean* ſolar time, when it is adjuſted to go 24 hours in a *mean* ſolar day. *See Art.* 127.

the planet it is then approaching a point 360° from it; hence 360° : hour angle :: $23h.\ 56'.\ 4'' \pm t$: time from the meridian.

102. The hour angle which we have hitherto found for the time at which a body riſes has been upon ſuppoſition that the body is upon the rational horizon at the inſtant it appears; but all bodies in the horizon are elevated by refraction 33′ above their true places; this therefore would make them appear when they are 33′ below the rational horizon, or 90° + 33′ from the zenith; alſo all the bodies in our ſyſtem are depreſſed below their true places by parallax, as will be afterwards explained, therefore from this cauſe they would not appear till they were elevated above the rational horizon by a quantity equal to their horizontal parallax, or when diſtant from the zenith 90° — hor. par. Hence from both cauſes together, a body becomes viſible when its diſtance ZV from the zenith = 90° + 33′ — hor. par. V being the place of the body when it becomes viſible, Z the zenith and P the pole; hence knowing ZV, alſo ZP the complement of latitude and PV the complement of declination, we can find the hour angle ZPV. A fixed ſtar has no parallax, therefore $ZV = 90°.\ 33'$. FIG. 13.

103. If the body ſenſibly alter its declination in a few hours, as the moon does, the time of its riſing may be thus found. Let w be the place of the moon on the meridian, v when in the horizon, and d the point when it becomes viſible, ſo that ew is the change of declination in the time from riſing to the meridian. Now from knowing the time (105) of paſſing the meridian, and the declination at noon, with the change of declination in the interval of the paſſages of the moon over the meridian by the Nautical Almanac, compute the change of declination in the interval between noon and the time of the moon's tranſit, and you will get the moon's declination at the time of its tranſit. To that declination compute the hour angle upon ſuppoſition that the declination continued the ſame as on the meridian, which will be nearly the angle wPd. From the Nautical Almanac find the change (v) of declination in the interval (t) of time from the moon's paſſage over the meridian till it returns to it again; then ſay, 360° : hour angle :: v : the change of declination in deſcribing that angle, which added to or ſubtracted from the declination at the time of paſſing the meridian gives very nearly the declination at riſing; to which compute the hour angle and convert it into time as before and ſubtract it from the time of paſſing the meridian, and it gives very nearly the time of riſing; and if greater accuracy ſhould be required, the operation may be repeated by taking this hour angle. FIG. 14.

Ex. To find at what time the moon roſe at Greenwich on July 1, 1767. The latitude of Greenwich is 51°. 28′. 40″, and (105) the moon paſſed the meridian at $4h.\ 2'.\ 9''$; now $t = 24h.\ 40'$, and $v = 5°.\ 28'$; hence $24h.\ 40' : 4h.\ 2'.\ 9'' :: 5°.\ 28' : 53'.\ 38''$ the change of declination in $4h.\ 2'.\ 9''$, which,

as

as the declination is decreasing, subtracted from 5°. 22′, the moon's north declination at noon, leaves 4°. 28′. 22″ for the moon's declination when it was on the meridian; hence we take $Pd = 85°.\ 31'.\ 38''$, also $PZ = 38°.\ 31'.\ 20''$; and as the moon's hor. parallax $= 54'.\ 21''$, and refraction $33'$, we have $Zd = 89°.\ 38'.\ 39''$, hence the angle $ZPd = 95°.\ 3'\ 50''$. Hence $360° : 95°.\ 3'.\ 50'' :: 5°.\ 28' : 1°.\ 26'.\ 37''$ the change of declination in the time of describing 95°. 3′. 50″, which added to 4°. 28′. 22″ gives 5°. 54′. 59″ for the declination at the time of rising, very nearly; hence $Pd = 84°.\ 5'.\ 1''$, therefore the angle $ZPd = 96°.\ 54'.\ 2''$; hence $360° : 96°.\ 54'.\ 2'' :: 24h.\ 40' : 6h.\ 38'.\ 22''$ the time of describing the angle ZPd, which subtracted from $4h.\ 2'.\ 9''$, the time when the moon was on the meridian, gives the time of rising $21h.\ 23'.\ 47''$, answering to $9h.\ 23'.\ 47''$ in the morning apparent time.

104. In determining the time when any body rises, or when it is at any known altitude or position, it has been supposed that we know the time at which it comes to the meridian; the determination of this circumstance must therefore be next explained.

105. Let a clock be adjusted to mean solar time, which we may therefore consider as the time from the sun's leaving the meridian till it returns to it again, where great accuracy is not required, the difference being only the variation of the equation of time in 24 hours. Let S and P be the places of the sun and a planet reduced to the equator; then the meridian sr approaches the sun at the rate of 15° in an hour; for when it leaves the sun at S it may be considered as approaching a point at that time 360° from it, and which it comes up to in 24 hours; hence if any other point were moving forwards with the velocity of the sun, the meridian would approach it at the same rate. Therefore if the planet at P move forwards with a different velocity from that of the sun, the interval of their passages over any meridian will be the same as if we supposed the sun to be at rest and the planet to move with its own proper motion *minus* that of the sun, the planet's motion in right ascension being *greater* than that of the sun. Let x be the difference of their motions in right ascension in 24 hours reduced into time, and $t = SP$ reduced also into time in like manner, the planet being at P at the time the meridian passes through the sun at S; and let v be the place of the planet when the meridian overtakes it, and e be the arc Pv in time; then the motions of the meridian will be 24 and $t + e$, and of the planet in the same times x and e; hence, as we may consider each motion as uniform, $24 : x :: t + e : e$, $\therefore 24 - x : x :: t : e = \frac{tx}{24 - x}$.

FIG. 12.

This is the case if the planet's motion be greater than the sun's, but if the sun's be greatest, then x itself becomes negative, and therefore $-x$ will be positive; hence $e = \frac{-tx}{24 + x}$; therefore $t + e = t \pm \frac{tx}{24 \mp x} = \frac{24t}{24 \mp x}$ the time from

from apparent noon when the planet paſſes the meridian, where the upper or lower ſign prevails according as the planet's or ſun's motion is greateſt. If the motion of the planet in right aſcenſion be retrograde, it is manifeſt that x is the *ſum* of the motions of the planet and ſun in 24 hours, for the bodies moving in oppoſite directions they approach each other with the *ſum* of their motions; let therefore v' be the place of the planet when it comes to the meridian, then the motion of the meridian from its paſſage through the ſun to the planet will be $t - e$; hence $24 : x :: t - e : e$, therefore $24 + x : x :: t : e = \frac{tx}{24+x}$, hence the time required $= t - \frac{tx}{24+x} = \frac{24t}{24+x}$. But as the diviſion by $24 \mp x$ is not ſo convenient as it would be by 24, therefore reſolve $\frac{24t}{24 \mp x}$ into $t \pm \frac{tx}{24} + \frac{tx^2}{24^2} \pm$ &c. where the two firſt terms will be ſufficient for all caſes except the moon, where it will be neceſſary to take the third. For a fixed ſtar x will repreſent the increaſe of the ſun's right aſcenſion in 24 hours, and the time required $= \frac{24t}{24+x} = t - \frac{tx}{24}$. By this method we find, very nearly, the time at which any body comes to the meridian, and hence, by the laſt articles, we may find the time of its riſing, or the time at any given altitude.

Ex. To find the time of the moon's paſſage over the meridian at Greenwich on July 1, 1767. The ſun's *AR.** when on the meridian that day was $6h.\ 40'.\ 25''$, and its daily increaſe $4'.\ 48''$; alſo the moon's *AR.* was $10h.38'.\ 8''$, and its daily increaſe $42'.\ 28''$. Hence $t = 10h.\ 36'.\ 8'' - 6h.\ 40'.\ 25'' = 3h.\ 55'.\ 43'' = 3{,}9285$ (Tab. 3.), alſo $x = 42'.\ 28'' - 4'.\ 48'' = 38'.\ 20'' = 0{,}6388$; hence $\frac{tx}{24} = 6'.\ 16''$; $\frac{tx^2}{24^2} = 10''$; therefore $t + \frac{tx}{24} + \frac{tx^2}{24^2} = 4h.\ 2'.\ 9''$ the apparent time of paſſing the meridian.

Where great accuracy of time is required from an obſerved altitude, the body made uſe of muſt be the ſun or a fixed ſtar. The method of finding the time by the ſun has been already explained (92); and the time by a ſtar may be found by the following method.

106. Find the ſtar's true altitude, and take its declination from the 7th of the Requiſite Tables, or from any other tables if it be not there; then in the triangle ZPx (x repreſenting the place of the ſtar) we have ZP the complement of

FIG. 8.

* *AR.* means right aſcenſion.

of latitude, Px the complement of declination and Zx the complement of the ſtar's altitude, to find the angle ZPx, the ſtar's diſtance from the meridian, which convert into time. Now the point of the equator which is upon the meridian at any time, is called the *mid-heaven*; therefore the angle ZPx meaſures the ſtar's diſtance from the mid-heaven. Hence, if the ſtar be to the *eaſt* of the meridian, *ſubtract* its diſtance from the meridian from its *AR.* (adding, if neceſſary, 24 hours to its *AR.*), and the difference is the *AR.* of the mid-heaven: But if ſtar be to the *weſt*, *add* them together (ſubtracting 24 hours from the ſum, if greater,) and the ſum gives the *AR.* of the mid-heaven*. Then find the ſun's *AR.* at the preceding noon at Greenwich from the Nautical Almanac, and from thence at noon at the given place by the 23d of the Requiſite Tables, and ſubtract it from the *AR.* of the mid-heaven (adding 24 hours to the latter†, if neceſſary), and the difference would be the apparent time from the preceding noon, or the eſtimate time, if the ſun had had no motion in that time; but as it has moved, find that motion by the 23d of the Requiſite Tables, and ſubtract it, and it gives the *apparent* time required.—Hence, if we apply the equation of time it gives the true time, which compared with the watch ſhows how much it is too faſt or too ſlow; and by repeating the obſervations, the rate of going of the watch may be determined; but this will be further explained in Chap. IV.

Ex. On April 14, 1780, lat. 48°. 56″. N. lon. 66°. W. the true altitude of Aldebaran weſt of the meridian was 22°. 17′. 50″; to find the apparent time.

Sun's *AR.* for noon at Greenwich by the Nautical Almanac	1*h*. 31′. 1″
Corrected for the Long. by the 23rd. of the Requiſite Tables	+ 41
Sun's *AR.* at noon at the given place - - - - - - -	1. 31. 42

FIG 15.

* That this is true for every poſition of the point aries and place of the ſtar, may be thus ſhown. Let EQ repreſent the equator, E the point on the meridian, Υ, Υ', Υ'', different poſitions of the point aries, in reſpect to the place A, A' of the ſtar referred to the equator, A on the weſtern ſide of the meridian, and A' on the eaſtern; B the point to which the ſun is referred; ΥEBQ the direction in which the right aſcenſion is meaſured. Now ſuppoſe the ſtar at A', to the eaſt of the meridian; then, 1. $\Upsilon A' - A'E = \Upsilon E$. 2. $\Upsilon'' A' - A'E = -\Upsilon'' E = -24h. + \Upsilon'' QE$, $\therefore \Upsilon'' A + 24h. - A'E = \Upsilon'' QE$. Now ſuppoſe the ſtar at A on the weſt ſide; then 1. $\Upsilon A + AE = \Upsilon E$. 2. $\Upsilon' QA + AE - 24h. = \Upsilon' QA + A\Upsilon' + \Upsilon' E - 24h. = \Upsilon' E$, becauſe $\Upsilon' QA + A\Upsilon' = 24h.$

† For, 1. $\Upsilon'' BQE - \Upsilon'' B = EQB$. 2. $\Upsilon E + 24h. - \Upsilon B = \Upsilon E + EA'B + EQB - \Upsilon B = EQB$, becauſe $\Upsilon E + EA'B = \Upsilon B$.

Alſo

Alſo by Requiſite Table 7. the ſtar's dec. is 16°. 3′ N. Hence $ZP = 41°.$ 4′, $Zx = 67°.\ 42'.\ 10''$, $xP = 73°.\ 57'$; hence by ſph. trig.

Px =	73°. 57′. 0″	arith. comp. of ſine 0.017304
ZP =	41. 4. 0	arith. comp. of ſine 0.182476
Zx =	67. 42. 10	
Sum =	182. 43. 10	
½ Sum =	91. 21. 35	ſine - - - - - 9.999874
Zx =	67. 42. 10	
Dif. =	23. 39. 25	ſine - - - - - 9.603425
		2)19.803079
		9.901539 the coſine

of 37°. 8′. 29″; hence the angle xPZ (or in FIG. 15. the arc AE) = 74°. 16′. 58″, or in time = 4h. 57′. 8″; hence

Star weſt of merid. -	4h. 57′. 8″	Eſtimate Time - - -	7h. 48′. 46″
Star's AR. by Req. Tab. 7.	4. 23. 20	Correc. from Req. Tab. 7.	— 1. 12
AR. of mid-heaven -	9. 20. 28	Apparent Time req^{d}. -	7. 47. 34
Sun's AR. at noon -	1. 31. 42		

107. The time of the paſſage of a ſtar over the meridian may be found (78) from taking the times at which it had equal altitudes on each ſide of the meridian, and biſecting the interval. If equal altitudes be taken at 8 and 11 o'clock, the ſtar was upon the meridian at half paſt 9 o'clock. But for the ſun this will want a correction, owing to its change of declination, on which account it is not at equal altitudes when equidiſtant from the meridian. If be be the diurnal arc deſcribed by the ſun in its aſcent to the meridian, and ed in its deſcent from it, and mn be drawn parallel to HOR, then the ſun is at equal altitudes at m and n, and the angle mPn, or the arc qr, meaſures the difference of the times at m and n from the meridian; when we therefore biſect the interval of the times at which the ſun was at m and n, we muſt correct it by half mPn, or half qr, in order to get the time at which it comes to the meridian. This correction is called the *equation of equal altitudes*. Now by MAUDUIT's trig. art. 297, or CRACKELT's Tranſlation, if d'' = the variation of the ſun's dec. in the interval of the obſervations, t = tan. lat. v = tan. decl. at noon, s = ſine, r = tan. of the hour angle from noon at the time of the obſervation, taking the half interval of times for the meaſure of that angle; then $\frac{1}{2}qr = \frac{1}{2}d'' \times \frac{t}{s} \pm \frac{v}{r}$, FIG. 16.

radius being unity; or as the value of d'' in time is $\frac{d''}{15''}$ ſeconds, eſtimated at the rate of 15° for 1 hour, or 15″. for 1 ſecond of time, therefore the correction $= \frac{d}{30} \times \frac{t}{s} \pm \frac{v}{r}$ ſeconds of time, where the ſign — is to be uſed when the lat. and decl. are both north or both ſouth, and + when one is north and the other ſouth. Now in north latitude, when the ſun approaches the north pole, or is in the 9th. 10th. 11th. 0th. 1ſt. 2nd. ſigns, it is manifeſt from the figure that the ſun, after paſſing the meridian, will not come to the ſame altitude as at the obſervation before, until it be at a greater diſtance from the meridian; therefore the middle point of time between the obſervations muſt be, when the ſun has paſſed the meridian, and the correction muſt be *ſubtracted*. When the ſun is in the other ſigns, receding from the north pole, it comes to the ſame altitude at a leſs diſtance from the meridian; therefore the middle point of time muſt be, before the ſun comes to the meridian, and conſequently the correction muſt be *added*. To facilitate this computation, Mr. WALES conſtructed and computed a ſet of tables which were publiſhed in the Nautical Almanac for 1773; theſe tables are called *Equation to correſponding altitudes*.

To find the Time the Sun is paſſing the Meridian, or the horizontal or perpendicular Wire of a Teleſcope.

FIG. 8. 108. Let mx be the diameter d'' of the ſun, eſtimated in ſeconds of a great circle; then, (as the minutes in mx, conſidered as a ſmall circle, muſt be greater in proportion as the radius is leſs, becauſe, when the arc is given, the angle is inverſely as the radius), ſin. Px, or cos. dec. rx : rad. :: ſeconds d'' in mx of a great circle : the ſeconds in mx of the ſmall circle ea which is equal to (13) the ſeconds in qr = the angle rPq, and therefore the angle $rPq = d''$ divided by cos. dec. (rad. being unity) $= d''$ × ſec. dec., which meaſures the time the ſun is paſſing over its diameter, and conſequently the time the diameter would be in paſſing over the meridian; hence (as in Art. 107) the time of paſſing the meridian $= \frac{d'' \times \text{ſec. dec.}}{15''}$

109. Hence qr, the ſun's diameter in right aſcenſion, is equal to d'' × ſec. dec. If therefore the ſun's diameter = 32′ = 1920″, and its dec. 20°, its diameter in right aſcenſion = 1920″ × 1,064 = 34′, 2″,88. The ſame will do for the moon, if d'' = its diameter.

110. By

110. By Art. 93. $qr = nx \times \frac{\text{rad.}^2}{\text{cos. lat.} \times \text{fin. azi.}} =$ (if $nx = d''$ the ſun's diam.) $d'' \times \frac{\text{rad.}^2}{\text{cos. lat.} \times \text{fin. azi.}}$; hence, as before, the time of deſcribing qr, or the time in which the ſun aſcends perpendicularly through a ſpace equal to its diameter, or the time of paſſing an horizontal wire, is equal to $\frac{d''}{15''} \times \frac{\text{rad.}^2}{\text{cos. lat.} \times \text{fin. azi.}}$ The ſame expreſſion muſt alſo give the time which the ſun is in riſing. If $d'' = 1980''$ the horizontal refraction, then d'' divided by $15'' = 132''$; hence refraction accelerates the riſing of the ſun by $132'' \times \frac{\text{rad.}^2}{\text{cos. lat.} \times \text{fin. azi.}}$.

111. The fin. nxm : fin. nmx :: mn : $nx = mn \times \frac{\text{fin. } nmx}{\text{fin. } nxm}$; hence (93) $qr = mn \times \frac{\text{fin. } nmx}{\text{fin. } nxm} \times \frac{\text{rad.}^2}{\text{fin. } nmx \times \text{cos. } rx} = mn \times \frac{\text{rad.}^2}{\text{fin. } nxm \times \text{cos. } rx}$; and if $mn = d''$, we find the time, in which the horizontal motion of the ſun is equal to its diameter, to be $\frac{d''}{15''} \times \frac{\text{rad.}^2}{\text{cos. } ZxP \times \text{cos. dec.}}$, which is therefore the time in which the ſun would paſs the vertical wire of a teleſcope.

On the Principles of Dialling.

112. As the apparent motion of the ſun about the axis of the earth is at the rate of 15° in an hour, very nearly, let us ſuppoſe the axis of the earth to project its ſhadow into the meridian oppoſite to that of the ſun, and then this meridian will move at the rate of 15° in an hour. Hence let $zPRpH$ repreſent a meridian on the earth's ſurface, POp its axis, z the place of the ſpectator, $HKRV$ a great circle of which z is the pole; draw the meridians $P1p$, $P2p$, &c. making angles with PRp of 15°, 30°, &c. reſpectively; then ſuppoſing PR to be the meridian into which the ſhadow of PO is projected at 12 o'clock, $P1$, $P2$, &c. are the meridians into which it is projected at 1, 2, &c. o'clock, and the ſhadow will be projected on the plane $HKRV$ in the lines OR, $O1$, $O2$, &c., and the arcs $R1$, $R2$, &c. will meaſure the angles $RO1$, $RO2$, &c. between the 12 o'clock line and the 1, 2, &c. o'clock lines. Now in the right angled triangle $PR1$, we have PR (84) the latitude of the place, and the angle $RP1 = 15°$; hence rad. : tan. 15° :: ſin. PR : tan. $R1$; in the ſame manner we may calculate the arcs $R2$, $R3$, &c. In this caſe we make the earth's axis the gnomon, and the ſhadow is projected upon the plane $HKRV$. But if we take a plane $abcd$ at z parallel to $HKRV$, and conſequently parallel to the horizon at

FIG. 17.

z, and draw zr parallel to POp, then on account of the great diſtance of the ſun we may conceive it to revolve about zt in the ſame manner as about Pp, and conſequently the ſhadow will be projected upon the plane $abcd$ in the ſame manner as the ſhadow of PO is projected upon the plane $HKRV$, and therefore the hour angles are calculated by the ſame proportion. This is an *horizontal* dial.

FIG. 18. 113. Now let $NLzK$ be a great circle perpendicular to $PRpHz$, and conſequently perpendicular to the horizon at z, and the ſide next to H is full ſouth. Then, for the ſame reaſon as before, if the angles $Np1$, $Np2$, &c. be 15°, 30°, &c. the ſhadow of pO will be projected into the lines $O1$, $O2$, &c. at 1, 2, &c. o'clock, and the angles $NO1$, $NO2$, will be meaſured by the arcs $N1$, $N2$, &c. Hence in the right angled triangle $pN1$, pN = the complement of the latitude, and the angle $Np1 = 15°$, therefore rad. : tan. 15° :: ſin. pN : tan. $N1$; in the ſame manner we find $N2$, $N3$, &c. Hence, for the ſame reaſon as for the horizontal dial, if $zabc$ be a plane coinciding with $NLzK$, and st be parallel to Op, st will project its ſhadow in the ſame manner on the plane $zabc$ as Op does on the plane $NLzK$, and therefore the hour angles from the 12 o'clock line are computed by the ſame proportion. This is a *vertical ſouth* dial. In the ſame manner the ſhadow may be projected upon a plane in any poſition, and the hour angles be calculated.

114. In order to fix an horizontal dial, we muſt be able to tell the exact time of the ſun's coming to the meridian; for which purpoſe, find the time (92) by the ſun's altitude when it is at the ſolſtices, becauſe then the declination does not vary, and ſet a well regulated watch to that time; then when the watch ſhews 12 o'clock, the ſun is on the meridian; at that inſtant therefore ſet the dial to 12'clock, and it ſtands right.

115. Hence we may eaſily draw a meridian line upon any horizontal plane. Suſpend a plumb line ſo that the ſhadow of it may fall upon the plane, and when the watch ſhows 12, the ſhadow of the plumb line is the true meridian. The common way is to deſcribe ſeveral concentric circles upon an horizontal plane, and in the center to erect a gnomon perpendicular to it with a ſmall round well defined head, like the head of a pin; make a point upon any one of the circles where the ſhadow of the head, by the ſun, falls upon it on the morning, and again where it falls upon the ſame circle in the afternoon; draw two radii from theſe two points, and biſect the angle which they form, and it will be a meridian line. This ſhould be done when the ſun is at the tropic, when it does not ſenſibly change its declination in the interval of the obſervation; for if it do, the ſun will not (107) be equidiſtant from the meridian at equal altitudes. This method is otherwiſe not capable of very great accuracy, as, from the ſhadow not being very accurately defined, it is not eaſy to ſay at what inſtant of time the ſhadow of the head of the gnomon is biſected by

the

the circle. If however ſeveral circles be made uſe of, and the mean of the whole taken, the meridian may be gotten with ſufficient accuracy for all common purpoſes.

116. To find whether a wall be full ſouth for a vertical ſouth dial, erect a gnomon perpendicular to it and hang a plumb line from it; then when the watch ſhows 12, if the ſhadow of the gnomon coincide with the plumb line, the wall is full ſouth.

CHAP.

CHAP. III.

TO DETERMINE THE RIGHT ASCENSION, DECLINATION, LATITUDE AND LONGITUDE OF THE HEAVENLY BODIES.

Art. 117. THE foundation of all Aſtronomy is to determine the ſituation of the fixed ſtars, in order to find, by a reference to ſuch fixed objects, the places of the other bodies at any given time, and from thence to deduce their proper motions. The poſitions of the fixed ſtars are found from obſervation, by finding their right aſcenſions and declinations by means of the tranſit teleſcope and aſtronomical quadrant, as explained in my *Treatiſe on Practical Aſtronomy*; and then by computation their latitudes and longitudes may be found.

118. Now as the earth revolves uniformly about its axis, the apparent motion of all the heavenly bodies, ariſing from this motion of the earth, muſt be uniform; and as this motion is parallel to the equator, the interval of the times, in which any two ſtars paſs over any meridian, muſt be in proportion to the arc of the equator intercepted between the two ſecondaries paſſing through them, becauſe (13) this arc of the equator contains the ſame number of degrees as the arc of any ſmall circle parallel to it and comprehended between the ſame ſecondaries; and therefore, if one increaſe uniformly, the other muſt. Hence the right aſcenſion of ſtars paſſing the meridian at different times will differ in proportion to the difference of the times of their paſſing; and as the clock is ſuppoſed to go uniformly, we have the following rule. As the interval of the times of the paſſage of any fixed ſtar over the meridian : the interval of the paſſage of any two ſtars :: 360° : their *apparent* difference of right aſcenſions; which corrected for their aberration in right aſcenſion, gives their *true* difference of right aſcenſions. By the ſame method we may find the difference of right aſcenſions of the ſun or moon, when they paſs the meridian, and a ſtar, and therefore if that of the ſtar be known, that of the ſun or moon will; which will be rendered more exact, if we compare them with ſeveral ſtars and take the mean; remembering to apply the ſtar's aberration in right aſcenſion to the *apparent*, in order to get the *true* difference. When we thus determine the ſun's right aſcenſion from that of a ſtar, the ſun's aberration, which in longitude is always 20″, is not here conſidered, becauſe the ſun's place in the tables is put down

down as affected by aberration; and the uſe of obſerving the ſun's right aſcenſion is to compare it with the tables in order to find their error.

119. Now to determine the right aſcenſion of a fixed ſtar, Mr. FLAMSTEAD propoſed a method, by comparing the right aſcenſion of the ſtar with that of the ſun when near the equinoxes, and having the ſame declination; and as this method has not been explained by any writers, we ſhall give a very full explanation thereof, together with an example. Let *AGCKE* be the equator, *ABCWE* the ecliptic, *S* the place of the ſtar, and *Sm* a ſecondary to the equator, and let the ſun be at *P*, very near to *A*, when it is on the meridian, and take $C\Upsilon = PA$, and draw *PL*, $\Upsilon\Omega$ perpendicular to *AGC*, and ΩL parallel to *AC*; then the ſun's declination is the ſame at Υ as at *P*. Obſerve the meridian altitude of the ſun when at *P*, and alſo the time of the paſſage of it's center over the meridian; obſerve alſo at what time the ſtar paſſes over the meridian, and then (118) find the apparent difference *Lm* of their right aſcenſions. When the ſun approaches near to Υ, obſerve its meridian altitude for ſeveral days, ſo that on one of them at *t* it may be greater and on the next day at *e* it may be leſs than the meridian altitude at *P*, ſo that in the intermediate time it may have paſſed through Υ; and drawing *tb*, *es* perpendicular to *AGCE*, obſerve on theſe two days the differences *bm*, *sm* of the ſun's right aſcenſion and that of the ſtar; draw alſo *sw* parallel to Ωo. Hence to find Ωb, we may conſider the variation both of the right aſcenſion and declination to be uniform for a ſmall time, and conſequently to be proportional to each other; hence *vb* (the change of meridian altitudes in one day) : *ob* (the difference of the meridian altitudes at *t* and Υ, or the difference of declinations) :: *sb* (the difference of *sm*, *bm* found by obſervation) : Ωb, which added to *bm*, or ſubtracted from it, according to the ſituation of *m*, gives Ωm, to which add *Lm*, or take their difference, according to circumſtances, and we get ΩL, which ſubtracted from *AGC*, or 180°, half the remainder will be *AL* the ſun's right aſcenſion at the firſt obſervation, to which add *Lm* and we get the ſtar's right aſcenſion at the ſame time. Inſtead of finding $b\Omega$, we might have found $s\Omega$, by taking $\Upsilon\Omega - es$ for the ſecond term, and from thence we ſhould have gotten Ωm. Thus we ſhould get the right aſcenſion of a ſtar, upon ſuppoſition that the poſition of the equator had remained the ſame, and the apparent place of the ſtar had not varied, in the interval of the obſervations. But the interſection of the equator with the ecliptic has a retrograde motion, called the Preceſſion of the Equinoxes; alſo the inclination of the equator to the ecliptic is ſubject to a variation, called the Nutation; and from the Aberration of the ſtar, its apparent place is continually changing. The effects of all theſe circumſtances in changing the right aſcenſion of the ſtar will be explained and inveſtigated in their proper places. Now Tables VII. and VIII. contain theſe corrections for 34 principal ſtars; that

FIG. 19.

that is, if the *mean* right aſcenſion of any ſtar be taken for the beginning of the year, and theſe corrections be applied to it, according to their ſigns, for any day, the reſult gives the *apparent* right aſcenſion of the ſtar for that day.

FIG. 20. 120. Let therefore $ABCE$ be the ecliptic, $AGCE$ the poſition of the equator at the firſt obſervation when the ſun was at P, and $agce$ the poſition of the equator at the time of the obſervation at the other equinox, and take $\Upsilon C = PA$, and draw ΥQ perpendicular to $AGCE$, as before, and draw Qq parallel to ABC, and tqr perpendicular to $AGCE$; let Ae be alſo perpendicular to $agce$. Now as the poſition of the equator and the apparent place of the ſtar are altered in the time between the two obſervations, let m be the point where a ſecondary from the apparent place of the ſtar to the equator at the firſt obſervation would cut it, and v the place at the ſecond obſervation, and draw vw perpendicular to $AGCE$; then Am is the apparent right aſcenſion of the ſtar at the firſt obſervation, and av at the ſecond. Alſo the ſun muſt be at t when it has the ſame declination tq at the ſecond obſervation as it had at the firſt, and conſequently qv is the apparent difference of right aſcenſions of the ſun at t and ſtar, which difference is found by obſervation in the ſame manner as the difference at Υ was before found, when the equator was fixed. Alſo as $Qq = Cc = Aa$, and the angle $qQr = cCQ = Aae$, we have $Qr = ae = Aa \times$ cos. Aae. Now if we put M for the *mean* right aſcenſion of the ſtar at the beginning of the year, and S for the ſum of all the corrections due at the time of the firſt obſervation, and s for the ſum due at the ſecond; then, from what we have already explained in the laſt article, $M + S = Am$, $M + s = av$, hence if we take the former from the latter, ſuppoſing s to be greater than S, we have $s - S = av - Am = ae + ev - Aw - wm$ (m lying beyond w); but $ev = Aw$; hence $s - S = ae - wm$, conſequently $wm = ae - \overline{s - S}$. Now qv, or rw, is known, hence we know rm, and as Qr is known, Qm will be known; and as we alſo know Lm, we get the value of QL,* with which we proceed, as before, to get the ſtar's right aſcenſion. The great advantage of this method is, that it does not depend upon any determination of the latitude of the place, declination of the ſun or accuracy in the diviſions of the inſtrument. If the latitude be known, we may find the declination from the meridian altitude, it being, from Art. 87, equal to the difference between the meridian altitude and the complement of latitude, and then one obſervation at the ſecond equinox will be ſufficient, becauſe the daily variation of the declination and right aſcenſion may be taken from the Nautical Almanac. Having thus determined the right aſcenſion of one ſtar, the right aſcenſion of all the heavenly bodies may from thence be found (118).

* In all theſe caſes, if you draw the figure and put the ſtar in its proper place, and put m and w in their proper ſituations, which may be done by obſerving whether ew or Am be the greater, you will immediately ſee what quantities are to be added together, and what ſubtracted. This figure is drawn for the Example.

If

If the right aſcenſion of a ſtar, which is not in theſe tables, ſhould be required, the corrections muſt be computed by the Rules which we ſhall give in their proper places. If the right aſcenſion of the ſtar be firſt computed without conſidering theſe corrections, it will be ſufficiently accurate to compute the corrections from, and then they may be applied.

Ex. Let it be required to find the right aſcenſion of *Pollux* on March 24, in the year 1768, from Dr. MASKELYNE's obſervations.

On March 24, Pollux paſſed the meridian at $7^h.\ 31'.\ 38''$, and on the 25, at $7^h.\ 31'.\ 37'',66$; on the ſame day the ſun paſſed at $0^h.\ 16'.\ 35'',5$; hence the apparent difference of the *AR's.* of the ſun and Pollux on the 24th, allowing for the error of the clock (122), was $7^h.\ 15'.\ 2'',46 = 108°.\ 45'.\ 36'',9 = Lm.$ Now on March 24,

Appar. zen. diſt. ⊙ L. L. - -	49°. 58′. 58″,7
Semidiam. - - - -	— 16. 4, 4
Appar. zen. diſt. ⊙ cen. - -	49 · 42 · 54, 3
Parallax - - - -	— 6, 7
Refr. cor. for Bar. and Ther. -	+ 1 . 10, 4
True zen. diſt. ⊙ cen. - -	49 · 43 · 58
True meridian altitude - -	40 . 16 . 2

To find when the ſun had the ſame meridian altitude, or declination, juſt before it came to the next equinox, let us take Sep. 18, on which we find,

Appar. zen. diſt. ⊙ L. L. - -	50°. 8′. 37″,8
Semidiam. - - - -	— 15 · 59, 4
Appar. zen. diſt. ⊙ cen. - -	49 · 52 · 38, 4
Parallax - - - -	— 6, 7
Refr. cor. for Bar. and Ther. -	+ 1 . 5, 8
True zen. diſt. ⊙ cen. - -	49 · 53 · 37, 5
True meridian altitude - -	40 . 6 . 22, 5

As this altitude is leſs than that on March 24, the inſtant of time when the ſun had the *ſame* declination as on the 24th muſt be *before* the 18th; therefore as the ſun on the 18th had gotten beyond that point where it's declination was the ſame as at *P*, we muſt, from the difference of the right aſcenſions of the ſun and ſtar obſerved on that day, ſubtract the increaſe of the ſun's right aſcenſion between the 18th and that point of time when the declination was the ſame as at *P*, in order to get the difference of the apparent right aſcenſions at

the time when the ſun's declination was the ſame as at *P*. We may alſo obſerve, that the difference of any two true meridian altitudes is the ſame as the difference of the declinations at the ſame times. Now as the ſun's altitude was not obſerved on the 17th, we will take the change of declination for that day from the Nautical Almanac, which is $23'.\ 20''$; alſo the increaſe of the ſun's *AR*. for that day was $3'.\ 36''$ in time, or $54'$ in ſpace. The difference of the true meridian altitudes, or the difference of declinations on March 24, and Sep. 18, was $9'.\ 39'',5$; hence $23'.\ 20'' : 9'.\ 39'',5 :: 54' : 22'.\ 21'',4$, the increaſe of the ſun's right aſcenſion from the time before the 18th at which the declination was the ſame as on March 24, to the 18th. On Sep. 18, Pollux paſſed the meridian at $7h.\ 30'.\ 39'',9$, and on the 19th at $7h.\ 30.\ 40''$. On the 18th the ſun paſſed at $11h.\ 44'.\ 53'',33$; therefore the apparent difference of the *AR*'s of the ſun and Pollux on that day, allowing for the error of the clock (122), was $4h.\ 14'.\ 13'',5 = 63°.\ 33'.\ 22'',5$, from which ſubtract $22'.\ 21'',4$ and we have $63°.\ 11'.\ 1'',1 = qv$. Now to get the correction in Table VIII. we muſt have the place of the moon's aſcending node, which, from the Lunar Tables, is found to be $9^s.\ 17°.\ 45'.\ 28''$ on March 24, and $9^s.\ 8°.\ 19'.\ 54''$ on Sep. 18. Hence,

March 24, Correction from Table VII. - - -		$+ 19'',2$
VIII. - - -		$+ 19,5$
		$+ 38,7 = S$
Sep. 18, Correction from Table VII. - - -		$+ 31'',3$
VIII. - - -		$+ 20$
		$+ 51,3 = s$

Hence $s - S = 12'',6$.

Prec. of Equin. from March 24, to Sep. 18, Table XVII. - - - -	$24'',9$
Variation of the equat. of equinoxes, Table XVIII.	$+ 0,7$
True Preceſſion in the interval - - - - -	$25,6 = Aa$
Cos. $23°.\ 28'$ - - - - - -	$,917$
	$23,4 = ae$

Hence $mw = 23'',4 - 12'',6 = 10'',8$; therefore $rm = rw - mw = qv - mw = 63°.10'.\ 49'',3$; to this add $\mathfrak{Q}r = 23'',4$, and we have $\mathfrak{Q}m = 63°.\ 11'.\ 12'',7$, which being added to $Lm = 108°.\ 45'.\ 36'',9$ we have $L\mathfrak{Q} = 171°.\ 56'.\ 49'',6$, which ſubtracted from 180°, half the difference is $4°.\ 1'.\ 35'',2 = AL$ the ſun's right

right aſcenſion on March 24, to which add $Lm = 108°.45'.36''{,}9$ and we get $112°.47'.12''{,}1$ the apparent right aſcenſion of Pollux at the ſame time; and if from this we ſubtract $38''{,}7$ the equation at that time, we get $112°.46'.33''{,}4$ for it's *mean* right aſcenſion. This concluſion differs about $3''$ from that determined by Dr. MASKELYNE in Table VI, from the mean of ſeven obſervations.

121. But the method made uſe of by Dr. MASKELYNE in ſettling the right aſcenſions of the ſtars, though founded upon the ſame principle as this of Mr. FLAMSTEAD, is different in its proceſs, and procured him the advantage of a greater number of obſervations, both of the ſun and ſtars, in the ſame time, and conſequently enabled him to fix the right aſcenſion of the ſtars with greater accuracy in a ſhorter time. He took α *Aquilæ* for his fundamental ſtar, and aſſumed it's right aſcenſion as ſettled by Dr. BRADLEY, reducing it to the time of his obſervations by the mean preceſſion, and afterwards making the following correction. By comparing a great many obſerved tranſits of ſuch ſtars as he thought proper to ſelect, with that of α Aquilæ, in various parts of the year, and applying the proper equations, he obtained their mean right aſcenſions relative to that of α Aquilæ aſſumed, or affected with the ſame error; and comparing the tranſits of the ſun near the equinoxes with thoſe of the above mentioned ſtars obſerved on the ſame day, he obtained the ſun's right aſcenſion relative to that of α Aquilæ aſſumed. From the obſerved zenith diſtances of the ſun on the ſame days, corrected for refraction, parallax and the error of the line of collimation, with the apparent obliquity of the ecliptic at the time, he deduced the ſun's right aſcenſions: and then by comparing the ſun's right aſcenſions deduced from the obſerved tranſits with thoſe deduced from his obſerved zenith diſtances at equal or nearly equal declinations of the ſame kind near both equinoxes, he deduced the error of the aſſumed right aſcenſion of α Aquilæ, which came out $3''{,}8$ additive. He obſerved further, that in the interval of 12 years, which paſſed between the ſettling of Dr. BRADLEY's Catalogue about 1755 and his own about 1767, the preceſſion in right aſcenſion was diminiſhed by $2''{,}16$ by the action of the planets. Therefore if this had been allowed in aſſuming the right aſcenſion of α Aquilæ from Dr. BRADLEY's determination, the correction of the right aſcenſion of α Aquilæ would have come out $5''{,}96$ additive, or at the rate of $\frac{1}{2}''$ a year, which agrees very well with the annual proper motion of α Aquilæ deduced from other obſervations.

122. The practical method of finding the right aſcenſion of a body from that of a fixed ſtar, by a clock adjuſted to ſidereal time, is thus. Let the clock begin its motion from $0h.0'.0''$ at the inſtant the firſt point of Aries is on the meridian; then, when any ſtar comes to the meridian, the clock would ſhow the

apparent right aſcenſion of the ſtar, the right aſcenſion being eſtimated in time at the rate of 15° an hour, provided the clock was ſubject to no error, becauſe it would then ſhow at any time how far the firſt point of Aries was from the meridian. But as the clock is neceſſarily liable to err, we muſt be able at any time to aſcertain what it's error is, that is, what is the difference between the right aſcenſion ſhown by the clock and the right aſcenſion of that point of the equator which is at that time on the meridian. To do this, we muſt, when a ſtar, whoſe apparent right aſcenſion is known, paſſes the meridian, compare it's apparent right aſcenſion with the right aſcenſion ſhown by the clock, and the difference will ſhow the error of the clock. For inſtance, let the apparent right aſcenſion of *Aldebaran* be $4^h.\ 23'.\ 50''$ at the time when it's tranſit over the meridian is obſerved by the clock, and ſuppoſe the time ſhown by the clock to be $4^h.\ 23'.\ 52''$, then there is an error of $2''$ in the clock, it giving the right aſcenſion of the ſtar $2''$ more than it ought. If the clock be compared with ſeveral ſtars* and the mean error taken, we ſhall have, more accurately, the error at the mean time of all the obſervations. Theſe obſervations being repeated every day, we ſhall get the rate of the clock's going, that is, how faſt it gains or loſes. The error of the clock, and the rate of its going, being thus aſcertained, if the time of the tranſit of any body be obſerved, and the error of the clock at the time be applied, we ſhall have the right aſcenſion of the body. This is the method by which the right aſcenſion of the ſun, moon and planets are regularly found in Obſervatories.

Ex. On April 27, 1774, the following obſervations were made at Greenwich: *α Serpentis* paſſed the meridian at $15^h.\ 31'.\ 28'',76$, the moon's ſecond limb paſſed at $15^h.\ 59'.\ 7'',75$, and *Antares* at $16^h.\ 13'.\ 55'',02$ ſidereal time; to find the moon's right aſcenſion.

* The ſtars uſed for this purpoſe at the Obſervatory at Greenwich are thoſe in Tab. VI. whoſe *AR*'s Dr. MASKELYNE ſettled to a very great degree of accuracy. As many of theſe as conveveniently can, are obſerved every day, in order to aſcertain the going of the clock, and for no other purpoſe.

Firſt,

First, to find the error of the clock by the tranſit of the ſtars.

Mean *AR.* of *α ſerpentis* at begin. of 1790 by Tab. VI.	15ʰ. 33′. 55″,84
Preceſſion in 16 years by Tab. VI. - - -	— 46, 94
Mean *AR.* at begin. of 1774 - -	15 . 33 . 8, 90
Cor. for aber. and prec. to April 27, by Tab. VII.	+ 2, 12
Cor. for nutation by Tab. VIII. - -	— 0, 23
App. *AR.* by the tables - - -	15 . 33 . 10, 79
App. *AR.* by the clock - - -	15 . 31 . 28, 76
Error of the clock by *α ſerpentis* too ſlow -	1 . 42, 03

Mean *AR.* of *Antares* at begin. of 1790 by Tab. VI.	16 . 16 . 33, 24
Preceſſion in 16 years by Tab. VI. - -	— 58, 45
Mean *AR.* at begin. of 1774 - -	16 . 15 . 34, 79
Cor. for aber. and prec. to April 27, by Tab. VII.	+ 2, 38
Cor. for nutation by Tab. VIII. - -	— 0, 09
App. *AR.* by the tables - - -	16 . 15 . 37, 08
App. *AR.* by the clock - - -	16 . 13 . 55, 02
Error of the clock by *antares* too ſlow -	1 . 42, 06

The mean of theſe two errors gives 1′. 42″,045 for the error at the middle between the times of the tranſits of the two ſtars, or at 15*h*. 52′. 41″,89. Now from knowing the error of the clock at this time, and the rate of it's going, we we muſt find the error at the time the moon paſſed, which may, in this caſe, be conſidered the ſame, the times being nearly equal. Hence

Moon paſſed the meridian by the clock -	15ʰ. 59′. 7″,75
Error of the clock, too ſlow - -	+ 1 . 42, 045
True *AR.* of the moon's 2d. limb -	16 . 0 . 49, 795
Do. in degrees - - - -	8°. 0°. 12′. 26″,9
Moon's ſemid. in *AR.* (109) - - -	— 17 . 13, 5
True *AR.* of the moon's center -	7 . 29 . 55 . 13, 4

The error of the clock is generally determined by a greater number of ſtars, when they can be obſerved; and the mean error from day to day gives the rate of it's going, from which we may find the error at any other time. For inſtance,

ſtance, on Auguſt 8, 1769, I found, from taking the mean of the errors of four ſtars, that the mean error of the clock was 2″,32, too faſt, at 16*h*. 21′. 18″, being the mean of all the times when the ſtars were obſerved; and on the 9th the error was 2″,09, too faſt, at 13*h*. 52′. 58″, the mean of all the times. Alſo *Jupiter* paſſed the meridian on the 9th at 14*h*. 49′. 10″,4. Now the interval between the 8*d*. 16*h*. 21′. 18″ and 9*d*. 13*h*. 52′. 58″ is 21*h*. 31′. 40″, in which time the clock loſt 0″,23; alſo the interval between 13*h*. 52′. 58″ and 14*h*. 49′. 10″,4 is 56′. 12″,4; hence 21*h*. 31′. 40″ : 56′. 12″,4 :: 0″,23 : 0″,009, which is what the clock loſt in the ſecond interval; therefore when Jupiter paſſed the meridian, the clock was 2″,09 — 0″,009 = 2″,08 too faſt, which ſubtracted from 14*h*. 49′. 10″,4 gives 14*h*. 49′. 8″,32, the apparent right aſcenſion of Jupiter. To the apparent *AR*. apply the aberration in *A.R*. and you get the *true AR*.

123. The right aſcenſion of the heavenly bodies being thus aſcertained, the next thing to be explained is, the method of finding their declinations. Take the apparent altitude of the body, when it paſſes the meridian, by an aſtronomical quadrant, as explained in my *Treatiſe on Practical Aſtronomy*; correct it for parallax and refraction, and for the error of the line of collimation of the inſtrument, if neceſſary, and you get the true meridian altitude, the difference between which and the altitude of the equator (87) (which is equal to the complement of the latitude, previouſly determined) is the declination required.

Ex. On April 27, 1774, the zenith diſtance of the moon's lower limb when it paſſed the meridian at Greenwich was 68°. 19′. 37″,3; it's parallax in altitude was 56′. 19″, 2, allowing for the ſpheroidical figure of the earth; the barometer ſtood at 29, 58, and the thermometer at 49; to find the declination.

Obſerved zenith diſtance of L. L. -	68°. 19′. 37″,3
Refr. cor. for bar. and ther. Tab. XIII. XIV.	+ 2. 23
	68 . 22 . 00, 3
Parallax - - - - -	— 56 . 19, 2
True zenith diſtance of L. L. -	67 . 25 . 41, 1
Semidiameter - - -	— 16 . 35
True zenith diſtance of the center - -	67 . 9 . 6, 1
Latitude - - - - - -	51 . 28 . 40
Declination *ſouth* - - -	15 . 40 . 26, 1

The *horizontal* parallax and ſemidiameter may be taken from the Nautical Almanac; and the parallax in altitude may be found, as will be explained when we come to treat of the Parallax, and then the correction is to be applied to it, and to the ſemidiameter, from Tables XV. and XVI.

124. To

124. To find the latitude and longitude from the right aſcenſion and declination, or the converſe, we have the following admirable Rules, given by Dr. MASKELYNE.

Given the Right Aſcenſion and Declination of an Heavenly Body, and the obliquity of the Ecliptic, to find it's Latitude and Longitude.

1. The *ſine of *AR.* + cotang. decl. —10, = cotang. of arc A, which call *north* or *ſouth*, according as the declination is *north* or *ſouth*. 2. Call the obliquity of the ecliptic *ſouth* in the 6 firſt ſigns of *AR*, and *north* in the 6 laſt. Let the ſum of arc A and obl. eclip. according to their titles, = arc B with it's proper title †. 3. The arith. comp. of cos. arc A + cos. arc B + tan. *AR.* — 10, = tan. of the *longitude*, of the ſame kind as *AR*, unleſs arc B be more than 90°, in which caſe, the quantity found of the ſame kind as *AR.* muſt be ſubtracted from 12 ſigns or 360°. 4. The ſine of longitude + tan. arc B — 10, = tan. of the required *latitude*, of the ſame title as arc B. N.B. If the longitude come out near 0°, or near 180°, for the ſine of long. in the laſt operation, ſubſtitute tan. long. + cos. long. —10, ‡; or the laſt operation will be, tan. long. + cos. long. + tan. arc B — 20, = tan. lat. The tan. long. is already given.

Given the Latitude and Longitude of an Heavenly Body, and the obliquity of the Ecliptic, to find its Right Aſcenſion and Declination.

1. Sine long. + cot. lat. —10, = cot. arc A, which call *north* or *ſouth*, according as the lat. is *north* or *ſouth*. 2. Call the obliquity of the ecliptic *north* in the firſt ſemicircle of longitude, and *ſouth* in the ſecond. Let the ſum of arc A and obl. eclip. according to their titles, = arc B with it's proper title. 3. The arith. comp. of cos. arc A + cos. arc B + tan. long. —10, = tan. of *right aſcenſion*, of the ſame kind as the longitude, unleſs arc B be more than 90°, in which caſe, the laſt quantity found of the ſame kind as the longitude, muſt be ſubtracted from 12 ſigns or 360°. 4. The ſine of *AR.* + tan. arc B — 10, = tan. of the required *declination*, of the ſame title as arc B. N.B. If *AR.* come out near 0°, or near 180°, for the ſine *AR.* in the laſt operation, ſub-

* By ſine, tang. &c. is meant log. ſine, log. tang. &c.

† If one be *north* and the other *ſouth*, the proper title is that belonging to the greater of the two, and in this caſe, arc B is their difference, one being conſidered as negative to the other.

‡ For the reaſon of this correction in extreme caſes, ſee Dr. MASKELYNE's excellent Introduction to TAYLOR's Logarithms.

ſtitute,

ſtitute, tan. *AR.* + cos. *AR.* −10,; or the laſt operation will be, tan. *AR.* + cos. *AR.* + tan. arc B −20, = tan. declination. The tan. *AR.* is already given.

DEMONSTRATION. Let *s* be the body, ♈ *C* the ecliptic, ♈ ♌ the equator, *sr*, *sn* perpendicular to ♈ *C*, ♈ ♌. Then rad. : ſin. ♈ *n* :: cot. *sn* : cot. *s* ♈ *n*, hence log. ſin. ♈ *n* + log. cot. *sn* −10, = log. cot. *s* ♈ *n* arc A. Hence *s* ♈ *n* ∓ ♌ ♈ *C* = *s* ♈ *r* arc B. Alſo

FIG. 21.

cos. *s* ♈ *n* : rad. :: tan. *n* ♈ : tan. *s* ♈

rad. : cos. *s* ♈ *r* :: tan. *s* ♈ : tan. *r* ♈

∴ cos. *s* ♈ *n* : cos. *s* ♈ *r* :: tan. *n* ♈ : tan. *r* ♈ = $\frac{\text{cos. } s \curlyvee r \times \text{tan. } n \curlyvee}{\text{cos. } s \curlyvee n}$, hence ar. co. log. cos. *s* ♈ *n* + log. cos. *s* ♈ *r* + log. tan. *n* ♈ −10, = log. tan. *r* ♈ the *longitude*. And rad. : ſin. *r* ♈ :: tan. *r* ♈ *s* : tan. *sr*; hence log. ſin. *r* ♈ + log. tan. *r* ♈ *s* −10, = log. tan. *sr* the *latitude*. And in whatever poſition we take *s*, theſe concluſions will give the rule as ſtated above. If we conſider ♈ *C* as the equator and ♈ ♌ the ecliptic, the demonſtration will do for the ſecond rule.

Ex. Given the true *A.R.* of the moon's center 7*s*. 29°. 55′. 13″,4. and it's declination 15°. 40′. 26″,1 *ſouth*, as determined in the two laſt Examples, to find it's latitude and longitude.*

By Dr. MASKELYNE's obſervations, the mean obliquity of the ecliptic at the beginning of the year 1784, was 23°. 28′. 0″,2, and as it's gradual diminution is at the rate of ½ a ſecond in a year, the mean obliquity at the beginning of 1774 was 23°. 28′. 5,2, which corrected by Tab. XI. XII. gives 23°. 27′. 55″,8 for the obliquity at the time of obſervation.

Sine of right aſc. -	7*s*. 29°. 55′. 13″,4	-	9.9371817
Cotan. of decl. - -	15. 40. 26, 1	-	10.5519183
Cotan. arc A *ſouth* -	17. 57. 57, 8	-	10.4891000
Obliq. ecl. *north* - -	23. 27. 55, 8		
Arc B *north* - -	5. 29. 58, 0	cos.	9.9979964
Arith. comp. of log. cos. arc A - - -			0.0217102
Tang. of right aſc. - - - -			10.2371744
Tang. of *longitude* - -	8*s*. 1° 2′. 7″,4	-	10.2568810
Sine of longitude - - - - -			9.9419678
Tang. of arc B - - - - - -			8.9835328
Tang. of *latitude* north -	4°. 48′. 54″,1	-	8.9255006

* In making trigonometrical calculations, it will ſave time, when the ſame arcs occur, to take out all their logarithms at once, to avoid the trouble of turning to them again. The Computer therefore, before he begins his operation, ſhould put it down in it's proper order, leaving it to be filled up by the logarithms; he will then ſee what arcs are repeated, and he may, at one opening of the tables, take out all their logarithms and put them down in their proper places.

In

In like manner the right aſcenſions and declinations of the fixed ſtars being found from obſervation, their latitudes and longitudes may be computed, and thus a catalogue of all the fixed ſtars may be made for any time. But as both the equator and ecliptic are ſubject to a change in their poſitions, the right aſcenſion, declination, latitude and longitude of all the fixed ſtars will vary. Hence if their annual variations be computed, as will be afterwards explained, their right aſcenſions, &c. may be found at any other time.

125. If the body be the ſun at s', whoſe right aſcenſion and declination are given, to find its longitude; then ſin. $s' \curlyvee n$: rad. :: ſin. $s'n$: ſin. $\curlyvee s'$, that is, *ſin. obl. ecl.* : *rad.* :: *ſin. decl.* : *ſin. longitude.* Or, cos. $s' \curlyvee n$: rad. :: tan. $\curlyvee n$: tan. $\curlyvee s'$, that is, *cos. obl. ecl.* : *rad.* :: *tan. right aſc.* : *tan. longitude.* The ſun being always in the ecliptic has no latitude.

To find the angle of Poſition.

126. Let p be the pole of the ecliptic $\curlyvee L$, P the pole of the equator $\curlyvee C$, S a ſtar, draw the great circles $pPLC$, pSD, $PSBA$, and (53) PSp is the angle of poſition. Now the angle PpS, or (12) DL, is the complement of longitude $\curlyvee D$; the angle pPS is the ſupplement of APC, or of AC (12), which is the complement of the right aſcenſion $\curlyvee A$ of the ſtar; pP is the obliquity of the ecliptic; PS is the complement of declination, and pS the complement of the latitude of the ſtar. Hence if the longitude and declination of a ſtar be given, we have, ſin. PS : ſin. PpS :: ſin. Pp : ſin. PSp, that is, *cos. ſtar's dec.* : *cos. it's long.* :: *ſin. obl. ecl.* : *ſin. angle of Poſition.* If the latitude and declination of the ſtar be given, we know pS and PS their complements, and Pp; hence ſin. $pS \times$ ſin. PS : rad.^{2} :: ſin. $\frac{1}{2} \times \overline{SP + Sp + Pp} \times$ ſin. $\frac{1}{2} \times \overline{SP + Sp - Pp}$: $\overline{\cos. \frac{1}{2} \angle PSp}^2$. Or of the right aſcenſion, declination, latitude and longitude of the ſtar, any two being known, we ſhall know three parts of the triangle PpS, and conſequently the angle PSp may be found. If S be the ſun, $pS = 90°$, and the triangle may be ſolved by the circular parts.

FIG. 22.

CHAP. IV.

ON THE EQUATION OF TIME.

Art. 127. HAVING explained, in the laſt Chapter, the practical methods of determining the place of any body in the Heavens, we come next to the conſideration of another circumſtance not leſs important, that is, the irregularity of time as meaſured by the ſun. The beſt meaſure of time which we have is a clock regulated by the vibration of a pendulum. But however accurately a clock may be made, it muſt be ſubject to go irregularly, partly from the imperfection of the workmanſhip, and partly from the expanſion and contraction of the materials by heat and cold, by which the length of the pendulum, and conſequently the time of vibration, will vary. As no clock therefore can be depended upon for keeping time accurately, it is neceſſary that we ſhould be able to aſcertain at any time, how much it is too faſt or too ſlow, and at what rate it gains or loſes. For this purpoſe it muſt be compared with ſome motion which is uniform, or of which, if it be not uniform, you can aſcertain the variation. The motions of the heavenly bodies have therefore been conſidered as moſt proper for this purpoſe. Now the earth revolving uniformly about it's axis, the apparent diurnal motion of the fixed ſtars about the axis muſt be uniform. If a clock therefore be adjuſted to go 24 hours from the paſſage of any fixed ſtar over the meridian till it returns to it again, it's rate of going may be at any time determined by comparing it with any fixed ſtar, and obſerving whether the interval continues to be 24 hours; if not, the difference ſhows how much it gains or loſes in that time. A clock adjuſted to go 24 hours in this interval is ſaid to be adjuſted to *ſidereal* time. But if we compare a clock with the ſun, and adjuſt it to go 24 hours from the time the ſun leaves the meridian on any day, till it returns to it the next day, which is a *true* ſolar day, the clock will not, even if it go uniformly, continue to agree with the ſun, that is, it will not ſhow 12 when the ſun comes to the meirdian.

FIG. 23. 128. For let P be the pole of the earth, $vwyz$ it's equator, and let the earth revolve about it's axis in the order of the letters $vwyz$; γDLE the celeſtial equator, and γCL the ecliptic, in which the ſun moves according to that direction. Let a, m, be the ſun when on the meridian of any place on two ſucceſſive days, and draw $Pvae$, $Prmh$, ſecondaries to the equator, and let the

ſpec-

ſpectator be at *s* on the meridian *Pv*, with the ſun at *a* on his meridian. Then when the earth has made one revolution about it's axis, *Psv* is come again into the ſame poſition; but the ſun having moved forward to *m*, the earth has ſtill to deſcribe the angle *vPr* in order to bring the meridian *Psv* into the poſition *Pr*, ſo that the ſun may be again in the ſpectator's meridian. Now the angle *vPr* is meaſured by the arc *eh*, which is the increaſe of the ſun's right aſcenſion in a *true* ſolar day; hence the length of a *true* ſolar day is *equal to the time of the earth's rotation about it's axis + the time of it's deſcribing an angle equal to the increaſe of the ſun's right aſcenſion in a true ſolar day.* Now if the ſun moved uniformly in the equator ♈ *DLE*, this increaſe *eh* would be always the ſame in the ſame time, and therefore the ſolar days would be always equal; but the ſun moves in the ecliptic ♈ *CL*, and therefore *if* it's motion were *uniform*, equal arcs *am* upon the ecliptic would not give equal arcs *eh* upon the equator.* But the motion of the ſun is not uniform, and therefore *am*, deſcribed in any given time, is ſubject to a variation, and which alſo muſt neceſſarily make *eh* variable. Hence the increaſe *eh* of the ſun's right aſcenſion in a day varies from two cauſes, that is, from the obliquity of the ecliptic to the equator, and from the unequal motion of the ſun in the ecliptic. Hence the length of a true ſolar day is ſubject to a continual variation; conſequently a clock adjuſted to go 24 hours for any one true ſolar day, would not continue to ſhow 12 when the ſun comes to the meridian, becauſe the intervals by the clock would continue equal (the clock being ſuppoſed neither to gain or loſe), but the intervals of the ſun's paſſage over the meridian would vary.

129. As the ſun moves through 360° of right aſcenſion in $365\frac{1}{4}$ days very nearly, therefore $365\frac{1}{4}$ days : 1 day :: 360°. : 59′. 8″,2 the increaſe of right aſcenſion in one day, if the increaſe were uniform, or it would be the increaſe in a *mean* ſolar day, that is, if the ſolar days were all equal. If therefore a clock be adjuſted to go 24 hours in a *mean* ſolar day, it cannot continue to coincide with the ſun, that is, to ſhow 12 when the ſun is on the meridian; but the ſun will paſs the meridian, ſometimes *before* 12 and ſometimes *after*. This difference is called the *Equation of Time*. A clock thus adjuſted is ſaid to be ad-

* For draw *mt* parallel to *eh*, and ſuppoſe *ma* to be indefinitely ſmall; then by plain trigon.
ma : *mt* :: rad. : ſin. *mat*, or ♈*ae*,
mt : *eh* :: cos. *ae* : rad. (13)
∴ *ma* : *eh* :: cos. *ae* : ſin. ♈*ae* :: (becauſe ſin. $\Upsilon ae = \frac{\cos. a\Upsilon e \times \text{rad.}}{\cos. ae}$) $\sqrt{\cos. a\Upsilon e^2}$: cos. *a*♈*e* × radius; hence the ratio of *ma* to *eh* is variable.

juſted to *mean ſolar time.** The time ſhown by the clock is called *true* or *mean* time, and that ſhown by the ſun is called *apparent* time. What we call *apparent* time the French call *true*.

130. A clock adjuſted to go 24 hours in a *mean* ſolar day, would coincide with an imaginary ſtar moving uniformly in the equator with the ſun's mean motion 59′. 8″,2 in right aſcenſion, if the ſtar were to ſet off from any given meridian when the clock is 12; that is, the clock would always ſhow 12 when the ſtar came to the meridian, becauſe the interval of the paſſages of this ſtar over the meridian would be a *mean* ſolar day. This ſtar therefore, if we reckon it's motion from the meridian in time at the rate of 1 hour for 15°, would always coincide with the clock; that is, when the clock ſhows 1 hour, the ſtar's motion would be 1 hour; when the clock ſhows 2 hours, the ſtar's motion would be 2 hours; and ſo on. Hence this ſtar may be ſubſtituted inſtead of the clock; therefore when the ſun paſſes the given meridian, the difference between it's right aſcenſion and that of the ſtar, converted into time, is the difference between the time when the ſun is on the meridian and 12 o'clock, or the equation of time; becauſe the given meridian paſſes through the ſtar at 12 o'clock, and it's motion in reſpect to that ſtar is at the rate of 15° in an hour (132).

FIG. 24. 131. Now to compute this equation of time, let $APLS$ be the ecliptic, ALv the equator, A the firſt point of aries, P the the ſun's apogee, S any place of the ſun, draw Sv perpendicular to the equator, and take $An = AP$. When he ſun ſets out at P, let the imaginary ſtar ſet out at n with the ſun's mean motion in right aſcenſion, or longitude, or at the rate of 59′. 8″,2 in a day, and when n paſſes the meridian let the clock be adjuſted to 12, as deſcribed in the laſt Article: Theſe are the correſponding poſitions of the clock and ſun as aſſumed by Aſtronomers. Take $nm = Ps$, and when the ſtar comes to m, the place of the ſun, if it moved uniformly with it's mean motion, would be at s, but at that time let S be the place of the ſun. Now let the ſun S, and conſequently v, be on the meridian; and then as m is the place of the imaginary ſtar at that inſtant, mv is the equation of time. The ſun's mean place is at s, and as $An = AP$, and $nm = Ps$, $\therefore Am = APs$, conſequently $mv = Av - Am = Av - APs$. Let a be the *mean* equinox, and draw az perpendicular to AL; then $Am = Az + zm = Aa \times \cos. aAz + zm = \frac{11}{12} Aa + zm$; hence $mv = Av - zm - \frac{11}{12} Aa$; but Av is the ſun's true right aſcenſion, zm is the mean right aſcenſion, or mean longitude, and $\frac{11}{12} Aa$ (Az) is the equation of the equinoxes in right aſcenſion; hence the equation of time

* As the earth deſcribes an angle of 360°. 59′. 8″,2 about it's axis in a *mean* ſolar day of 24 hours, and an angle of 360° in a *ſidereal* day, therefore 360°. 59′. 8″,2 : 360° :: 24*h*. : 23*h*. 56′. 4″,098 the length of a ſidereal day in mean ſolar time, or the time from the paſſage of a fixed ſtar over the meridian till it returns to it again.

is

is equal to *the difference of the ſun's true right aſcenſion, and it's mean longitude corrected by the equation of the equinoxes in right aſcenſion.* When *Am* is *leſs* than *Av*, *mean* time precedes *apparent*, and when *greater*, *apparent* time precedes *mean*; for as the earth turns about it's axis in the direction *Av*, or in the order of right aſcenſion, that body whoſe right aſcenſion is leaſt muſt come to the meridian firſt. That is, when the ſun's true right aſcenſion is *greater* than it's mean longitude corrected as above, we muſt *add* the equation of time to apparent, to get the mean time; and when it is *leſs*, we muſt *ſubtract*. To convert mean time into apparent, we muſt *ſubtract* in the *former* caſe and *add* in the *latter*. This Rule for computing the equation of time was firſt given by Dr. MASKELYNE in the *Phil. Tranſ.* 1764.

132. As a meridian of the earth, when it leaves *m*, returns to it again in 24 hours, it may be conſidered, when it leaves that point, as approaching a point at that time 360° from it, and at which it arrives in 24 hours. Hence, the relative velocity with which a meridian accedes to or recedes from *m* is at the rate of 15° in an hour. Therefore when the meridian paſſes through *v*, the arc *vm* reduced into time at the rate of 15° in an hour, gives the equation of time at that inſtant. Hence the equation of time is computed for the inſtant of *apparent* noon. Now the time of apparent noon in mean ſolar time, for which we compute, can only be known by knowing the equation of time. To compute therefore the equation on any day, you muſt aſſume the equation the ſame as on that day four years before, from which it will differ but very little, and it will give the time of apparent noon, ſufficiently accurate for the purpoſe of computing the equation. If you do not know the equation four years before, compute the equation for noon mean time, and that will give apparent noon accurately enough.

Ex. To find the equation of time on July 1, 1792, for the meridian of Greenwich, by MAYER's Tables.

The equation on July 1, 1788, was, by the *Nautical Almanac*, 3′. 28″, to be added to apparent noon, to give the correſponding mean time; hence, for July 1, 1792, at 0*h*. 3′. 28″ compute the true longitude *.

* The reaſon of this operation will appear, when we come to the conſtruction and uſe of the Solar Tables.

Epoch

	Mean. Long. ☉	Long. ☉'s Apog.	N°.1.	N°.2.	N°.3.	N°.4.
Epoch for 1792.	9ˢ. 10°. 50′. 0″,7	3ˢ. 9°. 23′. 46″	241	227	123	478
Mean Mot. July 1,	5. 29. 23. 16, 2	33	163	456	312	27
3′	7, 4					
28″	1, 1					
Mean Longitude	3. 10. 13. 25, 4	3. 9. 24. 19	404	683	435	505
Equat. of Center	— 1. 37, 1	3. 10. 13. 25, 4				
Equat. ☽ I.	+ 4, 5					
♃ II.	− 4, 7	49. 6, 4	Mean Anomaly.			
♀ III.	+ 3,65					
☌ IV.	− 0, 6					
True Longitude	3. 10. 11. 51,15					

With this true longitude and obliquity 23°. 27′. 48″,4 of the ecliptic, the true right aſcenſion of the ſun is found to be 3ˢ. 11°. 5′. 41″,25; alſo the equation of the equinoxes in longitude = − 0″,6; hence

The mean longitude	3ˢ. 10°. 13′. 25″,4
$\frac{11}{12}$ of − 0″,6	− 0,55
Mean longitude corrected	3. 10. 13. 24,85
True right aſcenſion	3. 11. 5. 41,25
Equation	52. 16, 4 which

converted into time gives 3′. 29″,1 for the true equation of time; which muſt be added to apparent to give the true time, becauſe the true right aſcenſion is greater than the mean longitude.

133. The ſun's apogee *P* has a progreſſive motion, and the equinoctial points *A*, *L*, have a regreſſive motion; the inclination alſo of the equator to the ecliptic is ſubject to a conſtant variation. Hence, the ſame Table of the equation of time cannot continue to ſerve for the ſame degree of the ſun's longitude. Alſo the ſun's longitude at noon at the ſame place is different for the ſame days on different years, and it is for apparent noon that the equation is computed. For theſe reaſons, the equation of time muſt be computed a-new for every year.

134. Whenever it is required to make any calculations from aſtronomical Tables, and the time given is apparent time, the equation of time muſt be applied to convert it into mean time, and for that time the computations muſt be made, the Tables being diſpoſed according to mean motions. Thus, if it were required

quired to find the ſun's place on any day at apparent noon, the equation of time that day at apparent noon muſt be applied to 12 o'clock, and then the ſun's place computed from the tables for that time. All the articles in the Nautical Almanac anſwering to noon are computed in the ſame manner.

135. A clock adjuſted to ſidereal time begins at o*h*. o′. o″ when the true equinox *A* is upon the meridian; therefore the diſtance of the meridian from *A* meaſures ſidereal time. A clock adjuſted to mean ſolar time begins at o*h*. o′. o″ when *m* is upon the meridian. Let *x* be a point of the equator through which the meridian paſſes at any time, then *Ax* is the ſidereal time; and let *t* be the place of the imaginary ſtar at the ſame inſtant, and *y* it's place when the meridian coincided with it; then (132) the arc *xt* is the meaſure of the time from the mean noon. Hence to get *xt*, ſubtract the ſun's mean right aſcenſion *Ay* in time at noon on the given day from the time *Ax* ſhown by the ſidereal clock, and you get *xy*, which is nearly the time *xt* from mean noon; from this ſubtract *ty*, the ſun's mean motion in right aſcenſion in the interval *xy* of ſidereal time, and you have *xt* the time from mean noon by a clock adjuſted to mean ſolar time. To facilitate this computation, Dr. MASKELYNE has given two Tables; Table XIX. ſhows the mean motion of the ſun in right aſcenſion for every day of the year; Table XX. is the mean motion of the ſun in right aſcenſion in time to hours and minutes of ſidereal time. Hence, from the Solar Tables, take the epoch of the ſun's mean longitude for the year, and convert it into time, and add it to the time in Table XIX. correſponding to the given day, and correct it by Table XXI, and it gives the ſun's mean longitude, or mean right aſcenſion, expreſſed in ſidereal time, reckoned from the true equinox, at the mean noon of the propoſed day: This ſubtracted from the propoſed ſidereal time, gives the mean time nearly, with which Table XX. is to be entered, and the number taken out of it, being the ſun's mean motion ſince the mean noon, ſubtracted from the mean time found nearly, will give the mean time correct. It is to be obſerved, that the mean time found nearly, or before it is corrected by Table XX, is a portion of ſidereal time, being the interval by the clock between the tranſit of the imaginary ſtar, and the propoſed inſtant; and therefore to ſhorten the operation, Table XX. is made to be entered with ſidereal time, inſtead of mean time, commonly uſed in Aſtronomical Tables. Dr. MASKELYNE alſo gave another Table of the epoch of the ſun's mean right aſcenſion in time for the beginning of the year; but as that can be taken from our Tables of the ſun's motion, the mean right aſcenſion and mean longitude being the ſame, it is not here given.

Ex. On July 1, 1790, the time by the ſidereal clock was 11*h*. 20′. 14″, to find the mean ſolar time.

Epoch

Epoch of ſun's *AR.* 1790 -	18*h*. 41′. 15″,9
Mean mot. in *AR.* to July 5, Tab. XIX.	12 . 13 . 19, 3
Equat. equin. Tab. XXI. -	+ 0, 66
⊙'s mean long. at mean noon -	6 . 54 . 35, 86
Sidereal time given - -	11 . 20 . 14
Mean time nearly - -	4 . 25 . 38, 14
Cor. by Tab. XX. - -	43, 5
Mean ſolar time - -	4 . 24 . 54, 64

Hence if the mean ſolar time be given, for inſtance, 4*h*. 24′. 54″,64, we may thus find the ſidereal time. To get the correction from Table XX, correſponding to mean time nearly, firſt get it for mean ſolar time, which is 42″,39, and add it to the mean ſolar time, and we have 4*h*. 25′. 37″,03, which is very near what we call, mean time nearly; correſponding therefore to this time, take out the correction from Table XX, which is 43′,5, and add it to the given mean ſolar time, and we get 4*h*. 25′. 38″,14 correctly for what we call mean time nearly; add this to 6*h*. 54′. 35″,86, the ſun's mean longitude at noon, and it gives 11*h*. 20′. 14″ the ſidereal time required.

136. Whenever the time is computed from the ſun's altitude, that time muſt be apparent time, becauſe we compute it from the time when the ſun comes to the meridian, which is noon, or 12 o'clock, apparent time. Hence alſo the time ſhown by a dial is apparent time, and will differ from the time ſhown by a well regulated watch or clock, by the equation of time. A clock or watch may therefore be regulated by a good dial, by applying the equation, as before directed, to the apparent time ſhown by the dial, and it will give the *mean* time, or that which the clock or watch ought to ſhow.

137. Mr. Wollaston has propoſed to regulate a watch or clock by a dial conſtructed to ſhow mean noon, or 12 o'clock by a watch or clock. A ray of light through a ſmall hole being let into a dark chamber upon the floor, draw a meridian upon the floor correſponding to the hole, on which therefore the ſun's rays will always fall when the ſun comes to the meridian. On each ſide of this line, for every day of the year, make a point where the image of the ſun is at 12 o'clock mean time, by a clock or watch regulated for that purpoſe; through all theſe points draw a curve, and then you may regulate your clock or watch by ſetting it to 12 when the image of the ſun falls on that curve. To prevent any miſtake, put the months againſt the different parts of the curve on which the ray falls in them. Or the ſame may be done on any horizontal plane, by erecting a piece of braſs, and making a ſmall hole for the ſun to ſhine through.

through. The curve may alſo be laid down by calculation, as Mr. WOLLASTON has ſhown; and if it be drawn with great care, it will be ſufficiently accurate for regulating all common clocks; and it has this advantage over that of correcting them by a common ſun dial, that as the months are put to the curve, you cannot eaſily make a miſtake; whereas in applying the equation of time to a dial, a perſon ignorant of theſe matters is very apt to apply it wrong.

138. The Equation of Time was known to, and made uſe of by PTOLEMY. TYCHO employed only one part, that which ariſes from the unequal motion of the ſun in the ecliptic; but KEPLER made uſe of both parts. He further ſuſpected, that there was a third cauſe of the inequality of ſolar days, ariſing from the unequal motion of the earth about it's axis. But the Equation of Time, as now computed, was not generally adopted till 1672, when FLAMSTEAD publiſhed a Diſſertation upon it, at the end of the works of HORROX.

CHAP. V.

ON THE LENGTH OF THE YEAR, THE PRECESSION OF THE EQUINOXES FROM OBSERVATION, AND THE OBLIQUITY OF THE ECLIPTIC.

Art. 139. FROM comparing the ſun's right aſcenſion every day with the fixed ſtars lying to the eaſt and weſt, the ſun is found conſtantly to recede from thoſe on the weſt, and approach to thoſe on the eaſt; and the interval of time from it's leaving any fixed ſtar till it returns to it again is called a *ſidereal* year, being the time in which the ſun completes it's revolution amongſt the fixed ſtars, or in the ecliptic. But the ſun, after it leaves either of the equinoctial points, returns to it again in a leſs time than it returns to the ſame fixed ſtar, and this interval is called a *ſolar* or *tropical* year, becauſe the time from it's leaving one equinox till it returns to it, is the ſame as from one tropic till it comes to the ſame again. This is the year on which the return of the ſeaſons depends.

On the Sidereal Year.

140. To find the length of a *ſidereal* year. On any day take the difference between the ſun's right aſcenſion when it paſſes the meridian and that of a fixed ſtar; and when the ſun returns to the ſame part of the heavens the next year, compare it's right aſcenſion with the ſame ſtar for two days, one when their difference of right aſcenſions is leſs and the other when greater than the difference before obſerved; and let D be the increaſe of the ſun's right aſcenſion in this interval of one day; then take the difference (d) between the differences of the ſun's and ſtar's right aſcenſions on the firſt of theſe two days and on the day when the obſervation was made the year before; and let t be equal to the exact time between the tranſits of the ſun over the meridian on the two days; then $D : d :: t$: the time from the paſſage of the ſun over the meridian on the firſt day to the inſtant when it had the ſame difference of right aſcenſion compared with the ſtar which it had the year before; the interval between theſe two times gives the length of a ſidereal year. The beſt time for theſe obſervations is about March 25, June 20, September 17, December 20, the ſun's motion in right aſcenſion being then uniform. Inſtead of obſerving the difference

difference of the right aſcenſions, you may obſerve that of their longitudes. If inſtead of repeating the ſecond obſervations the year after, there be an interval of ſeveral years, and you divide the obſerved interval of time when the difference of their right aſcenſions was found to be equal, by the number of years, you will have the length of a ſidereal year more exact. Or the length may be found thus.

141. Take the time (t) of a ſtar's tranſit over the meridian by a clock adjuſted to mean ſolar time; then the year after, take the time again on two days, one (m) when it paſſes the meridian *before*, and the other (n) *after* the time t; then $m-n : m-t :: 23h.\ 56'.\ 4''$: the time from m till the difference between the ſtar's and ſun's right aſcenſion was the ſame as at the firſt obſervation; and the interval of theſe two times is the length of a ſidereal year. *Caſſini's Elem. d'Aſtron.* pag. 202.

Ex. On April 1, 1669, at $0h.\ 3'.\ 47''$ mean ſolar time, M. PICARD obſerved the difference between the ſun's longitude and that of *Procyon* to be $3^s.\ 8^\circ.\ 59'.\ 36''$, which is the moſt ancient obſervation of this kind whoſe accuracy can be depended upon, ſee *Hiſt. Celeſte, par M. le Monnier*, pag. 37. And on April 2, 1745, M. de la CAILLE found, by taking their difference of longitudes on the 2d and 3d, that at $11h.\ 10'.\ 45''$ mean ſolar time, the difference of their longitudes was the ſame as at the firſt obſervation. Now as the ſun's revolution was known to be nearly 365 days, it is manifeſt that it had made in this interval 76 complete revolutions in reſpect to the ſame fixed ſtar in the ſpace of 76 years $1d.\ 11h.\ 6'.\ 58''$. Now in theſe 76 years there were 58 of 365 days, and 18 biſſextiles of 366 days; that interval therefore contains $27759d.\ 11h.\ 6'.\ 58''$, which being divided by 76 the quotient is $365d.\ 6h.\ 8'.\ 47''$ the length of a ſidereal year.

Ex. M. CASSINI obſerved the tranſit of *Sirius* over the meridian on May 21, 1717, to be at $2h.\ 38'.\ 58''$; on May 21, 1718, it paſſed at $2h.\ 40'$, and on the 22d at $2h.\ 36'$; to find the length of the ſidereal year.

In this caſe $t = 2h.\ 38'.\ 58''$, $m = 2h.\ 40'$, $n = 2h.\ 36'$, hence $4' : 1'.\ 2'' :: 23h.\ 56'.\ 4'' :: 6h.\ 10'.\ 59''$, which added to $2h.\ 40'$ the time it paſſed on May 21, 1718, gives $8h.\ 50'.\ 59''$ for the time on that day when the difference between the ſun's and ſtar's right aſcenſions was the ſame as on May 21, 1717. Hence this interval is $365d.\ 6h.\ 10'.\ 59''$ for the length of a ſidereal year. The mean of theſe two gives the length $365d.\ 6h.\ 9'.\ 53''$. But the length of a ſidereal year has generally been determined from the length of a tropical year, found as we ſhall now proceed to explain.

On the Tropical Year.

142. Obſerve the meridian altitude (a) of the ſun on the day neareſt to the equinox; then the next year take it's meridian altitude on two following days, one when it's altitude (m) is leſs than a, and the next when it's altitude (n) is greater than a, and $n - m$ is the increaſe of the ſun's declination in 24 hours; hence $n - m : a - m ::$ 24 hours : the interval from the firſt of the two days till the ſun has the ſame declination as at the obſervation the year before, becauſe at that time the ſun's declination increaſes uniformly. Hence we find the time when the ſun's place in the ecliptic had the ſame ſituation in reſpect to the equinoctial points which it had at the time of the obſervation the year before. Therefore this 4th term being added to the number of days between the two firſt obſervations, gives the length of a tropical year. If inſtead of repeating the ſecond obſervation the next year, there be an interval of ſeveral years, and you divide the interval between the times when the declination was found to be the ſame, by the number of years, you will get the time more exactly. *Caſſini's Elem. d'Aſtron.* pag. 204.

Ex. M. Cassini informs us, that on March 20, 1672, his Father obſerved the meridian altitude of the ſun's upper limb at the Royal Obſervatory at Paris, to be 41°. 43′; and on March 20, 1716, he himſelf obſerved the meridian altitude of the upper limb, to be 41°. 27′. 10″; and on the 21ſt to be 41°. 51′. Hence the difference of the two latter altitudes was 23′. 50″, and of the two former 15′. 50″; hence 23′. 50″ : 15′. 50″ :: 24 hours : 15*h*. 56′. 39″; therefore on March 20, 1716, at 15*h*. 56′. 39″ the ſun's declination was the ſame as on March 20, 1672. Now the interval between theſe two obſervations was 44 years, of which 34 conſiſted of 365 days each, and 10 of 366; therefore the interval in days was 16070; hence the whole interval between the equal declinations was 16070 days 15*h*. 56′. 39″, which divided by 44, gives 365*d*. 5*h*. 49′. 0″. 53‴ the length of a tropical year from theſe obſervations.

But when we determine the length of a tropical or ſolar year from the times of the equinoxes, it will want a correction to give the length of a mean tropical or mean ſolar year; becauſe, from the motion of the ſun's apogee, the equation of the orbit at the equinox is not the ſame in different years, which will affect the time of the return of the ſun to the ſame mean longitude; and therefore will make the apparent ſolar year different from the mean ſolar year. This correction therefore gives the time that would have elapſed between the equinoxes, if the apogee had been fixed; this is called the *mean* ſolar year. To apply this correction to the laſt Example, we proceed thus.

On

On March 20, 1672, the place of the ſun's apogee was 3ſ. 7°. 7′. 6″ by CASSINI, therefore the ſun's true anomaly was 8ſ. 22°. 52′. 54″; from which we find that the equation of the center, or the difference between the true and mean anomaly, was 1°. 54′. 42″, ſhowing how much the true anomaly exceeds the mean; ſubtract this from 0ſ. 0°. 0′. 0″ and we get 11ſ. 28°. 5′. 18″ for the mean longitude of the ſun at the time of the equinox. The place of ſun's apogee on March 20, 1716, was 3ſ. 7°. 52′. 23″, and therefore it's true anomaly was 8ſ. 22°. 7′. 37″, from which the equation of the center was 1°. 54′. 29″, which ſubtracted from 0ſ. 0°. 0′. 0″ gives 11ſ. 28°. 5′. 31″ for the mean longitude of the ſun at the equinox in 1716. Hence the ſun's mean place at the equinox in the ſpring 1716 is greater by 13″ than in 1672, and this anſwers to 5′. 16″ in time; in this interval of time therefore (44 years), there have been 44 mean revolutions + 5′. 16″; and conſequently 44 apparent ſolar years are greater by 5′. 16″ than 44 mean; divide this by 44, the number of years in the interval, and it gives 7″. 11‴ for the length of the *apparent* above the *mean* ſolar year. Now the length of the apparent ſolar year was determined to be 365*d*. 5*h*. 49′. 0″. 53‴; hence, from theſe obſervations, the length of the *mean* ſolar year is 365*d*. 5*h*. 48′. 53″. 42‴.

143. The length of a tropical year may alſo be found by obſerving the exact time of the equinoxes. To do this we muſt previouſly know the latitude of the place, from which we ſhall know the altitude of the point of the equator on the meridian, it being equal (87) to the complement of latitude. Take the meridian altitude of the ſun's center on two days, one when it is leſs than the complement of latitude and the other when greater; then the ſun muſt have paſſed the equator in the intermediate time. Take the difference (*D*) between theſe altitudes and it gives the increaſe of the ſun's declination in 24 hours; take alſo the difference (*d*) between the altitude on the firſt day and the complement of latitude, and then ſay, *D* : *d* :: 24 hours : to the time from noon on the firſt day till the ſun came to the equator. Repeat this when the ſun returns to the ſame equinox, and the interval of the times gives the length of a tropical year. If an interval of ſeveral years be taken, and you divide by the number, it will give the time more accurately. If we take a difference of two days, the third term muſt be 48*h*. The ſame may be done by one obſervation, if we know the rate at which the ſun changes it's declination in 24 hours, which at the equinox in ſpring time is found, by the mean of a great number of obſervations, to be 23′. 40″, and in the autumn to be 23′. 28″. *Caſſini's Elem. d' Aſtr.* pag. 207.

Ex. On March 20, 1672, the ſun's meridian altitude at the Royal Obſervatory at Paris was obſerved to be 41°. 25′. 56″, from which ſubtract 41°. 9′. 50″ the meri-

meridian altitude of the equator, and there remains 16′. 6″ for the ſun's declination; hence 23′. 40″ : 16′. 6″ :: 24 hours : 16*h*. 19′, the ſun's diſtance in time from the equinox, which, as the ſun was paſt the equinox, ſubtracted from the 20th gives the 19th day 7*h*. 41′ for the time of the equinox. And in 1731 the time of the equinox was found, in the ſame manner, to be on Mar. 20, at 14*h*. 45′. In this interval of 59 years there were 13 biſſextiles, and conſequently the whole number of days in the 59 years was 21548, and therefore the whole interval between the two equinoxes was 21549*d*. 7*h*. 4′, which divided by 59 gives the length of the *apparent* ſolar year 365*d*. 5*h*. 48′. 53″; from this ſubtract 7″, the variation of the equation of the orbit in the interval of the obſervations, and we have the mean length of the ſolar year 365*d*. 5*h*. 48′. 46″. The interval has here been taken between the *true* equinoxes, whereas we want to get the length of a tropical year between the *mean* equinoxes in order to get the length of a *mean* tropical year. But in taking a long interval of time, the difference, whether we take the true or mean equinox, will be inſenſible. Another correction might alſo be added, when we compare the modern obſervations with the ancient ones, on account of the preceſſion of the equinoxes being greater now than it was then. From the modern obſervations the length of a mean ſolar year appears to be 2″,6 leſs than that which is deduced from comparing the ſame obſervations with thoſe of HIPPARCHUS.

144. As the ſun's declination at the equinoxes changes about 24′ in 24 hours, an error of 10″ in the altitude of the ſun will cauſe an error of 10 minutes in the determination of the time of the equinox, and conſequently the ſame error in the length of the year, if it were determined by 2 obſervations at the interval only of 1 year; but if the interval were 60 years, the error would be only 10 ſeconds. As the accuracy therefore is very much increaſed by taking a long interval, let us compare the moſt ancient obſervations with the modern ones.

HIPPARCHUS, in the year 145 before J. C. found the time of the equinox to be on March 24, at 11*h*. 55′ in the morning at Alexandria. In the year 1735, at the Royal Obſervatory at Paris the time of the equinox was found to be on March 20, at 14*h*. 20′. 40″. Now the difference of the meridians between Paris and Alexandria is, in time, 1*h*. 51′. 46″, which, as Alexandria lies to the eaſt of Paris, being added to 14*h*. 20′. 40″ gives 16*h*. 12′. 26″ the time at Alexandria. Reduce this time to the Julian year, by ſubtracting 11 days by which the Gregorian is before the Julian, and we have the time of the equinox by this ſtyle, on March 10, at 4*h*. 12′. 26″ in the morning. Between theſe two obſervations there was an interval of 1880 Julian years, except 14*d*. 7*h*. 42′. 34″. In theſe years there were 470 biſſextiles and the reſt common Julian years of 365 days. Therefore if we divide 14*d*. 7*h*. 42′. 34″ by 1880 it gives 10′. 58″. 10‴, ſhowing how much the apparent ſolar year is leſs than 365

days

days 6 hours; hence the length of the *apparent* ſolar year is 365*d*. 5*h*. 49′. 1″. 50‴, to which add 6″. 30‴, being what the *apparent* is leſs than the *mean* ſolar year, found as before, and we get 365*d*. 5*h*. 49′. 8″. 20‴ the length of the *mean* ſolar year from theſe obſervations. The mean of 10 reſults from different obſervations made by HIPPARCHUS, compared with the modern ones, gives the length of the *mean* ſolar year 365*d*. 5*h*. 48′. 49″.

145. The length of the year may alſo be found by finding the time when the ſun comes to the tropic. For let *ADL* be the equator, *ASL* the ecliptic, *A* aries; find the time (119) when the ſun has the ſame declination *mv*, *nw* on each ſide of the tropic *S*, and at the ſame times find alſo the differences of it's right aſcenſion and that of a fixed ſtar *s*, the ſum or difference of which *wz*, *vz*, according to the poſition of *z*, meaſures the motion *vw* of the ſun in right aſcenſion, the half of which is *wD* (*SD*, *sz* being perpendicular to *AL*); hence we ſhall get *Dz* which is equal to $wD \pm wz$. Now to find when the ſun comes to *D*, obſerve it's right aſcenſion at *x*, either the day before or day after the ſolſtice, compared with the ſame ſtar, and you have *xz*, the difference between which and *Dz*, is *Dx*. Obſerve alſo the increaſe (*d*) of the ſun's right aſcenſion at that time in 24 hours; then $d : xD :: 24h.$: the time of the paſſage of the ſun from *x* to *D*, which added to or ſubtracted from the time at *x*, according as *vx* is leſs or greater than *vD*, gives the time when the ſun in right aſcenſion is at *D*, or when it is in the ſolſtice *S*. *Caſſini's Elem. d'Aſtron.* pag. 238. FIG. 25.

Ex. According to CASSINI on May 29, 1737, the altitude of the ſun's upper limb, when it paſſed the meridian, was 63°. 6′. On July 14, it's altitude on the meridian was 63°. 7′, and on the 15th it was 62°. 57′. 35″; it was therefore diminiſhed 9′. 25″ in one day, and on the 14th it's altitude was 1′ greater than on May 29; hence 9′. 25″ : 1′ :: 24*h*. : 2*h*. 32′. 55″ which added to the 14th gives 2*h*. 32′. 55′ for the time when the altitude, and conſequently the declination, was the ſame as on May 29. On the ſame May 29, the difference *vz* of the right aſcenſion of the ſun and *Sirius* was 32°. 9′. 8″. On July 14, the difference was 15°. 16′. 4″ when the ſun was on the meridian; and as the increaſe of the ſun's right aſcenſion was then 1°. 0′. 45″ in 24 hours, we have 24*h*. : 2*h*. 32′. 55″ :: 1°. 0′. 45″ : 6′ : 40″, which added to 15°. 16′. 4″ gives $zw = 15°. 22′. 44″$ the difference of the right aſcenſions of the ſun and Sirius on July 14, at 2*h*. 32′. 55″. But as Sirius paſſed the meridian before the ſun, *z* in this caſe will fall between *D* and *w*, and therefore $vw = vz + zw = 47°. 31′. 52″$, hence $Dw = 23°. 45′. 56″$, from which take *zw* and we get $Dz = 8°. 23′. 12″$ the diſtance of Sirius in right aſcenſion from the ſolſtice. Now on June 21, Sirius paſſed the meridian at 0*h*. 33′. 34″, at which time the difference *zx* of it's right aſcenſion

and

and that of the ſun was 8°. 23′. 30″, and conſequently $xD = 18''$, ſhowing what the ſun wants in right aſcenſion of the ſolſtice. Now taking the increaſe of the ſun's right aſcenſion at that time to be 62′,25 in 24 hours, we have 62′, 25 : 18″ :: 24*h*. : 6′. 56″, which added to 0*h*. 33′. 34″ gives 0*h*. 40′. 30″ on the 21ſt for the time of the ſolſtice. Hence, by finding the interval of two ſolſtices, we get the length of a tropical year. After getting the apparent ſolar year, we get the mean ſolar year, by applying to it the variation of the equation of the center, for the ſame reaſon that we made a ſimilar correction at the equinoxes.

M. Cassini, by comparing a ſolſtice obſerved at Athens on June 27, 481 years before J. C. with one obſerved at Paris on June 21, 1717, found the length of a mean ſolar year to be 365*d*. 5*h*. 49′. 48″. 39‴. By comparing one obſerved at Alexandria on June 24, 140 years after J. C. with one obſerved at Paris on June 20, 1732, the length was found 365*d*. 5*h*. 47′. 36″. By ſolſtices obſerved at Nuremberg in 1487, 1493, 1498, 1503, and one at Paris on 1731, the length is found to be 365*d*. 5*h*. 48′. 31″. By comparing 14 ſolſtices obſerved at Uranibourg with as many obſerved at Paris, he found the length of the mean ſolar year to be 365*d*. 5*h*. 48′. 52″. The accuracy of theſe obſervations appears from hence, that of the 14 determinations, only 1 differed 20″, 1 differed 15″, 1 differed 11″, and all the others leſs.

If we take a mean of all the mean ſolar years as determined by Cassini from the equinoxes, leaving out 2 which differ very much from all the reſt, we have the length of a mean ſolar year 365*d*. 5*h*. 48′. 51″½. If we do the ſame by thoſe determined from the ſolſtices, the length comes out 365*d*. 5*h*. 48′. 42″½; the mean of which gives 365*d*. 5*h*. 48′. 47″ the length of a mean ſolar year.

146. M. de la Lande, in a Piece entitled *Memoire ſur la veritable Longueur de l'Année Aſtronomique*, which gained the prize propoſed by the Royal Society at Copenhagen for the year 1780, has, by comparing a great number of the moſt diſtant obſervations, and thoſe which could be moſt depended upon for their accuracy, determined the length of the mean ſolar year to be 365*d*. 5*h*. 48′. 48″, differing only 1″ from our determination from Cassini.

To find the Preceſſion of the Equinoxes from Obſervation.

147. The ſun returning to the equinox every year before it returns to the ſame point of the Heavens, ſhows that the equinoctial points have a retrograde motion, which, as we ſhall prove, ariſes from the motion of the equator, cauſed by the attraction of the ſun and moon upon the earth in conſequence of it's heroidical figure. The effect of this is, that the longitude of the ſtars

muſt

muſt conſtantly increaſe; and hence by comparing the longitude of the ſame ſtars at different times, the motion of the equinoctial points, or the preceſſion of the equinoxes, may be found.

148. HIPPARCHUS was the firſt perſon who obſerved this motion, by comparing his own obſervations with thoſe which TIMOCHARIS made 155 years before. From this he judged the motion to be one degree in about 100 years; but he doubted whether the obſervations of TIMOCHARIS were accurate enough to deduce any concluſion to be depended upon. In the year 128 before J. C. he found the longitude of *Virgin's Spike* to be 5*s*. 24°; and in the year 1750 it's longitude was found to be 6*s*. 20°. 21′, the difference of which is 26°. 21′. In the ſame year he found the longitude of the *Lyon's Heart* to be 3*s*. 29°. 50′; and in 1750 it was 4*s*. 26°. 21′, the difference of which is 26°. 31′. The mean of theſe two gives 26°. 26′ for the increaſe of longitude in 1878 years, or 50″. 40‴ in a year, for the preceſſion. By comparing the obſervations of d'ALBATEGNIUS in the year 878 with thoſe made in 1738, the preceſſion appears to be 51″. 9‴. From a compariſon of 15 obſervations of TYCHO with as many made by M. de la CAILLE, the preceſſion is found to be 50″. 20‴. But M. de la LANDE, from the obſervations of M. de la CAILLE compared with thoſe in FLAMSTEAD's Catalogue, determines the ſecular preceſſion to be 1°. 23′. 45″, or 50″,25 in a year.

149. The preceſſion being given, and alſo the length of a tropical year, the length of a ſidereal year may be found by this proportion; 360°— 50″,25 : 360°. :: 365*d*. 5*h*. 48′. 48″ : 365*d*. 6*h*. 9′. 11″½ the length of the *ſidereal* year.

On the Anomaliſtic Year.

150. The year, called the *anomaliſtic* year, is ſometimes uſed by Aſtronomers, and is the time from the ſun's leaving it's apogee till it returns to it. Now the motion of the ſun's apogee is 1′ 2″ every year, in longitude, or in reſpect to the equinox, according to M. de la LANDE, therefore 1′. 2″— 50″,25 = 11″,75 the progreſſive motion of the apogee in a year, and hence the anomaliſtic muſt be longer than the ſidereal year by the time the ſun takes in moving over 11″,75 of longitude at his apogee; but when the ſun is in it's apogee, it's motion in longitude is 58′. 13″ in 24 hours; hence 58′. 13″ : 11″,75 :: 24 hours : 4′. 50″⅔, which added to 365*d*. 6*h*. 9′. 11½″ gives 365*d*. 6*h*. 14′. 2′⅙ for the length of the anomaliſtic year. M. de la LANDE determined this motion of the apogee, from the obſervations of M. de la HIRE, and thoſe of Dr. MASKELYNE. CASSINI made it the ſame. MAYER made it 1′. 6″ in his Tables.

On the Obliquity of the Ecliptic.

151. The method uſed by Aſtronomers to determine the obliquity of the ecliptic is that explained in Art. 86. by taking half the difference of the greateſt and leaſt meridian altitudes of the ſun. The following is the obliquity as determined by different Aſtronomers.

ERATOSTHENES 230 years *before* J. C.	23°. 51′. 20″
HIPPARCHUS 140 years *before* J. C. -	23 . 51 . 20
PTOLEMY 140 years *after* J. C. -	23 . 51 . 10
PAPPUS in the year 390 - -	23 . 30 . 0
ALBATEGNIUS in 880 - - -	23 . 35 . 40
ARZACHEL in 1070 - - -	23 . 34 . 0
PROPHATIUS in 1300 - - -	23 . 32 . 0
REGIOMONTANUS in 1460 - -	23 . 30 . 0
COPERNICUS in 1500 - - -	23 . 28 . 24
WALTHERUS in 1490 - - -	23 . 29 . 47
TYCHO in 1587 - - - -	23 . 29 . 30
CASSINI (the Father) in 1656 -	23 . 29 . 2
CASSINI (the Son) in 1672 - -	23 . 28 . 54
FLAMSTEAD in 1690 - - -	23 . 28 . 48
De la CAILLE in 1750 - - -	23 . 28 . 19
Dr. BRADLEY in 1750 - - -	23 . 28 . 18
MAYER in 1750 - - - -	23 . 28 . 18
Dr. MASKELYNE in 1769 - -	23 . 28 . 8,5
M. de la LANDE in 1786 - -	23 . 28 . 0

The obſervations of ALBATEGNIUS, an Arabian, are here corrected for refraction. Thoſe of WALTHERUS, M. de la CAILLE computed. The obliquity by TYCHO is here put down as correctly computed from his obſervations. Alſo the obliquity, as determined by FLAMSTEAD, is corrected for the nutation of the earth's axis. Theſe corrections M. de la LANDE applied.

152. It is manifeſt from the above obſervations, that the obliquity of the ecliptic keeps diminiſhing; and the irregularity which here appears in the diminution we may aſcribe to the inaccuracy of the ancient obſervations, as we know that they are ſubject to greater errors than the irregularity of this variation. If we compare the firſt and laſt obſervations, they give a diminution of 70″ in 100 years. If we compare the laſt with that of TYCHO, it gives 45″. The laſt compared with that of FLAMSTEAD gives 50″. If we compare that of Dr. MASKELYNE with Dr. BRADLEY's and MAYER's, it gives 50″. The com-

compariſon of Dr. MASKELYNE's determination, with that of M. de la LANDE, which he took as the mean of ſeveral reſults, gives 50″. We may therefore ſtate the ſecular diminution of the obliquity of the ecliptic, at this time, to be 50″, as determined from the moſt accurate obſervations. This reſult agrees very well with that deduced from theory, as will be ſhown when we come to treat of the phyſical cauſe of this diminution.

CHAP. VI.

ON PARALLAX.

Art. 153. THE center of the earth deſcribes that circle in the Heavens which is called the ecliptic; but as the ſame object would appear in different poſitions in reſpect to this circle, when ſeen from the center and ſurface, Aſtronomers always reduce their obſervations to what they would have been, if they had been made at the center of the earth, in conſequence of which, the places of the heavenly bodie are computed as ſeen from the ecliptic, and it becomes a fixed point for that purpoſe, on whatever part of the earth's ſurface the obſervations are made.

FIG. 26. 154. Let C be the center of the earth, A the place of the ſpectator on it's ſurface, S any object, ZH the ſphere of the fixed ſtars, to which the places of all the bodies in our ſyſtem are referred; Z the zenith, H the horizon; draw CSm, ASn, and m is the place ſeen from the center, and n from the ſurface. Now the plane SAC paſſing through the center of the earth muſt be perpendicular to it's ſurface, and conſequently it will paſs through the zenith Z; and the points m, n, lying in the ſame plane, the arc of parallax mn muſt lie in a circle perpendicular to the horizon, and hence the azimuth is not affected, if the earth be a ſphere. Now the parallax mn is meaſured by the angle mSn or ASC, and by trig. $CS : CA ::$ ſin. SAC or SAZ : ſin. ASC the parallax $= \frac{CA \times \text{sin. } SAZ}{CS}$. As CA is conſtant, ſuppoſing the earth to be a ſphere, the ſine of the parallax varies as the ſine of the apparent zenith diſtance directly, and the diſtance of the body from the center of the earth inverſely. Hence a body in the zenith has no parallax, and at s in the horizon it is the greateſt. If the object be at an indefinitely great diſtance, it has no parallax; hence the apparent places of the fixed ſtars are not altered by it. As n is the apparent place, and m is called the true place, the parallax depreſſes an object in a vertical circle. For the ſame body at different altitudes, the parallax varies as the ſine (s) of the apparent zenith diſtance; therefore if $p =$ the horizontal parallax, and radius be unity, the ſine of the parallax $= ps$. To aſcertain therefore the parallax at all altitudes, we muſt firſt find it at ſome given altitude.

155. *Firſt* method, for the *ſun*. ARISTARCHUS propoſed to find the ſun's parallax, by obſerving it's elongation from the moon at the inſtant it is

is dichotomized, at which time the angle at the moon is a right angle; therefore we ſhould know the angle which the diſtance of the moon ſubtend sat the ſun; which diminiſhed in the ratio of the moon's diſtance from the earth's center to the radius of the earth would give the ſun's horizontal parallax. But a very ſmall error in the time when the moon is dichotomized, (and it is impoſſible to be very accurate in this) will make ſo very great an error in the ſun's parallax, that nothing can be depended upon from it. VENDELINUS determined the angle of elongation when the moon was dichotomized to be 89°. 45′, from which the ſun's parallax was found to be 15″. But P. RICCIOLI found it to be 28″ or 30″ from like obſervations.

156. *Second* method. HIPPARCHUS propoſed to find the ſun's parallax from a lunar eclipſe, by the following method. Let *S* be the ſun, *E* the earth, *Ev* the length of it's ſhadow, *mr* the path of the moon in a central eclipſe. Obſerve the length of this eclipſe, and then, from knowing the periodic time of the moon, the angle *mEr*, and conſequently *nEr*, will be known. Now the horizontal parallax *ErB* of the moon being known, we have the angle $Evr = ErB - nEr$; hence we know $EAB = AES - Evr = AES - ErB + nEr$; that is, the ſun's horizontal parallax = the apparent ſemidiameter of the ſun − the horizontal parallax of the moon + the ſemidiameter of the earth's ſhadow where the moon paſſes through. The objection to this method is, the great difficulty of determining the angle *nEr* with ſufficient accuracy; for any error in that angle will make the ſame error in the ſun's parallax, the other quantities remaining the ſame. By this method PTOLEMY made the ſun's horizontal parallax 2′. 50″. TYCHO made it 3′. FIG. 27.

157. *Third* method, for the *moon*. Take the meridian altitudes of the moon, when it is at it's greateſt north and ſouth latitudes, and correct them for refraction; then the difference of the altitudes, thus corrected, would be equal to the ſum of the two latitudes of the moon, if there were no parallax; conſequently the difference between the ſum of the two latitudes and the difference of the altitudes will be the difference between the parallaxes at the two altitudes. Now to find from thence the parallax itſelf, let *S*, *s* be the ſines of the greateſt and leaſt apparent zenith diſtances, *P*, *p* the ſines of the correſponding parallaxes; then as, when the diſtance is given, the parallax varies (154) as the ſine of the zenith diſtance, $S : s :: P : p$, hence $S - s : s :: P - p : p = \frac{s \times \overline{P-p}}{S-s}$ the parallax at the greateſt altitude. This ſuppoſes that the moon is at the ſame diſtance in both caſes, but as this will not neceſſarily happen, we muſt correct one of the obſervations in order to reduce it to what it would have been, had the diſtance been the ſame. If the obſervations be made in thoſe places where the moon paſſes through the zenith in one of the obſervations, the diffe-

difference between the ſum of the two latitudes and the zenith diſtance at the other obſervation will be the parallax at that altitude.

FIG. 28. 158. *Fourth* method. Let a body P be obſerved from two places A, B in the ſame meridian, then the whole angle APB is the effect of parallax between the two places. The parallax (154) $APC = hor.\ par. \times$ ſin. PAL, taking APC for ſin. APC, and the parallax $BPC = hor.\ par. \times$ ſin PBM; hence $hor.\ par. \times$ ſin. $PAL +$ ſin. $PBM = APB$, $\therefore$ $hor.\ par. = APB$ divided by the ſum of theſe two ſines. If the two places be not in the ſame meridian it does not ſignify, provided we know how much the altitude varies from the change of declination of the body in the interval of the paſſages over the meridians.

Ex. On Oct. 5, 1751, M. de la CAILLE, at the Cape of Good Hope, obſerved *Mars* to be 1′. 25″,8 below the parallel of λ in aquarius, and at 25° diſtance from the zenith. On the ſame day at Stockholm, Mars was obſerved to be 1′. 57″,7 below the parallel of λ and at 68°. 14′ zenith diſtance. Hence the angle APB is 31″,9, and the ſines of the zenith diſtances being 0,4226 and 0,9287, the horizontal parallax was 23″,6. Hence if the ratio of the diſtance of the earth from Mars to it's diſtance from the ſun be found, we ſhall have the ſun's horizontal parallax. Now from comparing the altitudes of the northern limb of Mars with ſtars nearly in the ſame parallel obſerved on the ſame days at the Cape and at Greenwich, Bologna, Paris, Stockholm, Upſal, Hernoſand, the mean of the whole gave 10″,2 for the horizontal parallax of the ſun; and rejecting thoſe reſults which differed the moſt from the reſt, the mean was 9″,842. From the mean of another ſet of obſervations, the reſult was 9″,575. From the mean of ſeveral obſervations on *Venus* made in like manner, the parallax came out 10″,38. The mean of the three laſt gives 9″,93 for the horizontal parallax of the ſun. FLAMSTEAD, from an obſervation on Mars, concluded the ſun's parallax could not be more than 10″. MARALDI found the ſame. From the obſervations of POUND and Dr. BRADLEY, Dr. HALLEY found it never greater than 12″ nor leſs than 9″. CASSINI, from his obſervations on Mars, found it to be between 11″ and 15″. But the moſt accurate method of determining the ſun's parallax is from the tranſit of Venus over it's diſc, as will be explained when we treat on that ſubject.

159. If the earth be a ſpheroid, let E be the equator; draw GAv, HBr perpendicular to the ſurface, and compute the angles CAv or LAG, and CBr or MBH by the Rule which we ſhall give, when we treat of the figure of the earth; ſubtract theſe from the obſerved zenith diſtances PAG, PBH, and we have the angles PAL, PBM. Now $CP : CA ::$ ſin. CAP or $PAL :$ ſin. $APC = \frac{CA \times \text{ſin. } PAL}{CP}$, alſo $CP : CB ::$ ſin. CBP or $PBM :$ ſin. $BPC =$

CB

$\frac{CB \times \text{sin. } PBM}{CP}$; and as the parallax is very small, the sum of the two sines will be very nearly the sine of the sum, therefore the sine of $APB = \frac{CA \times \text{sin. } PAL + CB \times \text{sin. } PBM}{CP}$; hence $CP = \frac{CA \times \text{sin. } PAL + CB \times \text{sin. } PBM}{\text{sin. } APB}$.

160. *Fifth* method. Let EQ be the equator, P it's pole, Z the zenith, v the true place of the body and r the apparent place as depressed by parallax in the vertical circle ZK, and draw the secondaries Pva, Prb; then ab is the parallax in right ascension, and rs in declination. Now $vr : vs ::$ 1 (rad.) : sin. vrs or ZvP, and $vs : ab ::$ cos. va : 1 (13), hence $vr : ab ::$ cos. va : sin. ZvP, $\therefore ab = \frac{vr \times \text{sin. } ZvP}{\text{cos. } va}$; but $vr = hor.\ par. \times$ sin. vZ (154), and sin. vZ : sin. $ZP ::$ sin. ZPv : sin. $ZvP = \frac{\text{sin. } ZP \times \text{sin. } ZPv}{\text{sin. } vZ}$, therefore by substitution, $ab = \frac{hor.\ par. \times \text{sin. } ZP \times \text{sin. } ZPv}{\text{cos. } va}$. Hence for the same star, where the *hor. par.* is given, the parallax in right ascension varies as the sine of the hour angle. Also the $hor.\ par. = \frac{ab \times \text{cos. } va}{\text{sin. } ZP \times \text{sin. } ZPv}$. For the eastern hemisphere, the apparent place b lies on the equator to the east of a the true place, and therefore the right ascension is diminished by parallax; but in the western hemisphere, b lies to the west of a, and therefore the right ascension is increased. Hence if the right ascension be taken before and after the meridian, the whole change of parallax in right ascension between the two observations is the sum (s) of the two parts before and after the meridian, and is therefore $= \frac{vr}{\text{cos. } va} \times$ sum (S) of the sines of the two hour angles; and the $hor.\ par. = \frac{s \times \text{cos. } va}{\text{sin. } ZP \times S}$. On the meridian there is no parallax in right ascension. FIG. 29.

161. To apply this Rule, observe the right ascension of the planet when it passes the meridian, compared with that of a fixed star, at which time there is no parallax in right ascension; about 6 hours after, take the difference of their right ascensions again, and observe how much the difference (d) between the apparent right ascensions of the planet and fixed star has changed in that time. Next observe the right ascension of the planet for 3 or 4 days when it passes the meridian, in order to get it's true motion in right ascension; then if it's motion in right ascension in the above interval of time between the taking of the right ascensions of the fixed star and planet on and off the meridian be equal to d, the planet has no parallax in right ascension; but if it be not equal to d, the difference is the parallax in right ascension, and hence,

by

by the laſt Article, the horizontal parallax will be known. Or one obſervation may be made as long before the planet comes to the meridian, by which a greater difference will be obtained.

Ex. On Auguſt 15, 1719, *Mars* was very near a ſtar of the 5th magnitude in the eaſtern ſhoulder of aquarius, and at 9^h. 18′ in the evening, Mars followed the ſtar in 10′. 17″, and on the 16th at 4^h. 21′ in the morning it followed it in 10′. 1″, therefore in that interval, the apparent right aſcenſion of Mars had increaſed 16″ in time. But according to obſervations made in the meridian for ſeveral days after, it appeared, that Mars approached the ſtar only 14″ in that time, from it's proper motion, therefore 2″ in time, or 30″ in motion, is the effect of parallax in the interval of the obſervations. Now the declination of Mars was 15°, the co-latitude 41°. 10′, and the two hour angles 49°. 15′ and 56°. 39′; therefore the *hor. par.* $= \frac{30'' \times \text{cos. } 15^\circ}{\text{ſin. } 41^\circ.\ 10' \times \text{ſin. } 49^\circ.\ 15' + \text{ſin. } 56^\circ.\ 39'} = 27\frac{1}{2}''$. But at that time, the diſtance of the earth from Mars was to it's diſtance from the ſun as 37 to 100, and therefore the ſun's horizontal parallax comes out 10″,17.

162. When Dr. MASKELYNE was at St. Helena and Barbadoes, he made ſeveral obſervations of this kind on the moon, in order to determine her horizontal parallax; and he further obſerves, "that if the like obſervations were repeated in different parts of the earth, it would probably afford the beſt means, yet propoſed, for aſcertaining the true figure of the earth, as they would determine the ratio of the diameters of the parallels of latitude to each other, the horary parallaxes being in proportion thereto: For though the earth affords but a ſmall baſe at the moon, yet, by repeating theſe trials, and comparing the reſults, we may hope to attain that degree of exactneſs, which we could never expect from fewer obſervations."

163. But beſides the effect of parallax in right aſcenſion and declination, it is manifeſt that the latitude and longitude of the moon and planets muſt alſo be affected by it; and as the determination of this, in reſpect to the moon, is in many caſes, particularly in ſolar eclipſes, of great importance, we ſhall proceed to ſhow how to compute it, ſuppoſing that we have given the latitude of the place, the time, and conſequently the ſun's right aſcenſion, the moon's true latitude and longitude, with her horizontal parallax.

FIG. 30. 164. Let *HZR* be the meridian, ♈ *EQ* the equator, *p* it's pole; ♈ *C* the ecliptic, *P* it's pole; ♈ the firſt point of aries, *HQR* the horizon, *Z* the zenith, *ZL* a ſecondary to the horizon paſſing through the true place *r* and apparent place *t* of the moon; draw *Pt*, *Pr*, which produce to *s*, drawing the ſmall circle *ts* parallel to *ov*; let *rn* be perpendicular to *Pt*, and draw the ſmall circle *ra* parallel

parallel to *ov*; then *rs*, or *ta*, is the parallax in latitude, and *ov* the parallax in longitude. Draw the great circles $P\Upsilon$, $PZAB$, $Ppde$, and ZW perpendicular to pe; then as $\Upsilon P = 90°$, and $\Upsilon p = 90°$, Υ muſt (4) be the pole of $Ppde$, and therefore $d\Upsilon = 90°$; conſequently d is one of the ſolſticial points ☋ or ♑; alſo draw Zx perpendicular to Pr, and join $Z\Upsilon$, $p\Upsilon$. Now ΥE, or the angle ΥpE, or $Zp\Upsilon$, is the right aſcenſion of the mid-heaven, which is known (106); $PZ = AB$ (becauſe AZ is the complement of both) the altitude of the higheſt point A of the ecliptic above the horizon, called the nonageſimal degree, and ΥA, or the angle ΥPA is it's longitude. Now in the right angled triangle ZpW, we have Zp the co-latitude of the place, and the angle ZpW, the difference between the right aſcenſion of the mid-heaven ΥpE and Υd, to find pW; hence $PW = pW \pm pP$, where the upper ſign is to be taken when the right aſcenſion of the mid-heaven is leſs than 180°, and the under, when greater. Alſo in the triangles WZp, WZP, ſin Wp : ſin WP :: cot. WpZ : cot. WPZ, or tan. $AP\Upsilon$; and as we know Υo, or ΥPo, the true longitude of the moon, we know APo, or ZPx; alſo cos. WPZ, or ſin. APZ, : rad. 1 : tan. WP : tan. ZP. Hence in the triangle ZPr, we know ZP, Pr and the angle P, from which the angle ZrP or trs, and Zr may be found; for in the right angled triangle ZPx, we know ZP and the angle P, to find Px; therefore we know rx; and hence (as the ſines of the ſegments of the baſe of any triangle are inverſely as the tangents of the angles at the baſe adjacent to which they lie) we may find the angle Zrx, with which, and rx, we may find Zr the *true* zenith diſtance; to which, as if it were the *apparent* zenith diſtance, find the parallax (154) and add to it, and you will get very nearly the apparent zenith diſtance, correſponding to which, find the parallax rt; then in the right angled triangle rst, which may be conſidered as plane, we know rt and the angle r, to find rs the parallax in *latitude*; find alſo ts, which multiplied (108) by the ſecant of tv, the apparent latitude, gives the arc ov, the parallax in *longitude*.

Ex. On January 1, 1771, at 9*h*. apparent time, in lat. 53° N. the moon's true longitude was 3*s*. 18°. 27′. 35″, and latitude 4°. 5′. 30″ S. and it's horizontal parallax 61°. 9″; to find it's parallax in latitude and longitude.

The ſun's right aſcenſion was 282°. 22′. 2″ by the Tables, and it's diſtance from the meridian 135°; alſo (106) the right aſcenſion ΥE of the mid-heaven was 57°. 22′. 2″; hence the whole operation for the ſolution of the triangles may ſtand thus.

Tri.

Tri. *ZpW*	*ZpW*	= 32°. 37'. 58"	- -	cos.	9.9253864
	Zp	= 37 . 0 . 0	- - -	tan.	9.8871144
	pW	= 32 . 23 . 57	- - -	tan.	9.8025008
	Pp	= 23 . 28 . 0			
	PW	= 55 . 51 . 57			
Tri. *WpZ, WPZ*	*pW*	= 32 . 23 . 57	- -	*A.C.* sin.	0.2709855
	PW	= 55 . 51 . 57	- - -	sin.	9.9178865
	ZpW	= 32 . 37 . 58	- - -	cot.	10.1935941
	APγ	= 67 . 29 . 8	- - -	tan.	10.3824661
	oPγ	= 108 . 27 . 35			
	oPA	= 40 . 58 . 27			
Tri. *WPZ*	*APZ*	= 67 . 29 . 8	- - -	sin.	9.9655700
	WP	= 55 . 51 . 57	- -	tan. + 10	20.1688210
	ZP	= 57 . 56 . 36	- - -	tan.	10.2032510
Tri. *ZPx*	*ZP*	= 57 . 56 . 36	- - -	tan.	10.2032555
	ZPx	= 40 . 58 . 27	- - -	cos.	9.8779500
	Px	= 50 . 19 . 33	- - -	tan.	10.0812055
	Pr	= 94 . 5 . 30			
Tri. *ZPx, Zrx*	*rx*	= 43 . 45 . 57	- -	*A.C.* sin.	0.1600743
	Px	= 50 . 19 . 33	- - -	sin.	9.8863144
	ZPx	= 40 . 58 . 27	- - -	tan.	9.9387676
	Zrx	= 44 . 1 . 16	- - -	tan.	9.9851563
Tri. *Zrx*	*Zrx*	= 44 . 1 . 16	-	cos. + 10	19.8567795
	rx	= 43 . 45 . 57	- - -	tan.	9.9812846
	Zr	= 53 . 6 . 10	- - -	cot.	9.8754949
	Zr	= 53 . 6 . 10	- - -	sin.	9.9029362
	Hor. par. = 61'. 9" = 3669"		-	log.	3.5645477
	rt uncorrected = 2934" = 48'. 54"		-	log.	3.4674839
	App. zen. dist. *Zt* = 53°. 55'. 4" nearly			sin.	9.9075042
	Hor. par. = 61'. 9" = 3669".		-	log.	3.5645477
Tri. *trs*	Par. *rt* cor. = 2965" = 49'. 25"		-	log.	3.4720519
	trs = 44°. 1'. 16"		- - -	cos.	9.8567795
	rs par. in *lat.* = 2132" = 35'. 32"		-	log.	3.3288314

Tri.

Tri. *trs*	*rt* cor. = 2965″ - - - -	log.	3.4720519
	trs = 44°. 1′. 16″ - - -	ſin.	9.8419369
	ts = 2061″ = 34′. 21″ - - -	log.	3.3139888
	True lat. *ro* = 4°. 5′. 30″.		
	App. lat. *tv* = *ro* − *rs* = 4°. 41′. 2″ -	ſec.	10.0014528
	ov par. in *long.* = 2067″ = 34′. 27″ -	log.	3.3154416

The value of *tv* is *ro* − or + *rs*, according as the moon has N. or S. latitude. The Figure is drawn for north latitude, but the Example is for ſouth latitude.

This is the direct method of ſolving the problem from the triangles; but the operation may be rendered eaſier by the following Rule (the moſt convenient of any yet given) diſcovered by Dr. MASKELYNE, but communicated without the demonſtration. The inveſtigation here given, is by the Rev. Mr. BRINKLEY, A.M. Profeſſor of Aſtronomy at Dublin.

Let the height *H* of the nonageſimal degree, or *PZ*, and the angle *ZPr* (*n*), the moon's true diſtance from the nonageſimal, be computed as before. Put *P* = the parallax *ov* in longitude, *Q* = the parallax *at* in latitude, depreſſing the moon ſouthwards, *L* = the true latitude, *l* the apparent latitude, *h* the horizontal parallax. Now

$$\left.\begin{array}{l} P : rn :: \text{rad.} : \text{ſin. } Pr \\ rn : rt :: \text{ſin. } ntr : \text{rad.} \\ rt : h :: \text{ſin. } Zt : \text{rad.} \end{array}\right\} \therefore P : h :: \text{ſin. } ntr \times \text{ſin. } Zt : \text{ſin. } Pr,$$

radius being unity;

hence $P = \dfrac{h \times \text{ſin. } ntr \times \text{ſin. } Zt}{\text{ſin. } Pr} =$ (as ſin. $ntr \times$ ſin. $Zt =$ ſin. $ZPt \times$ ſin. PZ)

$\dfrac{h \times \text{ſin. } PZ \times \text{ſin. } ZPt}{\text{ſin. } Pr} = \dfrac{h \times \text{ſin. } H \times \text{ſin. } \overline{n+P}}{\text{coſ. } L}$, the parallax in *Longitude*.

Alſo, $tn : tr ::$ coſ. $rtn :$ rad. $::$ ſin. $rtn :$ tan. rtn

$tr : h ::$ ſin. $Zt :$ rad.

$\therefore tn : h ::$ ſin. $rtn \times$ ſin. $Zt :$ tan. $rtn \times$ rad. $::$ ſin. $PZ \times$ ſin. $ZPt :$ $\dfrac{\text{ſin. } ZPt}{\text{ſin. } Pt \times \text{cot. } ZP - \text{coſ. } Pt \times \text{coſ. } ZPt}$, ſubſtituting for the third and fourth terms their values; hence, $tn = h \times$ ſin. $PZ \times$ ſin. $Pt \times$ cot. $ZP - h \times$ ſin. $PZ \times$ coſ. $Pt \times$ coſ. $ZPt = h \times$ coſ. $H \times$ coſ. $l - h \times$ ſin. $H \times$ ſin. $l \times$ coſ. $\overline{n+P}$.

Now as the angle rPn is very ſmall, we have $an = \dfrac{rn^2}{2 \text{ tan. } Pr}$ * = (from the

* If we conceive two tangents to be drawn to *Pr* and *Pa* at *r* and *a*, and to meet, then *rn* may be conſidered as the ſine of *ra* to the length of theſe tangents as a radius, and therefore, by the property of the circle, $an = rn^2$ divided by twice the tangent.

firſt proportion above) $\frac{P^2 \times \text{ſin. } Pr^2}{2 \text{ tan. } Pr} = \frac{1}{2} P^2 \times \text{ſin. } Pr \times \text{coſ. } Pr = \frac{1}{2} P \times P \times$ ſin. $Pr \times$ coſ. $Pr =$ (as, from above, $P \times$ ſin. $Pr = h \times$ ſin. $PZ \times$ ſin. ZPt) $\frac{1}{2} P \times h \times$ ſin. $H \times$ ſin. $\overline{n+P} \times$ ſin. L, or ſin. l nearly; hence, $\mathfrak{Q} = ta = tn - an = h \times$ coſ. $H \times$ coſ. $l - h \times$ ſin. $H \times$ ſin. $l \times$ coſ. $\overline{n+P} - h \times$ ſin. $H \times \frac{1}{2} P \times$ ſin. $\overline{n+P} \times$ ſin. l. But as P is very ſmall, we may call $\frac{1}{2} P$ the ſine of $\frac{1}{2} P$, and it's coſine we may put $=$rad. $= 1$; hence, for coſ. $\overline{n+P}$ we may ſubſtitute coſ. $\overline{n+P} \times$ coſ. $\frac{1}{2} P$, and for $\frac{1}{2} P \times$ ſin. $\overline{n+P}$ we may put ſin. $\overline{n+P} \times$ ſin. $\frac{1}{2} P$; hence, $\mathfrak{Q} = h \times$ coſ. $H \times$ coſ. $l - h \times$ ſin. $H \times$ ſin. $l \times$ coſ. $\overline{n+P} \times$ coſ. $\frac{1}{2} P +$ ſin. $\overline{n+P} \times$ ſin. $\frac{1}{2} P =$ (becauſe by plane Trigonometry, coſ. $\overline{n+P} \times$ coſ. $\frac{1}{2} P +$ ſin. $\overline{n+P} \times$ ſin. $\frac{1}{2} P =$ coſ. $\overline{n+\frac{1}{2} P}$) $h \times$ coſ. $H \times$ coſ. $l - h \times$ ſin. $H \times$ ſin. $l \times$ coſ. $\overline{n+\frac{1}{2} P}$, the parallax in *Latitude*.

Now P enters into the expreſſion for the value of P, and as P is very ſmall, we muſt firſt ſuppoſe $P = \frac{h \times \text{ſin. } H \times \text{ſin. } n}{\text{coſ. } L}$, which will give a near value of P; then put that value into the numerator and you will get a very accurate value of P. Alſo in the expreſſion for $\mathfrak{Q}$, we have the apparent latitude, which cannot be known without knowing $\mathfrak{Q}$; hence we muſt firſt get a near value of $\mathfrak{Q}$ and apply it to the true latitude to get the apparent nearly; to do this, we may omit the ſecond part as being ſmall, on account of ſin. l being ſmall for the moon, and ſuppoſe $\mathfrak{Q} = h \times$ coſ. $H \times$ coſ. $l = h \times$ coſ. $H \times$ coſ. L nearly; or when the latitude is very ſmall, as is the caſe of the moon in ſolar eclipſes, we may ſuppoſe $\mathfrak{Q} = h \times$ coſ. H, from which we ſhall get the apparent latitude with ſufficient accuracy.

In the application of this Rule, regard muſt be had to the ſigns of the quantities; if $n + \frac{1}{2} P$ be greater than 90° it's coſine becomes negative, in which caſe $\mathfrak{Q}$ will be the *ſum* of the quantities, unleſs the apparent latitude l is ſouth, in which caſe, it's coſine will be negative, which makes the firſt term negative. In general, $\mathfrak{Q}$ will be the *ſum* of the two parts, when $n + \frac{1}{2} P$ and the moon's apparent diſtance from P are, one greater and the other leſs than 90°; otherwiſe $\mathfrak{Q}$ will be the *difference*. The parallax in longitude *increaſes* the longitude, if the body be to the *eaſt* of the nonageſimal degree, and *decreaſes* it, if it be to the *weſt*. This Rule is more correct than the other, becauſe in that we took the ſmall circle ts, inſtead of a great circle from t, as the perpendicular from t upon Pr produced. This error, for the moon, may ſometimes amount to about 2″. It may be corrected by applying an found above.

To apply this Rule to the laſt caſe, we have $H = 57°.\ 56'.\ 36''$, $n = 40°.\ 58'.\ 27''$, $L = 4°.\ 5'.\ 30''$ ſouth, $h = 61'.\ 9'' = 3669''$; hence

Log.

Log. h		3.5645477
Sin. H		9.9281518
Cos. L	*A.C.*	0.0011084
		3.4938079
Sin. n		9.8167176
Log. $2044'' = 34'.\ 4'' = P$ nearly		3.3105255
Therefore $n + P = 41°.\ 32'.\ 31''$; hence		3.4938079
Sin. $\overline{n+P}$		9.8216237
Log. $2067'' = 34'.\ 27''$ par. in *Longitude*		3.3154316
Log. h		3.5645477
Cos. H		9.7248963
Log. $2133'' = 35'.\ 33''$ par lat. nearly		3.2894440
	$4°.\ 5'.\ 30''$	
	$4.\ 41.\ 3$ app. lat. nearly.	
Log. h	3.5645477	
Cos. H	9.7248963	
Cos. $l = 4°.\ 41'.\ 3''$ nearly	9.9985470	first part of $\mathfrak{Q}$.
Log. $1941'' = 32'.\ 21''$	3.2879910	
Log. h	3.5645477	
Sin. H	9.9281518	
Sin. l	8.9120258	second part of $\mathfrak{Q}$.
Cos. $\overline{n + \frac{1}{2}P}$	9.8759399	
Log. $191'' = 3'.\ 11''$	2.2806652	
	$32.\ 21$	
	$35.\ 32$ par. in *Latitude*.	

The *ſum* of the two parts is here taken, becauſe Pt is greater than 90°, and $n + \frac{1}{2}P$ leſs than 90°.

165. Hitherto we have conſidered the effect of parallax, upon ſuppoſition that the earth is a ſphere; but as the earth is a ſpheroid, having the polar diameter ſhorter than the equatorial, it will be neceſſary to ſhow how the computations are to be made for this caſe. The following method is given by CLAIRAUT.

166. Let $EPQp$ be the earth, EQ the equatorial and Pp the polar diameters, O the place of the ſpectator, HCR the rational horizon, to which draw FIG. 31.

ZONK

$ZONK$ perpendicular; L the moon, join LO, LC, LK, and draw CV perpendicular to LK. Now to compare the apparent places ſeen from O and C, let us compare the places ſeen from O and K, and from K and C. Put $h =$ the horizontal parallax to the radius OC, or ON which is very nearly equal to it, on account of the ſmallneſs of the angle CON. Let $CO = 1$, and CN (the ſine of CON to that radius) $= a$, $t =$ tan. of the angle KCN the latitude of the place; then rad. $= 1 : t :: a : ta = NK$; hence as $h =$ the angle under which ON (which we may conſider as equal to unity) appears when ſeen directly at the moon, we have $h \times ta =$ the angle under which NK would appear; therefore $h \times 1 + ta =$ the horizontal parallax of OK; conſidering therefore K as the center of a ſphere and KO the radius, compute the parallax as before. Now as the planes of all the circles of declination paſs through Pp, in eſtimating the parallax either from K or O, the parallax in right aſcenſion muſt be the ſame, becauſe K and O lie in the plane of the ſame circle of declination; the only difference therefore between the effect of parallax at K and O muſt be in declination. Now at K, the angular diſtance of the moon from the pole P is LKP, and the angular diſtance from C is LCP; the difference of theſe two angles therefore, or CLK, is the difference between the parallax in declination at K and at C, and this angle CLK is *always* to be *added* to the polar diſtance ſeen from K to get the polar diſtance from C. Now $CLK = h \times CV$; but the angle VCK $(= LCE)$ is the moon's declination, therefore $CV = CK \times$ cos. dec. alſo $CK = \frac{CN}{\text{cos. } KCN} = \frac{a}{\text{cos. lat.}}$; hence $GLK = \frac{h \times a \times \text{cos. dec.}}{\text{cos. lat.}}$. This therefore is the equation of declination for the ſpheroid, to be applied to find the parallax in declination ſeen from C, after having calculated the effect of parallax in declination for a ſphere whoſe center is K and radius KO. There is no equation for the parallax in right aſcenſion. To find how this equation in declination will affect the latitude, let P be the pole of the equator, p the pole of the ecliptic, L the place of the moon ſeen from K, and b ſeen from C; then bL is the equation in declination; draw La perpendicular to pb, and ba is the equation in latitude, and the angle apL the equation in longitude. Now conſidering bL and ba as the variations of the two ſides Pb, pb, whilſt Pp and the angle P remain conſtant, we have * $bL : ba ::$ ſin. $Pb \times$ ſin. $pb :$ cos. $Pp -$ cos. $Pb \times$ cos. pb; hence $ba = bL \times \frac{\text{cos. } Pp - \text{cos. } Pb \times \text{cos. } pb}{\text{ſin. } Pb \times \text{ſin. } pb} = \frac{h \times a}{\text{cos. lat.}} \times \frac{\text{cos. } Pb}{\text{ſin. } pb} - \text{cos. } Pb \times \text{cotan. } pb =$

FIG. 32.

* See MAUDUIT's Trig. or CRACKELT's Tranſlation, Art. 254.

$h \times a$

$\frac{h \times a}{\text{cos. lat.}} \times \frac{\text{cos. } 23°.\ 28'}{\text{cos. moon's lat.}} - \text{sin. dec.} \times \text{tan. moon's lat.}$ But if CP be to CE as $1 : 1 + m$, and x, y, = the sine and cosine of the latitude of the place, then $a = 2m \times xy$, as shown in the Chapter on the Figure of the Earth; hence $ba =$ $2hmx \times \frac{\text{cos. } 23°.\ 28'}{\text{cos. moon's lat.}} - \text{sin. dec.} \times \text{tan. moon's lat.}$ The sign $-$ becomes $+$ if the declination and latitude of the moon be of different affections, that is, one south and the other north. The latitude here used is that seen from the center of the earth. This correction increases the moon's distance from the pole p of the ecliptic.

167. To find the correction of the longitude, or the angle Lpa, we have (13) $La = Lpa \times \text{sin. } pL$, hence $Lpa = \frac{La}{\text{sin. } pL}$; but $aL = bL \times \text{sin. } b$, and by spher. trig. sin. Pb . sin. p :: sin. Pp : sin. $b = \frac{\text{sin. } p \times \text{sin. } Pp}{\text{sin. } Pb}$; also Lb $= 2hmx$; hence $Lpa = 2hmx \times \frac{\text{sin. } p \times \text{sin. } Pp}{\text{sin. } Pb \times \text{sin. } pL} = 2hmx \times \frac{\text{cos. lon. } ☾ \times \text{sin. } 23°.\ 28'}{\text{cos. dec. } ☾ \times \text{cos. lat. } ☾}$ = (as the cos. of the moon's latitude may be considered equal to unity) $2hmx \times \frac{\text{sin. } 23°.\ 28'}{\text{cos. dec. } ☾} \times \text{cos. lon. } ☾$. In north latitude, we must *add* this correction to the parallax computed for K, when the moon is in the descending signs 3, 4, 5, 6, 7, 8, but *subtract* it, when in the ascending signs 0, 1, 2, 9, 10, 11, to have the longitude seen from C; and the contrary when the latitude of the place is south. FIG. 31.

168. According to the Tables of MAYER, the greatest parallax of the moon, (or when she is in her perigee and in opposition) is 61'. 32"; the least parallax (or when in her apogee and conjunction) is 53'. 52", in the latitude of Paris; the arithmetical mean of these is 57'. 42"; but this is not the parallax at the mean distance, because the parallax varies inversely as the distance, and therefore the parallax at the mean distance is 57'. 24", an harmonic mean between the two. M. de LAMBRE recalculated the parallax from the same observations from which MAYER calculated it, and found it did not exactly agree with MAYER's. He made the equatorial parallax 57'. 11",4. M. de la LANDE makes it 57'. 5" at the equator, 56'. 53",2 at the pole, and 57'. 1" for the mean radius of the earth, supposing the difference of the equatorial and polar diameters to be $\frac{1}{300}$ of the whole. From the formula of MAYER, the equatorial parallax is 57'. 11",4 with the following equations, according to M. de la LANDE.

57'.

57′. 11″,4	− 3′. 7″,7	cos. ano. ☾
	+ 10, 0	cos. 2 ano. ☾
	− 0, 3	cos. 3 ano. ☾
	− 37, 3	cos. arg. evection
	+ 0, 3	cos. 2 arg. evect.
	+ 26, 0	cos. 2 dist. ☾ à ☉
	− 1, 0	cos. dist. ☾ à ☉
	+ 0, 2	cos. 4 dist. ☾ à ☉
	+ 2, 0	cos. 2 (apo. ☾ − ☉)
	+ 0, 2	cos. 3 (apo. ☾ − ☉)
	+ 1, 0	cos. (arg. evect. + ano. ☉)
	+ 0, 8	cos. (2 arg. lat. − ano. ☾ cor.)
	− 0, 8	cos. (2 dist. ☾ à ☉ − ano. ☉)
	− 0, 7	cos. (2 dist. ☾ à ☉ + ano. ☉)
	+ 0, 6	cos. (arg. evect. − mean ano. ☾)
	+ 0, 4	cos. 2 (☿ − ☉), or 2 (☉ + sup. ☿)
	+ 0, 3	cos. mean ano. ☉
	+ 0, 2	cos. (mean ano. ☾ − mean ano. ☉)
	+ 0, 1	cos. (2 dist. ☉ à ☾ + mean. ano. ☾)

169. Let $r = \frac{1}{2}$ the semiaxis major, $\rho = \frac{1}{2}$ the semiaxis minor, $n =$ the sine, m the cosine of the angle OCE; then, from conics, the sine of the horizontal polar parallax : sine of the hor. parallax at $O :: \sqrt{r^2n^2 + \rho^2m^2} : r\rho$; hence the sine of the hor. par. at $O = \frac{r\rho}{\sqrt{r^2n^2 + \rho^2m^2}} \times$ the sine of the hor. polar parallax. If $r : \rho :: 230 : 229$, we have the following Table for the horizontal parallax for every degree of latitude, that at the pole being unity.

Lat,

Lat.	Hor. Par.	Lat.	Hor. Par.	Lat.	Hor. Par.
0°	100438	31°	100321	61°	100103
1	100438	32	100314	62	100097
2	100437	33	100307	63	100091
3	100436	34	100300	64	100085
4	100435	35	100293	65	100079
5	100434	36	100286	66	100073
6	100432	37	100279	67	100067
7	100430	38	100272	68	100062
8	100428	39	100265	69	100057
9	100426	40	100257	70	100052
10	100424	41	100250	71	100047
11	100421	42	100243	72	100042
12	100418	43	100235	73	100038
13	100415	44	100227	74	100034
14	100412	45	100219	75	100030
15	100408	46	100211	76	100026
16	100404	47	100203	77	100023
17	100400	48	100195	78	100020
18	100396	49	100187	79	100017
19	100391	50	100180	80	100014
20	100386	51	100173	81	100012
21	100381	52	100166	82	100010
22	100376	53	100159	83	100008
23	100371	54	100152	84	100006
24	100365	55	100145	85	100004
25	100359	56	100138	86	100003
26	100353	57	100131	87	100002
27	100347	58	100124	88	100001
28	100341	59	100117	89	100000
29	100335	60	100110	90	100000
30	100328				

Hence, by multiplying the polar parallax by the number corresponding to any latitude, it gives the horizontal parallax at that latitude. From the Theorem, the parallax may be very eaſily calculated for any other ratio of the diameters of the earth.

170. To find the mean diſtance Cs of the moon, we have AC, the mean radius (r) of the earth, : Cs, the mean diſtance (D) of the moon from the earth, :: ſin. $57'.\ 1'' = AsC$ (168) : radius :: 1 : 60,3; conſequently $D = 60{,}3r$; but $r = 3964$ miles; hence $D = 239029$ miles. FIG. 26.

171. According to M. de la LANDE, the horizontal ſemidiameter of the moon : it's horizontal parallax for the mean radius (r) of the earth :: $15'$: $54'.\ 57''{,}4$,

$54'.57''{,}4$, or very nearly as 3 : 11; hence the ſemidiameter of the moon is $\frac{3}{11}r = \frac{3}{11} \times 3964 = 1081$ miles; and as the magnitudes of ſpherical bodies are as the cubes of their radii, we have the magnitudes of the moon and earth as $3^3 : 11^3 :: 1 : 49$.

172. In the ſpheroid, beſides the parallax in right aſcenſion and declination, latitude and longitude, there is alſo a parallax in azimuth, and alſo a correction of the parallax in altitude. For the plane which is perpendicular to the ſurface at O, always paſſes through ON, and therefore the azimuth ſeen from O or N and from C muſt be different, except when the body is on the meridian, in which caſe the plane alſo paſſes through C; and the altitude ſeen from N muſt alſo be different from that ſeen from C. Hence, having compared the parallax between O and N in altitude, we ſhall want a correction for the difference between the altitudes and azimuths ſeen from N and C. Let therefore CN repreſent CN in FIG. 31. L the moon, LCR a plane perpendicular to the horizon, and then will NCR be the azimuth ſeen from C; draw NM perpendicular to CR, MS perpendicular to CL, and LR perpendicular to the horizon; and let m and n be the ſine and coſine of NCM, r the ſine of MCS, $a = CN$, the ſine of CON in FIG. 31. and c the coſine of LNR, and let $d =$ the diſtance of the moon; then $cd = RN$, $ma = MN$. Now the line CO in FIG. 31. or unity, at the diſtance d appears under an angle h when ſeen directly; hence $\frac{1}{d} : h :: \frac{ma}{cd}$: the angle $NRC = \frac{hma}{c}$ the difference of the azimuths ſeen from C and N. Alſo, as the arc parallel to the horizon between any two ſecondaries to it varies (13) as the coſine of the altitude, the arc of the difference of the azimuths at the altitude of the moon $= hma = h \times MN$. Now as the plane NML is perpendicular to CLM, and NM is extremely ſmall, the altitudes ſeen from N and M will not ſenſibly differ; hence the difference between the altitudes at N and C is the angle $CLM = h \times SM = h \times r \times CM = h \times r \times n \times a$. If the moon be to the *ſouth* of the prime vertical, we muſt *ſubtract* this correction from the altitude at N to get the altitude at C; if it be to the *north*, we muſt *add* the correction.

FIG. 33.

173. But the moſt elegant and ſimple method of finding the parallax in latitude and longitude on a ſpheroid, is the following, given by MAYER. The parallax at any place O in the ſpheroid is the ſame as on a ſphere whoſe radius is CO, and latitude OCE; ſubtract therefore the angle COK (found from the following Table) from the latitude OvE on the ſpheroid, and you get the angle OCE the latitude of the point O reduced to a ſphere. Alſo the horizontal parallax which is made uſe of muſt be adapted to the radius OC, by diminiſhing the equatorial horizontal parallax by a quantity correſponding to the difference between CE and CO. This diminution is alſo found in the

FIG. 31.

ſame

ſame Table. The latitude thus reduced, and the horizontal parallax thus found, are to be employed in computing the moon's parallaxes in longitude, latitude, right aſcenſion and declination, which will now be performed by the Rule (164) founded on the hypotheſis of the earth being a ſphere; for by means of the Table, both the baſe of the parallax and the latitude of the place are referred to the earth's center.

ARGUMENT.

Elevation of the Pole, and Equatorial Parallax.

Elev. of Pole.	Equatorial Parallax. 54′ | 57′ | 60′ Reduction of Parallax.			Reduct. of Elevat. of Pole.
0°	−0″,0	−0″,0	−0″,0	−0′. 0″
6	0, 2	0, 2	0, 2	3. 6
12	0, 6	0, 7	0, 7	6. 4
18	1, 4	1, 4	1, 5	8. 57
24	2, 3	2, 5	2, 6	11. 6
30	3, 5	3, 7	3, 9	12. 56
36	4, 9	5, 1	5, 4	14. 12
42	6, 3	6, 7	7, 0	14. 51
48	7, 7	8, 2	8, 6	14. 51
54	9, 2	9, 7	10, 2	14. 12
60	10, 5	11, 1	11, 7	12. 56
66	11, 7	12, 4	13, 0	11. 6
72	12, 7	13, 4	14, 1	8. 57
78	13, 4	14, 2	14, 9	6. 4
84	13, 9	14, 6	15, 4	3. 6
90	14, 1	14, 8	15, 6	0. 0

Ex. If the latitude on the ſpheroid be 63°, and the equatorial parallax be 56′, what are the reductions?

The reduction of the parallax is 11″,5, and of the elevation of the pole it is 55″; hence the reduced latitude is 62°. 59′. 5″, and the parallax 55′. 48″,5.

CHAP. VII.

ON REFRACTION.

Art. 174. WHEN a ray of light paſſes out of a vacuum into any medium, or out of any medium into one of greater denſity, it is found to deviate from it's rectilinear courſe towards a perpendicular to the ſurface of the medium into which it enters. Hence light paſſing out of a vacuum into the atmoſphere will, where it enters, be bent towards a radius drawn to the earth's center, the top of the atmoſphere being ſuppoſed to be ſpherical and concentric with the center of the earth; and as, in approaching the earth's ſurface, the denſity of the atmoſphere continually increaſes, the rays of light, as they deſcend, are conſtantly entering into a denſer medium, and therefore the courſe of the rays will continually deviate from a right line and deſcribe a curve; hence, at the ſurface of the earth, the rays of light enter the eye of the ſpectator in a different direction from what they would have entered, if there had been no atmoſphere; conſequently the apparent place of the body from which the light comes muſt be different from the true place. Alſo the refracted ray muſt move in a plane perpendicular to the ſurface of the earth; for conceiving a ray to come in that plane before it is refracted, then the attraction being always towards the perpendicular which lies in that plane, the ray muſt continue to move in that plane. Hence the refraction is always in a vertical circle. The ancients were not unacquainted with this effect. PTOLEMY mentions a difference in the riſing and ſetting of the ſtars in different ſtates of the atmoſphere; he makes however no allowance for it in his computations from his obſervations; this correction therefore muſt be applied, where great accuracy is required. ARCHIMEDES obſerved the ſame in water, and thought the quantity of refraction was in proportion to the angle of incidence. ALHAZEN, an Arabian Optician, in the ſixteenth century, by obſerving the diſtance of a circumpolar ſtar from the pole, both above and below, found them to be different, and ſuch as ought to ariſe from refraction. SNELLIUS, who firſt obſerved the relation between the angles of incidence and refraction, ſays, that WALTHERUS in his computation allowed for refraction; but TYCHO was the firſt perſon who conſtructed a Table for that purpoſe, which however was very incorrect, as he ſuppoſed the refraction at 45° to be nothing. About the year 1660, CASSINI publiſhed a new Table of refractions, much more correct than

than that of TYCHO; and ſince his time, Aſtronomers have employed much attention in conſtructing more correct Tables, the niceties of modern Aſtronomy requiring their utmoſt accuracy. We ſhall treat this ſubject, by firſt ſhowing the practical methods by which the quantity of refraction is determined at ſome certain altitudes, and then give the inveſtigation of the rules for the variation at different altitudes, from which a Table for the Refraction at all altitudes may be conſtructed.

175. *Firſt method.* Take the altitude of the ſun, or a ſtar whoſe right aſcenſion and declination are known, and note the time by the clock; obſerve alſo the times of their tranſits over the meridian; then find (92) the hour angle; hence in the triangle PZx, we know PZ and Px the complements of latitude and declination, and the angle xPZ, to find the ſide Zx, the complement of which is the altitude, the difference between which and the obſerved altitude is the refraction at that altitude. FIG. 8.

Ex. On May 1, 1738, at 5^h. 20′ in the morning, CASSINI obſerved the altitude of the ſun's center at Paris to be 5°. 0′. 14″, and the ſun paſſed the meridian at 12^h. 0′. 0″, to find the refraction, the latitude being 48°. 50′. 10″, and the declination was 15°. 0′. 25″. The ſun's diſtance from the meridian was 6^h. 40′, which gives 100° for the hour angle xPZ; alſo $PZ = 41°.\ 9'.\ 50''$ and $Px = 74°.\ 59'.\ 35''$; hence $Zx = 85°.\ 10'.\ 8''$, conſequently the true altitude was 4°. 49′. 52″. Now to 5°. 0′. 14″, the apparent altitude, add 9″ for the parallax, and we have 5°. 0′. 23″ for the apparent altitude corrected for parallax; hence $5°.\ 0'.\ 23'' - 4°.\ 49'.\ 52'' = 10'.\ 31''$ the refraction at the apparent altitude 5°. 0′. 14″.

176. *Second method.* Take the greateſt and leaſt altitude of a circumpolar ſtar which paſſes through, or very near, the zenith, when it paſſes the meridian above the pole; then the refraction being nothing in the zenith, we ſhall have the true diſtance of the ſtar from the pole at that obſervation, the altitude of the pole above the horizon being previouſly determined; but when the ſtar paſſes the meridian under the pole, we ſhall have it's diſtance affected by refraction, and the difference of the two obſerved diſtances above and below the pole gives the refraction at the apparent altitude below the pole.

Ex. M. de la CAILLE obſerved at Paris a ſtar to paſs the meridian within 6′ of the zenith, and conſequently at the diſtance of 41°. 4′ from the pole; hence it muſt paſs the meridian under the pole at the ſame diſtance, or at the altitude 7°. 46′; but the obſerved altitude at that time was 7°. 52′. 25″; hence the refraction was 6′. 25″ at that apparent altitude.

176. M.

177. *Third method.* M. de la CAILLE alſo employed obſervations made at Paris and at the Cape of Good Hope, in order to aſcertain the refraction. The method he made uſe of was this. The diſtance of the parallels of Paris and the Cape was found to be about 82°. 46′; the half of which is 41°. 23′; therefore a ſtar vertical to a parallel in the middle between Paris and the Cape, muſt be at the zenith diſtance of 41°. 23′ from each. Now the ſum of the apparent zenith diſtances of ſuch a ſtar was found to be 82°. 44′. 46″, which therefore is the diſtance of the two parallels, diminiſhed by the ſum of the two refractions at the zenith diſtance 41°. 23′, for refraction elevating a ſtar, muſt make the apparent zenith diſtance of each ſtar leſs than the true diſtance. Next, the apparent altitude of the pole at the Cape was obſerved to be 33°. 56′. 49″,1, and the altitude at Paris to be 48°. 52′. 27″,5, the ſum of theſe two apparent altitudes is 82°. 49′. 16″,6 the diſtance of the parallels increaſed by the ſum of the two refractions correſponding to the altitude of each pole. The difference of theſe two determinations is 4′. 30″,6 for the ſum of the four refractions. Now taking the refraction to be as the tangent of the zenith diſtance, he found the tangents of 41°. 23′, and of the complement of the altitudes of the two poles, and divided 4′. 30″,6 into four parts in the ratio of theſe tangents, making the refraction a fortieth part leſs at the Cape than at Paris, as he had obſerved it; hence he got 1′. 36″,5 for the refraction at the altitude 33°. 56′. 49″,1 at the Cape, and 58″,2 at the altitude 48°. 52′. 27″,5 at Paris; alſo 57″,2 for the refraction at the zenith diſtance 41°. 23′ at the Cape, and 58″,7 for the refraction at the zenith diſtance 41°. 23′ at Paris. The altitudes and zenith diſtances corrected by theſe refractions give 82°. 46′. 42″ for the true diſtance of the parallels of Paris and the Cape.

178. Having determined the refraction at the altitude 48°. 52′. 27″,5 at Paris, he calculated the refractions from that altitude up to the zenith, upon ſuppoſition that they were as the tangents of the zenith diſtances, and hence he knew the refractions at theſe altitudes at the Cape. Therefore, by taking the meridian altitudes of ſtars from 7° to 48° at Paris, and the correſponding meridian altitudes at the Cape, and correcting theſe latter for refraction, he got the refraction from 7° to 48° at Paris; for the ſum of the two true zenith diſtances was 82°. 46′. 42″, therefore knowing the true zenith diſtance at the Cape, the true zenith diſtance at Paris was known, the difference between which and the apparent zenith diſtance was the refraction. Thus M. de la CAILLE formed his Table of refractions. His method was very ingenious; but from more accurate obſervations ſince his time, it appears, that his refractions are a little too great. This Dr. MASKELYNE has clearly ſhown in the *Phil. Tranſ.* 1787. By comparing the ſum of the two apparent zenith diſtances of ſtars obſerved at a low altitude at Paris, and conſequently at an high altitude at the Cape, and at an high

high altitude at Paris, and therefore at a low altitude at the Cape, he found the refraction at the Cape to be a fortieth part leſs than at Paris.

179. *Fourth method.* BOSCOVICH propoſes to find the refraction by the circumpolar ſtars, only by knowing it's variation at different altitudes. Let a and a' be the apparent meridian zenith diſtances of a ſtar below and above the pole, x and x' the reſpective refractions; b and b' the apparent meridian zenith diſtances of another ſtar below and above the pole, z and z' the correſponding refractions; then the true diſtance will be $a+x$, $a'+x'$, and $b+z$, $b'+z'$; and as the diſtance of the pole from the zenith is equal to half the ſum of the greateſt and leaſt true zenith diſtances, $a+x+a'+x'=b+z+b'+z'$; hence (A) $x+x'-z-z'=b+b'-a-a'$. Now taking, at firſt, the refractions to be as the tangent of the zenith diſtances, we have tan. a : tan. a' :: x : x' = $\frac{x \text{ tan. } a'}{\text{tan. } a}$; for the ſame reaſon $z=\frac{x \text{ tan. } b}{\text{tan. } a}$, $z'=\frac{x \times \text{tan. } b'}{\text{tan. } a}$; ſubſtitute theſe into the equation (A), and we get $x=\frac{\overline{b+b'-a-a'} \times \text{tan. } a}{\text{tan. } a+\text{tan. } a'-\text{tan. } b-\text{tan. } b'}$; hence the other refractions are known. But as the refractions vary more accurately as the tangent of the zenith diſtance diminiſhed by three times the refraction, put $a-3x=m$, $a'-3x'=m'$, $b-3z=n$, $b'-3z'=n'$, and we have $x=\frac{\overline{b+b'-a-a'} \times \text{tan. } m}{\text{tan. } m+\text{tan. } m'-\text{tan. } n-\text{tan. } n'}$, the correct refraction at the apparent altitude a; hence we know $x'=\frac{x \text{ tan. } m'}{\text{tan. } m}$, $z=\frac{x \times \text{tan. } n}{\text{tan. } m}$ and $z'=\frac{x \text{ tan. } n'}{\text{tan. } m}$. The operation may be ſhortened, by taking $3x$, $3x'$, $3z$, $3z'$ from the common Tables. As $a+x$, $a'+x'$, are the true zenith diſtances of one of the ſtars below and above the pole, the true zenith diſtance of the pole will be one half of $a+x+a'+x'$, which is the complement of the latitude of the place.

Ex. The apparent zenith diſtance of η *Draconis* below and above the pole was obſerved to be 69°. 5′. 2″,4 and 13°. 8′. 27″,2; and of θ *Urſæ minoris* 53°. 2′. 57″,2 and 29°. 11′. 23″,2; to find the correſponding refractions, and the latitude of the place.

$a=69°$

$a = 69°.\ 5'.\ 2'',4$ $a' = 13.\ 8.\ 27,2$ $b = 53.\ 2.\ 57,2$ $b' = 29.\ 11.\ 23,2$ $a + a' = 82.\ 13.\ 29,6$ $b + b' = 82.\ 14.\ 20,4$ $c = 0.\ 0.\ 50,8$	$\tan. a = 2,616$ $\tan. a' = 0,233$ $2,849$ $\tan. b = 1,329$ $\tan. b' = 0,558$ $1,887$ $c' = 0,962$	$\frac{c}{c'} = 52'',807$ $x = 52,807 \times 2,616 = 138'',2$ $x' = 52,807 \times 0,233 = 12,3$ $z = 52,807 \times 1,329 = 70,2$ $z' = 52,807 \times 0,558 = 29,5$

$a = 69°.\ 5'.\ 2'',4$	$a' = 13°.\ 8'.\ 27'',2$	$b = 53°.\ 2'.\ 57'',2$	$b' = 29°.\ 11'.\ 23'',2$
$3x = 0.\ 6.\ 54,6$	$3x' = 0.\ 0.\ 36,9$	$3z = 0.\ 3.\ 30,6$	$3z' = 0.\ 1.\ 28,5$
$m = 68.\ 58.\ 7,8$	$m' = 13.\ 7.\ 50,3$	$n = 52.\ 59.\ 26,6$	$n' = 29.\ 9.\ 54,7$

$\tan. m = 2,6009$ $\tan. m' = 0,2333$ $2,8342$ $\tan. n = 1,3266$ $\tan. n' = 0,5581$ $1,8847$ $c' = 0,9495$	$\frac{c}{c'} = 53'',505$ $x = 53,505 \times 2,6009 = 139'',2$ $x' = 53,505 \times 0,2333 = 12,5$ Refraction at zenith dist. $69°.\ 5'.\ 2'',4$ is $139'',2$; at zenith dist. $13°.\ 8'.\ 27'',2$ is $12'',5$	$a + a' = 82°.\ 13'.\ 29'',6$ $x + x' = 0.\ 2.\ 31,7$ $a + a' + x + x' = 82.\ 16.\ 1,3$ $41.\ 8.\ 0,6$ $90.\ 0.\ 0$ Lat. of Place $48.\ 51.\ 59,4$ We may get the correct refractions z, z' in like manner.

180. *Fifth method.* Dr. MASKELYNE informs us in the *Phil. Tranſ.* 1787, that Dr. BRADLEY found his refractions in the following manner. He obſerved the pole ſtar, and other circumpolar ſtars, above and below the pole, and from thence deduced the apparent zenith diſtance of the pole. By the apparent and equal zenith diſtances of the ſun at the two equinoxes, having at the ſame time oppoſite right aſcenſions, as found by comparing (118) it's obſerved tranſits over the meridian with thoſe of fixed ſtars, he found the apparent zenith diſtance of the equator, which diminiſhed by parallax and added to the apparent zenith diſtance of the pole, gave a ſum leſs than 90° by the ſum of the two refractions belonging to the pole and meridian altitude of the equator*. Now he obſerved, that the difference of the refractions at theſe altitudes came out within 2″ or 3″, from the beſt Tables then extant, whether deduced ſolely from obſervations, or partly from obſervation and partly from theory. Hence, knowing the ſum and difference of the refractions, he knew the refraction at each alti-

* For the ſum of the two *true* zenith diſtances = 90°; but the true diſtance of each is diminiſhed by refraction, and therefore the ſum (after the correction for parallax) muſt be leſs than 90° by the ſum of the two refractions.

tude.

tude. He afterwards more accurately divided the ſum of the two refractions, by taking the parts in proportion to the tangents of the zenith diſtances. The apparent zenith diſtance of the equator, by the mean of 20 obſervations in 1746-47 he found to be 51°. 27′. 28″; and the mean apparent zenith diſtance of the pole, by obſervations made between 1750—52, was 38°. 30′. 35″; the ſum of which being 89°. 58′. 3″ the ſum of the two refractions is 1′. 57″; conſequently the polar refraction is $45\frac{1}{2}''$, and the equatorial $1'.\ 11\frac{1}{2}''$; therefore the latitude of Greenwich Obſervatory is 51°. 28′. $39\frac{1}{2}''$. Dr. BRADLEY here ſuppoſed the ſun's horizontal parallax to be $10\frac{1}{2}''$; but Dr. MASKELYNE obſerves, that had he taken it $8\frac{1}{4}''$, as determined from the two laſt tranſits of *Venus* over the ſun, the refraction at 45°, which he fixed at 57″, would have come out $56\frac{1}{2}''$, and the latitude of the obſervatory 51°. 28′. 40″. Dr. BRADLEY having thus ſettled the refraction at the altitude of the equator and pole, could calculate the refraction at all higher altitudes, or for all ſtars between the equator and pole, by taking it as the tangent of the zenith diſtances, which would be very accurate for all ſuch altitudes. Hence by taking the altitudes of the circumpolar ſtars above and below the pole, and knowing the refraction above, he immediately got the refraction at the lower altitudes; for knowing the refraction at the altitude above the pole, he knew the true altitude above, and knowing the altitude of the pole he got the true diſtance of the ſtar from the pole, which ſubtracted from the altitude of the pole, gave the true altitude below, the difference between which and the apparent altitude was the refraction. When the weight and temperature of the air remains the ſame, the Dr. found that the refraction varied as the tangent of the zenith diſtance diminiſhed by three times the refraction found by the common Rule; and having fixed the refraction at 45° (whoſe tangent, if radius = 1, is unity) to be 57″, if r = the refraction in the Tables, $\underline{z} =$ the apparent zenith diſtance, he got this proportion, r : 57″ :: tan. $z - 3r$: 1.* And by comparing the refractions in different temperatures of the air, and at different altitudes of the barometer, he inferred the following elegant Rule for determining the refraction at all altitudes: Put a = the altitude of the barometer in inches, $k°$ = the altitude of FAHRENHEIT's ther-

* The application of this Rule to find the refraction at all altitudes is thus. Let the apparent zenith diſtance be z, then the refraction will be nearly 57″ × tan. z, which put = r; and the correct mean refraction will be 57″ × tan. $\overline{z - 3r}$. If at very low altitudes it ſhould be required to have the refraction more correctly, put 57″ × tan. $\overline{z - 3r} = r'$, and the refraction becomes 57″ × tan. $\overline{z - 3r'}$. Let the refraction at the apparent zenith diſtance 70° be required. The tangent of 70° is 2,747; hence 57″ × 2,747 = 2′. 36″,6, which multiplied by 3 and ſubtracted from 70° gives 69°. 52′. 10″, the tangent of which is 2,728; therefore 57″ × 2,728 = 2′. 35″,5 the *mean* refraction at the apparent zenith diſtance 70°. In this manner Table XIII. was calculated.

mometer,

mometer, then the *true* refraction : $57'' :: \frac{a}{29,6} \times \tan. \overline{z - 3r} : \frac{h° + 350°}{400°}$. The very near agreement of this Rule with that given by MAYER, and their agreement with obſervations, are a ſtrong confirmation of the accuracy of each. This correction for the barometer and thermometer may be immediately found from Table XIV.—The Inſtrument invented by Mr. RAMSDEN, called a *Circular Inſtrument* (for a deſcription of which ſee my *Treatiſe on Practical Aſtronomy*), is admirably calculated to determine the quantity of refraction at all altitudes; for by taking the altitude and azimuth of a body whoſe declination is known, the true altitude may be immediately computed from the latitude of the place, declination of the body, and obſerved azimuth; hence the difference between the obſerved and computed altitudes gives the refraction at that apparent altitude. Having thus explained the practical methods of finding the refraction, we proceed next to give the Inveſtigations of the Rules which we have had occaſion to make uſe of.

FIG. 34. 181. Let ACn be the angle of incidence, ACm the angle of refraction, and conſequently mCn the quantity of refraction; let AT be the tangent of Am, mv it's ſine, nw the ſine of An, and draw rm parallel to vw; then as the refraction in air is very ſmall, we may conſider mrn as a rectilinear triangle, and hence, by ſimilar triangles, $Cv : Cm :: rn : mn = \frac{Cm \times rn}{Cv}$; but Cm is conſtant, and as the ratio of mv to nw is conſtant by the laws of refraction, their difference rn muſt vary as mv; hence mn varies as $\frac{mv}{Cv}$; but $AT = \frac{Cm \times mv}{Cv}$ which varies as $\frac{mv}{Cv}$, becauſe Cm is conſtant; hence the refraction mn varies as AT, the tangent of the apparent zenith diſtance of the ſtar, becauſe the angle of refraction ACm is the angle between the refracted ray and the perpendicular to the ſurface of the medium, which perpendicular is directed to the zenith. Whilſt therefore the refraction is very ſmall, ſo that rmn may be conſidered as a rectilinear triangle, this Rule will be ſufficiently accurate; otherwiſe we muſt uſe Dr. BRADLEY's Rule, the demonſtration of which is given by BOSCOVICH in his Works, Vol. II. but one of the principles, that the force with which the ray is attracted in paſſing through the air may be conſidered as uniform, is taken from Mr. SIMPSON's Solution in his Mathematical Diſſertations. We ſhall therefore firſt give his reaſons for this ſuppoſition.

182. After conſtructing his Table of refraction, he obſerves, that the only material objection which it is liable to is, it's being founded upon ſuppoſition, that the denſity of the air decreaſes uniformly, which appears contrary to experiment, whereby it is proved, that the denſity of the air decreaſes as the com-

compreſſing force decreaſes: But though this is true in air of the ſame temperature, yet it cannot be ſuppoſed to hold true in the earth's atmoſphere, ſince the upper region thereof is known to be much colder, and conſequently the elaſticity there is much leſs than at the earth's ſurface: But a convincing proof that this law of denſity cannot obtain in our atmoſphere is, that the mean horizontal refraction computed from it, according to the known refractive power and ſpecific gravity of the air, will be found to come out no leſs than 52′, which is greater by about $\frac{1}{3}$ of a degree than it ought to be, it being only 33′; whereas, if the ſame refraction be calculated upon the hypotheſis of the denſity decreaſing uniformly, and compared with obſervations, the difference will be much leſs. This latter hypotheſis will therefore beſt correſpond to the ſtate of our atmoſphere.

183. Let us therefore ſuppoſe the atmoſphere to be divided into an infinite number of lamina concentric with the center of the earth, and of an equal thickneſs, then the denſity of theſe lamina is ſuppoſed to decreaſe uniformly, for the reaſons above given, and therefore the difference of the denſities is conſtant. But when a ray of light paſſes out of one medium into another, it is attacted by a force which depends on the difference of their denſities, and therefore when the difference is conſtant the force is conſtant. Hence a ray of light deſcending through the atmoſphere may be ſuppoſed to be attracted by it in a direction perpendicular to the ſurface of the earth by a conſtant force.

184. Let C be the center of the earth, AM it's ſurface, ZF the top of the atmoſphere, FA the paſſage of the ray; draw the tangents SFH, IAG cutting each other in I, and let CH, CG be drawn perpendicular to them, and AL parallel to CF. Now the ſtate of the atmoſphere remaining the ſame, the ſine of incidence is to the ſine of refraction for each lamina in a given ratio, therefore by compoſition, the ſine of incidence CFH at F is to the ſine of refraction CAG at G in a given ratio. Hence if radius $= 1$, $\frac{CH}{CF}$ and $\frac{CG}{CA}$ will be theſe reſpective ſines; but the velocities at F and A are as CG to CH, which aſſume as 1 to $1+b$; and if $MF = e$, $CM = 1$, $\frac{CH}{CF} : \frac{CG}{CA} :: \frac{1+b}{1+e} : 1$; put $m = \frac{1+b}{1+e}$, $a =$ angle CAG, and then $1 : m ::$ ſin. a : ſin. $CFH = m \times$ ſin. a. FIG. 35.

Let $x =$ angle ACF, $r =$ angle GIH of refraction. In the quadrilateral figure $CAIF$, the angle $ACF + IFC =$ the ſum of the external angles $GIH + CAG$, becauſe $FIA + CAI$ added to each would make the ſum equal to four right angles; hence IFC or $CFH = CAG - ACF + GIH$, that is, $m \times$ ſin. $a =$ ſin. $\overline{a-x-r}$, therefore $1 : m ::$ ſin. a : ſin. $\overline{a-x-r}$; but by plain trigonometry,

the ſum of the ſines of two angles : their difference :: tan. of half the ſum of the angles : tan. of half their difference; hence $1+m : 1-m$ :: tan. $a-\frac{n}{2}.\overline{x-r}$: tan. $\frac{1}{2}.\overline{x-r}$, and as this ratio is conſtant, the tan. $a-\frac{1}{2}.\overline{x-r}$ varies as the tan. $\frac{1}{2}.\overline{x-r}$; but as the difference between x and r muſt be very ſmall, the tangent of $\frac{1}{2}.\overline{x-r}$ may be conſidered as equal to the angle itſelf $\frac{1}{2}.\overline{x-r}$; alſo a is the apparent zenith diſtance; hence the angle $\frac{n}{2}.\overline{x-r}$ varies as the tangent of the apparent zenith diſtance diminiſhed by $\frac{1}{2}.\overline{x-r}$. If therefore the ratio of x to r be conſtant, then $x-r$, and conſequently r itſelf, will vary as the tangent of the zenith diſtance diminiſhed by ſome multiple of r; for if $dr=x$, then $x-r=dr-r=\overline{d-1}\times r$; let therefore $1+m : 1-m$ :: tan. $a-\frac{1}{2}nr$: tan. $\frac{1}{2}nr$, and then the refraction r varies as tan. $\overline{a-\frac{1}{2}nr}$. On this ſuppoſition $\frac{1}{2}.\overline{x-r}=\frac{1}{2}nr$, or $x-r=nr$. That x is to r in a conſtant ratio may be thus proved.

185. Let us conceive AF to be an indefinitely ſmall part of the whole curve, taken any where, and AL (which is drawn parallel to FC) is the ſagitta of the curve. Put $v=$ the velocity through FA, $t=$ the time, $z=CF$, $\dot{z}=FM$, $\dot{x}=$ the angle FCA, $\dot{r}=$ the angle GIH, $f=$ the force in the direction FC. Now from the principles of Mechanics, $AF=vt$, and the ſagitta $LA=\frac{1}{2}\ddot{z}=ft^2$; hence the tangent AI (which $=\frac{1}{2}AF$) $=\frac{1}{2}vt$; alſo, as the arc varies as the angle multiplied into the radius, $AM=z\dot{x}$, and the ſine of ALI or $CFL=\frac{AM}{AF}=\frac{z\dot{x}}{vt}$; but $AI : AL$ :: ſin. ALI : ſin. AIL, that is, $\frac{1}{2}vt : ft^2$ :: $\frac{z\dot{x}}{vt}$: ſin. $\dot{r}$ or $\dot{r}$, hence $\frac{\ddot{r}}{\dot{x}}=\frac{2fz}{v^2}$. Now if we conſider the velocity and diſtance from the center as having but a very ſmall variation, and f to be conſtant (183), we may conſider $\frac{\ddot{r}}{\dot{x}}$ as conſtant, and conſequently $\dot{r}$ varies as $\dot{x}$, therefore r varies as x when AF is finite. Hence (184) r varies as the tan. $\overline{a-\frac{1}{2}nr}$.

186. Becauſe $1+m : 1-m$ :: tan. $\overline{a-\frac{1}{2}nr}$: tan. $\frac{1}{2}nr$ :: (by trig.) ſin. $a+$ ſin. $\overline{a-nr}$: ſin. $a-$ ſin. $\overline{a-nr}$, hence $m\times$ ſin. $a=$ ſin. $\overline{a-nr}=$ (by trig.) ſin. $a\times$ cos. $nr-$ ſin. $nr\times$ cos. $a=$ (becauſe nr being a very ſmall arc it's cos. $=\sqrt{1-n^2r^2}=1-\frac{1}{2}n^2r^2$, and the ſine $=$ arc very nearly) ſin. $a-$ ſin. $a\times\frac{1}{2}n^2r^2-nr\times$ cos. a, and by dividing by ſin. a we have $m=1-\frac{1}{2}n^2r^2-nr\times$ cot. a. Now let a' be any other apparent zenith diſtance, and r' the refraction, then, for the ſame reaſon, $m=1-\frac{1}{2}n^2r'^2-nr'\times$ cot. a'; make theſe values of m equal, and we get $\frac{1}{2}n=\frac{r'\times\text{cot. }a'-r\times\text{cot. }a}{r^2-r'^2}$. Now by Dr. BRADLEY's obſervations, if $a=60^\circ$, $r=1'.38''{,}4$; and if $a=90^\circ$, $r=33'$; hence $\frac{1}{2}n=2{,}996$; he therefore aſſumes $\frac{1}{2}n=3$; the refraction therefore varies as the tang. $a-3r$, that is, *the refrac-*

refraction varies as the tangent of the apparent zenith distance diminished by three times the refraction. SIMPSON makes $n = 5,5$, CASSINI $= 6,452$ and BOUGUER $= 6,645$. But Dr. BRADLEY's value is most to be depended upon, as best agreeing with observations, which we shall therefore follow.

187. Because $m = 1 - \frac{1}{2} n^2 r^2 - nr \times$ cot. a, therefore, as $\frac{1}{2} n^2 r^2$ is very small in respect to the other terms, $m = 1 - nr \times$ cot. a, hence $1 - m = nr \times$ cot. a. For the horizontal refraction, $a = 0$, $r = 33'$; therefore $m = 1 - \frac{1}{2} n^2 r^2 =$ cos. nr; hence if $n = 6$, we have $m =$ cos. $6r =$ cos. $3'.18'' = 0,9983$. Hence also (184) $x - y = nr = 6r$, according to Dr. BRADLEY, therefore $x = 7r$, or the angle which the refracted ray subtends at the center of the earth $= 7$ times the refraction.

188. Join CI, and let the angle $ACI = y$, then CIA or $CIG = a - y$, $CIH = a - y + r$, and their sines are as the perpendiculars CG, CH, which are inversely as the velocities at A and E, or as $1 : 1 + b$; hence $\overline{1 + b} \times$ sin. $\overline{a - y} =$ sin. $\overline{a - y + r} =$ sin. $\overline{a - y} \times$ cos. $r +$ sin. $r \times$ cos. $\overline{a - y} =$ (because r being very small it's cos. $= 1$, and it's sine $= r$) sin. $\overline{a - y} + r \times$ cos. $\overline{a - y}$, hence $1 + b = 1 + r \times$ cot. $\overline{a - y}$, and $b = r \times$ cot. $\overline{a - y}$. But if we make a approach to 90°, y will be very small when compared with a, therefore $b = r \times$ cot. a. If $a = 60°$, then $r = 1'.38'',4$ according to Dr. BRADLEY; hence $b = r \times$ cot. $a =$ sin. $r \times$ cot. $a = 0,0002755$; therefore the sine of incidence out of a vacuum into air at the mean density at the earth's surface is to the sine of refraction as 1,0002755 : 1. Mr. HAUKSBEE makes it as 1.000264 : 1 by experiment. As $b = r \times$ cot. a, therefore $6b = 6r \times$ cot. $a = 1 - m$ from the last Article, hence $b = \frac{1 - m}{6}$.

189. Having determined the values of b and m, we get, from the equation $\frac{1 + b}{1 + e} = m$, the value of $e = \frac{1 - m + b}{m} =$ (as $b = \frac{1 - m}{6}$) $\frac{7 - 7m}{6m} = 0,001942$ parts of the earth's radius $= 77,25$ miles, the altitude above the earth's surface at which the air begins to have any sensible effect on the rays of light to refract them.

190. The refraction varies as the tan. $\overline{a - 3r}$ at any altitude above the earth's surface; for the proof remains the same for whatever part of the curve you take from the top of the atmosphere. Hence we may find the refraction at any altitude, by making e denote it's distance from the top of the atmosphere; for by the last Article $m = \frac{7}{7 + 6e} =$ (by division, and neglecting all the powers of e above the first on account of their smallness) $1 - \frac{6e}{7} =$ (187) cos. $6r$; hence

the

the cos. of $6r$ being known, $6r$, and consequently r itself, the horizontal refraction in this case, will be known, and hence the refraction at any other altitude.

191. As (186) $m \times$ sin. $a =$ sin. $a - 6r$, according to Dr. BRADLEY, put $p =$ the complement of a, and let $m \times$ cos. $p =$ cos. q, then cos. $q = m \times$ cos. $p = m \times$ sin. $a =$ sin. $a - 6r =$ (as $a = 90° - p$) sin. $90° - p - 6r =$ cos. $p + 6r$, hence $p + 6r = q$, therefore $r = \frac{q - p}{6}$. This expression is accommodated to find the refraction below the horizon, when the observer is elevated above it, by making p negative. Hence the refraction below the horizon increases very fast, r being expressed by the sum of p and q.

192. In the horizon, cos. $6r = 1 - \frac{6r}{7}$, therefore $\frac{6r}{7} = 1 -$ cos. $6r =$ ver. sin. $6r = 18r^2$ by the property of the circle; consequently the horizontal refraction r varies as the square root of e. Hence if h be the altitude of the atmosphere, we know the horizontal refraction at any altitude $h - e$ above the horizon, for it will be to the horizontal refraction on the earth's surface as $\sqrt{e} : \sqrt{h}$. The horizontal refraction therefore being known, the refraction at any other altitude will be known.

FIG. 36. 193. Hence, upon the same principles, we have a very elegant method of finding the radius of curvature to the curve which the ray describes. Let AF be an indefinitely small part of the curve adjacent to A the surface of the earth, and conceive AZV to be a circle of curvature, O it's center, and QOK perpendicular to AV, which therefore must bisect AV. Then the angle $AIE = FOA = 2FVA = FKA$; but (187) $7AIE = FCA$, therefore $7FKA = FCA$, hence $AK = 7AC$ the radius of the earth, and therefore is a constant quantity for all angles IAE. Hence the center of the circle of curvature is always in the line QK. By trig. $AO : AK ::$ rad. $= 1 :$ sin. AOK or IAE, hence $AO = \frac{AK}{s.IAE} = \frac{7AC}{s.IAE}$, and as AC is constant, the radius of curvature varies inversely as the sine of the apparent zenith distance. Hence for horizontal refractions, the radius of curvature is equal to 7 times the radius of the earth. This agrees with the conclusions deduced by J. H. LAMBERT in his very elegant Treatise entitled, *Les Proprietés remarquables de la Route de la Lumière par les Airs*, which he has applied with so much success to terrestrial refractions, and which we shall now proceed to consider.

194. Suppose MF to be any object, and FA the curve described by a ray of light coming from F to A; then for so small a distance we may suppose FA to be

be circular. Let $m =$ ſin. FAE, then $AO = \frac{7AC}{m}$ is known. Now the effect of refraction in altering the apparent altitude is the angle between AI and the chord drawn to the arc FA; for the latter is the direction in which F would be ſeen if there were no refraction, and the former if ſeen by refraction; but this angle between the chord and tangent muſt be equal to $\frac{1}{2} FOA = \frac{FA}{2AO}$; but $FA = \frac{AM}{m}$, and $AO = \frac{7AC}{m}$; hence the refraction $= \frac{AM}{14AC} = \frac{1}{14}$ of the angle ACM. Hence any point ſituated in the line MF, and ſeen at A, has the ſame refraction, for it is independent of the altitude MF; conſequently any object ſituated in a line perpendicular to the earth will not have it's apparent length altered by refraction, becauſe each end will appear equally elevated by it. Hence alſo the terreſtrial refraction varies as the diſtance AM. If therefore MF be a mountain, and we want to find the altitude from the given diſtance AM, and the apparent angle of elevation MAI, we muſt firſt correct this angle by ſubtracting from it $\frac{1}{14}$ of ACM.

195. Hence we may readily find the diſtance at which an object of a given altitude whoſe top is depreſſed below the horizon, may be ſeen by refraction. For take $AK = 7AC$, and with the center K deſcribe the circle Ar, and the point r will be ſeen by refraction; draw $srvC$, and Av is the diſtance at which an object vr is viſible; draw alſo the tangent Ax. Now the angles ACv, AKr being very ſmall, and the arcs Av, Ar very nearly equal, $sr : sv :: AC : AK :: 1 : 7$, hence $vr : sv :: 6 : 7$, therefore $sv = \frac{7vr}{6}$; but $sv = \frac{Av^2}{2}$, the radius of the earth being unity; therefore $\frac{Av^2}{2} = \frac{7vr}{6}$, conſequently $Av = \sqrt{\frac{14vr}{6}} = \sqrt{\frac{7vr}{3}}$. Hence the diſtance at which an object can be ſeen, varies as the ſquare root of it's altitude. FIG. 37.

196. If yw be perpendicular to the ſurface of the earth and equal to vr, the object vr can be ſeen at y without refraction; but yw or $vr = \frac{Ay^2}{2}$, hence $Ay = \sqrt{2vr}$, therefore the diſtance at which an object can be ſeen by refraction : diſtance at which it could be ſeen without refraction :: $\sqrt{\frac{7vr}{3}} : \sqrt{2vr} :: \sqrt{7} : \sqrt{6}$, which is nearly as 14 : 13.

197. An eye at r ſees A in the direction of the tangent at r, and therefore it appears below the horizon at v by the angle formed by the two tangents to r and

r and v, or by the angle CrK. Now (195) Av, or the angle ACv, $= \sqrt{\frac{7vr}{3}}$, and $Kr : CK$ (:: 7 : 6) :: fin. rCK or rCA : fin. CrK :: (on account of the ſmallneſs of theſe angles) rCA, or $\sqrt{\frac{7vr}{3}}$; $\therefore CrK = \sqrt{\frac{12vr}{7}}$ the depreſſion of the point A below the horizon. Hence the depreſſion below the horizon varies as the ſquare root of the altitude.

198. Conſidering the arcs Av, Ar as equal on account of the ſmallneſs of the angle ACv, the ſagittas sv, sr will be inverſely as the radii; hence $sv : sr :: 7 : 1$; therefore $rv : sv :: 6 : 7$, and conſequently the point r appears to be elevated by a quantity equal to $\frac{1}{6}\,vr$ or $\frac{1}{7}\,sv$; but $sv = \frac{Av^2}{2}$, therefore sr $(= \frac{1}{7}\,sv) = \frac{Av^2}{14}$. Hence, as the refraction remains nearly the ſame for all objects near the horizon, this correction muſt be made in calculating the altitudes of ſuch objects from the apparent angles of elevation. All the above numbers are for the mean ſtate of the air.

199. Hence, we may find the altitude vr of a cloud at r, by obſerving the inſtant when it ceaſes to be enlightened by the ſun; for at that time calculate the depreſſion of the ſun below the horizon, and from it ſubtract the horizontal refraction and you will have the true depreſſion below the horizon, or the angle between As and a tangent to v, or the angle ACv; hence we know Av and conſequently vr. This ſuppoſes that the ray coming to the cloud is a tangent to the ſurface of the ſea, or to an horizontal plane at land.

FIG. 38. 200. Let SB be a ray of light falling on the atmoſphere at B and refracted in the curve BAE touching the earth at A, and emerging in the direction EF, meeting DC parallel to SB in F; to find CF. As Cv is a perpendicular upon the incident ray, and CA upon the refracted ray, they will be as the ſine of incidence to the ſine of refraction out of a vacuum into air of the ſame denſity as that at the earth's ſurface, or as 1,0002755 : 1; hence put $m = 1{,}0002755 = Cv = Cr$, n = the angle $CFr = Fsx = rCv = 2ACv$, or twice the horizontal refraction, and $CF = Cv \times$ coſec. $n =$ (if $n = 66'$) 53,1 radii of the earth. If the direction of the ray of light be not parallel to DC but to dCf, and the angle dCD be put $= x$, then the angle $rCf = n + x$, and $Cf = v \times$ coſec. $\overline{n + x}$.

201. If the line dC be ſuppoſed to join the centers of the ſun and earth, and the ray SB to come from the limb of the ſun, Cf will be the length of the total ſhadow of the earth, as all the umbra beyond f will have ſome rays of the ſun by refraction. Now let $x = 16'$ the ſun's ſemidiameter, and $Cf = v \times$ coſec. $82' = 41{,}94$ ſemidiameters of the earth, which being very little more than $\frac{2}{3}$ of the

the diſtance of the moon, it appears that in a total eclipſe of the moon, ſome rays from the ſun muſt fall upon it, which is the cauſe of it's being viſible in that ſituation.

202. Having thus fully explained the principles of refraction, and the methods of conſtructing the Tables for the mean refraction, it will be proper to give ſome account of the variations to which the air is ſubject, from a change of temperature and denſity, for which proper corrections are given, except when the obſervations are very near to the horizon, where changes frequently take place which cannot be altogether accounted for, and for which therefore no correction can be applied; they probably ariſe from exhalations of various kinds which are ſuddenly raiſed and ſuſpended in the air near to the earth's ſurface, the cauſes of which do not ſenſibly affect the barometer and thermometer. Hence all obſervations made very near to the horizon muſt be ſubject to a very conſiderable degree of uncertainty, and therefore Aſtronomers never uſe them when great accuracy is required.

203. TYCHO, when he conſtructed his Table of refraction, knew that it was ſubject to variation; but CASSINI and PICARD were the firſt who meaſured accurately the change. PICARD found, from the meridian altitudes of the ſun, that the refraction was greater in winter than in ſummer; he obſerved alſo, that it was greater in the night than in the day. And from obſerving the horizontal refraction of the upper limb of the ſun when it firſt appeared in the horizon, and then that of the lower limb, he found that in the time in which the ſun was riſing, the refraction was diminiſhed 25″. BOUGUER obſerved in America, that the refractions in the night were greater than in the day, by about $\frac{1}{6}$ or $\frac{1}{7}$. Dr. NETTLETON meaſured the altitude of an hill in a clear day; and repeating the obſervations in a cloudy day when the air was ſomewhat groſs and heavy, he found the angle conſiderably greater. He alſo obſerved that the altitudes of ſome of the hills which he meaſured appeared greater in the morning before ſun riſe and late in the evening, than at noon in a clear day. At the time of the great froſt at Paris in 1740, MONNIER obſerved, when the thermometer was 10° below the freezing point, that at the apparent altitude 4°. 44½′ the refraction was 11′. 15″; but when the mercury ſtood at 24° above the freezing point, the refraction at the ſame altitude was found to be only 9′. 20″; hence there was a difference of 1′. 55″ for 36°. of the thermometer. The barometer was at 28 inches. From theſe differences of refractions in ſummer and winter, in the day and night, it might be conjectured that the refractions would be greater towards the north, where it is colder. But the French Academicians in the year 1737, at Tornea on the borders of Lapland, where they were ſent to meaſure a baſe in order to determine the length of a degree of latitude,

found that the refractions agreed with thoſe at Paris. M. de la CAILLE however found that the refractions at the Cape of Good Hope were about $\frac{1}{40}$ leſs than at Paris; from which ſmall difference he concluded that a Table of refractions might be conſtructed which would anſwer very accurately for every part of the temperate zone. In the torrid zone M. BOUGUER found the horizontal refraction to be 27′; at 6° high, 7′. 4″; and at 45° high, 44″. Admitting therefore the refraction to be leſs in climates warmer than at Paris, we may conclude that it muſt be greater in thoſe which are colder, and that it was from want of a ſufficient number of obſervations, or from their inaccuracy, that the Academicians in Lapland did not find it ſo.

204. The refraction being thus found to vary in different ſtates of the air, the next enquiry is, what allowance muſt be made for any variation of the temperature and weight of the air, from any ſtandard which we may make the mean. Dr. BRADLEY made 29,6 inches the mean ſtandard for the barometer; and as Mr. HAUKSBEE had determined from experiment that the refraction was in proportion to the denſity of the air, it muſt alſo be as the altitude of the mercury in the barometer. Now in the mean ſtate of the air, that is, when the barometer is at 29,6 inches, and FAHRENHEIT's thermometer at 50°, the refraction (180) : 57″ :: tan. $\overline{z-3r}$: 1; hence at any altitude (a) of the mercury, the refraction : 57″ :: $a \times$ tan. $\overline{z-3r}$: 29,6. The refraction, thus corrected for the variation of the weight of the air, agrees very well with obſervations. The next thing to be done is, to find how the refraction varies in different temperatures. M. de la CAILLE found that the refraction was diminiſhed $\frac{1}{27}$ part from an increaſe of 10° in the altitude of the mercury in the thermometer of REAUMUR. MAYER obſerved that the refraction varied about $\frac{1}{22}$ part for 10° of variation. M. BONNER made ſome experiments in order to determine the variation of refraction ariſing from that of the temperature; calling the refraction unity for the altitude 10° of the thermometer, he found the refraction to be 0,92 at the altitude 30°, or diminiſhed $\frac{1}{25}$ for a variation of 10°; and at 8° below 0° he found the refraction to be 1,085 or $\frac{1}{21,18}$ for a variation of 10°. The mean of theſe differ but very little from the determination of MAYER. The obſervations, upon which Dr. BRADLEY formed his rate of variation, have never been publiſhed. He uſed FAHRENHEIT's thermometer, and fixed the mean temperature at 50°; and if $h°$ be any other altitude, he found that

that the refraction varied in the ratio of 400° : $h°+350°$, or 1 : $\frac{h°+350°.}{400°}$ Hence, allowing for the variation of temperature and weight, he found, the *true* refraction : $57''$ :: $\frac{a}{29,6}$ × tan. $\overline{z-3r}$: $\frac{h°+350°}{400°}$. And this agrees very accurately with the Rule deduced by MAYER.

205. When the ſun is in the horizon, the rays in paſſing very obliquely through the atmoſphere are ſo far ſeparated, that M. BOUGUER, in a Work entitled *Traité d'Optique ſur la Gradation de la Lumière*, has concluded from experiment, that the intenſity of light is 1354 times leſs than when the ſun is in the zenith. M. de MAIRAN thinks that the weakneſs of the ſun's rays in the former caſe is principally to be attributed to the quantity of vapours with which the lower parts of the atmoſphere are always filled.

206. It is owing to the atmoſphere that we have any twilight in the morning and evening, which ariſes both from refraction and reflection of the ſun's rays. It may be explained thus. Let *AB* be the ſurface of the earth, *Sm* a ray of light coming from the ſun, and beginning to be refracted at *m*; let it deſcribe the curve *mBn* touching the earth at *B*, and at *n* let it be reflected into the curve *nA*, touching the earth at *A*, the place of the ſpectator; in this poſition therefore of the ſun, the twilight juſt appears; draw the tangents *Avz*, *mz*, *Bv*, and join *vnC*. Then *AO* (the radius of curvature to the arc *An*) = 7*AC* (193), conſidering *An* as a circle; from which it will differ but very little. Now ſuppoſe twilight to begin when the ſun is 18° below the horizon, that being about the quantity found by computing the ſun's depreſſion from the obſerved time at which the twilight begins; it varies however in different ſeaſons; hence the angle $z = 162°$; but the difference between the angle *z* and the angle *BvA* is the refraction through *mB*, or 33′; therefore the angle $AvB = 162°.\ 33'$, and $AvC = 81°.\ 16\frac{1}{2}'$, conſequently $ACv = 8°.\ 43\frac{1}{2}'$, and hence $nCO = 171°.\ 16\frac{1}{2}'$; alſo $On : Oc :: 7 : 6$; hence $On = 7 : Oc = 6$ :: ſin. $nCO = 171°.\ 16\frac{1}{2}'$: ſin. $CnO = 7°.\ 28\frac{1}{4}'$, therefore $nOC = 1°.\ 15\frac{1}{4}'$; hence ſin. *nCO* : ſin. *nOC* :: $On = 7 : Cn = 1,01$, from which take $Cx = 1$, and we have $nx = 0,01 = 39,64$ miles. But (189) the ray begins to be refracted at the altitude of 77,25 miles; hence the reflection takes place at about half the altitude at which the refraction begins. This is upon ſuppoſition that the rays come to the ſpectator after one reflection. If we ſuppoſe them to come after 2, 3 or 4 reflections, the altitudes *nx* will be about 12, 5,4 and 3 miles reſpectively, and the denſities of the air 10,75, 2,9 and 1,8 leſs than at the earth's ſurface. Which of theſe is moſt probable, may admit of ſome doubt. That air at the altitude of 39,64 miles, where it is 2700 leſs denſe than at the earth's ſurface, ſhould have the power of reflecting rays ſo copiouſly, is almoſt incredible. And

FIG. 39.

why ſhould that particular denſity reflect, when it is not the boundary of the atmoſphere, it having been ſhown that light is refracted at twice that altitude? It appears more probable that the reflection ariſes from the vapours and exhalations of various kinds with which the lower parts of the atmoſphere are charged; for the twilight laſts till the ſun is further below the horizon in the evening, than it is in the morning when it begins; and it is longer in ſummer than in winter. Now in the *former* caſe, the heat of the day has raiſed the vapours and exhalations; and in the *latter*, they will be more elevated from the heat of the ſeaſon; therefore, upon ſuppoſition that the reflection is made by them, the twilight ought to be longer in the evening than in the morning, and longer in ſummer than in winter.

207. Another effect of refraction is that of giving the ſun and moon an oval appearance, by the refraction of the lower limb being greater than that of the upper, whereby the vertical diameter is diminiſhed. For ſuppoſe the diameter of the ſun to be $32'$, and the lower limb to touch the horizon, then the mean refraction at that limb is $33'$, but the altitude of the upper limb being then $32'$, it's refraction is only $28'.\ 6''$, the difference of which is $4'.\ 54''$, the quantity by which the vertical diameter appears ſhorter than that parallel to the horizon. When the body is not very near the horizon, the refraction diminiſhing nearly uniformly, the figure of the body is very nearly that of an ellipſe. Now it is proved in that article where the diminution of weight of a body upon the ſurface of a ſpheroid is inveſtigated, that the diameter (D) of an ellipſe, which is nearly a circle, is diminiſhed, in going from the major to the minor axis, as the ſquare of the ſine (s) of the angle which it makes with the major axis; hence if $d =$ the diminution of the vertical diameter, $\text{rad.}^2 : s^2 :: d :$ the diminution of the diameter D. Thus we may find the diameter in any poſition; and in caſes where extreme accuracy is required, ſuch as meaſuring with a micrometer the diſtance of Venus or Mercury on the ſun's diſc from it's limb, this circumſtance may be conſidered.

CHAP.

CHAP. VIII.

ON THE SYSTEM OF THE WORLD.

Art. 208. WHEN any effect or phænomenon is diſcovered by experiment or obſervation, it is the buſineſs of Philoſophy to inveſtigate it's cauſe. But there are very few, if any, enquiries of this kind, where we can be led from the cauſe to the effect by a train of mathematical reaſoning, ſo as to pronounce with certainty upon the cauſe. Sir I. NEWTON therefore, in his PRINCIPIA, before he treats on the Syſtem of the World, has laid down the following Rules to direct us in our reſearches into the conſtitution of the univerſe.

RULE I. No more cauſes are to be admitted than what are ſufficient to explain the phænomenon.

RULE II. Of effects of the ſame kind, the ſame cauſes are to be aſſigned, as far as it can be done.

RULE III. Thoſe qualities which are found in all bodies upon which experiments can be made, and which can neither be increaſed nor diminiſhed, may be looked upon as belonging to all bodies.

RULE IV. In Experimental Philoſophy, propoſitions collected from phænomena by induction, are to be admitted as accurately or nearly true, until ſome reaſon appears to the contrary.

The principles, upon which the application of theſe Rules is admitted, are, the ſuppoſition that the operations of nature are performed in the moſt ſimple manner, and regulated by general laws. And although their application may, in many caſes, be very unſatisfactory, yet in the inſtances to which we ſhall here want to apply them, their force is little inferior to that of direct demonſtration, and the mind reſts equally ſatisfied as if the matter could be ſtrictly proved.

209. The diurnal motion of all the heavenly bodies may be accounted for, either by ſuppoſing the earth to be at reſt, and all the bodies daily to perform their revolutions in circles parallel to each other; or by ſuppoſing the earth to revolve about one of it's diameters as an axis, and the bodies themſelves to be fixed, in which caſe their apparent diurnal motions would be the ſame. If we

ſuppoſe

ſuppoſe the earth to be at reſt, all the fixed ſtars muſt make a complete revolution, in parallel circles, every day. But it will be ſhown in a future part of this Work, that the neareſt of the fixed ſtars cannot be leſs than 400000 times further from us than the ſun is, and that the ſun's diſtance from the earth is not leſs than 93 millions of miles. Alſo from the diſcoveries which are every day making by the improvement of teleſcopes, it appears that the heavens are filled with an almoſt infinite number of ſtars, to which the number viſible to the naked eye bears no proportion, and whoſe diſtances are, probably, incomparably greater than what we have ſtated above. But that an almoſt infinite number of bodies, moſt of them inviſible except by the beſt teleſcopes, at almoſt infinite diſtances from us and from each other, ſhould have their motions ſo exactly adjuſted, as to revolve in the ſame time, and in parallel circles, and all this without their having any central body, which is a phyſical impoſſibility, is an hypotheſis, which, by the Rules we have here laid down, is not to be admitted, when we conſider, that all the phænomena may be ſolved ſimply by the rotation of the earth about one of it's diameters. If therefore we had no other reaſon, we might reſt ſatisfied that the apparent diurnal motions of the heavenly bodies are produced by the earth's rotation. But we have other reaſons for this ſuppoſition. Experiments prove that all the parts of the earth have a gravitation towards each other. Such a body therefore, the greateſt part of whoſe ſurface is a fluid, muſt, from the equal gravitation of it's parts, form itſelf into a perfect ſphere. But it appears from menſuration, that the earth is not a perfect ſphere, but a ſpheroid, having the equatorial longer than it's polar diameter. Now if we ſuppoſe the earth to revolve, the parts moſt diſtant from the axis muſt, from their greater velocity, have a greater tendency to fly off, and therefore that diameter which is perpendicular to the axis muſt be increaſed. That this muſt be the conſequence appears from taking an iron hoop and making it revolve ſwiftly about one of it's diameters, and that diameter will be diminiſhed and the diameter perpendicular to it increaſed. The figure of the earth muſt therefore have ariſen from it's rotation, which is further confirmed from the following conſideration. There can be but one diameter about which the earth can revolve, which can ſolve all the phænomena of the apparent revolution of the heavenly bodies; for if the diameter about which the earth is ſuppoſed to revolve were changed, it would change the ſituation of all the bodies in reſpect to the horizon and zenith; now *that* diameter about which the earth muſt revolve, in order to ſatisfy all the phænomena, is the diameter which, from menſuration, is found to be the ſhorteſt. Another reaſon for the earth's rotation is from analogy. The planets are opaque and ſpherical bodies like to our earth; now all the planets, on which ſufficient obſervations have been made to determine the matter, are found to revolve about an axis, and the equatorial diameters of ſome of them are viſibly greater than their polar.

When

When theſe reaſons, all upon different principles, are conſidered, they amount to a proof of the earth's rotation about it's axis, which is as ſatisfactory to the mind as the moſt direct demonſtration could be. Theſe however are not all the proofs which might be offered; the ſituations and motions of the bodies in our ſyſtem neceſſarily require this motion of the earth.

210. Beſides this apparent diurnal motion, the ſun, moon, and planets have another motion; for they are obſerved to make a complete revolution amongſt the fixed ſtars, in different periods. But whilſt they are performing theſe motions in reſpect to the fixed ſtars, they do not always appear to move in the ſame direction, or in that direction in which their complete revolutions are made, but ſometimes appear ſtationary, and ſometimes to move in a contrary direction. We will here briefly deſcribe and conſider the different ſyſtems which have been invented, in order to ſolve theſe appearances. PTOLEMY ſuppoſed the earth to be perfectly at reſt, and all the other bodies, that is, the ſun, moon, planets, comets and fixed ſtars, to revolve about it every day; but that, beſides this diurnal motion, the ſun, moon, planets and comets had a motion in reſpect to the fixed ſtars, and were ſituated, in reſpect to the earth, in the following order; the Moon, Mercury, Venus, the Sun, Mars, Jupiter, Saturn. Theſe revolutions he firſt ſuppoſed to be made in circles about the earth placed a little out of the center, in order to account for ſome irregularities of their motions; but as their retrograde motions and ſtationary appearances could not thus be ſolved, he ſuppoſed them to revolve in epicycloids, in the following manner. Let *ABC* be a circle, *S* the center, *E* the earth, *abcd* another circle whoſe center *v* is in the circumference of the circle *ABC*. Conceive the circumference of the circle *ABC* to be carried round the earth every 24 hours according to the order of the letters, and at the ſame time let the center *v* of the circle *abcd* have a ſlow motion in the oppoſite direction, and let a body revolve in this circle in the direction *abcd*; then it is manifeſt, that by the motion of the body in this circle and the motion of the circle itſelf, the body may deſcribe ſuch a curve as is repreſented by *klmnop*; and if we draw the tangents *El*, *Em*, the body would appear ſtationary at the points *l* and *m*, and it's motion would be *retrograde* through *lm*, and then *direct* again. Now to make Venus and Mercury always accompany the Sun, the center *v* of the circle *abcd* was ſuppoſed to be always very nearly in a right line between the earth and ſun, but more nearly ſo for Venus than for Mercury, in order to give each it's proper elongation. This ſyſtem, although it will account for all the apparent motions of the bodies, yet it will not ſolve the phaſes of Venus and Mercury; for in this caſe, in both conjunctions with the ſun they ought to appear dark bodies, and to loſe their light both ways from their greateſt elongations; whereas it appears from obſervation, that in

FIG. 40.

one

one of their conjunctions they ſhine with a full face. This ſyſtem therefore cannot be true.

211. The ſyſtem received by the Egyptians was this: The Earth is immoveable in the center, about which revolve, in order, the Moon, Sun, Mars, Jupiter and Saturn; and about the Sun revolve Mercury and Venus. This diſpoſition will account for the phaſes of Mercury and Venus, but not for the apparent motions of Mars, Jupiter and Saturn.

212. The next ſyſtem which we ſhall mention, though poſterior in time to the true, or *Copernican Syſtem*, as it is uſually called, is that of TYCHO BRAHE, a Poliſh Nobleman. He was pleaſed with the Copernican ſyſtem, as ſolving all the appearances in the moſt ſimple manner; but conceiving, from taking the literal meaning of ſome paſſages in Scripture, that it was neceſſary to ſuppoſe the earth to be abſolutely at reſt, he altered the ſyſtem, but kept as near to it as poſſible. And he further objected to the earth's motion, becauſe it did not, as he conceived, affect the motion of comets obſerved in oppoſition, as it ought; whereas, if he had made obſervations on ſome of them, he would have found that their motions could not otherwiſe have been accounted for. In his ſyſtem, the earth is placed immoveable in the center of the orbits of the ſun and moon, without any rotation about an axis; but he made the ſun the center of the orbits of the other planets, which therefore revolved with the ſun about the earth. By this ſyſtem, the different motions and phaſes of the planets may be ſolved, the latter of which could not be, by the Ptolemaic ſyſtem; and he was not obliged to retain the epicycloids in order to account for their retrograde motions and ſtationary appearances. One obvious objection to this ſyſtem is, the want of that ſimplicity by which all the apparent motions may be ſolved, and the neceſſity that all the heavenly bodies ſhould revolve about the earth every day; alſo, it is phyſically impoſſible that a large body, as the ſun, ſhould revolve about a much ſmaller body, as the earth, at reſt; if one body be much larger than another, the center about which they revolve muſt be very near to the large body; this will be proved when we come to the principles of *phyſical* Aſtronomy. And this argument holds alſo againſt the Ptolemaic ſyſtem. It appears alſo from obſervation, that the plane in which the ſun muſt, upon this ſuppoſition, diurnally move, paſſes through the earth only twice in a year. It cannot therefore be any force in the earth which can retain the ſun in it's orbit, for it would move in a ſpiral continually changing it's plane. In ſhort, the complex manner in which all the motions are accounted for, and the phyſical impoſſibility of ſuch motions being performed, is a ſufficient reaſon for rejecting this ſyſtem; eſpecially when we conſider, in how ſimple a manner all theſe motions may be accounted for, and demonſtrated from the common principles of motion. Some of TYCHO's followers ſeeing the abſurdity of ſuppoſing,

poſing all the heavenly bodies daily to revolve about the earth, gave a rotatory motion to the earth, in order to account for their diurnal motion; and this was called the *Semi-Tychonic Syſtem*; but the objections to this ſyſtem are, otherwiſe, juſt the ſame.

213. The ſyſtem which is now univerſally received is called the *Copernican*. It was formerly taught by PYTHAGORAS, who lived about 500 years before J. C. and PLILOLAÜS, his diſciple, maintained the ſame; but it was afterwards rejected, till revived by COPERNICUS. Here the Sun is placed in the center of the Syſtem, about which the other bodies revolve in the following order; Mercury, Venus, the Earth, Mars, Jupiter, Saturn, and the Georgian Planet, which was lately diſcovered by Dr. HERSCHEL; beyond which, at immenſe diſtances, are placed the fixed ſtars; the moon revolves about the earth, and the earth revolves about an axis. This diſpoſition, and theſe motions of the bodies, ſolve, in the moſt ſimple manner, not only all the phaſes, and the direct and retrograde motions, but alſo every other irregularity belonging to them, and which motions may alſo be accounted for upon phyſical principles. We may alſo further obſerve, that the ſuppoſition of the earth's motion is neceſſary, in order to account for a ſmall apparent motion which every fixed ſtar is found to have, and which cannot otherwiſe be accounted for. The harmony of the whole will be as ſatisfactory a proof of the truth of this Syſtem, as the moſt direct demonſtration could be; this Syſtem therefore we ſhall aſſume.

CHAP. IX.

ON KEPLER'S DISCOVERIES.

Art. 214. KEPLER was the firſt who diſcovered the figures of the orbits of the planets to be ellipſes, having the ſun in one of the foci. PTOLEMY ſuppoſed that the orbits of the planets were circles, having the earth, not in the center C, but at ſome other point S; and taking $CB = CS$, he ſuppoſed that they revolved with an uniform angular velocity about B, called the *Punctum æquantis*. This was his ſuppoſition to account for the equation of the planet's orbit, or the *firſt* inequality of it's motion; but it was ſupported neither by obſervation or demonſtration. TYCHO altered this hypotheſis, by placing B at a different diſtance from C, by which he found his computations would agree with his obſervations within a few minutes. Notwithſtanding which, KEPLER ſuſpected the hypotheſis could not be true; for, from the goodneſs of TYCHO's obſervations, he believed that there could not have been ſo great a difference between the computations and obſervations, if it were true. But in reſpect to the orbit of the ſun, or rather of the earth, the ancients, and alſo TYCHO, believed the motion was equable about the center C. From the equation of the orbit, TYCHO computed the excentricity SB, which taken from AS gave a quantity BA different from the radius AC at firſt ſuppoſed; whence he concluded that the ſun was not always at the ſame diſtance from C. This induced KEPLER* to ſuſpect that the center was not the point about which the motion was equal, but that it biſected the excentricity. To determine this point he proceeded thus.

FIG. 41.

FIG. 42.

215. Let B be the point about which the motion is equable, S the ſun, take $BC = SC$, and let D and E the places of the earth when the planet *Mars* is at the ſame point M of it's orbit. On May 18, 1585, and January 22, 1591, he took the two places of Mars, found from obſervation, and by calculation reduced it's places to May 30, and January 20, in the ſame reſpective years, at which times the longitude of Mars ſeen from B, as calculated by TYCHO, was $6^s.\ 13^\circ.\ 28'$, and therefore he knew that Mars was in the ſame point of it's orbit; and the angles MBD, MBE were each $64^\circ.\ 23\frac{1}{2}'$. Now the longitudes of Mars on May 30, and January 20, were, by obſervation, $5^s.\ 6^\circ.\ 37'$

* See his Work, *De motibus Stellæ Martis.*

and

and 7ˢ. 21°. 34′, the differences between which and 6ˢ. 13°. 28′, the heliocentric longitude before calculated, are 36°. 51′ and 38°. 6′ for the angles *BMD*, *BME*; conſequently *BD* is leſs than *BE*, and therefore *B* is not the center of the circle. KEPLER next calculated the value of *BC* and found it to be 1837, *AC* being 100000. Now TYCHO had found from his obſervations, that the whole diſtance *BS* from the ſun to the center of equality was 3584, therefore it's half was 1792, which being ſo nearly equal to 1837, KEPLER immediately concluded that *C* biſected the excentricity.

216. Having found that the center of the earth's orbit biſected the excentricity, he proceeded to examine the ſame in the orbit of Mars, in the following manner. Let *S* be the ſun, *C* the center of the circle, *B* the point about which the motion is equable; and let *D*, *E*, *F*, *G* be 4 places of Mars obſerved in oppoſition; he then propoſed the following Problem. To find the angles *FBA*, *FSA* ſuch, that the four points *D*, *E*, *F*, *G* may be in the circumference of the circle, and *C* in the center between *B* and *S*. He reſolved this by aſſuming the diſtance *SB* and the angles *FBA*, *FSA*, and thence calculated all the other parts, to find whether all the angles formed about *S* were together equal to four right ones. He made 70 ſuppoſitions before he got one to agree with obſervation, the calculation of every one of which was extremely long and tedious: *Si te hujus laborioſæ methodi pertæſum fuerit, jure mei te miſereat, qui eam ad minimum ſeptuagies ivi cum plurima temporis jactura, et mirari deſines hunc quintum jam annum abire, ex quo Martem aggreſſus ſum, quamvis annus* 1603 *pene totus opticis inquiſitionibus fuit traductus*; pag. 95. Having thus determined the excentricity of the orbit of Mars, he calculated 12 oppoſitions obſerved by TYCHO, none of which differed more than 1′. 47″; but he found that the hypotheſis agreed neither with the latitude obſerved in oppoſition, nor with the longitude out of oppoſition, which differed ſometimes 8′ from obſervation. The circle which ſo well repreſented the 12 oppoſitions had it's excentricity *SB* = 18564, but he found *SC* = 11332 and *CB* = 7232, the mean diſtance of the earth from the ſun being 100000. From the want of agreement between the obſerved and computed latitudes in oppoſition, and the longitudes out of oppoſition, and from *SB* not being biſected in *C*, KEPLER was perſuaded that the orbit of Mars was not a circle. He therefore computed, in the following manner, three diſtances of Mars from the ſun, with the correſponding heliocentric longitudes, by which he could determine both the figure and magnitude of it's orbit. FIG. 43.

217. Let *S* be the ſun, *M* Mars, *D*, *E*, two places of the earth when Mars was in the ſame point *M* of it's orbit. When the earth was at *D*, he obſerved the difference between the longitudes of the ſun and Mars, or the angle *MDS*; in like manner he obſerved the angle *MES*. Now the places *D*, *E* of the earth FIG. 44.

earth in it's orbit being known, the diſtances DS, ES and the angle DSE will be known; hence in the triangle DSE, we know DS, SE, and the angle DSE, to find DE and the angles SDE, SED; hence we know the angles MDE, MED; therefore in the triangle MDE, we know DE, and the angles MDE, MED, to find MD; and laſtly, in the triangle MDS, we know MD, DS, and the angle MDS, to find MS, the diſtance of Mars from the ſun. He alſo found the angle MSD, the difference of the heliocentric longitudes of Mars and the earth. By this method, KEPLER, from obſervations made on Mars when in aphelion and perihelion (for he had determined the poſition of the line of the apſides, by a method which we ſhall afterwards explain, independent of the form of the orbit), determined the former diſtance from the ſun to be 166780, and the latter 138500, the mean diſtance of the earth from the ſun being 100000; hence the mean diſtance of Mars was 152640 and the excentricity of it's orbit 14140. He then determined, in like manner, three other diſtances, and found them to be 147750, 163100, 166255. He next calculated the ſame three diſtances, upon ſuppoſition that the orbit was a circle, and found them to be 148539, 163883, 166605; the errors therefore of the circular hypotheſis were 789, 783, 350. But he had too good an opinion of TYCHO's obſervations to ſuppoſe that theſe differences might ariſe from their inaccuracy; and as the diſtance between the aphelion and perihelion was too great, upon ſuppoſition that the orbit was a circle, he knew that the form of the orbit muſt be an oval; *Itaque planè hoc eſt: Orbita planetæ non eſt circulus, ſed ingrediens ad latera utraque paulatim, iterumque ad circuli amplitudinem in perigæo exiens, cujuſmodi figuram itineris ovalem appellitant*, pag. 213. And as of all ovals the ellipſe appeared to be the moſt ſimple, he firſt ſuppoſed the orbit to be an ellipſe, and placed the ſun in one of the foci; and upon calculating the above obſerved diſtances, he found they agreed together. He did the ſame for other points of the orbit, and found that they all agreed; and thus he pronounced the orbit of Mars to be an ellipſe, having the ſun in one of it's foci. Having determined this for the orbit of Mars, he conjectured the ſame to be true for all the other planets, and upon trial he found it to be ſo. Hence he concluded, *That the ſix primary Planets revolve about the Sun in ellipſes, having the Sun in one of the foci.*

A TABLE

A TABLE of the relative *mean* distances of the Planets from the Sun, according to different Authors.

Planets	KEPLER	STREET	HALLEY	M.de la LANDE	*Log. Dist.*
Mercury	38806	38710	38710	38710,	9,5878221
Venus	72413	72333	72333	72333,24	9,8593379
Earth	100000	100000	100000	100000	0,0000000
Mars	152349,5	152369	152369	152369,27	0,1828973
Jupiter	520000	520110	520098	520279,2	0,7162364
Saturn	951003,5	953800	954007,4	954072,4	0,9795813

The relative mean distance of the *Georgian* Planet from the sun is 1918352, according to M. de la PLACE.

The logarithms are here put down upon supposition that the mean distance of the earth from the sun is unity, this being the case in all Astronomical Tables. The mean distances are nearly as 4, 7, 10, 15, 52, 95, 192.

218. Having thus discovered the relative mean distances of the planets from the sun, and knowing their periodic times, he next endeavoured to find if there was any relation between them, having had a strong passion for finding analogies in nature. He saw that the more distant a planet was from the sun the slower it moved, so that on a double account the periodic times of the more distant planets would be increased. Saturn, for example, is $9\frac{1}{2}$ times further from the sun than the earth is, and the circle described by Saturn is so much greater in proportion; and as the earth revolves in 1 year, if their velocities were equal, the periodic time of Saturn would be $9\frac{1}{2}$ years; whereas it's periodic time is near 30 years. The periodic times therefore of the planets increase in a greater ratio than their distances, but in a less ratio than the squares of their distances; for upon that supposition the periodic time of Saturn would be about $90\frac{1}{4}$ years. On March 8, 1618, he began to compare the powers of these quantities, and at that time he took the squares of the periodic times and compared them with the cubes of the mean distances, but, from some error in the calculation, they did not agree. But on May 15, having made the last computations again, he discovered his error, and found an exact agreement between them.

them. Thus he diſcovered the famous Law, *That the ſquares of the periodic times of all the planets are as the cubes of their mean diſtances from the ſun.* Sir. I. NEWTON afterwards proved that this is a neceſſary conſequence of the motion of a body in an ellipſe about the focus. *Prin. Phil. Lib.* I. *Sec.* 2. *Pr.* 15.

219. KEPLER alſo diſcovered from obſervation, that the velocities of the planets, when in their apſides, are inverſely as their diſtances from the ſun; whence it followed, that they deſcribe, in theſe points, equal areas about the ſun in equal times. And although he could not prove, from obſervation, that the ſame was true in every point of the orbit, yet he had no doubt but that it was ſo. He therefore applied this principle to find the equation of the orbit (as will be explained in the next Chapter), and finding that his calculations agreed with obſervations, he concluded it was true in general, *That the planets deſcribe about the ſun equal areas in equal times.* This diſcovery was, perhaps, the foundation of the PRINCIPIA, as it probably might ſuggeſt to Sir I. NEWTON the idea, that the propoſition was true in general, which he afterwards proved it to be. Theſe important diſcoveries are the foundation of all Aſtronomy.

220. He alſo ſpeaks of *Gravity* as a power which is mutual between all bodies; and tells us, that the earth and moon would move towards each other, and meet at a point as much nearer to the earth than the moon, as the earth is greater than the moon, if their motions did not hinder it. He further adds, that the tides ariſe from the gravity of the waters towards the moon. That the reader may have a better conception of his ideas on this ſubject, I ſhall here give his own Words.

Vera doctrina de gravitate his innititur axiomatibus.

Omnis ſubſtantia corporea, quatenus corporea, apta nata eſt quieſcere omni loco, in quo ſolitaria ponitur, extra orbem virtutis cognati corporis.

Gravitas eſt affectio corporea, mutua inter cognata corpora ad unitionem ſeu conjunctionem (quo rerum ordine eſt et facultas magnetica) ut multo magis terra trahat lapidem, quàm lapis petit terram.

Gravia (ſi maximè terram in centro mundi collocemus) non feruntur ad centrum mundi, ut ad centrum mundi, ſed ut ad centrum rotundi cognati corporis, telluris ſcilicet. Itaque ubicunque collocetur ſeu quocunque tranſportetur tellus facultate ſuâ animali, ſemper ad illam feruntur gravia.

Si terra non eſſet rotunda, gravia non undiquaque ferrentur recta ad medium terræ punctum, ſed ferrentur ad puncta diverſa à lateribus diverſis.

Si duo lapides in aliquo loco mundi collocarentur propinqui invicem, extra orbem virtutis tertii cognati corporis; illi lapides ad ſimilitudinem duorum magneticorum corporum coirent loco intermedio, quilibet accedens ad alterum tanto intervallo, quanta eſt alterius moles in comparatione.

Si luna et terra non retinerentur vi animali, aut aliâ aliquâ æquipollenti, quælibet in ſuo circuitu; terra aſcenderet ad lunam quinquageſimâ quartâ parte intervalli, luna deſcenderet ad terram quinquaginta tribus circiter partibus intervalli: ibique jungerentur: poſito tamen, quòd ſubſtantia utriuſque ſit unius et ejuſdem denſitatis.

Si terra ceſſaret attrahere ad ſe aquas ſuas; aquæ marinæ omnes elevarentur, et in corpus lunæ influerent.

Orbis virtutis tractoriæ, quæ eſt in lunâ, porrigitur uſque ad terras, et prolectat aquas ſub zonam torridam, quippe in occurſum ſuum quacunque in verticem loci incidit, inſenſibiliter in maribus incluſis, ſenſibiliter ibi ubi ſunt latiſſimi alvei oceani, aquiſque ſpatioſa reciprocationis libertas, quo facto nudantur littora zonarum et climatum lateralium, et ſi qua etiam ſub torrida ſinus efficiunt reductiores oceani propinqui. Itaque aquis in latiori alveo oceani affurgentibus, fieri poteſt, ut in anguſtioribus ejus ſinubus, modo non nimis arctè concluſis, aquæ præſente lunâ etiam aufugere ab eâ videantur: quippe ſubſidunt, foris ſubtractâ copiâ aquarum. *See the Introduction to the abovementioned Work.*

CHAP.

CHAP. X.

ON THE MOTION OF A BODY IN AN ELLIPSE ABOUT THE FOCUS.

Art. 221. AS the orbits which are deſcribed by the primary planets revolving about the ſun are ellipſes having the ſun in one of the foci, and each deſcribes about the ſun equal areas in equal times, we next proceed to deduce, from theſe principles, ſuch conſequences as will be found neceſſary in our enquiries reſpecting their motions. From the equal deſcription of areas about the ſun in equal times, it appears * that the planets move with unequal angular velocities about the ſun. The propoſition therefore, which we here propoſe to ſolve, is, given the periodic time of a planet, the time of it's motion from it's aphelion, and the excentricity of it's orbit, to find it's angular diſtance from the aphelion, or it's *true* anomaly, and it's diſtance from the ſun. This was firſt propoſed by KEPLER, and therefore goes by the name of KEPLER'S PROBLEM. He knew no direct method of ſolving it, and therefore did it by very long and tedious tentative operations.

FIG. 46. 222. Let $AGQB$ be the ellipſe deſcribed by the body about the ſun at S in one of it's foci, AQ the major, GB the minor axis, A the aphelion, Q the perihelion, P the place of the body, $AVGE$ a circle, C it's center; draw NPI perpendicular to AQ, join PS, NS and NC, on which produced let fall the perpendicular ST. Let a body move uniformly in the circle from A to D with the *mean* angular velocity of the body in the ellipſe, whilſt the body moves in the ellipſe from A to P; then the angle ACD is the *mean*, and the angle ASP the *true* anomaly; and the difference of theſe two angles is called the *Equation of the planet's*

FIG. 45. * For if APQ be an ellipſe deſcribed by a planet about the ſun at S in the focus, the indefinitely ſmall area PSp deſcribed in a given time will be conſtant; draw Pr perpendicular to Sp; and, as the area SPp is conſtant for the ſame time, Pr varies as $\frac{1}{Sp}$; but the angle pSP varies as $\frac{Pr}{Sp}$, and therefore it varies as $\frac{1}{Sp^2}$; that is, in the *ſame* orbit, the angular velocity of a planet varies inverſely as the ſquare of it's diſtance from the ſun. For *different* planets, the areas deſcribed in the ſame time are not equal, and therefore Pr varies as $\frac{\text{area } SPp}{Sp}$, conſequently the angle pSP varies as $\frac{\text{area } SPp}{Sp^2}$; that is, the angular velocities of *different* planets are as the areas deſcribed in the ſame time directly and the ſquares of their diſtances from the ſun inverſely.

center,

center, or *Proſthapheresis*. Let $p =$ the periodic time in the ellipſe or circle (the periodic times being equal by ſuppoſition), and $t =$ the time of deſcribing AP or AD; then, as the bodies in the ellipſe and circle deſcribe equal areas in equal times about S and C reſpectively, we have

area ADC : area of the circle :: t : p,
area of the ellipſe : area ASP :: p : t,
alſo, area of the circle : area of the ellipſe :: area ASN : area ASP

$\therefore$ area ADC : area ASP :: area ASN : area ASP, hence $ADC = ASN$; take away the area ACN which is common to both, and the area $DCN = SNC$; but $DCN = \frac{1}{2} DN \times CN$, and $SNC = \frac{1}{2} ST \times CN$; therefore $ST = DN$. Now if t be given, the arc AD will be given; for as the body in the circle moves uniformly, we have $p : t :: 360° : AD$. Thus we always find the mean anomaly at any given time, knowing the time when the body was in the aphelion; hence if we can find ST, or ND, we ſhall know the angle NCA, called the *excentric* anomaly, from whence, by one proportion (223), we ſhall be able to find the angle ASP the *true* anomaly. The Problem is therefore reduced to this; to find a triangle CST, ſuch that the angle $C +$ the degrees of an arc equal to ST may be equal to the given angle ACD. This may be expeditiouſly done by trial in the following manner, given by M. de la CAILLE in his Aſtronomy. Find what arc of the circumference of the circle $ADQE$ is equal to CA, by ſaying, $355 : 113 :: 180°. : 57°. 17'. 44'',8$ the number of degrees of an arc equal in length to the radius CA; hence $CA : CS :: 57°. 17'. 44'',8$: the degrees of an arc equal to CS. Aſſume therefore the angle SCT, multiply it's ſine into the degrees in CS, and add it to the angle SCT, and if it equal the given angle ACD, the ſuppoſition was right; if not, add or ſubtract the difference to or from the firſt ſuppoſition, according as the reſult is leſs or greater than ACD, and repeat the operation, and in a very few trials you will get the accurate value of the angle SCT. The degrees in ST may be moſt readily obtained by adding the logarithm of CS to the logarithm of the ſine of the angle SCT and ſubtracting 10 from the index, and the remainder will be the logarithm of the degrees of ST. Having found the value of AN, or the angle ACN, we proceed next to find the angle ASP.

223. Let v be the other focus, and put $AC = 1$; then by Eucl. B. II. P. 12. $SP^2 - Pv^2 = vS^2 + 2vS \times vI = \overline{vS + 2vI} \times vS = \overline{2Cv + 2vI} \times 2SC = 2CI \times 2SC$; hence $SP + Pv : 2CI :: 2SC : SP - Pv$, or $2 : 2CI :: 2SC : \overline{2 - SP}$, or $1 : CI :: SC : SP - 1$, hence $SP = 1 + CS \times CI = 1 + CS \times \cos. \angle ACN$. * By plain trigon. $\frac{1 - \cos. ASP}{1 + \cos. ASP} = \overline{\tan. \frac{1}{2} ASP}^2$. But SP, or $1 + CS \times \cos. ACN$: rad. $= 1 :: SI$, or $CS + CI$, or $CS + \cos. ACN$:

* See MAUDUIT's Trig. or CRACKELT's Tranſlation, Ch. I. Th. 6.

cos. $ASP = \frac{CS + \text{cos. } ACN}{1 + CS \times \text{cos. } ACN}$. Hence $\overline{\text{tan. } \frac{1}{2} ASP}^2 \left(= \frac{1 - \text{cos. } ASP}{1 + \text{cos. } ASP}\right) =$ $\frac{1 + CS \times \text{cos. } ACN - CS - \text{cos. } ACN}{1 + CS \times \text{cos. } ACN + CS + \text{cos. } ACN} = \frac{1 - CS + \text{cos. } ACN \times \overline{CS - 1}}{1 + CS + \text{cos. } ACN \times \overline{CS + 1}} =$ $\frac{S\mathcal{Q} - \text{cos. } ACN \times S\mathcal{Q}}{SA + \text{cos. } ACN \times SA} = \frac{1 - \text{cos. } ACN}{1 + \text{cos. } ACN} \times \frac{S\mathcal{Q}}{SA} =$ (by the above theorem in trig.) $\overline{\text{tan. } \frac{1}{2} ACN}^2 \times \frac{S\mathcal{Q}}{SA}$; therefore $\sqrt{SA} : \sqrt{S\mathcal{Q}} :: \text{tan. } \frac{1}{2} ACN : \text{tan. } \frac{1}{2} ASP$, consequently we get ASP the *true* anomaly.

Ex. Required the true place of *Mercury* on Auguſt 26, 1740, at noon, the equation of the center, and it's diſtance from the ſun.

By M. de la Caille's Aſtronomy, Mercury was in it's aphelion on Auguſt 9, at $6h.\ 37'$. Hence on Auguſt 26, it had paſſed it's aphelion $16d.\ 17h.\ 23'$; therefore $87d.\ 23h.\ 15'.\ 32''$ (the time of one revolution) : $16d.\ 17h.\ 23'$ :: $360° : 68°.\ 26'.\ 28''$ the arc AD, or mean anomaly. Now (according to this Author) $CA : CS :: 1011276 : 211165$ (222) :: $57°.\ 17'\ 44'',8 : 11°.\ 57'\ 50'' = 43070''$, the value of CS reduced to the arc of a circle, the log. of which is 4,6341749. Alſo $68°.\ 26'.\ 28'' = 246388''$. Aſſume the angle SCT to be $60° = 216000''$, and the operation (222) to find the angle ACN will ſtand thus.

* By plain trigon. $\frac{1 - \text{cos. } ASP}{1 + \text{cos. } ASP} = \text{tan. } \frac{1}{2} ASP$. But SP, or

4,6341749

4,6341749		
9,9375306	log. of ..... 216000 = a	
4,5717055	.....	37300
		253300
		246388
		6912 = b
4,6341749		
9,9287987	..... 209088 = $a-b$ = 58°. 4′. 48″ = c	
4,5629736	.....	36557
		245645
		246388
		743 = d
4,6341749		
9,9297694	..... 209831 = $c+d$ = 58°. 17′. 11″ = e	
4,5639443	.....	36639
		246470
		246388
		82 = f
4,6341749		
9,9296626	..... 209749 = $e-f$ = 58°. 15′. 49 = g	
4,5638375	.....	36630
		246379
		246388
		9 = h;

hence as the difference between the value deduced from the assumption and the true value is now diminished about 9 times every operation, the next difference would be 1″; if therefore we add h to g, and then subtract 1″, we get 58°. 15′. 57″ for the true value of the angle ACN, the *excentric* anomaly. Hence (223) find the *true* anomaly ASP, from the proportion there given, by logarithms thus.

Log. tang. 29°. 7′. 58″½	9,7461246
½ Log. $S\mathfrak{Q}$ = 800111	2,9515751
.....	12,6976997
½ Log. SA = 1222441	3,0436141
Log. tang. 24°. 16′. 15″	9,6540856

Hence the *true* anomaly is 48°. 32′. 30″. Now the aphelion A was in 8ˢ. 13°. 54′. 30″; therefore the true place of Mercury was 10ˢ. 2°. 27′. Hence 68°. 26′. 28″ − 48°. 32′. 30″ = 19°. 53′. 58″ the *equation of the center*. Alſo $SP = 1 + CS \times \cos. \angle ACN = 1{,}10983$ the diſtance of Mercury from the ſun, the radius of the circle, or the mean diſtance of the planet, being unity. Thus we are able to compute, at any time, the place of a planet in it's orbit, and it's diſtance from the ſun; and this method of computing the *excentric* anomaly appears to be the moſt ſimple and eaſy of application of all others, and capable of any degree of accuracy.

224. As the bodies at D and P departed from A at the ſame time, and will coincide again at $\mathfrak{Q}$, $AD\mathfrak{Q}$, $AP\mathfrak{Q}$ being performed in half the time of a revolution; and as at A the planet moves with it's leaſt angular velocity (by the Note to Art. 221.), therefore from A to $\mathfrak{Q}$, or in the *firſt* 6 ſigns of anomaly, the angle ACD will be greater than ASP, or the *mean* will be greater than the *true* anomaly; but from $\mathfrak{Q}$ to A, or in the *laſt* 6 ſigns, as the planet at $\mathfrak{Q}$ moves with it's greateſt angular velocity, the *true* will be greater than the *mean* anomaly.

225. When the excentricity, and conſequently the angle NCD, is very ſmall, as in the orbits of Venus and the Earth, ND, conſidered as very nearly a ſtraight line, will be equal and parallel to $S\mathfrak{T}$, therefore SD is parallel to CN and conſequently the angle $NCD = CDS$. Now in the triangle DCS, we know the two ſides DC, CS, and the included angle DCS, the ſupplement of DCA; hence we can find the angle CDS or DCN. If the angle DCN do not exceed 1½° the concluſion will be accurate to a ſecond; and if it be greater, this method will give a near value of it, and conſequently we ſhall get a near value of the angle ACN to begin the operation with in the method already explained, which will be better, perhaps, than gueſſing at firſt. In our example, the angle DSC, or $S\mathfrak{T}C$ nearly, would, by this calculation, have been found 58°. 13′. 1″, whence $S\mathfrak{T}$ would firſt have been found 10°. 10′. 12″, and after two more operations the accurate value would have been obtained. When the angle DCN is not very ſmall, M. Cassini, in his Elements of Aſtronomy, page 144, has given the following method of finding it.

226. Draw

226. Draw Dz perpendicular to ST, and Tz is the ſine of the arc DN, conſequently Sz is the difference between the arc DN and it's ſine, or it may be conſidered as the difference between the arc of the angle CDS and it's ſine; compute therefore the angle CDS. (225), and by the following Table take out the difference between the arc and it's ſine, and ſay $SD : Sz ::$ rad. : ſin. SDz, which ſubtract from the angle SDC and you have the angle zDC, or the alternate angle DCN. The reſt of the operation is the ſame as before.

A TABLE ſhowing the Difference between the Arcs of a Circle and their Sines, Radius being 10000000.

Arc	Dif.	Arc	Dif.	Arc	Dif.	Arc	Dif.
1°. 00	9	4°. 00	567	7°. 00	3037	10°. 00	8848
10	15	10	641	10	3259	10	9299
20	23	20	720	20	3492	20	9755
30	31	30	807	30	3734	30	10235
40	42	40	900	40	3989	40	10730
50	56	50	1000	50	4255	50	11241
2. 00	71	5. 00	1108	8. 00	4532	11. 00	11767
10	90	10	1222	10	4822	10	12312
20	113	20	1344	20	5122	20	12873
30	139	30	1474	30	5435	30	13450
40	169	40	1613	40	5761	40	14042
50	203	50	1759	50	6100	50	14654
3. 00	239	6. 00	1913	9. 00	6450	12. 00	15278
10	281	10	2077	10	6815	10	15921
20	328	20	2255	20	7194	20	16585
30	380	30	2432	30	7585	30	17266
40	437	40	2625	40	7985	40	17964
50	499	50	2827	50	8404	50	18680

Ex. To find the *true* anomaly of Mercury, the *mean* being 60°.

Let the mean diſtance of Mercury be 100000, and the excentricity CS will be 20878, according to CASSINI; hence in the triangle DCS, $DC = 100000$, $CS = 20878$, and the angle $DCS = 120°$, therefore $DC = 111905$, and the angle $SDC = 9°. 17'. 52''$, correſponding to which we find, in the Table, the value of $Sz = 7120$;

= 7120; hence 1 : 1905 : : 71* : : rad. : sin. 1′. 11″, which ſubtracted from 9°. 17′. 52″ leaves 9°. 15′. 41″ for the angle DCN, which ſubtracted from 60° gives 50°. 44′. 19″ for the angle NCA. Hence

Log. tan. 25°. 44′. 19″	9,6759392
½ Log. SQ = 79122	2,4491486
	12,1250878
½ Log. SA = 120878	2,5411738
Log. tan. 20°. 59′. 18″	9,5839140

Therefore the true anomaly is 41°. 58′. 36″. Hence alſo the equation of the center is 18°. 1′. 24″.

Theſe indirect methods of finding the equation of the planet's center are, in general, more ready for practice than any of the direct methods.

227. The method aſcribed by ſome Writers to SETH WARD, Profeſſor of Aſtronomy at Oxford, and publiſhed in 1654, although, as M. de la LANDE obſerves, it is given both by WARD and MERCATOR to BULLIALDUS, is leſs accurate than theſe we have already given; yet as it may, in many caſes, ſerve as a uſeful approximation, it deſerves to be mentioned. He aſſumed the angular velocity about the other focus v to be uniform, ‖ and therefore made it

FIG. 47.

* By the Table 7120 is the difference, if the radius be 10000000, but as the radius here is 100000 the difference will be only 71.

† For the utmoſt exactneſs we ſhould take δz correſponding to 9°. 15′. 41″ inſtead of 9°. 17′. 52″, but the difference is too ſmall to be worth notice.

FIG. 48.

‖ That this is not true may be thus ſhown. With the center S and radius $SW = \sqrt{AC \times CE}$ deſcribe the circle zW, then the area of this circle = area of the ellipſe; let a body, moving uniformly in it, make one revolution in the ſame time the body does in the ellipſe; and let the bodies ſet off at the ſame time from A and z, and deſcribe AP, zv, in the ſame time; then the $\angle zSv$ is the *mean*, and ASP the *true* anomaly. Draw pS indefinitely near to PS, and Pr, po perpendicular to Sp, FP; then $Pr = po$. Now the $\angle PFp$ varies as $\frac{po}{PF} = \frac{Pr}{PF}$; but in a given time the area PSp is given, $\therefore Pr$ varies as $\frac{1}{PS}$; hence the $\angle PFp$, deſcribed in a given time, varies as $\frac{1}{PF \times PS}$, which is not a conſtant quantity. Alſo $PS : PF :: \angle PFp \left(\frac{1}{PF \times PS}\right) : \angle PSp \left(\frac{1}{PS^2}\right)$. And by the Note to Art. (221) as equal areas are deſcribed in equal times in the circle and ellipſe about S, the angular velocity about S in the circle becomes $\frac{1}{SW^2}$. Hence the angular velocity about F is greater or leſs than the mean angular velocity, according as $PF \times PS$ is leſs or greater than SW^2, or than $AC \times CE$. Alſo the angular velocity about F is the ſame in ſimilar points of the ellipſe in reſpect to the center, or at equal diſtances from the center.

repreſent

represent the *mean* anomaly. Produce vP to r, and take $Pr = PS$; then in the triangle Svr, $rv + vS : rv - vS :: \tan. \frac{1}{2}\angle vSr + vrS : \tan. \frac{1}{2}\angle vSr - vrS$; now $\frac{1}{2}.rv + vS = \frac{1}{2}A\mathcal{Q} + \frac{1}{2}vS = AS$, and $\frac{1}{2}.rv - vS = \frac{1}{2}A\mathcal{Q} - \frac{1}{2}vS = S\mathcal{Q}$, also tan. $\frac{1}{2}.\angle vSr + vrS = \tan. \frac{1}{2}\angle AvP$, and $\frac{1}{2}.\angle vSr - vrS =$ (as $Pr = PS$) $\frac{1}{2}.\angle vSr - PSr = \frac{1}{2}\angle ASP$; hence *the aphelion distance : perihelion distance :: tan. of* $\frac{1}{2}$ *the mean anomaly : tan.* $\frac{1}{2}$ *true anomaly.* This is called the *simple elliptic hypothesis*, and was used by Dr. HALLEY in constructing his *Tabula pro expediendo calculo æquationis centri Lunæ.* In the orbit of the earth, the error is never greater than 17″; in the orbit of the moon, it may be 1′. 35″. By this hypothesis, for 90° from aphelion and perihelion, the computed place is *backwarder* than the true; and for the other part it is *forwarder*.

228. Although the indirect methods above explained are, in general, the best for practice, yet as the Reader may wish to see the direct method of solving the Problem, we shall give that of Dr. KEILL, as being the most simple, and which may frequently be applied with advantage. Let the arc $ND = y$, $e =$ the sine of AD, $f =$ the cosine, $SC = g$. Then by trigonometry, the sine of $NA = y - \frac{y^3}{2.3} +$ &c. and cosine $= 1 - \frac{y^2}{2} + \frac{y^4}{2.3.4} -$ &c. hence the sine of $AN = e - fy - \frac{ey^2}{2} + \frac{fy^3}{2.3} + \frac{ey^4}{2.3.4} -$ &c. Also rad. $= 1$: sin. AN or $\angle SCT :: SC = g : ST$ or ND or $y = ge - gfy - \frac{gey^2}{2} + \frac{gfy^3}{2.3} + \frac{gey^4}{2.3.4} -$ &c. hence $ge = y + gfy + \frac{gey^2}{2} - \frac{gfy^3}{2.3} - \frac{gey^4}{2.3.4} +$ &c. Put $ge = z$, $1 + gf = a$, $\frac{ge}{2} = b$, $\frac{gf}{2.3} = c$, $\frac{ge}{2.3.4} = d$, &c. hence $z = ay + by^2 - cy^3 - dy^4 +$ &c. and by the reversion of series, $y = \frac{z}{a} - \frac{bz^2}{a^3} + \frac{2b^2 + ac}{a^5} \times z^3 - \frac{5abc - 5b^3 + a^2d}{a^7} \times z^4$ &c. but $b = \frac{ge}{2} = \frac{z}{2}$, $d = \frac{z}{2.3.4}$, &c. therefore $y = \frac{z}{a} - \frac{z^2}{2a^3} + \frac{cz^3}{a^4} - \frac{5cz^5}{2a^6}$ &c. If the arc AN be greater than 90° and less than 270°, f becomes negative, and therefore gf or e will be negative; hence $y = \frac{z}{a} - \frac{z^2}{2a^3} - \frac{cz^3}{a^4} + \frac{5cz^5}{2a^6}$ &c. Now to reduce the value of y into degrees, we know that an arc equal to radius, or unity, is equal to 57,29578 degrees $= r$; hence $1 : r :: \frac{z}{a} - \frac{z^2}{2a^3} + \frac{cz^3}{a^4} -$ &c. : the degrees of the arc

FIG. 46.

$y =$

$y = \frac{rz}{a} - \frac{rz^2}{2a^2} + \frac{rcz^2}{a^4}$ - &c. For the orbit of the earth, the firſt term will be ſufficient, not differing from the truth the ten thouſandth part of a degree. In other caſes it may be neceſſary to take more terms.

Ex. Let the excentricity of the earth's orbit be 0,01691, the mean diſtance being $= 1$, and the mean anomaly 30°; to find the true anomaly.

Log. of g	8,2281436
Log. ſin. of $e = 30°$	9,6989700
Log. of r	1,7581226
Log. of rge, or rz	9,6852362
Log. of a	0,0063137
Log. of $\frac{rz}{a}$	9,6789225

..., the natural number correſponding to which, being a decimal, is 0°, 47744 = 28′. 38″ = y, which is true to a ſecond; therefore $AN = 29°. 31'. 22''$; hence

Log. tan. 14°. 45′. 41″	9,4207651
½ Log. $S\mathfrak{Q} = 98309$	2,4962966
	11,9170617
½ Log. $SA = 101691$	2,5031412
Log. tan. 14°. 32′. 25″	9,4139205

Hence the *true* anomaly is 29°. 4′. 50″; conſequently the equation of the center is 55′. 10″.

229. When P and D are very near A, the variation of PS will be very ſmall; now by the Note to Art. 221. the angular velocity of P at A about S : angular velocity of D about C :: $\frac{\text{area deſ. by } P}{SP^2}$: $\frac{\text{area deſ. by } D}{DC^2}$, and therefore the angular velocities will be nearly in a given ratio ſo long as P is near to A; hence the difference of the angular velocities muſt vary nearly as the angular velocities themſelves; that is, the equation of the center varies nearly as the angular velocity of P about S, or as the true anomaly.

230. The *greateſt* equation of the center may be eaſily found from the Note Art. 227, giving the dimenſions of the orbit. For as long as the angular velocity of the body in the circle is greater than that in the ellipſe about S, the equation will keep increaſing, the bodies ſetting out from A and z; and when they become equal, the equation muſt be the greateſt; this therefore happens when

FIG. 48.

when $\frac{1}{SP^2}=\frac{1}{SW^2}=\frac{1}{AC\times CE}$, or when $AC\times CE=SP^2$, hence SP is known. Let SW repreſent this value of SP; then as we know SW, $FW(=2AC-SW)$ will be known, and as SF is known, we can find the angle FSW the *true* anomaly.* Hence (223) $\sqrt{SQ}:\sqrt{SA}$ :: tan. $\frac{1}{2}$ *true* anom. : tan. $\frac{1}{2}$ *excen.* anom. ACN or tan. $\frac{1}{2}$ SCT; hence as we know SC, we can find ST or ND; and to convert that into degrees, ſay, rad. $=1$: ND :: $57°.\ 17'.\ 44'',8$: the degrees in ND, which added to the angle ACN gives ACD the *mean* anomaly, the difference between which and the *true* anomaly is the *greateſt* equation. Thus we may find the equation at any other time, given SP. Or the cos. ACN may in general be found thus. By Art. 223. $SP=1+CS\times$ cos. ACN; hence cos. $ACN=\frac{SP-1}{CS}$; conſequently *log. cos.* $ACN=$ *log.* $\overline{SP-1}$ – *log.* CS. FIG. 46.

231. The excentricity, and conſequently the dimenſions of the orbit, may be found from knowing the greateſt equation. For (230) the greateſt equation is when the diſtance is a mean between the ſemi-axis major and minor, and therefore in orbits nearly circular, the body muſt be nearly at the extremity of the minor axis, and conſequently the angle NCA or SCT will be nearly a right angle, therefore ST is nearly equal to SC; alſo NSA will be very nearly equal to PSA. Now the angle $NCA-NSA$ or $PSA=SNC$, and $DCA-NCA=DCN$; add theſe together and $DCA-PSA=DCN+SNC$, which (as NC is nearly parallel to DS) is nearly equal to $2DCN$; that is, the difference between the *true* and *mean* anomaly, or the equation, is nearly equal to twice the arc DN, or twice ST, or very nearly twice SC. Hence, $57°.\ 17'.\ 44'',8$: half the greateſt equation :: rad. $=1$: SC the excentricity. But if the orbit be conſiderably excentric, to this excentricity compute the greateſt equation; and then, as the equation varies very nearly as SC, ſay, as the computed equation : excentricity found :: given greateſt equation : true excentricity.

Ex. If we ſuppoſe, with M. de la CAILLE, that Mercury's greateſt equation is $24°.\ 3'.\ 5''$; then $57°.\ 17'.\ 44'',8$: $12°.\ 1'.\ 32'',5$ :: 1 : ,209888 the excentricity very nearly. Now the greateſt equation computed from this excentricity

* Let $FW+WS+FS=2AC+FS=2M$; then by M. CAGNOLI's excellent Treatiſe on Trigonometry, Art. 234. the ſine of $\frac{1}{2}$ $FSW=\sqrt{\frac{\overline{M-SF}\times\overline{M-SW}}{SF\times SW}}$; hence log. ſine of $\frac{1}{2}$ FSW $=\frac{1}{2}\times\overline{\text{log. }\overline{M-SF}+\text{log. }\overline{M-SW}-\text{log. }SF-\text{log. }SW}$.

is 23°. 54′. 28″,5; hence 23°. 54′. 28″,5 : 24°. 3′. 5″ :: ,209888 : ,211165 the true excentricity. M. de la LANDE makes the greateſt equation 23°. 40′, and the excentricity ,207745.

232. The converſe of this Problem, that is, given the excentricity and true anomaly to find the mean, may be very readily and directly ſolved. The excentricity being given, the ratio of the major to the minor axis is known,* which is the ratio of NI to PI; hence the angle ASP being given, we have $PI : NI$ :: tan. ASP : tan. ASN; therefore in the triangle NCS, we know NC, CS and the angle CSN, to find the angle SCN, the ſupplement of which is the angle ACN or SCT; hence in the right angled triangle STC, we know SC and the angle SCT, to find ST, which is equal to ND the arc meaſuring the equation, which may be found by ſaying, Radius : ST :: 57°. 17′. 44″,8 : the degrees in ND, which added to ACN gives ACD the mean anomaly.

To find the hourly Motion of a Planet in it's Orbit, having given the mean *hourly Motion.*

233. The hourly motion of a planet in it's orbit is found immediately from the Note to Art. 227; for it appears from thence, that the angles PSp, WSw, deſcribed by the body at P in the ellipſe and the body W in the circle in the ſame time are as $SW^2 : SP^2$, or as $AC \times CE : SP^2$, hence $PSp = WSw \times \frac{AC \times CE}{SP^2}$ the hourly motion of a planet in it's *orbit*, the angle WSw being the *mean* motion of the planet in an hour. For extreme accuracy, SP muſt be taken at the middle of the hour. Thus we may eaſily compute a Table of the hourly motions of the planets in their orbits.

FIG. 48.

To find the hourly Motion of a Planet in Latitude and Longitude.

FIG. 49.

234. Let AD be the ecliptic, AE the orbit of the planet; and let Bm repreſent the hourly motion in the orbit; draw the great circles BC, mo perpendicular to AD, and the ſmall circle Bn parallel to AD. Now by plane trigonom.

$Bm : Bn$ :: rad. : ſin. Bmn or ABC, and

$Bn : Co$ :: cos. BC : rad. (13)

$\therefore Bm : Co$ :: cos. BC : ſin. ABC; but ſin. $ABC = \frac{\text{cos. } A}{\text{cos. } BC}$; and (233) if $a =$

* For as AC, CS are known, we have $GC = \sqrt{SG^2 - SC^2} = \sqrt{AC^2 - SC^2}$.

the

the ſemi-axis major, b the ſemi-axis minor of the orbit, $x =$ the diſtance of the planet from the ſun, $v =$ the *mean* hourly motion, then $Bm = v \times \frac{ab}{x^2}$; hence $v \times \frac{ab}{x^2} : Co :: \text{cos. } BC : \frac{\text{cos. } A}{\text{cos. } BC}$, therefore $Co = v \times \frac{ab}{x^2} \times \frac{\text{cos. } A}{\text{cos. } BC^2} = v \times \frac{ab}{x^2} \times \frac{\textit{cos. incl. orb. to ecl.}}{\textit{cos. lat.}^2}$ the hourly motion in *Longitude*.

Alſo $Bm : mn ::$ rad. : cos. Bmn or $ABC ::$ (becauſe cos. $B =$ cotan. $AB \times$ tan. BC divided by rad.) rad^2. : cotan. $AB \times$ tan. BC; hence (radius being unity) $mn = v \times \frac{ab}{x^2} \times$ cotan. $AB \times$ tan. $BC = v \times \frac{ab}{x^2} \times$ *cot. plan. diſt. node* $\times$ *tan. lat.* the hourly motion in *Latitude*. Hence we may conſtruct Tables of the hourly motions of the planets both in longitude and latitude.

CHAP. XI.

ON THE OPPOSITIONS AND CONJUNCTIONS OF THE PLANETS.

Art. 235. THE place and time of the oppoſition of a ſuperior planet, or conjunction of an inferior, are the moſt important obſervations for determining the elements of the orbit, becauſe at that time the obſerved is the ſame as the true longitude, or that ſeen from the ſun; whereas if obſervations be made at any other time, we muſt reduce the obſerved to the true longitude, which requires the knowledge of their relative diſtances, and which, at that time, are ſuppoſed not to be known. They alſo furniſh the beſt means of examining and correcting the Tables of the planets motions, by comparing the computed with the obſerved places.

236. To determine the time of oppoſition, obſerve, when the planet comes very near to that ſituation, the time at which it paſſes the meridian, and alſo it's right aſcenſion (118 or 122); take alſo it's meridian altitude; do the ſame for the ſun, and repeat the obſervations for ſeveral days. From the obſerved meridian altitudes find the declinations, and from the right aſcenſions and declinations compute (124) the latitudes and longitudes of the planet, and the longitudes of the ſun. Then take a day when the difference of their longitudes is nearly 180°, and on that day reduce the ſun's longitude, found from obſervation when it paſſed the meridian, to the longitude found at the time (t) the planet paſſed, by finding from obſervation, or computation, at what rate the longitude then increaſes. Now in oppoſition the planet is retrograde, and therefore the difference between the longitudes of the planet and ſun increaſe by the ſum of their motions. Hence the following Rule; As the ſum of their daily motions in longitude : the difference between 180° and the difference of their longitudes reduced to the ſame time (t), (ſubtracting the ſun's longitude from that of the planet to get the difference reckoned from the ſun according to the order of the ſigns) :: 24h. : interval between that time (t) and the time of oppoſition. This interval added to or ſubtracted from the time (t), according as the difference of their longitudes at that time was greater or leſs than 180°, gives the time of oppoſition. If this be repeated for ſeveral days and the mean of the whole taken, the time will be had more accurately. And if the time of oppoſition found from obſervation be compared with the time by computation from the Tables, the difference will be the error of the Tables, which may ſerve as a means of correcting them.

Ex.

Ex. On October 24, 1763, M. de la LANDE obſerved the difference between the right aſcenſion of β *Aries* and *Saturn*, which paſſed the meridian at 12*h*. 17′. 17″ apparent time, to be 8°. 5′. 7″, the ſtar paſſing firſt. Now the apparent right aſcenſion of the ſtar at that time was 25°. 24′. 33″,6, hence the apparent right aſcenſion of Saturn was 1ˢ. 3°. 29′. 40″,6 at 12*h*. 17′. 17″ apparent time, or 12*h*. 1′. 37″ mean time. On the ſame day he found from obſervation of the meridian altitude of Saturn, that it's declination was 10°. 35′. 20″ N. Hence from the right aſcenſion and declination of Saturn it's longitude is found to be 1ˢ. 4°. 50′. 56″, and latitude 2°. 43′. 25″ ſouth. At the ſame time the ſun's longitude was found by calculation to be 7ˢ. 1°. 19′. 22″, which ſubtracted from 1ˢ. 4°. 50′. 56″ gives 6ˢ. 3°. 31′. 34″; hence Saturn was 3°. 31′. 34″ beyond oppoſition, but being retrograde muſt afterwards come into oppoſition. Now, from the obſervations made on ſeveral days at that time, Saturn's longitude was found to decreaſe 4′. 50″ in 24 hours, and by computation the ſun's longitude increaſed 59′. 59″ in the ſame time, the ſum of which is 64′. 49″; hence 64′. 49″ : 3°. 31′. 34″ :: 24*h*. : 78*h*. 20′. 20″, which added to October 24, 12*h*. 1′. 37′ gives 27*d*. 18*h*. 21′. 57″ for the time of oppoſition. Hence we may find the longitude of Saturn at the time of oppoſition, by ſaying, 24*h*. : 78*h*. 20′. 20″ :: 4′. 50″ : 15′. 47″ the retrograde motion of Saturn in 78*h*. 20′. 20″, which ſubtracted from 1ˢ. 4°. 50′. 56″ leaves 1ˢ. 4°. 35′. 9″ the longitude of Saturn at the time of oppoſition. In like manner we may find the ſun's longitude at the ſame time, in order to prove the oppoſition; hence 24*h*. : 78*h*. 20′. 20″ :: 59′. 59″ : 3°. 15′. 47″, which added to 7ˢ. 1°. 19′. 22″, the ſun's longitude at the time of obſervation, gives 7ˢ. 4°. 35′. 9″ for the ſun's longitude at the time of oppoſition, which is exactly oppoſite to that of Saturn. Hence alſo we may find the latitude of Saturn at the ſame time, by obſerving in like manner the daily variation, or by computation from the Tables after the elements of it's motions are known and the Tables conſtructed; by which it appears, that in the interval between the time of obſervation and oppoſition the latitude had increaſed 6″, and conſequently the latitude was 2°. 43′. 31″.

237. This is the method which is now made uſe of to determine the time of oppoſition of the planets. The method uſed by TYCHO, HEVELIUS and FLAMSTEAD was the ſame, except that they determined the latitude and longitude of the planet from obſerving it's diſtance from two known fixed ſtars, in the following manner. Let *P* be the pole of the ecliptic, *a* and *b* the two ſtars, *m* the planet; then obſerve *ma*, *mb*. Alſo *Pa*, *Pb* the complements of the latitudes of *a* and *b*, and the angle *aPb* the difference of their longitudes, are known, from which find *ab* and the angle *Pab*; then in the triangle *amb* we know all the ſides to find the angle *mab*, which added to or ſubtracted from the angle *Pab*, according to the poſition of *m*, gives the angle *Pam*; hence in the FIG. 50.

the triangle *Pam* we know *Pa*, *am* and the angle *Pam*, to find *Pm* the complement of the planet's latitude, and the angle *aPm* the difference between the longitudes of the planet and the ſtar *a*. Thus alſo may the place of any new phænomenon, as a comet, be determined, if you have not an opportunity of obſerving it's right aſcenſion and declination, which however is the moſt accurate method.

238. The place and time of conjunction of an inferior planet may be found in like manner, when the elongation of the planet from the ſun, near the time of conjunction, is ſufficient to render it viſible; the moſt favourable time therefore muſt neceſſarily be when the geocentric latitude of the planet at the time of conjunction is the greateſt. In the year 1689, Venus was in it's inferior conjunction on June 25, and it was obſerved on 21, 22, and 28; from which obſervations it's conjunction was found to be at 13*h*. 46′ apparent time at Paris, in longitude ♋ 4°. 53′. 40″, and latitude 3°. 1′. 40″ north. The time and place of the ſuperior conjunction may be alſo thus obſerved, when the ſtate of the air is very favourable; for as Venus is then about ſix times as far from the earth as at it's inferior conjunction, it's apparent diameter and the quantity of light which we receive from it are ſo ſmall, as to render it difficult to be perceived. But the moſt accurate method of obſerving the time of an inferior conjunction both of Venus and Mercury is from obſervations made upon them in their tranſits over the ſun's diſc. This we ſhall explain, when we come to treat on that ſubject.

CHAP.

CHAP. XII.

ON THE MEAN MOTIONS OF THE PLANETS.

Art. 239. THE determination of the *mean* motions of the planets, from their conjunctions and oppositions, would very readily follow, if we knew the place of the aphelia and excentricity of their orbits; for then we could (223) find the equation of the orbit, and reduce the *true* to the *mean* place. The mean places being determined at two points of time give the mean motion corresponding to the interval between the times. But the place of the aphelion is best fixed from the mean motion. To determine therefore the mean motion, independent of the place of the aphelion, we must seek for such oppositions or conjunctions as fall very nearly in the same point of the Heavens; for then the planet being nearly in the same point of it's orbit, the equation will be very nearly the same at each observation, and therefore the comparison between the true places will be nearly a comparison of their mean places. If the equation should differ much in the two observations, it must be considered. Now by comparing the modern observations, we shall be able to get nearly the time of a revolution; and then by comparing the modern with the ancient observations, the mean motion may be very accurately determined; for any error, by dividing it amongst a great number of revolutions, will become very small in respect to one revolution. As this will be best explained by an example, we shall give one from M. CASSINI (*Elem. d'Astron.* pag. 362), with the proper explanations as we proceed.

Ex. On September 16, 1701, *Saturn* was in opposition at 2*h*. when the place of the sun was ♍ 23°. 21′. 16″, and consequently Saturn in ♓ 23°. 21′. 16″, with 2°. 27′. 45″ south latitude. On September 10, 1730, the opposition was at 12*h*. 27′ and Saturn in ♓ 17°. 53′. 57″, with 2°. 19′. 6″ south latitude. On September 23, 1731, the opposition was at 15*h*. 51′ in ♈ 0°. 30′. 50″, with 2°. 36′. 55″ south latitude. Now the interval of the two first observations was 29 years (of which 7 were bissextiles) wanting 5*d*. 13*h*. 33′; and the interval of the two last was 1*y*. 13*d*. 3*h*. 24′. Also the difference of the places of Saturn in the two first observations was 5°. 27′. 19″, and in the two last it was 12°. 36′. 53″. Hence in 1*y*. 13*d*. 3*h*. 24′ Saturn had moved over 12°. 36′. 53″; therefore 12°. 36′. 53″ : 5°. 27′. 19″ :: 1*y*. 13*d*. 3*h*. 24′ : 163*d*. 12*h*. 41′ the time of moving

moving over 5°. 27′. 19″ very nearly, becauſe Saturn, being nearly in the ſame part of it's orbit, will move nearly with the ſame velocity; this therefore *added* to the interval of time between the two firſt obſervations (becauſe at the ſecond obſervation Saturn wanted 5°. 27′. 19″ of being up to the place at the firſt obſervation) gives 29 common years 164*d*. 23*h*. 8′ for the time of one revolution. Hence ſay, 29*y*. 164*d*. 23*h*. 8′ : 365*d*. :: 360° : 12°. 13′. 23″. 50‴ the *mean* annual * motion of Saturn in a common year of 365 days, that is, the motion in a year if it had moved uniformly. If we divide this by 365 we ſhall get 2′. 0″. 28‴ for the mean daily motion of Saturn. If we had taken the mean annual motion of Saturn anſwering to 12°. 36′. 53″ in 1*y*. 13*d*. 3*h*. 24′, it would have been found 12°. 10′. 35″, which differing only about 3′ from the true motion, it follows that Saturn was then moving with it's mean velocity, very nearly, and conſequently was very near it's mean diſtance. The mean motion thus determined will be ſufficiently accurate to determine the number of revolutions which the planet muſt have made when we compare the modern with the ancient obſervations, in order to determine the mean motion more accurately.

The moſt ancient obſervation which we have of the oppoſition of Saturn was on March 2, in the year 228, before J. C. at one o'clock in the afternoon

FIG. 51.

* If *a* be the mean place of a planet in it's orbit, and *b* the mean place at the interval of a year (*ab* being the order of the ſigns), then *ab* is called the mean *annual* motion, the number of complete revolutions being rejected, if the planet have made one or more revolutions. Hence, if to the mean place of a planet at the beginning of any year we add the mean annual motion, it gives the mean place at the beginning of the next year, rejecting 360° if the ſum be greater. The mean annual motion is that belonging to a common year of 365 days; therefore for a biſſextile we muſt add the mean motion of 1 day in order to get the mean annual motion for that year. In like manner, if *a* and *b* be the mean places at the interval of 100 years containing 25 biſſextiles, *ab* is called the mean *ſecular* motion; which added to the mean place of a planet at the beginning of any year, gives the mean place at the end of the 100th year from that time. For inſtance, the mean annual motion of *Mars* is 6ˢ. 11°. 17′. 10″; and it's mean place at the beginning of 1789, was 9ˢ. 17°. 22′. 29″; to this therefore add 6ˢ. 11°. 17′. 10″ and (rejecting 360°) we get 3ˢ. 28°. 39′. 39″, the mean place at the beginning of 1790. As the mean daily motion of Mars is 31′. 27″, the mean annual motion in a year of 366 days is 6ˢ. 11°. 48′. 37″. Now in a biſſextile, the year begins on January 1, at noon, but in the common years it begins on December 31, at noon, by the civil account; therefore the year *preceding* the biſſextile has 366 days in the Aſtronomical Tables. Hence at the beginning of 1787, the mean place of Mars being 8ˢ. 24°. 16′. 43″, and the next year being biſſextile, if we add 6ˢ. 11°. 48′. 37″ it gives 3ˢ. 6°. 5′. 20″ for the mean place at the beginning of 1788. The mean ſecular motion of Mars is 2ˢ. 1°. 42′. 10″, which added to 11ˢ. 22°. 2′. 49″ the mean place of Mars at the beginning of the year 1400, will give 1ˢ. 23°. 44′. 59″ the mean place at the beginning of the year 1500. If the 100 years contain only 24 biſſextiles, as may ſometimes happen, the mean ſecular motion will be 2ˢ. 1°. 10′. 43″. But of this we ſhall have to ſay more, when we treat of the Conſtruction of the Tables of the Planets motions.

in

in the meridian of Paris, Saturn being then in ♍ 8°. 23′, with 2°. 50′ north lat. On February 26, 1714, at 8*h*. 15′, Saturn was found in oppoſition in ♍ 7°. 56′. 46″, with 2°. 3′ north lat. From this time we muſt ſubtract 11 days, in order to reduce it to the ſame ſtyle as at the firſt obſervation, and conſequently this oppoſition happened on February 15, at 8*h*. 15′. Hence the difference between theſe two places was only 26′. 14″. Alſo the oppoſition in 1715 was on March 11, at 16*h*. 55′, Saturn being then in ♍ 21°. 3′. 14″, with 2°. 25′ north lat. Now between the two firſt oppoſitions there were 1942 years (of which 485 were biſſextiles) wanting 14*d*. 16*h*. 45′, that is, 1943 common years and 105*d*. 7*h*. 15′ over. Alſo the interval between the times of the two laſt oppoſitions was 378*d*. 8*h*. 40′, during which time, Saturn had moved over 13°. 6′. 28″; hence 13°. 6′. 28″ : 26′. 14″ :: 378*d*. 8*h*. 40′ : 13*d*. 14*h*. which added to the time of the oppoſition in 1714, gives the time when the planet had the ſame longitude as at the oppoſition in 228 before J. C. This quantity added to 1943 common years 105*d*. 7*h*. 15′ gives 1943*y*. 118*d*. 21*h*. 15′, in which interval of time Saturn muſt have made a certain complete number of revolutions. Now having found, from the modern obſervations, that the time of one revolution muſt be nearly 29 common years 164*d*. 23*h*. 8′, it follows that the number of revolutions in the above interval was 66; dividing therefore that interval by 66 we get 29*y*. 162*d*. 4*h*. 27′ for the time of one revolution. From comparing the oppoſitions in the years 1714 and 1715, the true movement of Saturn appears to be very nearly equal to the mean movement, which ſhows that the oppoſitions have been obſerved very near the mean diſtance; conſequently the motion of aphelion cannot have cauſed any conſiderable error in the determination of the mean motion. Hence the mean annual motion is 12°. 13′. 35″. 14‴, and the mean daily motion 2′. 0″. 35‴. Dr. HALLEY makes the annual motion to be 12°. 13′. 21″. M. de PLACE makes it 12°. 13′. 36″,8. As the revolution here determined is that in reſpect to the longitude of the planet, it muſt be a *tropical* revolution. Hence to get the ſidereal revolution, we muſt ſay, 2′. 0″. 35‴ : 24′. 42″. 20‴ (the preceſſion in the time of a tropical revolution (148)) :: 1 day : 12*d*. 7*h*. 1′. 57″ which added to 29*y*. 162*d*. 4*h*. 27′ gives 29*y*. 174*d*. 11*h*. 28′. 57″ the length of a ſidereal year of Saturn.

240. In the ſame manner that we have determined the time of a tropical revolution of Saturn from thoſe oppoſitions which happen nearly in the ſame point of the heavens, we may determine the periodic time of *Jupiter* and *Mars*; we ſhall therefore ſelect ſuch obſervations from CASSINI, as may be proper for this purpoſe.

In 1699, *Jupiter* was in oppoſition at Paris on June 14, at 10*h*. 8′ in ♐ 23°. 52′. 40″, with 0°. 23′. 7″ north lat. In 1710 the oppoſition happened on May

17, at 18*h*. 24′ in ♐ 26°. 47′. 47″, with 1°. 4′. 50″ north lat. In 1711 the oppoſition was on June 20, at 6*h*. 37′ in ♐ 28°. 36′ with 0°. 15′. 50″ north lat. From theſe obſervations, the time of a mean revolution comes out 11*y*. 313*d*. 16*h*. 54′. Now the moſt ancient oppoſition is that obſerved by PTOLEMY on May 15, 133 years after J. C. at 23*h* 3′, Jupiter being in ♏ 23°. 22′. 22″. On May 12, 1698, it happened at 5*h*. 46′ in ♏ 22°. 20′. 32″. On June 14, 1699, it happened at 10*h*. 8′ in ♐ 23°. 52′. 42″. From theſe obſervations, proceeding as for Saturn, the time of a *tropical* revolution comes out 11*y*. 315*d*. 10*h*. But from the mean of ſeveral obſervations CASSINI determined it to be 11*y*. 315*d*. 14*h* 36′. Hence it's mean annual motion is 30°. 20′. 31″. 50‴. In his Tables he makes it 30°. 20′. 34″. Dr. HALLEY, in his Tables, makes it 30°. 20′. 38″. M. de la PLACE makes it 30°. 20′. 31″,7.

In 1715 *Mars* was in oppoſition on April 21, at 11*h*. in ♏ 1°. 9′. 30″. On June 11, 1717, the oppoſition happened at 9*h*. 11′ in ♐ 20°. 17′. 15″. Now in this time, which was 2 years (one of which was a biſſextile) and 50*d*. 22*h*. 11′, Mars had made one revolution and 49°. 27′. 45″ over; hence, from theſe two obſervations, we ſhall get a ſufficient approximation to the time of a revolution, by ſaying, 360°+49°. 27′. 45″ : 360° :: 781*d*. 22*h*. 11′ : 687*d*. 11*h*. 15′ the time of a revolution. Now, from the obſervations of PTOLEMY, it appears that Mars was in oppoſition on December 13, at 11*h*. 48′ at Paris, 130 years after J. C. in ♊ 21°. 22′. 50″. In 1709 Mars was in oppoſition on January 4, at 5*h*. 48′ in ♋ 14°. 18′. 25″. Between theſe obſervations there was an interval of 1578*y*. 11*d*. 18*h*, and conſequently the time of a *tropical* revolution comes out 686*d*. 22*h*. 16′. From the mean of ſeveral reſults CASSINI makes it 686*d*. 22*h*. 18′. Hence the mean annual motion is 6ˢ. 11°. 17′. 9″$\frac{1}{2}$. Dr. HALLEY makes it 6ˢ. 11°. 17′. 10″ in his Tables; and M. de la LANDE makes it the ſame. The mean motions thus found may be conſidered as ſufficiently accurate to ſettle the place of the aphelion and excentricity of the orbit; after which the periodic time may be determined with greater accuracy. Taking therefore the place of the aphelion and excentricity of Jupiter and Mars as we ſhall afterwards ſettle it, we will proceed to ſhow how we may correct the periodic time already found, by allowing for the difference of the equations at the different obſervations.

On May 15, 133 years after J. C. *Jupiter* was in oppoſition at 23*h*. 3′ in the meridian of Paris, in ♏ 23°. 22′. 22″; and the equation of the orbit being 5°. 12′. 46″, the *mean* place was ♏ 28°. 35′. 8″. On May 12, 1698, at 5*h*. 46′ in the evening, Jupiter was in oppoſition in ♏ 22°. 20′. 32″, and the equation being 3°. 51′. 21″, the *mean* place was ♏ 26°. 11′. 53″; hence the difference between

between the mean places was 2°. 23′. 15″, the time of deſcribing which was 28*d*. 17*h*. 15′ according to the mean motion already determined; this added to the time of oppoſition on May 12, 1698, gives June 10, 11*h*. 1′ at which time the mean place was the ſame as at the firſt obſervation. Hence the interval of theſe obſervations divided by 132, the number of revolutions, gives 11*y*. 315*d*. 12*h*. 54′ for the time of a mean *tropical* revolution. From the mean of this and two other obſervations, CASSINI finds the time to be 11*y*. 315*d*. 12*h*. 33′; and conſequently it's mean annual motion 30°. 20′. 33″. 56‴. *Elem. d'Aſtron.* p. 431.

On December 13, 130 years after J. C. *Mars*, from the obſervations of PTOLEMY was in oppoſition at 11*h*. 48′ in ♈ 21°. 22′. 50″; and the equation being 7°. 2′. 44″, the mean place was ♈ 14°. 20′. 6″. On December 11, 1691, at 3*h*. 14′ the oppoſition happened in ♈ 19°. 55′. 16″; and the equation being 10°. 16′. 14″, the mean place was 9°. 39′. 2″. Now the difference of theſe mean places was 4°. 41′. 4″, the time of deſcribing which is 8*d*. 22*h*. 31′. which added to December 11, 3*h*. 14′ gives December 20, 1*h*. 45′, when the mean place was the ſame as at the firſt obſervation. Now the interval of theſe times was 1561 years (of which 390 were biſſextiles) wanting 3*d*. 10*h*. 3′; which divided by 830, the number of revolutions, gives 686*d*. 22*h*. 18′. 39″ for the time of a mean *tropical* revolution; conſequently the mean annual motion is 6′. 11°. 17′. $9''\frac{1}{2}$. This correction is not neceſſary to be applied to our determination of the periodic time of Saturn; for as it was obſerved near the mean diſtance, where the equation is a maximum, the variation of $\frac{1}{2}°$ in the place would not cauſe any ſenſible variation in the equation.

241. In the ſame manner we may determine the time of a tropical revolution of *Venus*, by comparing the time of two conjunctions, firſt getting an approximation in order to be ſure of the number of revolutions. Now in 1709, on June 22, at 6*h*. apparent time, Venus was in ſuperior conjunction in ♋ 0°. 56′. 30″; and in 1705 on June 21, at 22*h*. an inferior conjunction happened in ♋ 0°. 36′. 52″. In this interval Venus muſt have made $6\frac{1}{2}$ revolutions and 19′. 38″; therefore the time of one revolution is found to be $224\frac{2}{3}$ days nearly. To get the time more accurately, we muſt take two conjunctions at a greater interval of time, and allow for the difference of the equations at the times of obſervation. Now in 1639 on December 4, at 6*h*. 11′ mean time, Venus was in conjunction in ♈ 12°. 31′. 44″ on the ecliptic, and ♈ 12°. 31′. 37″ on it's orbit; and the equation being 40′. 26″, gives it's mean place ♈ 13°. 12′. 3″. In the conjunction 1716, on Auguſt 28, at 16*h*. 37′ mean time, Venus was in

♓ 5°. 49′. 53″ on it's orbit; and the equation being 25′. 11″, gives the mean place ♓ 6°. 15′. 4″. Now in this interval of time, which has been 76 common years and 286*d*. 10*h*. 26′, there have, from what has been ſhown above, been 125 revolutions and 8ˢ. 23°. 3′. 1″; hence 125 revolutions 8ˢ. 23°. 3′. 1″ : 360°. :: 76*y*. 286*d*. 10*h*. 26′ : 224*d*. 16*h*. 41′. 40″ the time of a mean *tropical* revolution. Hence the mean annual motion is 7ˢ. 14°. 47′. 28″. *Elem. d'Aſtron.* page 562.

242. CASSINI propoſes alſo to find the time of a revolution, by comparing the ancient obſervations with the modern ones made when Venus was not in conjunction, for the ancient Aſtronomers could make no obſervations in conjunction for want of teleſcopes. For example. On December 25, 136, at 4*h*. Venus ſeen from the earth appeared in 1ˢ. 20°. 13′. 45″; and on December 17, 1594, at 4*h*. 30′ it was ſeen in 1ˢ. 23°. 1′. 36″, advanced 2°. 47′. 51″ beyond the firſt obſervation; and as Venus ran through this ſpace in 1*d*. 17*h*. 54′, CASSINI concluded that on December 15, 1594, at 10*h*. 36′ Venus was in the ſame place as at the firſt obſervation, the interval of which times was 1458 common years 354*d*. 6*h*. 36′, in which Venus had made 2370 revolutions; hence the time of one revolution is 224*d*. 16*h*. 39′. 4″. This method would be accurate, provided the earth was at the ſame point at both times, and the orbit of Venus was fixed. Hence the mean annual motion is 7ˢ. 14°. 47′. 45″. CASSINI in his Tables makes it 7ˢ. 14°. 47′. 29″. Dr. HALLEY makes it 7ˢ. 14°. 47′. 28″. M. de la LANDE makes it 7ˢ. 14°. 47′. 30″.

243. The periodic time of *Mercury* may be very accurately determined from it's tranſits over the ſun's diſc; for as they have frequently been obſerved, we have an opportunity of chuſing ſuch as will give us a very accurate concluſion. From the obſervations of the conjunction of Mercury on November 6, 1631, CASSINI found the time of the conjunction to be at 19*h*. 50′, and the true place of Mercury 1ˢ. 14°. 41′. 35″. On November 9, 1723, at 5*h*. 29′, the conjunction was in 1ˢ. 16°. 47′. 20″, only 2°. 5′. 45″ beyond the place at the firſt obſervation. Now according to the Tables of CASSINI, this difference is juſt equal to the motion of the aphelion of Mars in the ſame time; conſequently Mercury was in the ſame place in it's orbit at each time, and therefore the equation was the ſame. Alſo the conjunctions happening very nearly at the ſame time of the year, the equation of time was very nearly the ſame, and therefore the difference of the apparent times is the ſame as of the true. Hence in the interval of 92 years (of which 22 were biſſextiles) and 2*d*. 9*h*. 39′, Mercury (from firſt finding nearly the time of a revolution by 2 conjunctions near each other) is found to have made 382 revolutions 2°. 5′.45″; hence, by proportion, the time of a tropical revolution is 87*d*. 23*h*. 14′. 20″,9; and the mean annual motion comes out 1ˢ. 23°. 43′. 11″. 39‴. CASSINI, in his Tables, makes

makes it 1ˢ. 23°. 43′. 11″. Dr. HALLEY makes it 1ˢ. 23°. 43′. 2″; and M. de la LANDE, 1ˢ. 23°. 43′. 3″.

On the Secular Motions of Jupiter and Saturn.

244. The time of a revolution of Saturn deduced from the modern obſervations comes out greater than that deduced from a compariſon of the modern with the ancient obſervations. If therefore the modern obſervations could be depended upon to give the time of a revolution nearer than that difference, it would prove that the length of Saturn's year is increaſing. Now although obſervations made at a ſmall interval of time, could not be ſufficient to eſtabliſh this point, yet from a compariſon of our obſervations with thoſe made by TYCHO, it appears that this is the caſe. The length of the year therefore when aſcertained for one time will afterwards want a correction, and the quantity of this correction is called the *Secular Equation.*

245. KEPLER firſt obſerved this circumſtance, from examining the obſervations of REGIOMONTANUS and WALTHERUS; for he conſtantly found Jupiter forwarder and Saturn backwarder than they ought to have been from the mean motions determined from the obſervations of PTOLEMY and TYCHO. He ſaid the ſame of Mars; but M. de la LANDE obſerves, that he cannot find there is any ſecular equation wanted for that planet. FLAMSTEAD alſo obſerved, that in all the beſt Aſtronomical Tables the mean motions of Saturn were too ſwift, and of Jupiter too ſlow; whence it came to paſs, that the computations gave thoſe conjunctions which happened when the planets were *direct*, ſome days ſooner, and when *retrograde*, ſome days later than the time from obſervation; *Phil. Tranſ.* N°. 149. HEVELIUS alſo obſerved the ſame. M. MARALDI perceived alſo that the mean motions of Saturn, if we ſuppoſe them uniform, would not agree both with the obſervations of TYCHO and thoſe of this age. Dr. HALLEY, in his Aſtronomical Tables, applied a ſecular equation of $9°\frac{1}{4}$ for 2000 years to Saturn, and 3°. 40′ to Jupiter, but he does not give the obſervations from which he deduced theſe concluſions. M. de la LANDE, from comparing the oppoſitions in the years 1594, 1595, 1596 and 1597 with thoſe in 1713, 1714, 1715, 1716 and 1717, found the mean motion of Saturn to be 12°. 13′. 19″. 14‴ which is 16″ in a year leſs than that given by CASSINI; and the duration of the revolution greater by near 4 days. He choſe thoſe oppoſitions which happened near the mean diſtance (as CASSINI did alſo), becauſe the true and mean motions being then equal, the concluſions would be more accurate. He alſo choſe other oppoſitions at the diſtance of about 120 years, and when

when Jupiter and Saturn were in ſimilar ſituations, ſo that no error was to be apprehended from their mutual attraction, this being the ſame in each caſe. Now if with the mean motion found in 120 years, the place of Saturn, from where it is now found to be, be computed for the time of the obſervation before mentioned in the year 228 before J. C. the longitude will be found to be too great by 7°; this therefore is the ſecular equation for 2000 years, according to this mean motion. But from other obſervations he concluded the mean motion to be 12°. 13′. 26″,558. With this mean motion he finds the ſecular equation to be 47″ in the firſt century from which this motion was deduced; for with this mean motion and ſecular equation, the calculations beſt agree with the ancient obſervations. From the theory of attraction it appears, that, ſuppoſing the aphelion of Saturn and Jupiter to be fixed, the ſecular equation varies as the ſquare of the time, which M. de la LANDE thinks may be deduced from this conſideration, that the velocity loſt by Saturn in conſequence of the cauſe, which produces the equation, being ſo very ſmall, may be conſidered equal in equal times; whence from the principle of the law of falling bodies, the ſpace loſt muſt vary as the ſquare of the time. Now from five obſervations of PTOLEMY, he found the ſecular equation for the firſt 100 years to be 47″; hence $100^2 : t^2 :: 47''$: the ſecular equation for t years. But the logarithm of 47 *minus* the logarithm of 100^2 is 7,6720979; hence, if to this conſtant logarithm we add twice the logarithm of t, we ſhall have the logarithm of the ſecular equation for t years from the commencement of the 100 years, to be ſubtracted from the mean longitude.

246. But beſides the ſecular equation, the mean motion of Saturn is alſo ſubject to other irregularities which are found to follow from the attraction of Jupiter. Dr. HALLEY, in his Aſtronomical Tables, obſerves that Jupiter from his oppoſition in 1677, to that in 1689, was found, from indubitable obſervations, to be 12′ ſlower than in the preceding or ſubſequent revolutions. Alſo the periodic time of Saturn between the years 1668 and 1698 was nearly a week ſhorter than it's mean revolution; and the periodic time between 1689 and 1719 was nearly as much greater, ſo that between the two revolutions there was a difference of more than 13 days. This Dr. HALLEY ſuppoſes to ariſe from the attraction of the greater bodies in the ſyſtem being different in different poſitions. For he obſerves, that in 1683 there was a conjunction of Jupiter and Saturn, when, from the poſition of the apſides, the planets approached neareſt to each other, and Saturn was moſt urged towards the ſun and Jupiter from it; ſo that Jupiter's velocity being increaſed and it's force to the ſun diminiſhed, it's orbit was increaſed and conſequently it's periodic time; on the contrary, Saturn's velocity being diminiſhed and it's force to the ſun increaſed, it's orbit, and conſequently it's periodic time, was diminiſhed. Now,

ſays

ſays he, if the ſame thing ſhould happen again, that is, if a conjunction ſhould take place again in the ſame point of the Heavens, and the ſame effects ſhould follow, we may hope that it can be accounted for from the Laws of gravity; but if, in like circumſtances, the ſame effects are not found to take place, other extraneous cauſes are to be ſought for. But M. de la PLACE has diſcovered, that theſe inequalities, as well as the ſecular equations, may be repreſented by an equation, from Jupiter's attraction, of 48′, which depends on 5 times the longitude of Saturn minus twice that of Jupiter, of which the period is 918 years. For this we muſt employ the mean annual motion of 12°. 13′. 36″,81. Thus all the irregularities of Saturn's motion are confined to a certain period, after which they all return again. In the years 1701 and 1760 the errors of Dr. HALLEY's Tables were $8'\frac{1}{2}$ and $21'\frac{1}{2}$, according to M. de la LANDE, ſo that the motion of Saturn was greater by 13′, and it's periodic time was ſhorter $6\frac{1}{2}$ days, than in it's revolution between 1686 and 1745. Now the mean anomaly in 1701 and 1760 was 3°. 1′, and the angle at the ſun between Jupiter and Saturn was 19° in 1701 and 30° in 1760, ſo that the error in the mean motion could not ariſe from any diſſimilar ſituations of Saturn in it's orbit, by which the elements of the motions might err; nor from the different ſituations of Jupiter, that difference not being ſufficient to cauſe ſuch an error.

247. The motion of Jupiter requires alſo a ſecular equation, as Dr. HALLEY obſerved, who made it 3°. 49′. 24″ for 2000 years, or 34″,4 for the firſt century, ſuppoſing it to increaſe as the ſquare of the time. M. MARALDI alſo obſerved, that the modern obſervations gave the motion of Jupiter greater than the ancient. M. de la LANDE found by comparing the obſervations made 240 years before J. C. with thoſe in the year 508, that Jupiter's ſecular motion in 83 years was 2′. 04″. And comparing the obſervations in 508 with thoſe in 1503 and 1504, we find nearly the ſame reſult. But if we compare the conjunction of Jupiter with *Regulus* on October 12, 1623, with the like obſervation made in 1706, we find it 21′ for 83 years. Dr. HALLEY, in his Tables, fixed it at 12′. 26″ for 83 years, which makes the revolution 8 hours ſhorter than that deduced from the ancient obſervations. The oppoſitions from 1689 to 1698 compared with thoſe in 1749, give a mean motion equal to that in the Tables of CASSINI; which Tables give the place of Jupiter 1′ too much in 508. Theſe concluſions indicate a great irregularity in Jupiter's motions; and this irregularity is further confirmed, if we conſider that M. WARGENTIN makes the ſecular equation for the firſt 100 years to be 18″; M. BAILLY makes it $12''\frac{1}{3}$; and M. de la LANDE fixes

fixes it at $30''\frac{1}{2}$ for the firſt 100 years, or 3°. 23′. 20″ for 2000 years, admitting it to increaſe as the ſquare of the time, which agrees nearly with Dr. HALLEY's determination. M. de la GRANGE, from the theory of gravity, finds it to be 3′. 18″, which, as M. de la LANDE obſerves, agrees very well with the obſervations from 1590 to 1762, but not with the ancient obſervations. EULER determined it from theory to be 2′. 23″. M. de la LANDE ſays, that his own ſecular equation, with the mean ſecular motion of 5°. 6°. 27′. 30″, agree as nearly as poſſible to all the obſervations. M. de la PLACE found in 1786 an inequality of 20′ from the attraction of Saturn, the period of which equation is 918 years, as in Saturn. Thus he made the ſecular equation diſappear, it being only an irregularity whoſe period is 918 years. This ſuppoſes a ſecular motion of 5°. 6°. 17′. 33″. The ſecular equation being determined for 100 years, it may be found for any other time, as it was for Saturn, by taking it in proportion to the ſquare of the time.

The longitude of the ſun requires a ſecular equation of 12′ for 2500 years, ariſing from the diminution of the preceſſion of the equinoxes, according to M. de la LANDE.

REVO-

According to M. de la LANDE.

Planets	*Secular motion*	*Tropical Revolution*	*Sidereal Revolution*	*Mot. diur. trop.*
Mercury	2ˢ. 14°. 4′. 20″	87ᵈ. 23ʰ. 14′. 32″,7	87ᵈ. 23ʰ. 15′. 43″,6	4°. 5′. 32″,57
Venus	6. 19. 12. 25	224. 16. 41. 27, 5	224. 16. 49. 10, 6	1. 36. 7, 8
Earth	0. 0. 46. 0	365. 5. 48. 48	365. 6. 9. 11, 6	0. 58. 8, 33
Mars	2. 1. 42. 10	686. 22. 18. 27, 4	686. 23. 30. 35, 6	0. 31. 26, 66
Jupiter	5. 6. 17. 33	4330. 14. 39. 2	4332. 14. 27. 10, 8	0. 4. 59, 26
Saturn	4. 23. 31. 36	10746. 19. 16. 15, 5	10759. 1. 51. 11, 2	0. 2. 0, 6

According to Dr. HALLEY.

Planets	*Tropical Revolution*	*Sidereal Revolution*	*Sec. movement*
Mercury	87ᵈ. 23ʰ. 14′. 34″,4	87ᵈ. 23ʰ. 15′. 45″,5	2ˢ. 14°. 2′. 23″
Venus	224. 16. 41. 30, 6	224. 16. 49. 14, 5	6. 19. 11. 52
Mars	686. 22. 18. 18, 8	686. 23. 30. 34, 7	2. 1. 42. 20
Jupiter	4330. 8. 35. 4	4332. 8. 28. 1, 1	5. 6. 28. 11
Saturn	10750. 13. 14. 42, 1	10762. 20. 33. 41, 1	4. 23. 6. 0
	or 29. 321.	or 29. 321.	or 11. 315.
	or 11. 315.	or 11. 315.	or 29. 165.
	or 1ʸ. 321.	or 1ʸ. 321.	

The ſecular motion is in reſpect to the equinox.

The ſecular motion of the *Georgian* Planet in reſpect to the equinox is 2ˢ. 13°. 16′. 55″; it's tropical revolution is 83*y*. 52*d*. 4*h*. it's ſidereal revolution is 83*y*. 150*d*. 18*h*; and it's tropical diurnal motion is 42″,678026.

Dr. HALLEY made the length of a tropical year 365*d*. 5*h*. 48′. 55″; FLAMSTEAD and Sir I. NEWTON made it 57″,5; MAYER 51″; and M. de la CAILLE in his Tables 49″.

CHAP. XIII.

ON THE GREATEST EQUATION, EXCENTRICITY AND PLACE OF THE APHELIA OF THE ORBITS OF THE PLANETS.

Art. 248. HAVING determined the mean motions of the planets, we proceed next to ſhow the method of finding the greateſt equation of their orbits, and from thence the excentricity and place of their aphelia. For although, in order to determine the mean motions very accurately, theſe things were ſuppoſed to be known, yet without them the mean motions may be ſo nearly aſcertained, that theſe elements may from thence be very accurately ſettled.

FIG. 52. 249. Let A be the aphelion, S the focus; take SW a mean proportional between the ſemi-axis major and minor, then (230) when the planet comes to the points V and W the equation is the greateſt; at which times let the mean places be at v and w, then the difference between the true and mean motions from V to W is the ſum of the angles VSv, WSw, or $2WSw$, the half of which is the greateſt equation. Now to find when this happens, obſerve the true places of the planet when at V and W, take the difference of the two places, and compute the mean motions for the ſame time, and half the difference is the greateſt equation. But as it is impoſible to fix upon the times when the planet is accurately at V and W, ſeveral obſervations muſt be made about each time, and comparing them by two and two, find thoſe where the difference between the true and mean motions is the greateſt, and half the difference is the greateſt equation. The obſerver will eaſily find when the planet is got near to the mean diſtance, by comparing his obſervations for ſeveral days, and obſerving whether the true motion be nearly equal to the mean motion. Hence if we biſect the interval it will give the place A of the aphelion. Having found the greateſt equation, the excentricity will be known (231). Or the greateſt equation may be found thus. Having made two obſervations near to V and W, find the equation correſponding, and from thence the place of the aphelion and excentricity; then compute for the two times of obſervation the equations correſponding, and alſo the greateſt equation; and the difference between half the ſum of the computed equations for the times of obſervation and the computed greateſt equation ſhows the error ariſing from the obſervation; which added to the equation found from obſervation gives the greateſt equation.

To

Ex. To find the greateſt equation of the ſun. From the obſervations of M. de la CAILLE in 1751, on

October 7, ſun's place obſerved was	6ˢ.	13°.	47′.	13″,7
March 28, 1752	0.	8.	9.	25. 5
	5.	24.	22.	11, 8
Mean motion by calculation	5.	20.	31	43. 2
	3.	50.	28,	6

The half of which 1°. 55′. 14″,3 is the greateſt equation, if no correction be required. But if we take the place of the aphelion and excentricity from this equation, conſidered as the greateſt, and calculate the equations for theſe two times, half the difference will be the ſuppoſed greateſt equation; compute alſo the greateſt equation, and we ſhall find that theſe differ by 18″,6, which ſhows that the greateſt equation deduced from theſe two obſervations differs from the greateſt equation itſelf by that quantity; this therefore added to 1°. 55′. 14″,3 gives 1°. 55′. 32″,9 for the greateſt equation. From the mean of ſeveral obſervations M. de la CAILLE makes it 1°. 55′. 32″.

In the year 1717 on March 21, the ſun's place on the meridian at Paris, by CASSINI's Aſtronomy page 191, was in ♈ 0°. 47′. 28″ and on September 23, in ♎ 0°. 15′. 50″. Hence the true motion in 185*d*. 23*h*. 45′ was 5ˢ. 29°. 28′. 22″, and the mean motion in that time was 6ˢ. 3°. 19′. 12″, half the difference of which is 1°. 55′. 25″. By thus comparing the obſervation on September 23, 1717, with the obſervation on March 21, 1718, the equation comes out 1°. 55′. 16″$\frac{1}{2}$. If we compare the obſervation on March 28, 1717, with that on September 27, following, the equation comes out 1°. 55′. 37″$\frac{1}{2}$. And if we compare the obſervation on March 28, 1718, with that on September 27, 1717, the equation is found to be 1°. 56′. 3″$\frac{1}{2}$. The mean of all theſe is 1°. 55′ 35″$\frac{1}{2}$ for the greateſt equation, differing only 3″$\frac{1}{2}$ from the other; but CASSINI, in his Tables, makes it 1°. 55′. 51″. In the Tables of MAYER it is 1°. 55′. 31″,6. M. de LAMBRE, from the obſervations of Dr. MASKELYNE, makes it 1°. 55′. 30″,9 in 1780; for on account of the diminution of the excentricity of the earth's orbit, the greateſt equation is ſubject to a diminution.

250. To find the place of the aphelion *A*, obſerve the interval of time from *m* to *n*, two oppoſite points in the orbit; and if that be equal to half the anomaliſtic revolution, or the time from *A* to ♑, the points *m* and *n* muſt co- FIG. 53.

incide with A and $\mathcal{Q}$; for the whole area can only be bifected by the line $A S \mathcal{Q}$ paſſing through S, and conſequently the time of half a revolution about S can never be equal to half the time of one whole revolution but from A to $\mathcal{Q}$, the areas being in proportion to the times (219). Now the difference (d) between the times from A to $\mathcal{Q}$ and from m to n muſt, by taking away the time from m to $\mathcal{Q}$ which is common to both, be equal to the difference between the times through Am and $\mathcal{Q}n$. Put $t =$ the time from A to m, and let m and n be the angular velocities about S in 24 hours at A and $\mathcal{Q}$; then $n : m :: t : \frac{mt}{n} =$ the time through $\mathcal{Q}n$, the time of deſcribing equal angles being inverſely as the angular velocities; hence $t - \frac{mt}{n} = d$, conſequently $n - m : m :: d : t$. Now if the obſervation be made at m when the ſun is paſt A, the time through $m\mathcal{Q}n$ muſt be leſs than the time from A to $\mathcal{Q}$, becauſe the area ASm being greater than $\mathcal{Q}Sn$, the area $Am\mathcal{Q}$ deſcribed about S muſt be greater than that of $m\mathcal{Q}n$; and the contrary if m be on the other ſide of A.

Ex. On December 30, 1743, at $0h.\ 3'.\ 7''$ mean time, M. de la CAILLE found the ſun's longitude to be ♑ $8°.\ 29'.\ 12''\frac{1}{2}$; and on June 30, 1744, at $0h.\ 3'$ it was ♋ $8°.\ 51'.\ 1''\frac{1}{2}$; the interval of theſe two places is $180°.\ 21'.\ 49''$. Now reckoning, with M. de la CAILLE, the annual progreſſive motion of the apogee of the earth's orbit to be $1'.\ 3''$, the diſtance of the apogee from the perigee is $180°.\ 0'.\ 31''\frac{1}{2}$; but the ſun had deſcribed $180°.\ 21'.\ 49''$, which exceeds $180°.\ 0'.\ 31''\frac{1}{2}$, half an anomaliſtic revolution, by $21'.\ 17''\frac{1}{2}$; and the ſun's motion on June 30, being $57'.\ 12''$ in 24 hours, $57'.\ 12'' : 21'.\ 17''\frac{1}{2} :: 24h. : 8h.\ 56'$ the time of deſcribing $21'.\ 17''\frac{1}{2}$, which ſubtracted from June 30, $0h.\ 3'$ gives June 29, $15h.\ 7'$ when the ſun was in ♋ $8°.\ 29'.\ 43''$ at the diſtance of $180°.\ 0'.\ 31''\frac{1}{2}$ from the place where it was on December 30, at $0h.\ 3'.\ 7''$; the interval of theſe two times is $182d.\ 15h.\ 3'.\ 53''$, which being leſs than $182d.\ 15h.\ 7'.\ 1''$, half the time of an anomaliſtic revolution (150), by $3'.\ 8''$ ($= d$), the ſun was not come to it's apogee on June 29, $15h.\ 7'$. Now the ſun's motion on June 30, was $57'.\ 12''$ in a day $= m$, and on December 30, $61'.\ 12'' = n$; hence $4' : 57'.\ 12'' :: 3'.\ 8'' : 44'.\ 48''$, which added to June 29, $15h.$

$7'$

7′ gives June 29, 15*h*. 51′. 48″ when the ſun was in it's apogee, at which time the ſun's place was in ♋ 8°. 31′. 21″, which therefore was the place of the apogee.

251. To find the excentricity, we have (231) 57°. 17′. 44″,8 : 57′. 45″,5 (the half of 1°. 55′. 30″,9 the greateſt equation according to M. de LAMBRE) :: 1 : ,01681 the excentricity, the mean diſtance being unity. As the orbit is very nearly a circle, the correction is unneceſſary.

252. The above method of finding the place of the aphelion from the greateſt equation is very applicable to the caſe of the ſun and moon, but it cannot be applied with the ſame ſucceſs to the planets, becauſe they do not revolve about the earth, and therefore their velocities near the apſides, in reſpect to the ſun, cannot be obtained in like manner. M. CASSINI (*Elem. d'Aſtron.* pag. 366.) therefore propoſes the following method. Having found the greateſt equation, by obſerving the angle deſcribed between the mean diſtances B and D through the aphelion A, obſerve the planet at r near to A, and the angle BSr will be the true angle deſcribed between B and r; then from the time of deſcribing this angle compute the mean motion; and if the difference between the true and mean motions be *equal* to the greateſt equation, then r is the aphelion; if it be *leſs*, the planet is not got to it's aphelion. Make then another obſervation at m, and if the difference between the true and mean motions be now *greater* than the equation, the planet is got beyond A. Hence ſay, as the ſum of the equations at r and m : the equation at r :: the angle rSm : the angle rSA the diſtance of the point r from the aphelion; for (229) when the diſtance from the aphelion is ſmall, the equation varies very nearly as the true anomaly. This may be corrected, if neceſſary, by calculating, from the place of the aphelion, whether the body be found at r and B when it ought. And to find the time of coming to the aphelion, ſay, as the ſum of the equations at r and m : the equation at r :: time of deſcribing rm : time of deſcribing rA.

Ex. To find the greateſt equation, place of the aphelion and excentricity of the orbit of *Saturn*. Between the oppoſitions in 1686 and 1687 Saturn had moved through 12°. 38′. 20″, and it's mean motion in that interval being 12°. 39′. 34″, Saturn was then very near it's mean diſtance. Now Saturn was in oppoſition in

1686,

1686, March 16, 10*h*. 28′ in	5ˢ. 26°. 47′. 6″
1701, September 16, 2*h*. in	11. 23. 21. 16
Interval 15*y*. 186*d*. 15*h*. 32′	5. 26. 34. 10
Mean motion in this interval	6. 9. 36. 0
	13. 1. 50
Greateſt equation	6. 30. 55

To find the place of the aphelion, and the time of coming to it. Saturn was in oppoſition in

1686, March 16, 10*h*. 28′ in	5ˢ. 26°. 47′. 6″
1693, June 9, 19*h*. 32′ in	8. 19. 54. 41
Interval 7*y*. 87*d*. 9*h*. 4′	2. 23. 7. 35
Mean motion in this interval	2. 28. 29. 27
	5. 21. 52
Greateſt equation	6. 30. 55
Equation at *r*	1. 9. 3

Hence Saturn was not come to it's aphelion in oppoſition 1693. Now the oppoſition happened in

1686, March 16, 10*h*. 28′ in	5ˢ. 26°. 47′. 6″
1694, June 21, 19*h*. 30′ in	9. 1. 6. 40
Interval 8*y*. 99*d*. 10*h*.	3. 4. 19. 34
Mean motion in this interval	3. 11. 6. 51
	6. 47. 17
Greateſt equation	6. 30. 55
Equation at *m*	16. 22

Therefore Saturn had paſſed it's aphelion in oppoſition 1694. Hence 1°. 9′. 3″ + 16′. 22″ = 1°. 25′. 25″ : 1°. 9′. 3″ :: 11°. 12′ (the angle deſcribed between the oppoſitions in 1693 and 1694) : 9°. 3′. 20″, which added to 8ˢ. 19°. 54′. 41″ gives 8ˢ. 28°. 58′ for the place of the aphelion. And to find the time, we have 1°. 25′. 25″ : 1°. 9′. 3″ :: 376*d*. 23*h*. 58′ (the time between the oppoſitions in 1693 and 1694) : 305*d*. 16*h*. which added to 1693, June 9, 19*h*. 32′ gives 1694, April 11, 11*h*. 32′ the time when Saturn was in it's aphelion. Dr. HALLEY, in his Tables, makes the greateſt equation 6°. 32′. 4″. CASSINI makes it 6°. 31′. 40″. M. de LAMBRE makes it 6°. 26′. 42″ in 1750, and

ſuppoſes

ſuppoſes that it is diminiſhed 1″,1 in a year, according to the determination of M. de la PLACE. From the mean of ſix excentricities, determined (231) from the greateſt equation, CASSINI found the excentricity to be ,56515, the mean diſtance of the earth from the ſun being unity.

253. The ſame method may be applied to find the greateſt equation, place of the aphelion and excentricity of *Jupiter's* orbit, although we cannot ſo readily meet with obſervations made in the proper places, becauſe we have fewer oppoſitions of Jupiter in one revolution than of Saturn. The following however are proper for our purpoſe (*Elem. d'Aſtron.* page 423.) In 1723, on June 25, at 4*h*. Jupiter was in oppoſition in ♍ 3°. 21′. 22″, near it's mean diſtance; on December 22, 1728, at 3*h*. 9′ the true place of Jupiter in oppoſition was ♋ 1°. 8′. 2″. The difference of theſe places is 5ˢ. 27°. 46′. 40″; and the mean motion being 5ˢ. 16°. 50′. 15″, the difference is 10°. 56′. 25″, the half of which is 5°. 28′. 12″,5 the greateſt equation from theſe obſervations. On September 5, 1725, at 14*h*. 44′ Jupiter was in oppoſition in ♐ 13°. 18′; this compared with the oppoſition in 1723, gives 2ˢ. 9°. 56′. 38″ for the true motion of Jupiter in the interval; and the mean motion being 2ˢ. 6°. 47′. 24″, the difference is 3°. 9′. 14″, which ſubtracted from 5°. 28′. 12″ gives 2°. 18′. 58″ the equation at *r*. On October 13, 1726, at 6*h*. Jupiter was in ♈ 20°. 4′. 10″ in oppoſition; this compared with the oppoſition in 1723, gives 3ˢ. 16°. 52′. 48″ for the true motion in the interval; and the mean being 3ˢ. 10°. 15′. 39″, the difference is 6°. 37′. 9″, from which ſubtract 5°. 28′. 12″ and the remainder is 1°. 8′. 57″ the equation at *m*. Hence 2°. 18′. 58″ + 1°. 8′. 57″ = 3°. 27′. 55″ : 1°. 8′. 57″ :: 36°. 46′. 10″ (the angle deſcribed between the oppoſitions in 1725 and 1726) : 12°. 15′, which ſubtracted from ♈ 20°. 4′. 10″ gives ♈ 7°. 49′. 10″ the place of the perihelion. The time of oppoſition is alſo found by ſaying, 3°. 27″. 55″ : 1°. 8′. 57″ :: 372*d*. 15*h*. 16′ (the interval of the oppoſitions in 1725 and 1726) : 134*d*. 5*h*. 5′, which ſubtracted from the oppoſition in 1726 on October 13, at 6*h*. gives the time at which Jupiter was in it's perihelion to be on June 1, 0*h*. 55′. Alſo the excentricity is found to be 0,04774, the mean diſtance of Jupiter from the ſun being unity. It muſt be here obſerved, that the accuracy of this method depends upon the proximity of *r* and *m* to the aphelion or perihelion. CASSINI, in his Tables, makes the greateſt equation 5°. 31′. 17″. Dr. HALLEY makes it 5°. 31′. 36″. M. de LAMBRE finds it to be 5°. 30′. 37″,7 in 1750, and to increaſe 55″,36 in 100 years.

As in the ancient obſervations of *Mars* mentioned by PTOLEMY, there are only three which were made in oppoſition, and as they are not in proper places for the application of the laſt method, we ſhall give another Rule to determine the greateſt equation, the place of the aphelion and the excentricity,

from any three heliocentric places of a planet, and it's mean motion. This is reſolved in the following manner, firſt upon the ſuppoſition of the *ſimple elliptic hypotheſis* (227), and then correcting it.

FIG. 54. 254. Let S be the ſun, B, C, D three places of the planet obſerved in oppoſition, F the other focus, A the aphelion, $\mathfrak{Q}$ the perihelion; with the center F and radius FM equal to the major axis deſcribe a circle, and produce FB, FC, FD to the circumference, and join SG, SH, SE. Now the angles BSC, CSD are known from obſervation; alſo upon ſuppoſition of the *ſimple elliptic hypotheſis*, equal angles are deſcribed about F in equal times; therefore the angles BFC, CFD are known, by taking them to four right angles as the intervals of time between the firſt and ſecond, ſecond and third obſervations to the periodic time. Now as $FG = FB + BS$, therefore $SB = BG$; for the ſame reaſon $SC = CH$ and $SD = DE$. Hence $2FGS = FBS = BFA - BSA$; alſo $2FHS = FGS = CFA - CSA$; therefore $2FGS + 2FHS = BFC - BSC$; hence $FGS + FHS$ is known; but $FGS = BFA - GSA$, and $FHS = CFA - HSA$; therefore $FGS + FHS = BFC - GSH$, whence GSH is known. For the ſame reaſon HSE is known. Hence the angles GSH, HSE, GSE, and BFC, CFD, BFD are known. Produce ES to L, and join HL, HG, GL, and aſſume SH of any value in order to get the relative values of the other parts of the figure. Then in the triangle SHL, we know SH, the angle HSL (which is the ſupplement of HSE) and the angle HLS (which is half the angle HFE); hence we know SL; therefore in the triangle SLG, we know SL, the angle LSG (which is the ſupplement of GSE) and the angle SLG (the half of EFG); hence we know SG; therefore in the triangle GSH, we know GS, SH and the angle GSH; hence we know HG and the angle SHG; therefore in the iſoſceles triangle HFG, we know HG and the angle HFG; hence we know $FH = FC + CS$ the major axis, and the angle GHF, which taken from the angle SHG leaves the angle SHF which is therefore known; therefore in the triangle SHF, we know SH, HF and the angle SHF, from whence we know SF twice the excentricity, and the angle HSF, from which take the angle HSC (which $= SHF$) and we get the angle CSA, the diſtance of the aphelion A from the obſervation at C.

255. This method, being the *ſimple elliptic hypotheſis*, ſuppoſes that the angles deſcribed about F are proportional to the times, which will be ſufficiently accurate for orbits whoſe excentricity is ſmall, as that of the earth and Venus; for the orbits of the other planets it may be thus corrected.

256. Having determined, from the three obſerved places m, n, r, of the planet, the place of the aphelion and the excentricity from the *ſimple elliptic hypotheſis*, with the diſtances a, b, c, of the planet from the aphelion ſo found, calculate (232) the equation upon the true or KEPLER's hypotheſis, and you will get the

the mean anomalies a', b', c' upon the true hypotheſis. Then with theſe mean anomalies a', b', c', find the true anomalies a'', b'', c'', upon the ſimple elliptic hypotheſis, and the difference between a and a'', b and b'', c and c'' ſhows the difference of the places upon the two hypotheſes. To the place of the aphelion firſt found add the diſtances a'', b'', c'', and you get the places of the planet in the ſimple elliptic hypotheſis anſwering to the true place upon KEPLER's hypotheſis. Then with theſe three places compute, as at firſt, the place of the aphelion and excentricity upon the ſimple elliptic hypotheſis, and you will have the diſtances A, B, C, from the aphelion upon the ſimple elliptic hypotheſis, to theſe apply the differences of the two hypotheſes before found, adding or ſubtracting them according as the ſimple elliptic hypotheſis gave the place leſs or greater than KEPLER's hypotheſis, and you will have the diſtances from the aphelion upon the true or KEPLER's hypotheſis; ſubtract theſe from the correſponding places m, n, r of the planet obſerved, and you will have the place of the aphelion once corrected, and alſo the excentricity. In like manner the correction may be made as often as may be found neceſſary. *Elem. d'Aſtron.* page 184.

In 1694 on January 17, at 4^h. 20′. Mr. FLAMSTEAD obſerved the place of *Mars* to be in ♋ 28°. 12′; in 1698 on March 26, at 17^h. 55′ in ♎ 7°. 4′. 18″, and in 1702 on July 8, at 12^h. 58′ in ♍ 16°. 10′. 23″. Theſe obſervations reduced (268) to the orbit of Mars give the three places in ♋ 28°. 12′. 34″, ♎ 7°. 3′. 26″, and ♍ 16°. 11′. 9″. Hence, by KEPLER's hypotheſis, the place of the aphelion is found to be in ♏ 0°. 39′. 2″ with the excentricity ,09292, the ſemi-axis major being unity; and the greateſt equation 10°. 39′. 29″. *Elem. d'Aſtron.* page 474.

The ſame method may be applied to *Venus* from the conjunctions obſerved in the years 1715, 1716 and 1718; from which it appears, that the places of Venus ſeen from the ſun upon the ecliptic were in 1715 on January 26, at 8^h. 34′. mean time, in ♌ 6°. 22′. 58″; in 1716 on Auguſt 28, at 16^h. 36′. 42″ in ♐ 5°. 49′. 2″; and in 1718 on April 8, at 10^h. 15′. 11′. in ♎ 18°. 42′. 13″; which places reduced to the orbit of Venus will be ♌ 6°. 25′. 52″, ♐ 5°. 49′. 53″ and ♎ 18°. 39′. 24″. Hence, by the ſimple elliptic hypotheſis, the true place of the aphelion in 1716 is found to be ♒ 6°. 50′; the greateſt equation 49′. 8″; and the excentricity 0,00715. As the orbit of Venus differs but very little from a circle, there is no occaſion for any correction. *Elem. d'Aſtron.* page 562. CASSINI, in his Tables, makes the greateſt equation 49′. 6″. Dr. HALLEY makes it 48′. M. de la LANDE makes it 47′. 20″.

Upon the ſame principle we may deduce the place of the aphelion, excentricity and equation of the orbit of *Mercury*; but as the proper obſervations for

this purpoſe happen at a conſiderable diſtance of time from each other, it will be proper to allow for the motion of the aphelion in the intervals, which CASSINI aſſumes (from what he was beſt able to collect from the obſervations before made) at 1′. 20″ in a year, by which means the motion is reduced to the orbit as immoveable. In 1661 on May 3, at 4*h*. 48′. 28″ mean time, the true place of Mercury was found to be in ♏ 13°. 33′. 27″ in reſpect to the ecliptic, and 13°. 33′. 10″ on it's orbit. In 1690 on November 9, at 18*h*. 6′ it was in ♉ 18°. 20′. 46″ in reſpect to the ecliptic, and 18°. 22′. 28″ on it's orbit. In 1697 on November 2, at 17*h*. 42′ it was in ♉ 11°. 33′. 50″ in reſpect to the ecliptic, and 11°. 32′. 30″ on it's orbit. Now between the two firſt obſervations the motion of the aphelion was, by ſuppoſition, 39′. 20″; and between the firſt and laſt it was 48′. 40″; theſe ſubtracted from the ſecond and third obſervations give the places in the orbit ♉ 17°. 43′. 8″ and ♉ 10°. 43′. 50″ in reſpect to the firſt obſervation, the orbit being ſuppoſed at reſt. Hence by ſubtracting ♏ 13°. 33′. 10″ from ♉ 17°. 43′. 8″, we have 6ˢ. 4°. 9′. 58″ for the ſum of the two true anomalies of Mercury between the firſt and ſecond obſervations, the aphelion lying between the two obſerved places; and by ſubtracting ♉ 10°. 43′. 50″ from ♉ 17°. 43′. 8″, we have 6°. 59′. 18″ for the difference of the true anomalies between the ſecond and third obſervations. Alſo if we ſubtract 39′. 20″ from 6ˢ. 26°. 20′. 35″ the mean motion between the two firſt obſervations, and 48′. 40″ from 6ˢ. 21°. 51′. 7″ the mean motion between the firſt and third obſervations, we ſhall have 6ˢ. 25°. 41′. 15″ and 6ˢ. 21°. 2′. 27″ for the ſum of the mean anomalies in theſe intervals; hence 4°. 38′. 48″ is the mean anomaly correſponding to the two laſt obſervations, anſwering to 6°. 59′. 18″ of true anomaly. Hence, from the *ſimple elliptic* hypotheſis, the aphelion of Mercury at the ſecond obſervation is found to be in ♐ 10°. 51′. 50″, excentricity 0,21574, the mean diſtance being unity; and the greateſt equation 24°. 55′. 4″. This corrected ſeveral times gives the true place of the aphelion on November 9, 1690 in ♐ 12°. 22′. 25″, the excentricity 0,20878 and the greateſt equation 24°. 3′. CASSINI, in his Tables, makes it 24°. 2′. 58″. Dr. HALLEY makes it 23°. 42′. 36″. M. de la LANDE makes it 23°. 40′.

257. Beſides theſe methods of determining the poſition and excentricity of the planetary orbits, we ſhall explain another method, which may be ſometimes very ſucceſsfully uſed, and is moreover ſtrictly geometrical. By Art. 217, we may find the diſtance of a planet from the ſun in any point of it's orbit. The Problem therefore is, given in length and poſition three lines drawn from the focus of an ellipſe, to determine the ellipſe.

FIG. 55. 258. Let *SB*, *SC*, *SD* be the three lines; produce *CB*, *CD*, and take *SB* : *SC* :: *EB* : *EC*, and *SC* : *SD* :: *CF* : *DF*, then $SC - SB : SC :: BC : EC = \frac{SC \times BC}{}$

$\frac{SC \times BC}{SB - SB}$, and $SC - SD : SC :: DC : CF = \frac{SC \times DC}{SC - SD}$. Join FE, and draw DK, CI, BH perpendicular to it. Now by similar triangles, $IC : HB :: EC : EB ::$ (by con.) $SC : SB$; alſo, $IC : KD :: CF : DF :: SC : SD$. Hence the proportion of IC, HB, KD is the ſame as SC, SB, SD, conſequently EF is the directrix of the ellipſe paſſing through B, C, D. Through S draw $AS\mathfrak{Q}G$ perpendicular to FE; take $GA : AS :: CI : CS$, and $G\mathfrak{Q} : S\mathfrak{Q} :: CI : CS$; then $CI + CS : CS :: GS : S\mathfrak{Q} = \frac{CS \times GS}{CI + CS}$; alſo $AS = \frac{CS \times GS}{CI - CS}$, and $A, \mathfrak{Q}$ will be the vertices of the conic ſection.

259. *Calculation.* In the triangles SBC, SCD we know two ſides and the included angles, they being the diſtances of the obſerved places upon the orbit; hence we can find BC, CD and the angles BCS, SCD, and conſequently BCD. Hence (258) we know CE and CF, and the angle ECF being alſo known, the angle CEF can be found. Therefore in the right angled triangle CIE, CE and the angle E are given, hence CI is known. Join SI; then in the triangle SIC we know CI, CS and the angle $SCI (= ECI - BCS)$; hence we know SI and the angles CIS, CSI, and hence the angle SIG is known; therefore in the right angled triangle SIG, we know SI and the angle SIG, from whence SG is found. Hence (258) we know $SA, S\mathfrak{Q}$, half the difference of which is the excentricity, and their ſum $= A\mathfrak{Q}$. Laſtly, in the triangle BSO (O being the other focus) we know all the ſides, to find the angle BSA, the diſtance of the aphelion from the obſerved place B.

In the year 1740 on July 17, Auguſt 26, September 6, M. de la Caille found three diſtances of Mercury (the mean diſtance being 10000) as follows; SB 10351,5, $SC = 11325,5$, $SD = 9672,166$, the angle $BSC = 3^s. 27°. 0'. 35''$ and $CSD = 44°. 40'. 4''$. Hence $BCS = 29°. 55'. 5''$, $BC = 18941$, $SCD = 56°. 49'$, $CD = 8124,5$, $BCF = 86°. 44'. 5''$, $CE = 215004$, $CF = 55647$, $CEF = 14°. 41' 44''$, $CI = 54543$, $CSI = 124°. 47'. 45''$, $CIS = 9°. 49'. 4''$, $SI = 47281$, $SIG = 80°. 10'. 56''$, $SG = 46589$, $SP = 8010,5$, $SA = 12209$, $SO = 4198,5$, hence the excentricity $= 2099,75$, $BSA = 71°. 37'. 23''$ or $2^s. 11°. 37'. 23''$ which added to $6^s. 2°. 13'. 51''$, the poſition of SB, gives $8^s. 13°. 51'. 14''$ for the place of the aphelion. Hence the greateſt equation is $24°. 3'. 5''$.

260. Or from the ſame data the place of the aphelion and excentricity may be thus found. Put the ſemi-axis major $= 1$, $SB = a$, $SD = b$, $SC = c$, the angle $BSD = v$, $BSC = u$, $BS\mathfrak{Q} = x$, $OS = 2e$, half the parameter $= r$. Then by a well known property of the ellipſe, $a = \frac{r}{1 + e \cos. x}$, $b = \frac{r}{1 + e.\cos. v + x}$, $c = \frac{r}{1 + e.\cos. u + x}$; hence $r = a + ae.\cos. x = b + be.\cos. v + x = c$

$= c + ce.\cos.\overline{u+x}$; therefore $\frac{b-a}{a.\cos.x - b.\cos.\overline{v+x}} = e = \frac{c-a}{a.\cos.x - c.\cos.\overline{u+x}}$; now for $\cos.\overline{v+x}$ and $\cos.\overline{u+x}$ ſubſtitute $\cos.v.\cos.x - \sin.v.\sin.x$ and $\cos.u.\cos.x - \sin.u.\sin.x$, and we ſhall have

$$\frac{b-a}{a.\cos.x - b.\cos.v.\cos.x + b.\sin.v\ \sin.x} = \frac{c-a}{a.\cos.x - c.\cos.u.\cos.x + c.\sin.u\ \sin.x};$$

divide each denominator by $\cos.x$, and we have $\frac{b-a}{a - b.\cos.v + b.\sin.v.\tan x} = \frac{c-a}{a - c.\cos.u + c.\sin.u\tan.x}$; hence $\tan.x = \frac{b.\overline{c-a}.\cos.v - c.\overline{b-a}.\cos.u + a.\overline{c-b}}{b.\overline{c-a}.\sin.v - c.\overline{b-a}.\sin.u}$, which gives the place of the perihelion. Hence we know $e = \frac{c-a}{a.\cos.x - c.\cos.\overline{u+x}}$ the excentricity; conſequently $1-e$ and $1+e$ the perihelion and aphelion diſtances are known. This Theorem was firſt given by E. Prosperin, *Aſtron. Obſervatore reg.* in the *Nova Acta reg. ſoc. ſcien. Upſalienſis*, Vol. III. Mr. Robinson afterwards demonſtrated it by another method in the *Edin. Tranſ.* 1788, not knowing that it had been publiſhed before. The *Species* of the ellipſe being thus determined it's major axis may be thus found. Compute the mean anomaly correſponding to the angle *CSB*, then ſay, as that mean anomaly : 360° :: the time of deſcribing the angle *CSB* : the periodic time. The periodic time being known, the major axis is found (218) by Kepler's Rule.

261. Having explained the different methods of finding the place of the aphelion, excentricity and greateſt equation; it will be proper to explain the methods of examining at any time theſe elements in order to apply ſuch corrections as may be found neceſſary. M. de la Lande propoſes to examine the place of the aphelion by two obſervations, one near the aphelion and the other near the mean diſtance, ſuppoſing the equation of the center to be known. Calculate for each obſervation the equation of the center from the ſuppoſed place of the aphelion, and take the *difference* of the equations, if the two obſervations be on the *ſame* ſide of the aphelion, but the *ſum* if on *different* ſides; and the difference or ſum of the equations will ſhow how much the true motion differs from the mean, the mean being known from the known interval of the obſervations. Hence if the difference *calculated* agree with the difference obſerved, the place of the aphelion was rightly aſſumed; but if the true motion by calculation differ more from the mean motion than the true motion by obſervation does, the place of the aphelion was too near to or too far from the obſervation made near the mean diſtance. Aſſume therefore another place for the aphelion, as you may judge proper from the circumſtances, and trying again, you will

ſoon

ſoon find the true place. For at the mean diſtance, the equation being a maximum it alters there but a very little for ſome time; therefore the whole difference ariſes principally from the equation at the obſervation near the aphelion; conſequently the alteration of the place of the aphelion will deſtroy that difference.

262. M. de la LANDE propoſes another method of examining the place of the aphelion of the orbits of *Venus* and *Mercury*, by means of their greateſt elongations when at their mean diſtances. Let E be the earth, V the place of the greateſt elongation and A the aphelion according to the Tables to be examined; a the true place of the aphelion. Now the planet being near it's mean diſtance, it's computed heliocentric longitude will not be ſenſibly altered by a ſmall alteration of the aphelion, but it's diſtance from the ſun will be moſt altered; we may therefore ſuppoſe the obſerved place to be at v; hence the difference (d) of the obſerved and computed longitudes is the angle VEv. For any aſſumed alteration ASa (m) of the aphelion compute the variation Vv of the diſtance, and thence find the correſponding angle VEv(n), and we have $n : d :: m :$ the alteration of the aphelion from the place A in the Tables in order to make the obſerved and computed places agree. The aphelion is too backward by the angle ASa, when the perihelion is in inferior conjunction and the computed longitude is leſs than the obſerved, or when the aphelion is in inferior conjunction and the computed longitude is greater than the obſerved. In all other caſes the aphelion is too forward. FIG. 56.

On May 24, 1764, at $8^h.\ 7'.\ 50''$ true time, M. de la LANDE obſerved the greateſt elongation of Mercury at v, at about $9^s.\ 8°$ of anomaly in going from ſuperior to inferior conjunction, to be $22°.\ 51'.\ 12''$, and it's longitude to be $2^s.\ 26°.\ 50'.\ 35''$. Now by Dr. HALLEY's Tables it's longitude at V computed at that time was $2^s.\ 26°.\ 51'.\ 49''$, which was $1'.\ 14''$ greater than that obſerved. Now in the orbit of Mercury, an angle ASa of $1°$ anſwers to an angle VEv of very nearly $5'$; hence to find the angle ASa correſponding to $VEv = 1'.\ 14''$, ſay, $5' : 1° :: 1'.\ 14'' : 14'.\ 48''$ the angle ASa; therefore the place of the aphelion in Dr. HALLEY's Tables was $14'.\ 48''$ too backward, and the place thus corrected is found to agree with obſervation.

263. We have now fully explained the different methods of finding the place of the aphelion, excentricity and greateſt equation; but as it may appear, by comparing the computations with obſervations that the elements may not be accurate, M. de la CAILLE has given the following method of correcting them, which will be beſt underſtood by an Example; we ſhall therefore give that publiſhed by himſelf in the *Hiſtoire de l'Académie Royale des Sciences* for the year 1750, upon the elements of the theory of the ſun. Let $AIOM$ be the earth's orbit, FIG. 57.

orbit, *S* the ſun, *M*, *I*, *O** three places of the earth obſerved on March 29, July 6, and October 3, in the year 1749; *A* the aphelion, ſuppoſed to be in 3ˢ. 8°. 38′. 51″ on January 1, 1749, and it's annual motion 1′. 3″. The ſun's mean longitude at the ſame time was ſuppoſed to be 9ˢ. 10°. 15′. 6″. By obſervation, M. de la CAILLE found the angle $ISM = 95°.\ 27'.\ 7''$, $ISO = 85°.\ 58'.\ 34''$, theſe being the differences of the three true anomalies; and the correſponding mean anomalies were 97°. 34′. 26″, and 87°. 42′. 26″. Now we firſt make two ſuppoſitions for the excentricity, and aſſume two true anomalies for the point *M*, and from thence calculate the angle *ISM* and compare it with the obſervation.

	Firſt Hypotheſis	*Second Hypotheſis*
Excentricity ſuppoſed	0,01681	0,016825
Firſt aſſumed true anomaly of *M*	89°. 50′. 0″,0	89°. 50′. 0″,0
Hence the true anomaly of *I*	5 · 37 · 7	5 · 37 · 7
Mean anomaly of *M* by calculation	91 · 45 · 34, 6	91 · 45 · 50, 9
Mean anomaly of *I* by calculation	5 · 48 · 34, 5	5 · 48 · 36, 1
Sum of the two mean anomalies ex hyp.	97 · 34 · 9, 1	97 · 34 · 27
Sum of the two mean anomalies from obſ.	97 · 34 · 26	97 · 34 · 26
Difference, or error of the hypotheſis	− 16, 9	+ 1
Second aſſumed true anomaly of *M*	89 · 40 · 0, 0	89 · 40 · 0, 0
Hence the true anomaly of *I*	5 · 47 · 7	5 · 47 · 7
Mean anomaly of *M* by calculation	91 · 35 · 34, 7	91 · 35 · 50, 9
Mean anomaly of *I* by calculation	5 · 58 · 54, 9	5 · 58 · 56, 6
Sum of the two mean anomalies ex hyp.	97 · 34 · 29, 6	97 · 34 · 47, 5
Sum of the two mean anomalies from obſ.	97 · 34 · 26	97 · 34 · 26
Difference, or error of the hypotheſis	+ 3, 6	− 21, 5

Hence we have the following proportion for each hypotheſis; *As the ſum of the errors (or difference when they have the ſame ſign)* : *the leaſt error* :: *the difference of the two true anomalies, ſuppoſe in M,* : *the quantity to be applied to the aſſumed anomaly in M, anſwering to the leaſt error*; this quantity is to be added or ſubtracted according as the ſign of the error was − or +. With the aſſumed anomaly in *M* thus corrected, and the ſame excentricity, we proceed as before;

* Two of the obſervations ought to be near the mean diſtance, and one near the apſides, or two near the aphelion and one near the mean diſtance, as ſuch obſervations will add to the accuracy of the concluſion. Two obſervations near the apſides will beſt determine the place of the aphelion, and two near the mean diſtance will give moſt accurately the equation. The obſervations may be made at any intervals of time, provided we know the motion of the aphelion, ſo as to be able to reduce the obſervations to what they would have been if the orbit had been fixed. The longitudes ſhould alſo be reduced (268) to the orbit of the planet. The time of the planet's revolution is alſo ſuppoſed to be known, in order to find (239) it's mean motion.

Cor-

	First Hypotheſis	*Second Hypotheſis*
Corrected anomaly of *M*	89°. 41′. 46″,0	89°. 50′. 26″,7
Hence the anomaly of *I*	5 . 45 . 22,	5 . 36 . 43,
Mean anomaly of *M* by calculation . .	91 . 37 . 19, 7	91 . 46 . 14, 9
Mean anomaly of *I* by calculation . . .	5 . 57 . 6, 3	5 . 48 . 11, 2
Sum of the two mean anomalies ex hyp. .	97 . 34 . 26,	97 . 34 . 26, 1
Sum of the two mean anomalies from obſ. .	97 . 34 . 26,	97 . 34 . 26
Difference, or error of the hypotheſis . .	0 . 0 . 0	+ 0, 1

We have therefore two ſuppoſitions of the excentricity which anſwer to the two obſervations in *M* and *I*. We muſt therefore next ſee how theſe hypotheſes will agree with the obſervations in *I* and *O*. Aſſuming therefore the anomalies in *I* as above, we proceed thus.

	First Hypotheſis	*Second Hypotheſis*
The mean anomaly of *I* was found . .	5°. 57′. 6″,3	5°. 48′. 11″,2
Mean anomaly anſwering to the angle *ISO*	87 . 42 . 26	87 . 42 . 26
Hence the mean anomaly of *O* is . . .	93 . 39 . 32, 3	93 . 30 . 37, 2
True anomaly of *I*	5 . 45 . 22, 0	5 . 36 . 43, 0
The angle *ISO* by obſervation	85 . 58 . 34	85 . 58 . 34
Hence the true anomaly of *O*	91 . 43 . 56	91 . 35 . 17
Hence the mean anomaly of *O* from obſ. .	93 . 39 . 24, 5	93 . 31 . 2, 6
Difference from that which we want to find	− 7, 8	+ 25, 4

Hence we have the following proportion; *As the ſum of the errors (or difference when they have the ſame ſign)* : *the leaſt error* (which here belongs to the firſt hypotheſis) :: *the difference between the two ſuppoſed excentricities* : *the quantity to be applied to the firſt excentricity*; hence 33″,2 : 7″,8 :: 0,00004 : 0,0000094; now one excentricity giving a reſult −, and the other +, the true excentricity muſt be between them; hence 0,01681 + 0,0000094 = 0,0168194 the excentricity.

Again, *As the ſum of the ſame errors* : *the leaſt error* :: *the difference of the two true anomalies, ſuppoſe in M,* : *the quantity to be applied to the true anomaly in M anſwering to the leaſt difference*; hence 33″,2 : 7″,8 :: 8′. 39″ :: 2′. 2″, which added to 89°. 41′. 46″ gives 89°. 43′. 48″ the true anomaly of *M*. But the obſerved place of *M* was 8°. 55′. 21″ of longitude; hence the place of the aphelion on March 29, was 3ˢ. 8°. 39′. 9″. And if we allow 15″ for the motion of the apogee in reſpect to the equinoctial points from January 1, we ſhall have the true place of the ſun's apogee on January 1, 1749, to be 3ˢ. 8°. 38′. 54″. From the mean of ſeveral obſervations, M. de la Caille found the apogee at that time to be 3ˢ. 8°. 39′. 40″, and the excentricity 0,016293.

264. All the epochs in our Aſtronomical Tables are reckoned from noon on December 31, in the common years, and from January 1, in the biſſextiles.

Hence to find the epoch of the mean longitude, from the place of the aphelion and the true longitude of the planet at the time, you have the diſtance of the planet from the aphelion, or the true anomaly, from which find the mean anomaly and add it to the place of the aphelion, and you have the mean longitude at that time. Then take the interval from that time to that of the epoch, and find the mean motion correſponding, and add it to the mean longitude, and you have the mean longitude at the epoch. If you know the time when the planet paſſes the aphelion, you have then only to add to the place of the aphelion the mean motion from that time to the time of the epoch, becauſe at the aphelion the true and mean longitudes are the ſame.

Ex. On June 29, 1744, at $15^h.\ 57'.\ 46''$ the ſun was found in it's aphelion in $3^s.\ 8^\circ.\ 31'.\ 55''$. From that time to the laſt day of December at noon is 184 days $8^h.\ 2'.\ 14''$, in which time the mean motion is $6^s.\ 1^\circ.\ 40'.\ 21''$, which added to $3^s.\ 8^\circ.\ 31'.\ 55''$ gives $9^s.\ 10^\circ.\ 13'.\ 16''$ for the mean longitude on December 31, 1744, at noon, as deduced from this one obſervation. From the mean of ſeveral obſervations, M. de la CAILLE makes the mean longitude at the beginning of 1749, to be $9^s.\ 10^\circ.\ 15'.\ 17'',5$.

On February 15, 1743, at $19^h.\ 17'.\ 40''$ true time, the mean anomaly of *Mars*, according to M. de la LANDE, was $11^s.\ 25^\circ.\ 6'.\ 42''$, and the place of the aphelion $5^s.\ 1^\circ.\ 20'.\ 39''$, the ſum of which is $4^s.\ 26^\circ.\ 27'.\ 21''$ the mean longitude of Mars at that time; and the mean motion of Mars from that time to January 1, 1744, (that being leap year) was $15^s.\ 17^\circ.\ 16'.\ 53''$; hence we find the mean longitude for January 1, 1744, to be $10^s.\ 13^\circ.\ 45'.\ 14''$. In like manner we find the epochs of the mean longitudes of all the planets.

MEAN LONGITUDE FOR THE BEGINNING OF 1750.

Planets	M. CASSINI	Dr. HALLEY	M. de la LANDE
Sun	$9^s.\ 10^\circ.\ 0'.\ 35''$	$9^s.\ 10^\circ.\ 0'.\ 13''$	$9^s.\ 10^\circ.\ 0'.\ 35'',5$
Mercury	8. 13. 19. 5	8. 13. 7. 45	8. 13. 11. 15
Venus	1. 16. 19. 21	1. 16. 19. 23	1. 16. 20. 48
Mars	0. 21. 58. 43	0. 21. 58. 30	0. 21. 58. 47
Jupiter	0. 4. 0. 59	0. 4. 5. 17	0. 3. 42. 29
Saturn	7. 20. 41. 56	7. 20. 26. 24	7. 21. 20. 22

PLACE

PLACE OF THE APHELIA FOR THE BEGINNING OF 1750.

Planets	M. Cassini	Dr. Halley	M. de la Lande
Mercury	8ˢ. 13°. 41′. 18″	8ˢ. 13°. 27′. 12″	8ˢ. 13°. 33′. 58″
Venus	10. 7. 38. 0	10. 7. 18. 31	10. 7. 46. 42
Earth	3. 8. 27. 23	3. 8. 28. 43	3. 8. 37. 16
Mars	5. 1. 36. 9	5. 1. 31. 38	5. 1. 28. 14
Jupiter	6. 10. 14. 33	6. 10. 33. 46	6. 10. 21. 4
Saturn	8. 29. 13. 31	8. 29. 39. 58	8. 28. 9. 7

EXCENTRICITY OF THE ORBITS,

THE MEAN DISTANCE OF THE EARTH FROM THE SUN BEING 100000.

Planets	M. Cassini	Dr. Halley	M. de la Lande
Mercury	8092,5	7970	7955,4
Venus	517	504,985	498
Earth	1690	1691,9	1681,395
Mars	14155	14170	14183,7
Jupiter	25060	25078,6	25013,3
Saturn	54320	54381,4	53640,42
Georgian	* * *	* * *	90804

GREATEST EQUATIONS.

* * *	1740,	1719,	1750,
Planets	M. Cassini	Dr. Halley	M. de la Lande
Mercury	24°. 2′. 58″	23°. 42′. 36″	23°. 40′. 0″
Venus	0. 49. 6	0. 48. 0	0. 47. 20
Earth	1. 55. 51	1. 56. 20	1. 55. 36, 5
Mars	10. 39. 19	10. 40. 2	10. 40. 40
Jupiter	5. 31. 17	5. 31. 36	5. 30. 38, 3
Saturn	6. 31. 40	6. 32. 4	6. 26. 42
Georgian	* * *	* * *	5. 27. 16

The place of the aphelion of the *Georgian* Planet in 1788, was 11°. 16°. 19′. 30″, and mean longitude 3°. 14°. 49′. 14″, according to M. de la Lande.

265. The greateſt equations, and conſequently the excentricities of the orbits, are ſubject to a variation, ariſing from the mutual attractions of the planets. M. de la Grange, in the *Berlin Acts* for 1782, has calculated the variation of the greateſt equations for each, from the attraction of the others, and has found it for 100 years to be as in the following Table.

* * *	Mercury	Venus	Earth	Mars	Jupiter	Saturn
By Mercury	* * *	− 9″,02	− 0″,80	+ 0″,22	* * *	* * *
— Venus	+3″,04	* * *	+ 4, 18	+ 0, 22	* * *	* * *
— Earth	+0, 58	− 9, 02	* * *	+ 3, 66	* * *	* * *
— Mars	−0, 22	− 0, 64	− 4, 94	* * *	− 0″,02	* * *
— Jupiter	−1, 26	− 6, 16	−16, 02	+31, 68	* * *	−1′.50″,6
— Saturn	+0, 02	− 0, 14	− 0, 08	+ 1, 30	+56, 28	* * *
Whole Change	+2, 16	−24, 98	−17, 66	+37, 08	+56, 26	−1. 50,6

In

In this Table, the quantity of matter in Venus is ſuppoſed to be 1,31, that of the Earth being unity; but the denſity, and conſequently the quantity of matter, in Venus is ſubject to ſome uncertainty. If any other quantity of matter be aſſumed, the numbers in the horizontal line oppoſite to Venus will vary in the ſame ratio. The equation of the *Georgian* Planet is diminiſhed 0″,01 by Jupiter, and 0″,1 by Saturn, according to M. de la GRANGE.

A new Method of correcting the Elements of the Orbit of a Planet.

266. LEMMA. If any quantity z be aſſumed, and the value of any function of it be computed; then if z be increaſed by any very ſmall quantity v, the *variation* of the ſame function will be in proportion to v. This is a propoſition well known to Mathematicians.*

267. Given three obſerved heliocentric longitudes of a planet, the times of obſervations, and it's periodic time; alſo the place of the aphelion of it's orbit, and it's excentricity are ſuppoſed to be very nearly known; to correct theſe two elements. Let S be the ſun, M, I, O the three given obſerved places of the planet, A the eſtimated place of the aphelion, and SC the ſuppoſed excentricity. As the intervals of time of the planet's motion from M to I and from I to O are known, and the periodic time is given, the mean anomalies between M and I, I and O will be known (222); call theſe p and q reſpectively; and as the points M, I, O, A are given, the angles ASM, ASI, ASO are known, the three true anomalies; compute therefore (232) the three correſponding mean anomalies, and from thence we ſhall know the mean anomalies between M and I, I and O; call theſe P and Q. Then if $P=p$, and $Q=q$, the computed agree with the obſerved places, and conſequently the place of the aphelion and the excentricity were rightly aſſumed. But if P be not equal to p, let it be, for inſtance, leſs by m, and let Q be leſs than q by n. Now increaſe the place of the aphelion by a very ſmall quantity x, and compute the mean anomalies between M and I, I and O again, and let the correſponding errors be m' and n'. Hence from increaſing the place of the aphelion by x, the alteration of the mean anomalies between M and I, I and O will be $m \pm m'$ and $n \pm n'$ reſpectively, according as the errors are of a different or of the ſame kind. Increaſe

FIG. 57.

* To give the Reader an idea on which the truth of this depends, let A be the reſult of the firſt computation. Then for z ſubſtitute $z + v$, and compute again the ſame quantity with this new value of z. Now as v is very ſmall, we may reject all it's powers above the firſt, conſequently the ſecond reſult will be $A \pm mv$, m being ſome known coefficient; becauſe when $v = o$ the two reſults muſt be the ſame. Hence the variation mv of the firſt reſult is in proportion to v.

the excentricity by a very ſmall quantity y, and let the errors of the mean anomalies between M and I, I and O be m'' and n''; then will $m \pm m''$ and $n \pm n''$ be the correſponding alterations of the mean anomalies from the increaſe y of excentricity. Let x' and y' be the alterations neceſſary to be made to the firſt aſſumed place of the aphelion and the excentricity, in order to correct the errors m and n. Then (266) $x : x' :: m \pm m' : \frac{x' \times \overline{m \pm m'}}{x}$ the change of mean anomaly between M and I from the alteration x'; alſo $y : y' :: m \pm m'' : \frac{y' \times \overline{m \pm m''}}{y}$ the change which ariſes from the alteration y'. But we want to increaſe the mean anomaly between M and I which ariſes from the firſt aſſumed place of the aphelion and the excentricity, by the quantity m; hence we muſt aſſume $\frac{x' \times \overline{m \pm m'}}{x} + \frac{y' \times \overline{m \pm m''}}{y} = m$. For the ſame reaſon, $x : x' :: n \pm n' : \frac{x' \times \overline{n \pm n'}}{x}$ the change of mean anomaly between I and O from the alteration x'; alſo $y : y' :: n \pm n'' : \frac{y' \times \overline{n \pm n''}}{y}$ the change ariſing from the alteration y'. But we want to increaſe the mean anomaly between I and O from the firſt aſſumption, by the quantity n; hence we muſt aſſume $\frac{x' \times \overline{n \pm n'}}{x} + \frac{y' \times \overline{n \pm n''}}{y} = n$. Put $\frac{m \pm m'}{x} = a$, $\frac{m \pm m''}{y} = b$, $\frac{n \pm n'}{x} = c$, $\frac{n \pm n''}{y} = d$, and we have $ax' + by' = m$, $cx' + dy' = n$; hence $x' = \frac{dm - bn}{da - bc}$ and $y' = \frac{cm - an}{cb - ad}$, the corrections to be made to the firſt aſſumed place of the aphelion and the excentricity in order to make the computed agree with the obſerved mean anomalies. Thus we correct at once the two elements. If P or Q be greater than p or q, then, as the aſſumed place of the aphelion and the excentricity give the mean anomalies between M and I, I and O too great by m or n, it is manifeſt that the alteration which we want to produce, by altering theſe two elements, is to diminiſh the computed mean anomalies by m or n; to effect which, we muſt aſſume the alterations equal to $-m$ or $-n$. Regard muſt alſo be had to the ſigns of $m \pm m'$, $m \pm m''$, $n \pm n$, $n \pm n''$, by conſidering, whether the aſſumed variations x and y have produced an increaſe or decreaſe of the mean anomalies between M and I, I and O, and writing them poſitive or negative accordingly. The circumſtance of any particular caſe will immediately point out theſe matters.

To

To find the Reduction of a Planet to the Ecliptic.

268. Let γC be the ecliptic, AB the orbit of a planet, N the aſcending node, γC the order of the ſigns, P the place of the planet, and Pm perpendicular to γC; then Nm, reckoned from N according to the order of the ſigns, is called the *argument of latitude*, becauſe the latitude Pm depends upon Nm; hence to get the argument of latitude, we muſt always ſubtract the place of the node from the place of the planet reduced to the ecliptic, adding 12 ſigns to the latter if it be the leaſt. Take $NA = N\gamma$, and the longitude of a planet upon it's orbit is computed from the point A; hence the longitude on the orbit is $AP = AN + NP$; and the longitude on the ecliptic is $\gamma m = \gamma N + Nm = AN + Nm$; the difference of theſe longitudes is the difference between NP and Nm, which difference applied to the longitude of the planet upon the ecliptic, adding it to or ſubtracting it from, according as Nm is leſs or greater than NP, that is, as Nm is between 0° and 90° or 180° and 270°, or between 90° and 180° or 270° and 360°, gives the longitude upon it's orbit. This difference is called the *Reduction*. FIG. 58.

269. To find the reduction, put $c =$ the coſine of the angle PNm, $t =$ the tangent of Nm the argument of latitude; then the cotan. $PN = \frac{\text{rad.} \times c}{t}$; hence 10, + log. c − log. t = log. cotan. PN; and the difference between PN and Nm is the reduction required.

Ex. Let the inclination of the orbit of *Mercury* be 7°, and the argument of latitude 30°. 17′. 48″; then

7°. 0′. 0″	cos. + 10, =	19,9967507
30. 17. 48	tan. =	9,7666171
30. 29. 1	cot.	10,2301336
0. 11. 13	the Reduction.	

In the Tables of the planet's motions, a Table of reductions is given, which applied to NP gives Nm; or applied to the longitude of a planet on it's orbit gives the longitude upon the ecliptic; but if applied with a contrary ſign to the longitude on the ecliptic it gives the longitude on it's orbit. In like manner a reduction may be applied to the ſun's longitude to find it's right aſcenſion, or the contrary.

CHAP.

CHAP. XIV.

ON THE MOTION OF THE APHELIA OF THE ORBITS OF THE PLANETS.

Art. 270. HAVING explained in the laſt Chapter the methods of finding the place of the aphelia of the orbits of the planets, we proceed next to determine their motion, ariſing from their mutual attraction, which is immediately done by comparing the places as ſettled by the ancient and modern obſervations; or by comparing the length of an anomaliſtic with that of a tropical or ſidereal revolution.

271. To find the motion of the *Earth's* apogee. HIPPARCHUS, 140 years before J. C. determined it's place to be $2^s.\ 5°\frac{1}{2}$; and by the obſervations of WALTHERUS in 1496, the place was found to be $3^s.\ 3°.\ 57'.\ 57''$; from theſe obſervations, the motion of the apogee is $1'.\ 2''\frac{3}{4}$ in a year in reſpect to the equinoctial points. M. de la CAILLE determined the place of the apogee for the beginning of the year 1749 to be $3^s.\ 8°.\ 39'$; which compared with the obſervation of WALTHERUS gives $1'.\ 6''$ for the yearly motion. In the year 1588, TYCHO determined the place of the apogee to be $3^s.\ 5°.\ 30'$; and KEPLER in the ſame year determined it's place to be $3^s.\ 5°.\ 32'$. Theſe compared with the obſervation of CASSINI in the year 1738, who determined it's place to be then in $3^s.\ 8°.\ 19'.\ 8''$, give about $1'.\ 7''$ for the annual motion. M. de la CAILLE determined the length of the anomaliſtic year to be $26'.\ 35''$ longer than the tropical year, which makes the motion of the apogee to be $1'.\ 5'',5$ in a year. KEPLER made it $1'.\ 2''$; RICCIOLUS, $1'.\ 2''.\ 4'''.\ 4''''$ in a year. MAYER in his Tables makes it $1'.\ 6''$. Dr. HALLEY makes it $1'.\ 1''$; and CASSINI about $1'.\ 1'',25$. M. de la LANDE in his Tables makes it $1'.\ 2''$ as computed by M. de LAMBRE from Dr. MASKELYNE's obſervations in 1788; and this determination is moſt to be depended upon, as made by ſo eminent an Aſtronomer, from obſervations which are acknowledged to be the beſt that have been ever made. Theſe motions are in reſpect to the equinox. If we aſſume it to be $1'.\ 2''$, and the preceſſion of the equinoxes to be $50''\frac{1}{4}$, we ſhall have the *real* motion of the apogee to $11''\frac{3}{4}$ in a year.

272. To determine the motion of the aphelion of *Saturn*. The place of the aphelion in 1694 was $8^s.\ 28°.\ 58'$; but from three oppoſitions obſerved in the years 127, 133 and 136, it's place for the year 132 was $7^s.\ 24°.\ 14'.\ 29''$, which

makes

makes the annual motion 1′. 20″. TYCHO found the place of the aphelion on December 19, 1590, to be 8ˢ. 25°. 40′. 51″, which compared with the obſervation in 132 gives 1′. 18″,5 for the annual motion. The ſame obſervation of TYCHO compared with the place of the perihelion on December 12, 1708, in 8ˢ. 28°. 25′. 10″, gives 1′. 23″,5 for the annual motion. If the ſame obſervation of TYCHO be compared with the place of the aphelion in April 1694 in 8ˢ. 28°. 58′ it gives 1′. 55″ for the annual motion. CASSINI conjectured from all this, that the motion of the aphelion was quicker now than formerly. He alſo found the perihelion in 1708 not ſo forward by a degree as it ought, when compared with the place of the aphelion in 1694 at the annual movement of 1′. 20″; from whence he ſuſpected that the orbit had a librating motion, and that there ought to be an equation employed between the two points. The irregularities of Saturn however, as we have before obſerved, are ſo great, that we need not wonder at theſe differences. KEPLER makes it 1′. 16″. CASSINI ſuppoſes it to be 1′. 18″, and Dr. HALLEY 1′. 20″. M. de la GRANGE, from calculating the diſturbing force of each planet upon the other, has determined the annual motion of the aphelion to be 1′. 6″,3. M. de la PLACE makes it 1′. 6″,07, which M. de la LANDE has employed in his Tables.

273. To determine the motion of *Jupiter's* aphelion. According to the obſervations of PTOLEMY, the aphelion was in ♍ 14°. 38′ in the year 136; but in 1720 it was in ♎ 9°. 47′; this gives 57″. 11‴ for the annual motion. In the year 1590, the place of the aphelion, calculated from the obſervations of TYCHO, was found to be in ♎ 6°. 30′. 43″; this compared with the obſervation in 1720, gives 1′. 30″ for the annual motion. If we compare the places in 136, and 1590, they give 54″ for the annual motion. This induced CASSINI to think, that the motion of the aphelion is accelerated; or that it was ſubject to ſome irregularities; he ſtates the motion at 57″. 24‴. KEPLER makes it 47″. Dr. HALLEY makes it 72″. M. JEAURAT computed the place of the aphelion in 1590 to be in ♎ 7°. 49′. 19″, and in 1762 in ♎ 10°. 36′. 41″; from which he found the annual motion to be 58″,4. EULER, from the theory of attraction, found it to be 55″. M. de la GRANGE, 57″,2. M. WARGENTIN ſays, that an annual motion of 62″ beſt agrees with obſervation. M. de la LANDE has employed 56″,73 in his laſt Tables, according to the theoretical determination of M. de la PLACE.

274. To determine the motion of the aphelion of *Mars*. From three oppoſitions obſerved by PTOLEMY, the place of the aphelion in 135 was found to be 3ˢ. 29°. 24′; and by the obſervations made at Greenwich in 1691, 1696 and in 1700, the place was found to be in 5ˢ. 0°. 31′. 34″ in 1696; hence the annual motion of the aphelion is 1′. 11″. 47‴. 20⁗. KEPLER makes it 1′. 7″. Dr. HALLEY makes it 1′. 12″. From comparing the place in 1748 in 5ˢ. 1°. 26′. 10″ with the place in 1592 in 4ˢ. 28°. 49′. 50″, the motion is 1′. The

mean

mean of theſe determinations is 1′. 7″,5. M. de la LANDE ſuppoſes it to be 1′. 7″.

275. To determine the motion of the aphelion of *Venus*. CASSINI has found, from computing the place of the aphelion from the ancient obſervations, a difference of 15°, from which uncertainty it is more difficult to determine it's annual motion. However, the place, computed from the obſervations in 136, 138 and 140, (and which he thinks are the moſt to be depended upon) was found in 138 to be in ♐ 21°. 29′; this compared with the obſervations in 1715, 1716 and 1718 when it was found to be in ♒ 6°. 50′ in 1716, the annual motion is found to be 1′. 42″. 50‴. From comparing the place in 1596 in ♒ 1°. 54′ with the place in 1716 in ♒ 6°. 50′, the motion is 2′. 28″. HORROX fixed the place of the aphelion in 1639 in ♒ 5°; this compared with the place in 1716, gives 1′. 26″ for the motion. M. de la LANDE employed the ſame method to ſettle the place of the aphelion of Venus as for Mercury, which we have explained in Art. 262. By comparing the place of the aphelion in his firſt Tables with the place in KEPLER's Tables, the annual motion comes out 2′. 41″,5. CASSINI makes it 1′. 26″, and Dr. HALLEY 56″,5. KEPLER makes it 1′. 18″. Amidſt ſo much uncertainty, M. de la LANDE thinks it better to depend upon the theory, which, according to M. de la GRANGE, makes it 48″,5, and which M. de la LANDE employs in his Tables. On account of the ſmall excentricity, this uncertainty of the place of the aphelion is not of ſo much conſequence, as an error of 1° in the place of the aphelion will never produce an error of 1′ in the heliocentric longitude.

276. To determine the motion of the aphelion of *Mercury*. From the obſervations of the paſſages of Mercury over the ſun in 1661, 1690 and 1697, CASSINI determined the place of the aphelion on November 9, 1690, to be in 8ˢ. 12°. 22′. 25″; and upon ſuppoſition that the motion of the aphelion was 1′. 20″ in a year, he found that it repreſented the paſſages very well in 1631, 1672, 1723 and 1736. But as theſe paſſages were nearly at the ſame point of the orbit, it does not ſufficiently eſtabliſh 1′. 20″ to be the true motion, as it might anſwer to the ſame points nearly, but not to other parts of the orbit. We ought not therefore to be ſurpriſed, ſays M. de la LANDE, that a motion of 52″,5 by Dr. HALLEY anſwers equally well to the ſame obſervations. KEPLER makes it 1′. 45″. M. de la LANDE found, by the greateſt equation, that on May 6, 1753, the place of the aphelion was 8ˢ. 13°. 55′. From comparing this place with the place computed from 8 obſervations of PTOLEMY, (rejecting 6 others, 2 of which did not appear to be reconcilable with each other, and 4 were too near the aphelion) he found the motion to be 1′. 10″ in a year, which he conſtructed his firſt Tables upon; obſerving however at the ſame time, that this motion does not agree perfectly with the obſervations in this century. He has

has ſince found that a motion of 56″,25 will beſt agree with obſervation; and this he has aſſumed in his laſt Tables. M. de la GRANGE makes it 57″ by theory. The motions of the aphelia here determined are their motions in longitude; if therefore we ſubtract 50″,25 (the annual preceſſion of the equinoxes) from each, we ſhall get their real motions.

MOTION OF THE APHELIA IN ONE HUNDRED YEARS.

Planets	M. CASSINI	Dr. HALLEY	M. de la LANDE
Mercury	2°. 13′. 20″	1°. 27′. 37″	1°. 33′. 45″
Venus	2. 23. 20	1. 34. 13	1. 21. 0
Earth	1. 42. 55	1. 41. 7	1. 43. 35
Mars	1. 59. 38	1. 56. 40	1. 51. 40
Jupiter	1. 35. 42	2. 0. 0	1. 34. 33
Saturn	2. 9. 44	2. 13. 20	1. 50. 7

According to the calculations of M. de la GRANGE, the aphelion of the *Georgian* Planet is progreſſive 3″,17 in a year, from the action of Jupiter and Saturn; conſequently it's motion in longitude is 50″,25 + 3″,17 = 53″,42. He has alſo calculated the effect of each planet in diſturbing the aphelia of the reſt. The following Table contains the annual effect.

ANNUAL MOTION OF THE APHELIA.

. . .	Mercury	Venus	Earth	Mars	Jupiter	Saturn
By Mercury	. . .	− 4″,30	− 0″,42	0″,02	0″,00	0″,00
— Venus	4″,14	. . .	+ 5, 20	0, 70	0, 01	0, 00
— Earth	0, 84	− 5, 06	. . .	1, 92	0, 01	0, 00
— Mars	0, 04	+ 1, 18	+ 1, 54	. . .	0, 00	0, 00
— Jupiter	1, 56	+ 6, 38	+ 6, 79	12, 31	. . .	15, 99
— Saturn	0, 08	+ 0, 08	+ 0, 19	0, 70	6, 56	. . .
Real motion	6, 66	− 1, 72	13, 30	15, 65	6, 58	15, 99
Preceſſion	50, 25	50, 25	50, 25	50, 25	50, 25	50, 25
Mot. in long.	56, 91	48, 53	63, 55	65, 90	56, 83	66, 24

M. de la GRANGE here ſuppoſes, as before, the denſity of Venus to be 1,31, but M. de la LANDE. makes it only 0,95; for this denſity therefore, the ſecond horizontal line muſt be diminiſhed in the ratio of 1,31 to 0,95.

KEPLER makes the earth's apogee to have coincided with the equinoctial point ♈, on July 24, in the year 3993 before J. C. which, according to ſome Authors, is about the time of the Creation. At the ſame time he makes the aphelion of Saturn to be ♌ 24°. 28′. 6″; of Jupiter ♋ 23°. 34′. 18″; of Mars ♉ 15°; of Venus ♎ 0°. 0′. 0″; of Mercury ♋ 0°. 0′. 0″; and the apogee of the Moon ♎ 0°. 0′. 0″.

CHAP.

CHAP. XV.

ON THE NODES AND INCLINATIONS OF THE ORBITS OF THE PLANETS TO THE ECLIPTIC.

Art. 277. FROM obſerving the courſe of the planets for one revolution, their orbits are found to be inclined to the ecliptic, for they appear only twice in a revolution to be in the ecliptic; and as it is frequently requiſite to reduce their places in the ecliptic, aſcertained from obſervation, to the correſponding places in their orbits, it is neceſſary to know the inclinations of their orbits to the ecliptic, and the points of the ecliptic where their orbits interſect it, called the *Nodes*. But previous to this, we muſt ſhow the method of reducing the places of the planets ſeen from the earth to the places ſeen from the ſun, and how to compute the heliocentric latitudes.

278. Let E be the place of the earth, P the planet, S the ſun, γ the firſt point of aries; draw Pv perpendicular to the ecliptic, and produce ES to a. Compute,* at the time of obſervation, the longitude of the ſun ſeen at a, and you have the longitude of the earth at E, or the angle γSE; compute alſo the longitude of the planet, or the angle γSv, and the difference of theſe two angles is the angle ESv of *commutation*. Obſerve the place of the planet in the ecliptic; and the place of the ſun being known, we have the angle vES of elongation in reſpect to longitude; hence we know the angle SvE, which meaſures the difference of the places of the planet ſeen from the earth and the ſun; therefore the place of the planet ſeen from the earth being known, the place ſeen from the ſun will be known. Alſo tan. PEv : rad. :: vP : Ev

rad. : tan. PSv :: vS : vP

$\therefore$ tan. PEv : tan. PSv :: vS : Ev :: ſin. SEv : ſin. ESv; that is, *the ſine of elongation in longitude : ſin. of the difference of the longitudes of the earth and planet :: tan. of the geocentric latitude : tan. of the heliocentric latitudes.* When the latitude is ſmall, vS : Ev very nearly as PS : PE, which, in oppoſition, is very nearly as PS : $PS - SE$. Or we may, as Mr. BUGGE obſerves, compute (223) the values of PS and SE, which we can do with more accuracy than we can compute the angles SEv and ESv. The *curtate* diſtance Sv of the planet from the ſun may be found, by ſaying, rad. : cos. PSv :: PS : Sv. FIG. 59.

* The method of making theſe computations will be ſhown in the ſecond Volume of this Work.

279. *Firſt* method, to find the place of the node. The moſt ſimple method, when it can be applied, is to obſerve when the planet has no latitude, and then reduce (278) the apparent place to the place ſeen from the ſun, and it gives the place of the node.

280. *Second* method. The place of the node may be determined by finding two equal heliocentric latitudes on each ſide of the node, and the middle point between the longitudes found at the ſame times, is the place of the node.

FIG. 60. 281. *Third* method. Find the planet's heliocentric latitudes juſt before and after it has paſſed the node, and let a and b be the places in the orbit, m and n the places reduced to the ecliptic; then the triangles amN, bnN (which we may conſider as rectilinear) being ſimilar, we have $am + bn : mn :: am : mN$, that is, *the ſum of the two latitudes : the difference of the longitudes :: either latitude : the diſtance of the node from the longitude correſponding to that latitude.* Or if we take the two latitudes ſeen from the earth, it will be very nearly as accurate when the obſervations are made in oppoſition. If the diſtance of the obſervations ſhould exceed a degree, this Rule will not be ſufficiently accurate, in which caſe we muſt make our computations for ſpherical triangles thus. Put $mn = a$, $bn = \beta$, $am = b$, $nN = x$; then by ſph. trig. $\frac{\text{fin. } \overline{a-x}}{\text{tan. } b} = \text{cot. } N = \frac{\text{fin. } x}{\text{tan. } \beta}$; but fin. $\overline{a - x}$ = fin. $a \times$ cos. $x -$ fin. $x \times$ cos. a; hence $\frac{\text{fin. } a \times \text{cos. } x - \text{fin. } x \times \text{cos. } a}{\text{tan. } b} = \frac{\text{fin. } x}{\text{tan. } \beta}$, $\therefore \frac{\text{fin. } a \times \text{tan. } \beta}{\text{tan. } b + \text{cos. } a \times \text{tan. } \beta} = \frac{\text{fin. } x}{\text{cos. } x} =$ tan. x. This Rule is given by Mr. BUGGE, Profeſſor of Aſtronomy in the Univerſity of Copenhagen. See the *Phil. Tranſ.* 1787.

FIG. 61. 282. *Fourth* method. Let P be the pole of the ecliptic EC, am, bn two heliocentric latitudes of the planet, and produce ma, bn to P; then the angle at P is the difference of longitudes; and in the triangle aPb, we know aP, bP and the angle aPb, to find the angle b; therefore in the right angled triangle Nbn, we know bn and the angle b, to find Nn; and as the longitude of n is known, the longitude of the node N will be known.

Ex. To the *third* method. Mr. BUGGE obſerved the right aſcenſion and declination of *Saturn*, and from thence deduced (124, 278) the following heliocentric longitudes and latitudes.

1784, Apparent Time	Heliocentric longitude	Heliocentric latitude
July 12, at $12^h.\ 3'.\ 1''$	$9^s.\ 20^\circ.\ 37'.\ 29''$	$0^\circ.\ 3'.\ 13''$ N.
20, — $11.\ 29.\ 9$	$9.\ 20.\ 51.\ 53$	$0.\ 2.\ 41$
Aug. 1, — $10.\ 38.\ 25$	$9.\ 21.\ 13.\ 17$	$0.\ 1.\ 34$
8, — $10.\ 9.\ 0$	$9.\ 21.\ 26.\ 2$	$0.\ 0.\ 56$
21, — $9.\ 14.\ 59$	$9.\ 21.\ 49.\ 27$	$0.\ 0.\ 2$
27, — $8.\ 50.\ 19$	$9.\ 22.\ 0.\ 12$	$0.\ 0.\ 27$ S.
31, — $8.\ 33.\ 47$	$9.\ 22.\ 7.\ 32$	$0.\ 0.\ 50$
Sept. 5, — $8.\ 13.\ 45$	$9.\ 22.\ 16.\ 28$	$0.\ 1.\ 21$
15, — $7.\ 33.\ 45$	$9.\ 22.\ 34.\ 32$	$0.\ 1.\ 59$
Oct. 8, — $6.\ 4.\ 23$	$9.\ 23.\ 16.\ 15$	$0.\ 3.\ 35$

In computing theſe heliocentric latitudes and longitudes, Mr. BUGGE added the corrections for the perturbations, after the principles of M. LAMBERT, in the *Memoirs de Berlin*, 1783.

From the obſervations on Auguſt 21 and 27, by conſidering the triangles as plane, $x = 44'',5$; from thoſe on 21 and 31, $x = 42'',5$; and from thoſe on Auguſt 21, and September 5, $x = 40''$; the mean of theſe is $x = 42''$; Mr. BUGGE makes $x = 41''$, probably by taking the mean of a greater number, or computing from conſidering them as ſpherical triangles; hence the heliocentric place of the deſcending node was $9^s.\ 21^\circ.\ 50'.\ 8'',5$. Now on Auguſt 21, at $9h.\ 12'.\ 26''$ true time, *Saturn's* heliocentric longitude was $9^s.\ 21^\circ.\ 49'.\ 27''$, and on 27, at $8h.\ 49'.\ 23''$ true time, it was $9^s.\ 22^\circ.\ 0'.\ 12''$; therefore in five days $23h.\ 36'.\ 57''$ Saturn moved $10'.\ 45''$ in longitude; hence $10'.\ 45'' : 41'' :: 5d.\ 23h.\ 36'.\ 57'' : 9h.\ 7'.\ 44''$ the time of deſcribing $41''$ in longitude, which therefore added to Auguſt 21, $9h.\ 12'.\ 26''$, gives Auguſt 21, $18h.\ 20'.\ 10''$ the time when Saturn was in it's node.

283. To determine the inclination of the orbit, we have *bn* the latitude of the planet, and *nN* it's diſtance upon the ecliptic from the node; hence ſin. *nN* : tan. *bn* :: rad. : tan. of the angle *N*. But the obſervations which are near the node muſt not be uſed to determine the inclination, as a very ſmall error in the latitude will make a conſiderable error in the angle. If we take the obſervation on July 20, it gives the angle $2^\circ.\ 38'.\ 15''$; if we take that on October 8, it gives the angle $2^\circ.\ 22'.\ 13''$; the mean of theſe is $2^\circ.\ 30'.\ 14''$ the inclination of the orbit to the ecliptic. To get the inclination accurately, we muſt, after having ſettled the place of the node, obſerve a latitude and longitude at a conſiderable diſtance from it. From the obſervations of Dr. MASKELYNE, M. de LAMBRE found the place of the node on July 12, 1784, to be $3^s.\ 21^\circ.\ 48'.\ 15''$. FIG. 60.

On

On December 12, 1704, at 18*h*. 50′ at Paris, *Jupiter* was obſerved in oppoſition in 2ˢ. 21°. 26′. 22″ with 28′. 10″ ſouth latitude; and on January 14, 1706, at 16*h*. 2′ it was in oppoſition in 3ˢ. 24°. 40′. 40″ with 29′. 56″ north latitude ſeen from the earth. Now at the firſt and ſecond obſervations, the diſtance of Jupiter from the ſun was to the diſtance of the earth as 51144 to 9839, and 52566 to 9840; hence (278) 51144 : 41305 :: 28′. 10″ : 22′. 45″; and 52566 : 42726 :: 29′. 56″ : 24′. 20″ the latitudes ſeen from the ſun at the reſpective oppoſitions; alſo the difference of the two longitudes was 33°. 14′. 18″; hence (281) 22′. 45″ + 24′. 20″ : 22′. 45″ :: 33°. 14′. 18″ : 16°. 3′. 36″, which added to 2ˢ. 21°. 26′. 22″ gives 3ˢ. 7°. 29′. 58″ the place of the aſcending node from theſe obſervations, according to M. Cassini. It is difficult to determine accurately the place of Jupiter's node on account of the ſmall inclination of it's orbit. M. de Lambre, from obſervations in 1775, 1776, 1777, 1782 and 1783, found the longitude of the node in 1783, to be 3ˢ. 8°. 14′.

On May 3, 1700, at 12*h*. 24′, Mr. Flamstead found the latitude of *Mars* to be 10′. 9″ north; and on May 10, at 11*h*. 48′ to be 10′. 13″ ſouth. Now as the correſponding longitudes are not given we muſt proceed thus. The time between the two obſervations was 6*d*. 23*h*. 24′; hence 10′. 9″ + 10′. 13″ : 10′. 9″ :: 6*d*. 23*h*. 24′ : 3*d*. 11*h*. 40′, which added to the time of the firſt obſervation gives May 7, 0*h*. 4′ for the time when the planet was in it's node, at which time, by calculation, it's place was in ♏ 17°. 23′. 13″. Now the place of the planet computed at the time of oppoſition was in ♏ 18°. 5′; conſequently the difference 41′. 47″ ſhows how much the computed place at the time of paſſing the node wanted of the computed place at the time of oppoſition, or the difference of the two places at thoſe times; but the obſerved place in oppoſition was in ♏ 18°. 6′, from which therefore ſubtract 41′. 47″ and we have ♏ 17°. 24′. 13″ for the true place of the deſcending node. In this manner we may always correct a computed place, if we have an obſerved place near to it. In the *Phil. Tranſ.* for 1790, Mr. Bugge makes the place of the aſcending node to be 1ˢ. 17°. 54′. 24″ for December 7, 1783, which is 10′. 35″ greater than the place by M. Cassini, 23′. 27″ greater than by Dr. Halley, and 2″ leſs than by M. de la Lande in his laſt Tables.

On June 11, 1705, at 1*h*. 11′, the latitude of *Venus* was 5′. 35″ north; and on June 12, it was 7′. 35″ ſouth at 1*h*. 5′. By calculation the true places of Venus ſeen from the ſun at thoſe times was ♐ 13°. 22′. 37″, and ♐ 14°. 57′. 32″, the motion of Venus was therefore 1°. 34′. 55″ in this interval; hence 5′. 35″ + 7′. 35″ : 5′. 35″ :: 1°. 34′. 55″ : 40′. 15″, which added to the place at the firſt obſervation gives ♐ 14°. 2′. 52″ for the place of the node. Mr. Bugge in the *Phil. Tranſ.* 1790, determined the place of the deſcending node of Venus on Auguſt 25, 1786, to be 8ˢ. 14°. 44′. 38″, which is 3′. 53″ leſs than by M.

CASSINI, 1′. 59″ greater than by Dr. HALLEY, and 36″ less than by M. de la LANDE in his last Tables.

In like manner, the place of the node of *Mercury* may be determined; but the best method of finding the place of the nodes of Venus and Mercury is from their transits over the sun's disc, as will be explained when we treat on that subject.

LONGITUDES OF THE NODES FOR 1750.

Planets	M. CASSINI	Dr. HALLEY	M. de la LANDE
Mercury	1ˢ. 15°. 25′. 20″	1ˢ. 15°. 21′. 58″	1ˢ. 15°. 20′. 43″
Venus	2. 14. 27. 45	2. 14. 23. 42	2. 14. 26. 18
Mars	1. 17. 45. 45	1. 17. 56. 21	1. 17. 38. 38
Jupiter	3. 7. 49. 57	3. 8. 15. 49	3. 7. 55. 32
Saturn	3. 22. 1. 4	3. 21. 20. 5	3. 21. 32. 22

M. de la PLACE found the place of the node of the *Georgian* Planet in 1788 to be 2ˢ. 12°. 47′.

To find the Inclination of the Orbits of the Planets to the Ecliptic.

284. *First* method. The most simple method is to observe the latitude of the planet when it is 90° from it's node, and then reduce (278) the latitude seen from the earth to that seen from the sun, and you have the inclination.

285. *Second* method. Observe the latitude and longitude of the planet at any other time when it is at some distance from the node, and reduce them (278) to the latitude and longitude seen from the sun; then the place of the node being known, the distance of the planet in longitude from the node will be known; and in the triangle *bnN*, we know *bn*, *nN*, therefore sin. *nN* : tan. *bn* :: rad. : sin. of the angle *bNn*; the further the planet is from the node, the smaller will be the error in the angle, any given error being made in the latitude. FIG. 61.

286. *Third*

FIG. 62. 286. *Third* method. Let P be the place of a planet in it's orbit, Nn the line of the nodes, E the earth in that line; draw Pv perpendicular to the ecliptic, and Pr, vr perpendicular to Nn; then (13) the angle Prv is the inclination of the orbit. Now $rv : vP ::$ rad. : tan. Prv

$vP : vE ::$ tan. PEv : rad.

$\therefore rv : vE ::$ tan. PEv : tan. Prv; but $rv : vE ::$ fin. vEr : rad. hence, *fin.* vEr : *rad.* :: *tan.* PEv : *tan.* Prv, that is, *the fine of the difference of the longitudes of the ſun and planet ſeen from the earth* : *rad.* :: *tan. of the geocentric latitude* : *tan. of the inclination.*

Ex. On January 11, 1747, at 18*h*. 6′. 33″, M. de la CAILLE obſerved the longitude of *Saturn* to be 6ˢ. 26°. 12′. 52″, and the ſun was then in 9ˢ. 21°. 47′ in the node of Saturn, or at leaſt within about 12′ of it; alſo the obſerved latitude was 2°. 29′. 18″ north; hence by the third method, fin. 85°. 34′. 8″ : rad. :: tan. 2°. 29′. 18″ : tan. 2°. 29′. 45″ the inclination. CASSINI, from the mean of 7 determinations, makes it 2°. 30′. 33″. M. de la LANDE from Dr. MASKELYNE's obſervations in 1775, 1776, 1777, makes it 2°. 30′; in his Tables he makes it 2°. 29′. 50″ for 1780. M. de LAMBRE found it 2°. 29′. 55″ for 1750.

On March 28, 1661, *Jupiter* was, according to HEVELIUS, in ♎ 8°. 58′ in oppoſition to the ſun, diſtant only about 1°. 30′ from it's greateſt diſtance from it's node, and with 1°. 38′. 25″ apparent ſouth latitude. Now the diſtance of Jupiter from the earth was to it's diſtance from the ſun as 44537 to 54535; hence, by the firſt method, 54535 : 44537 :: fin. 1°. 38′. 25″ : fin. 1°. 20′. 23″ the heliocentric latitude, or the inclination of the orbit; for the diſtance of 1°. 30′ from the greateſt diſtance of the node will not cauſe an error of more than 2″ in the inclination. From the oppoſition of Jupiter on April 6, 1768, M. de la LANDE found the inclination to be 1°. 19′. 4″, Jupiter being then at it's greateſt latitude; he makes it 1°. 18′. 56″ for 1780 in his Tables. M. de LAMBRE makes it 1°. 19′. 2″ for 1750.

On March 27, 1694, at 7*h*. 4′. 40″ at Greenwich, Mr. FLAMSTEAD determined the right aſcenſion of *Mars* to be 115°. 48′. 55″, and it's declination 24°. 10′. 50″ north; hence (124) the geocentric longitude was ♋ 23°. 26′. 12″, and

FIG. 59. lat. 2°. 46′. 38″. Let S be the ſun, E the earth, P Mars, v the place reduced to the ecliptic. Now the true place of Mars (by calculation) ſeen from the ſun was ♌ 28°. 44′. 14″, and the place of the ſun was ♈ 7°. 34′. 25″; hence, ſubtracting the place of the ſun from the place of Mars ſeen from the earth, we have the angle vES between the ſun and Mars 105°. 51′. 47″; and the place of the earth being ♎ 7°. 34′. 25″, take from it the place of Mars, and we have the

the angle $ESv = 38°.\ 50'.\ 11''$; hence, (278) fin. $105°.\ 51'.\ 47''$: fin. $38°.\ 50'.\ 11''$ :: tan. $PEv = 2°.\ 46'.\ 38''$: tan. $PSv = 1°.\ 48'.\ 36''$. Now the place of the node was in ♉ $17°.\ 15'$, which ſubtracted from ☊ $28°.\ 44'.\ 14''$ gives $101°.\ 29'.\ 14''$ for the diſtance vN of Mars from it's node; hence fin. $vN = 101°.\ 29'.\ 14''$: tan. $Pv = 1°.\ 48'.\ 36''$ :: rad. : tan. $PNv = 1°.\ 50'.\ 50''$ the inclination of the orbit. Mr. BUGGE makes the inclination to be $1°.\ 50'.\ 56'',56$, for March, 1788. M. de la LANDE makes it $1°.\ 51'$ for 1780.

The inclination of the orbit of *Venus* ♀, may be very accurately determined, when Venus is about 90° from it's node N, and in it's inferior conjunction; becauſe at that time it being about three times nearer to the earth than to the ſun S, any error in taking the apparent latitude will not cauſe an error of above one third part thereof in the inclination. Let E be the earth, and draw Vr perpendicular to the ecliptic. On September 2, 1700, the latitude of Venus, in inferior conjunction, was obſerved at Paris to be $8°.\ 40'.\ 15''$ S. and it's longitude ſeen from the ſun was $11^s.\ 10°.\ 20'.\ 20''$, conſequently it was $86°.\ 22'$ from it's node. Now at that time, SV was to SE as 72769 to 100750; hence $72769 : 100750$ :: fin. $SEV = 8°.\ 40'.\ 15''$: fin. $EVS = 167°.\ 57'.\ 7''$; therefore the angle ESV, or Vr, is $3°.\ 22'.\ 38''$, and as $rN = 86°.\ 22'$, fin. $86°.\ 22'$: tan. $3°.\ 22'.\ 38''$ :: rad. : tan. $VNr = 3°.\ 23'.\ 5''$. By a like obſervation on Auguſt 28, 1716, the inclination was found to be $3°.\ 23'.\ 10''$. Mr. BUGGE makes it $3°.\ 23'.\ 38'',6$ in 1784. M. de la LANDE, from two obſervations in 1780 and 1782, makes it $3°.\ 23'.\ 35''$ for 1780. FIG. 63.

On July 16, 1731, at $10^h.\ 32'.\ 47''$ in the morning, M. CASSINI determined the place of *Mercury* ſeen from the earth to be ♋ $3°.\ 2'.\ 35''$, with $2°.\ 2'.\ 20''$ ſouth latitude. Let S be the ſun, E the earth, v the place of Mercury at M reduced to the ecliptic, N the node. By calculation, the true place of Mercury ſeen from the ſun was ♓ $25°.\ 54'.\ 9''$, and the place of the node N was ♉ $15°.\ 10'$, conſequently $vN = 49°.\ 15'.\ 51''$. Now the ſun was in ♋ $23°.\ 13'.\ 12''$, from which take the apparent place of Mercury ♋ $3°.\ 2'.\ 35''$, and we have the angle $SEv = 20°.\ 19'.\ 37''$. Subtract the place of the earth ♑ $23°.\ 13'.\ 12''$ from the true place of Mercury ♓ $25°.\ 54'.\ 9''$, and we have the angle $ESv = 62°.\ 40'.\ 57''$; hence the ſine $SEv = 20°.\ 10'.\ 37''$: ſine $ESv = 62°.\ 40'.\ 57''$ :: tan. $vEM = 2°.\ 2'.\ 20''$: tan. MSv, or Mv, $= 5°.\ 15'.\ 30''$; and ſine $Nv = 49°.\ 15'\ 51''$: tan. $Mv = 5°.\ 15'.\ 30''$ :: rad. : tan. $vNM = 6°.\ 51'.\ 58''$ the inclination. He fixes it at 7°. M. le GENTIL obſerved Mercury in the meridian on October 5, 1750, and found it's apparent longitude $217°.\ 18'.\ 19''$, with $2°.\ 50'.\ 23''$ ſouth latitude; alſo the place of the ſun was $6^s.\ 12°.\ 8'.\ 52'',5$, and the angle EvS of commutation $78°.\ 31'.\ 23'',5$; hence the heliocentric latitude was $6°.\ 31'\ 23''$, and from thence the inclination $7°.\ 1'$. Dr. HALLEY makes it $6°.\ 59'.\ 20''$. M. de la LANDE employs 7° in his Tables. FIG. 64.

INCLINATION OF THE ORBITS.

Planets	KEPLER	Dr. HALLEY	M. CASSINI	M. de la LANDE
Mercury	6°. 54′. 0″	6°. 59′. 20″	7°. 0′. 0″	7°. 0′. 0″
Venus	3. 22. 0	3. 23. 20	3. 23. 20	3. 23. 35
Mars	1. 50. 30	1. 51. 0	1. 50. 54	1. 51. 0
Jupiter	1. 19. 20	1. 19. 10	1. 19. 30	1. 18. 56
Saturn	2. 32. 0	2. 30. 10	2. 30. 36	2. 29. 50

This determination of M. de la LANDE is for the year 1780. He makes the inclination of the orbit of the *Georgian* Planet to be 46′. 20″.

287. But the inclination of the orbits are ſubject to a variation, ariſing from their mutual attractions, as we ſhall afterwards explain. This variation is too ſmall to be determined with ſufficient accuracy from obſervations; but by theory, M. de la GRANGE has found it to be as follows; for Saturn −23″,11; for Jupiter −27″,19; for Mars +3″,45; for Venus +4″,47; for Mercury +20″,43; this is the variation in 100 years.

On the Motion of the Nodes.

288. The motion of the nodes is found, by comparing their places at two different times; or it may be determined by theory, as we ſhall afterwards explain.

PTOLEMY mentions, that in the year 136 *Saturn* was at it's greateſt north latitude at the beginning of Libra, and conſequently the node muſt have been in the beginning of Capricorn; now in the year 1700 it was in ♑ 21°. 13′. 30″; hence it had advanced 21°. 13′. 30″ in 1564 years, or at the rate of 48″. 51‴ in a year, and 1°. 21′. 26″ in 100 years. But as a variation of ſeveral degrees in the place of the node would have but a very ſmall effect on the latitude when near it's greateſt, the obſervation of PTOLEMY cannot be depended upon for this purpoſe. On March 1, 228 before J. C. Saturn was obſerved, by the Chaldeans, to be about 5′ above the ſtar in the ſouth ſhoulder of Virgo,

marked

marked γ by BAYER; from this M. CASSINI found the place of the node to be $2^s.\ 21^\circ$, which compared with the place in 1720, gives $56''.\ 26'''$ for the yearly motion. BULLIALDUS mentions an occultation of Saturn by the moon in the year 503, from whence he found the place of the node to be $3^s.\ 12^\circ.\ 36'.\ 21''$; in the year 1769, M. de la LANDE found the place to be $3^s.\ 21^\circ.\ 40'.\ 47''$; this gives $25''.\ 48'''$ for the yearly motion of the nodes. TYCHO-BRAHE obſerved Saturn very near it's node on December 29, 1592, from whence M. CASSINI found the place of the node to be $3^s.\ 20^\circ.\ 21'.\ 5''$; this obſervation compared with the place of the node in 1700, determined to be $3^s.\ 21^\circ.\ 13'.\ 30''$, gives $29''.\ 24'''$ for the annual motion. From four obſervations of M. CASSINI (which M. de la LANDE thinks are moſt to be depended upon) reduced to the year 1700, the place of the node appears then to have been in $3^s.\ 21^\circ.\ 11'.\ 20''$; and comparing this with the place in 1769, the annual motion is $25'',6$. M. de LAMBRE makes it $33'',35$. M. de la GRANGE makes it $29''$, from the theory of attraction. M. de la LANDE makes it $31'',7$ in his Tables.

M. CASSINI found the place of the node of *Jupiter* in 1705, to be in $3^s.\ 7^\circ.\ 37'.\ 50''$. According to PTOLEMY, the place of the node in his time was in the beginning of Cancer; this gives $17''$ for the annual motion. By an obſervation on September 26, 508, in which Jupiter was in conjunction with *Regulus*, M. CASSINI computed the motion to be $24''.\ 37'''$ from the ſame obſervation. M. le GENTIL calculated the places of the node from the obſervations of GASSENDI, Dr. HALLEY and himſelf, to be, in 1633, in $3^s.\ 6^\circ.\ 4'.\ 50''$; in 1716, in $3^s.\ 7^\circ.\ 37'.\ 30''$; and in 1753, in $3^s.\ 8^\circ.\ 21'.\ 25''$. The two laſt obſervations give $66''$ for the annual motion; the firſt and laſt give alſo $66''$; but theſe motions are too great, as they will not agree with other obſervations. From the mean of ſeveral obſervations made at Paris between 1692 and 1730, it comes out $34''$. M. de LAMBRE makes it $35'',7$, which M. de la LANDE has aſſumed in his Tables. M. de la GRANGE makes it $31''$ by theory.

The place of the node of *Mars* on October 28, 1595, was found, from the obſervations of TYCHO, to be in $8\ 16^\circ.\ 24'.\ 33''$; and on November 13, 1721, M. CASSINI found it to be in $8\ 17^\circ.\ 29'.\ 49''$; theſe give $31''.\ 4'''$ for the annual motion of the nodes. By comparing the ſame obſervation of TYCHO with thoſemade at Paris and Greenwich in the year 1700, the former gives $38''.\ 15'''$, and the latter $34''.\ 16'''$. In the year 139, PTOLEMY ſays the greateſt north latitude of Mars was at the end of Cancer, which gives the place of the node at the end of Aries; this compared with the place in 1721 gives $39''.\ 50'''$. M. CASSINI tkinks this latter is not much to be depended upon, and therefore takes the mean of the others, which gives $34''.\ 32'''$ for the annual motion. Mr. BUGGE makes it $28'',2$. M. de LAMBRE makes it $28''$, which M. de la

LANDE employs in his Tables. M. de la GRANGE makes it 25″,4 by theory.

The place of the node of *Venus* in it's tranſit over the ſun in 1769, was found by M. de la LANDE to be 2ˢ. 14°. 36′. 20″, with a probable error of not more than 30″. Dr. HORNSBY calculated the place of the node in it's tranſit in 1639, from the obſervations of HORROX, and found it to be 2ˢ. 13°. 27′. 50″, which gives 31″,7 for it's annual motion. TIMOCHARES, on October 11, 271 years before J. C. obſerved ϖ in the ſouth wing of Virgo to be eclipſed by Venus; from this obſervation, M. CASSINI found the place of the node to be 1ˢ. 24°. 2′; this compared with the place in 1698 in 2ˢ. 14°. 1′. 45″, gives 36″,5. The obſervations in 1639 and 1698 make it 34″; and as this agrees very nearly with the reſults from the obſervations in 1705, 1710 and 1731, M. CASSINI fixed the motion at 34″. M. de la CAILLE, on December 21, 1746, found the place of the node to be 2ˢ. 14°. 23′. 10″; this compared with the place of the node obſerved by M. de la HIRE on October 31, 1692, gives 38″ for the annual motion. Mr. BUGGE makes it 30″,37. M. de la LANDE makes it 31″, which he uſes in his Tables. M. de la GRANGE makes it 30″,55 by theory.

The place of the node of *Mercury* on November 7, 1631, was found, from the obſervation of GASSENDI, to be in ♉ 13°. 30′. 47″; and on November 11, 1736, it was found to be in ♉ 15°. 14′. 5″; this gives the annual motion 59″. 2‴. According to the obſervations of HEVELIUS, the true place of the node on May 3, 1661, was in ♉ 14°. 19′; this compared with the obſervation in 1736, gives the annual motion 43″. 42‴; the mean of theſe is 51″. 22‴. This is M. CASSINI's determination. M. le GENTIL, by comparing the place of the node in 1753 in ♉ 15°. 24′. 14″ with the place in 1677 in ♉ 14°. 21′. 3″, found the motion to be 50″,21. M. de la LANDE, by comparing the places of the node of Mercury found from it's tranſits over the ſun, makes it 43″, and theſe obſervations are moſt to be depended upon. He employs this in his Tables. M. de la GRANGE finds it to be 41″,3 by theory.

289. This motion of the nodes is in reſpect to the equinox; if therefore we ſubtract from each 50″,25 the preceſſion of the equinoxes, it will give the motion in reſpect to the fixed ſtars, or the real motion. The motion in the following Table is in reſpect to the equinoxes.

MOTION

MOTION OF THE NODES IN ONE HUNDRED YEARS.

Planets	M. Cassini	Dr. Halley	M. de la Lande
Mercury	1°. 24′. 40″	1°. 23′. 20″	1°. 12′. 10″
Venus	0. 56. 40	0. 51. 40	0. 51. 40
Mars	0. 56. 40	1. 3. 20	0. 46. 40
Jupiter	0. 40. 9	1. 23. 20	0. 59. 30
Saturn	1. 35. 11	0. 30. 0	0. 55. 30

The *Georgian* Planet has not been diſcovered long enough to determine the motion of it's nodes from obſervation. M. de la Grange has found the annual motion to be 12″,5 by theory. But if we take the denſity of Venus according to M. de la Lande, it will be 20″. 40″, which he uſes in his Tables.

Thus we have determined all the elements neceſſary for computing the place of a planet in it's orbit at any time; but to facilitate the operation, which would be extremely tedious if we had only the elements thus given, Aſtronomers have conſtructed Tables of their motions, by which their places at any time may be very readily computed. The conſtruction and uſe of theſe Tables, we ſhall explain in the Introduction to the Tables in the ſecond Volume.

CHAP.

CHAP. XVI.

ON THE GEORGIAN PLANET.

Art. 290. ON March 13, 1781, between ten and eleven o'clock in the evening, as Dr. HERSCHEL was examining the ſmall ſtars near the feet of *Gemini*, he obſerved one conſiderably larger than the reſt, but it not being quite ſo brilliant, he ſuſpected that it might be a comet; in conſequence of which he obſerved it with different magnifying powers, from 227 with which he diſcovered it, to 2010; and found that it's apparent magnitude increaſed in proportion, contrary to what takes place in the fixed ſtars. He therefore meaſured it's diſtance from ſome of the neighbouring fixed ſtars, and comparing it's diſtance from them for ſeveral nights, he found that it moved at the rate of about $2\frac{17}{4}''$ in an hour. On this, Dr. HERSCHEL wrote immediately to the Royal Society, that other Aſtronomers might join in obſerving it; upon which it was found and obſerved by Dr. MASKELYNE, who almoſt immediately declared, that he ſuſpected it to be a Planet; and on April 1, he wrote an account of this diſcovery to the Aſtronomers at Paris, ſo that it was ſoon obſerved by all the Aſtronomers in Europe. Mr. LEXELL was then in England, and applied himſelf to compute the orbit, upon ſuppoſition that it was a comet; he therefore, according to the uſual manner in ſuch a caſe, ſuppoſed the orbit to be a parabola, and aſſumed ſeveral perihelion diſtances 6, 8, 10, 12, 14, 16 and 18 times the earth's diſtance from the ſun; and found that any perihelion diſtance between 14 and 18 would anſwer very well to the obſervations. BOSCOVICH printed a Memoir on the ſubject, in which he ſhowed that there were four different parabolas in which the body might move, and yet the computed places would agree with the obſervations which had then been made. Other Aſtronomers however found that a circular orbit, whoſe radius was about 18 times the diſtance of the ſun from the earth, would agree better with the obſervations; and this confirmed Dr. MASKELYNE's opinion that it was a planet. Upon ſuppoſition therefore of a circular orbit, M. de la LANDE proceeded to inveſtigate it's magnitude from the following obſervations. *Mem. de l'Acad. Roy. des Sci.* 1779.

Time

Time of obſervation . . .	April 25, 1781, at 9^h. 47′.	July 31, 1781, at 15^h. 33′.	Dec. 12, 1781, at 10^h. 10′.
Right aſcenſion obſerved	2^s. 25°. 15′. 27″	3^s. 1°. 7′. 49″	3^s. 1°. 23′. 31″
North declination obſ.	23. 35. 34	23. 40. 25	23. 42. 47
Longitude	2. 25. 39. 17	3. 1. 2. 7	3. 1. 16. 28
Latitude north	11. 36	12. 24	14. 54
Nutation in longitude	+10	+ 8	+ 7
Aberration in longitude	+19	+21	−18
Sun's longitude from the mean equinox	1. 5. 58. 53	4. 9. 7. 13	8. 21. 21. 50
Log. of the ſun's diſtance	0,003196	0,006272	9,992993

291. From theſe obſervations, M. de la LANDE proceeded thus to find the circular orbit. He aſſumed the radius of the orbit, and then calculated the heliocentric places of the planet at the times of the firſt and laſt obſervation; conſequently the angle deſcribed by the planet about the ſun in that interval of 231 days 23′ was known; and hence the time of the whole revolution was known by proportion, upon ſuppoſition that the orbit was circular. Next, knowing the radius of the orbit compared with the mean diſtance of the earth from the ſun, he calculated the periodic time by KEPLER's Rule (218); but as this time did not agree with that before found, he varied his ſuppoſition of the diſtance, until he found they agreed, in which caſe the radius of the orbit was found to be 18,931 times the mean diſtance of the earth from the ſun, and the duration of the revolution 82,37 years. This circular orbit therefore agreed to the firſt and laſt obſervations; and by computing from it the place at the ſecond obſervation, he found that it differed only 5″ from the obſerved place, which difference might eaſily ariſe from the unavoidable errors in the obſervation. He then calculated 32 other obſervations made by Dr. MASKELYNE, MONNIER, MESSIER, MECHAIN, d'AGELET, LEVESQUE and himſelf, and found they all agreed very well, except in April 1781, and July, Auguſt and September 1782, the laſt differing move than two minutes. He then proceeded, as before, to find what radius would anſwer to the obſervation on April 25, 1781, and on July 21, 1782, at 15^h. at Paris, when the lon-

longitude obſerved was 3ˢ. 4°. 42′. 39″; this radius he found to be 18,893, and the periodic time 82,12 years. But by uſing this radius, he found the calculations to differ 1′. 27″ from the place obſerved in oppoſition in December 1781. This indicated an irregularity in the motion of the planet; but the irregularity was too ſmall, and the obſervations too near together, to afford proper data for the inveſtigation of the orbit. M. de la LANDE proceeded to determine the place of the node and inclination of the orbit; but on account of the ſmall motion in latitude, great accuracy could not at that time be expected. The geocentric latitudes obſerved on April 25, and December 12, 1781, were 11′. 36″ and 14′. 54″ north, which give the heliocentric latitudes 11′. 59″ and 14′. 8″; and the motion in longitude being 2°. 46′. 3″ between the obſervations, he found the place of the node to be 2ˢ. 12°. 54′, and inclination of the orbit 0°. 46′. Again, the obſerved geocentric latitudes on April 16, 1781, and March 26, 1782, were 11′. 48″ and 15′. 5″, and hence the heliocentric latitudes were found to be 12′. 7″ and 15′. 10″; and the motion in longitude between the obſervations being 4°. 7′. 44″, the place of the node was found to be 2ˢ. 12°. 2′, and the inclination 0°. 44′. He further obſerves, that the planet was ſtationary 11 days before Dr. HERSCHEL firſt obſerved it, and therefore if his obſervations had been made 11 days ſooner, he would not have perceived any motion, and the diſcovery might, M. de la LANDE thinks, have been loſt. It is probable however, that if this had happened, the diſcovery would have been made; for from the ſingularity of it's appearance, which alone made Dr. HERSCHEL pay attention to it, he would undoubtedly have continued to obſerve it, till he had diſcovered it's motion, which muſt very ſoon have been perceived.

It having been found that the motion did not agree to that of any one circle, the next enquiry was to determine the ellipſe in which it moved, ſuppoſing that, like the other planets, it revolves in ſuch a curve, having the ſun in one of it's foci.

292. The methods of finding the orbit of a planet as deſcribed in Chap. XIII. are by three heliocentric places and the times between, or by three diſtances from the ſun and the angles between. The firſt method may be applied from three obſerved oppoſitions; and to apply the other we muſt have five; but as the latter method is direct, and alſo ſo very ſimple when compared with the former, we ſhall prefer that, as there are now obſervations ſufficient for it; if we had wanted the elements of the orbit before there had been ſufficient data for the latter, we muſt have uſed the former method. By this, Mr. ROBISON, Profeſſor of Natural Philoſophy in the Univerſity of Edinburgh, has inveſtigated the elements of the orbit, in the *Edinb. Tranſ.* Vol. I. 1788; we ſhall therefore fully explain the principles and computations as given by him; the method is capable of great accuracy, ſo far as the obſervations are accurate, and may be

eaſily

easily understood by those who are well acquainted with only the elementary parts of Mathematics and Philosophy.

The observations upon which the investigation is founded are as follows;

True Time at Edinburgh.	Longitude.	N. Lat.
Dec. 21, 1781, ... 17h. 44′. 33″	3s. 0°. 52′. 11″	15′. 7″
—— 26, 1782, ... 8. 56. 56	3. 5. 20. 29	18. 56
—— 31, 1783, ... 0. 46. 24	3. 9. 50. 52	22. 10
Jan. 3, 1785, ... 17. 28. 56	3. 14. 23. 2	25. 40
—— 8, 1786, ... 10. 39. 31	3. 18. 57. 5	28. 52

293. We have here the times of five successive oppositions, as deduced from observations, and the corresponding heliocentric longitudes and latitudes. Hence the the longitude of the node on January 1, 1786, was 2s. 12°. 48′. 45″, and inclination of the orbit 46′. 26″. The place of the node and the inclination of the orbit being determined, the places of the planet reduced (268) to the orbit will be known, and thus we may find the arcs described in the orbit itself between the above oppositions.

294. Mr. Robison next took the opposition on December 31, 1783, for an epoch to which the other observations were to be reduced. The interval between this and the preceding opposition was 369*d*. 15*h*. 49′. 28″; from this opposition he counted back the same interval of time; and in like manner he counted forwards from the epoch two equal intervals; thus he got four equal intervals of time, to which times he found the places of the planet upon it's orbit; and upon comparing their differences, he discovered that they had irregularities not consistent with the motion of a body in an ellipse; these therefore must have arisen from some inaccuracies in the observations; and as, upon account of the small intervals of the places, such errors would be the cause of great errors in the elements of the orbit, it was necessary to correct these inaccuracies, so as to give the differences such a law, as near as possible, that they ought to have.

295. The next consideration was, upon what principle this correction was to be made; and this was, by finding, as nearly as possible, about what part of the ellipse the planet was in at the time of the above observations, and then by observing in similar parts of the ellipses described by the other planets, what law the first and second differences of the angles described in equal times observe. The places of the planet in the ecliptic at five points of time being known, it's place at any other point of time may be very accurately found by interpolation. Now on March 6, 1782, at 6*h*. 14′. 56″ mean time (at which time the planet

was ſtationary), it's apparent longitude upon the ecliptic was obſerved to be 2ˢ. 28°. 49′. 27″; the heliocentric longitude was alſo found by interpolation; hence the diſtance of the planet from the ſun came out 18,9053, the earth's diſtance from the ſun being unity. By interpolating the place of the planet for March, 7*d*. 6*h*. 14′.56″, it was found to have moved 43″,4365 in 24 hours; but a planet revolving about the ſun in a circle whoſe radius is 18,9053, will have it's diurnal motion = 43″,1647. Now the angular velocity of a body in an ellipſe is to the angular velocity in a circle at the ſame diſtance, in the ſubduplicate ratio of half the latus rectum to the diſtance; hence the planet's diſtance from the ſun was leſs than half the latus rectum. Alſo by a like proceſs for April 1781 it appears, that at that time the angular motion of the planet exceeded, by a very little, the angular motion of a body in a circle at the ſame diſtance; therefore it's diſtance from it's perihelion could be but a very little leſs than 90°. We find moreover, that the angular velocity of the planet about the ſun was continually accelerated at the time of the above obſervations, and therefore the planet was approaching it's perihelion. Now by examining the Tables of the planet's motions in ſimilar ſituations, it appears that, in equal intervals of time, the firſt differences decreaſe very ſlowly, and the ſecond differences increaſe very ſlowly. Mr. Robison therefore gave to the firſt differences a very ſmall diminution, and to the ſecond differences a very ſmall increaſe, and this correction was made without altering any of the longitudes more than 3″; for the firſt obſervation had it's longitude diminiſhed 1″, the ſecond and third increaſed 2″,5, and the fourth and fifth diminiſhed by 3″, and this muſt be allowed to be within the limits of probability. The times correſponding to the above mentioned equal intervals, and the correſponding corrected longitudes, cleared from the effects of aberration and nutation, and reduced to the orbit, and the epoch of 1783, are as follows:

True time at Greenwich.	Longitude.
Dec. 21, 1781, 17ʰ. 20′. 17″	3ˢ. 0°. 53′. 50″
—— 26, 1782, 9. 9. 45	3. 5. 21. 16,5
—— 31, 1783, 0. 59. 13	3. 9. 50. 37, 5
Jan. 3, 1785, 16. 48. 41	3. 14. 21. 52
—— 8, 1786, 8. 38. 9	3. 18. 54. 58

Theſe

These give the following intercepted arcs, with their first and second differences:

$4°.27'.26'',5$

$1'.54'',5$

$4.29.21$ $1''$

$1.53,5$

$4.31.14,5$ 2

$1.51,5$

$4.33.6$

From these data the elliptic orbit of the planet is to be constructed.

296. Let ACP be the orbit, P the perihelion, S the focus, A, B, C, D, E the places of the planet at the five oppositions; and draw the chords and the radii. Now we may conceive the chords AC, CE to be bisected by the radii SB, SD in x and g. For supposing them to be bisected, the triangle $ASx = CSx$, and the triangle $BxC = BxA$, by Euclid B. I. P. 38. And the ellipse being nearly a circle, Sx is nearly perpendicular to CA, and therefore the chords BC, BA, and consequently the two segments, will be very nearly equal; and each being also extremely small compared with the triangles CSB, ASB, the sectors CSB, ASB will be very nearly equal, and hence the times from A to B, and from B to C, may be considered as equal, without any sensible error, and therefore B will be the place of the planet at the second observation. In like manner, D will be the place at the fourth observation. FIG. 65.

297. Let the given angles $ASB = u$, $BSC = v$, $CSD = x$, $DSE = y$, $ASC = w$, $CSE = z$; then $AS : Ax ::$ sin. AxS : sin. u, and Cx, or $Ax : CS ::$ sin. v : sin. CxS or AxS; hence $AS : CS ::$ sin. v : sin. u; in like manner, $ES : CS ::$ sin. x : sin. y; thus we know the ratio of AS, CS, ES, and the angles between them, consequently the species and position of the ellipse may (257) be found. The error arising from the supposition of the chords being bisected, is here so extremely small, that it may safely be neglected; however, as Mr. ROBISON has shown how it may be corrected, I shall explain the method, as it may, upon other occasions, be necessary.

298. Bisect AE in F, AC in H, CE in G, and draw SHh, SFc, SGd, OFk, OGv, O being the center of the ellipse. Since the angles kOv, cSd are very small, the triangles cFk, dGv are nearly similar, and cF, dG being considered as versed sines, they will be very nearly as the squares of the chords; hence the area $cFk : dGv :: cF^2 : dG^2 :: AE^4 : CE^4$. Now by the property of the ellipse, the area $EFk = AkF$, also $EFS = AFS$; hence $SFkE = AkFS$; add Fkc to both, and $ScE = AkFS + Fkc = ScA + 2Fkc$; therefore $ScE - ScA = 2Fkc$; but as $SCE = SCA$, therefore $ScE + ScA = 2SCc$, consequently $Fkc = SCc$. For the same reason, $dGv = SDd$; but as SD, SC are

very nearly equal, $Cc : Dd$ :: area SCc : SDd :: Fkc : dGv :: AE^4 : CE^4. And as the arcs AC, CE are very ſmall and nearly equal, therefore $\phi F = Cc$, and $Gg = dD$ very nearly; alſo $AE : CE$ :: 2 : 1 very nearly; hence $\phi F : Gg$ :: 16 : 1. For the ſame reaſon, $\phi F : Hx$ :: 16 : 1 nearly.

299. Let $ABCDE$ be the true ellipſe; take $Se : SC$ :: ſin. x : ſin. y, and $Sa : SC$ :: ſin. v : ſin. u, and Se, Sa are the values of the firſt and laſt radii, as determined in Art. 297. conſequently Ee, Aa are the errors to be found.

Now $SC : Cg$ :: ſin. g : ſin. x
And $Cg : Eg :: Cg : Eg$
Alſo $Eg : SE$ :: ſin. y : ſin. g
$\therefore$ $SC : SE :: Cg \times$ ſin. $y : Eg \times$ ſin. x
But $Se : SC$ :: ſin. x : ſin. y
$\therefore$ $Se : SE :: Cg : Eg$
$\therefore$ $Ee : SE :: Cg - Eg : Eg :: 2gG : Eg$.

In like manner, $Aa : SA :: 2xH : Ax$ :: (becauſe the arcs AC, CE are very nearly equal) $2gG : Eg$, and hence, as $SE = SA$ nearly, $Ee = Aa$ nearly.

Now $SE : E\phi$ :: ſin. ϕ : ſin. z
And $E\phi : A\phi :: E\phi : A\phi$
Alſo $A\phi : SA$ :: ſin. w : ſin. ϕ
$\therefore$ $SE : SA :: E\phi \times$ ſin. $w : A\phi \times$ ſin. z
Aſſume $SA : So$ :: ſin. z : ſin. w
$\therefore$ $SE : So :: E\phi : A\phi$
$\therefore$ $SE : Eo :: E\phi : A\phi - E\phi$, or $2\phi F$.

But as EA is nearly $= 2AC$, $E\phi = 2Eg$ nearly; alſo $2\phi F = 32Gg$;

Hence $SE : Eo :: 2Eg : 32Gg :: Eg : 16Gg$
But $Ee : SE :: 2Gg : Eg$
$\therefore$ $Ee : Eo :: 2Gg : 16Gg :: 1 : 8$.

Make $Sa : S\varepsilon$ :: ſin. z : ſin. w, and then $Sa : S\varepsilon :: SA : So$, therefore $Aa : o\varepsilon :: Sa : S\varepsilon$; and as $S\varepsilon = Sa$ nearly, therefore $Aa = o\varepsilon$ nearly; but $Aa = Ee$ nearly, conſequently $Ee = o\varepsilon$ nearly; but $Eo = 8Ee$, hence $e\varepsilon = 6Ee$, and therefore $Ee = \frac{e\varepsilon}{6}$ nearly.

In like manner, find a point α as ε was found, by taking $Se : S\alpha$ :: ſin. w : ſin. z, and a point o' as o was found, by taking $SE : So'$ :: ſin. w : ſin. z, and by

by the ſame reaſoning it will appear, that $Aa = \frac{dx}{6}$. Hence we have the following conſtruction to obtain the three radii. Take CS of any value; aſſume $SC : Sa ::$ ſin. $u :$ ſin. v, $SC : Se ::$ ſin. $y :$ ſin x, $Se : S\alpha ::$ ſin. $w :$ ſin. z, and $Sa : S\varepsilon ::$ ſin. $z :$ ſin. w. Then make $SA = Sa + \frac{a\alpha}{6}$, and $SE = Se - \frac{e\varepsilon}{6}$, and SA, SE will be the other two radii. Hence by Art. 257. the angle ESP = 2°. 4°. 14′. 53″; and the excentricity = 0,9006, the mean diſtance of the Earth being unity. Alſo (232) the mean anomalies correſponding to the true anomalies OSA, OSE will be known. Therefore the difference of theſe two mean anomalies : 360° :: time from A to E : the time of a ſidereal revolution; and the ſquare of a ſidereal year : ſquare of this ſidereal revolution :: 1 : the cube of the planet's mean diſtance from the ſun. Hence we deduce the following elements.

Mean diſtance		19,08247
Excentricity		0,9006
Periodic time		83,359 years
Mean anomaly at E		4°. 0°. 32′. 51″
Long. of aphelion	for epoch Dec. 31. 1783	11. 23. 9. 51
Long. of the node		2. 12. 46. 14
Inclination of the orbit		0. 46. 25
Equation of the center		5. 26. 56,6

300. Theſe elements, ſays Mr. ROBISON, are as accurate as the obſervations on which they are founded can give them; and agree at preſent (1788) very well with the obſervations, the differences being as often as much in defect as in exceſs; but as the obſervations were made ſo near together, it cannot be expected that this agreement will laſt for a long time. As they may be found to vary from obſervations, they may be corrected by Art. 267. without computing them over again. The ſtar N°. 964. obſerved by MAYER in 1756, is not now to be found; and by computing the place of this planet for the time of his obſervation, Mr. ROBISON found the planet to be only 3′. 52″ weſtward of the ſtar, and 1″ northward, from which he ſuſpected that it might have been this planet which MAYER obſerved. It will appear however that this was not the caſe. It was alſo conjectured by ſome Aſtronomers, that the ſtar N°. 34, *Tauri*, of the Britiſh Catalogue, was the new Planet; but Mr. ROBISON, thinks this conjecture by no means to be admitted, as it cannot be made to agree with the elements. Mr. ROBISON has computed Tables of this planet's motion, and obſerves, that the deviations from obſervations made near the vernal ſtations are in defect, whilſt thoſe near the autumnal ſtations are in exceſs.

Hence

Hence it may be preſumed, that the mean diſtance and periodic time are ſomewhat too ſmall, and the aphelion too forward. This he did not perceive till after he had computed his Tables, and, he obſerves, the taſk was too tedious to make the computations a-new. He therefore publiſhes them, not in the perſuaſion that they are perfect, but becauſe they are more conſiſtent with obſervations than thoſe of M. de la PLACE, and ORIANI, the only ones which he had then ſeen.

The Elements given by M. de la PLACE, *are,*

Mean longitude 1784	3ˢ. 14°. 43′. 18″
Aphelion	11. 17. 6. 44
Node	2. 12. 46. 47
Equation	5. 21. 3,2
Inclination	46. 16
Secular motion of the aphelion	1. 28. 0
——————— node	26. 10
Mean diſtance	19,18352

301. M. de la LANDE, in the *Hiſtoire de l'Academie Royal des Sciences*, 1787, has corrected theſe elements, after determining two diſtances from the ſun, the angle, and time between. We ſhall explain the manner in which he has reduced the Problem to theſe data. To examine more accurately the motion of this Planet, he ſettled, from the beſt obſervations, the places of thoſe fixed ſtars with which the Planet had been compared.

FIG. 66. 302. Let S be the ſun, E and F the places of the earth when the Planet was in quadratures at H and K. Now in the quadrature before oppoſition, the geocentric longitude computed was found to be greater than that by obſervation, and in the quadrature after oppoſition, to be leſs. Draw SGH, SIK, and ſuppoſe G and I to be the computed places; then as the difference between the true and computed diſtances from the ſun cannot ſenſibly vary between the two quadratures, we may ſuppoſe $GH = IK$, and conſequently the angle HEG, $= KFI$; and as the difference between the true and computed angular velocities will not ſenſibly vary, we may ſuppoſe the true places to be at H and K, when the computed are at G and I. Hence, on the contrary, when the angles HEG, KFI are obſerved to be equal, the true places will be at H and K, and the computed ones at G and I. Now the diſtance SG compared with SE being given, and the angle SEG a right angle, if we aſſume the angle $HEG = 10''$, we ſhall find $GH = 0{,}017$. At the quadratures at E on November 21, 1788, the

the error *HEG* was found to be 23″, and the error *KFI* in the preceding quadrature May 8, was 20″; we will therefore take the mean 21″,5 for each error; hence 10″ : 0,017 :: 21″,5 : 0,03655 the quantity by which you muſt augment the computed diſtance in order to get the true diſtance. M. de la LANDE makes it 0,04. Now from the poſition of *E* and *S* in reſpect to *G*, as the computed geocentric longitude of *G* was diminiſhed 1″,5, the correſponding computed heliocentric longitude will be diminiſhed by about the ſame quantity; ſubtract therefore 1″,5 from the computed heliocentric longitude, and you will have the true heliocentric longitude. Repeat the ſame for any other quadrature, and you will get the two diſtances from the ſun, with the angle and time between. In this manner M. de la LANDE found for October 8, 1781, at 17*h*. 9′ the true heliocentric longitude to be 2ˢ. 29°. 58′. 43″, and the diſtance 18,947; and for March 31, 1788, at 7*h*. 38′ to be 3ˢ. 29°. 5′. 46″, and 18,562. From which he corrected the elements in the Tables in the following manner.

303. Let *SP*, *S*☋ be the two given diſtances. *Firſt hypotheſis.* Aſſume the mean diſtance as in the Tables; alſo a certain excentricity *SC*, and place *A* of the aphelion for the *firſt ſuppoſition.* Find from thence the two true anomalies at *P* and ☋; and then compute the correſponding mean anomalies, and the diſtance *SP*, which will probably be greater or leſs than that which is given. Change the place of *A*, until *SP* comes out the ſame. Compute again from this new place of the aphelion, the two mean anomalies, and ſee how their difference agrees with that given in the Tables, and you have the error of the Tables. FIG. 67.

Second ſuppoſition. Change the excentricity *SC*, and compute again the difference of the mean anomalies, and you will have the error from this ſuppoſition. Then by the Rule of three, find the excentricity which gives no error, and you have an *hypotheſis* which repreſents the firſt diſtance and the two anomalies.

Calculate from hence the ſecond true anomaly, which ought to be the ſame as that which we had after correcting the place of the aphelion; if not, the excentricity muſt be varied until they agree.

Compute, upon the firſt hypotheſis, the value of *S*☋, and obſerve the error.

Second hypotheſis. Change the mean diſtance. This gives another difference of mean anomalies. Then, as in the firſt hypotheſis, vary the place of the aphelion and excentricity, until you repreſent the firſt diſtance, *SP* and the two anomalies; but the ſecond diſtance probably does not agree with obſervation.

From hence, ſays M. de la LANDE, by the progreſs of the errors of the two hypotheſes, I form a third hypotheſis which will repreſent each diſtance, and the mean motion between. Hence he deduced the following elements.

Mean

Mean longitude in 1784	3ˢ. 14°. 49′. 14″
Aphelion	11. 16. 19. 30
Equation	5. 26. 47
Secular motion	2. 9. 11. 11
Mean diſtance	19,2033
Tropical revolution	30637 days.
Sidereal revolution	30737 days.

Theſe elements are formed from taking into conſideration the diſturbing forces of Jupiter and Saturn, which M. de la LANDE found neceſſary; for otherwiſe the diſtances would be repreſented very well, when the longitude varied conſiderably. Theſe elements give the place of the planet different from the place of the ſtar obſerved by MAYER in 1756 by 11′, from which we may conclude that that ſtar was not the Georgian Planet.

304. To correct the place of the node, M. de la LANDE found the heliocentric longitudes on September 28, 1781, and March 8, 1788, to be 2ˢ. 29°. 51′. 24″ and 3ˢ. 28°. 48′. 49″; and latitudes 13′. 38″ and 33′. 20″,5. But the preceſſion of the equinoxes being 5′. 24″ in the interval, it muſt be ſubtracted from the difference of the longitudes to get the true motion in longitude, which was therefore 28°. 52′. 1″; and the laſt latitude was diminiſhed 3″ for the ſame reaſon, but augmented 1″ on account of the motion of the node; hence the place of the node at the beginning of 1781 was 2ˢ. 12°. 44′. 12″, and the inclination 46′. 20″.

305. M. de la GRANGE makes the annual motion in longitude of the aphelion to be 53″,42; and that of the node to be 12″,5. But if the denſity of Venus be ſuch as is ſuppoſed by M. de la LANDE, the annual motion of the nodes will be 20″. 40‴.

Dr. HERSCHEL has diſcovered two ſatellites belonging to the Georgian Planet; the elements of the orbits of which we ſhall give, when we treat on the Secondary Planets.

CHAP.

CHAP. XVII.

ON THE APPARENT MOTIONS AND PHASES OF THE PLANETS.

Art. 312. AS all the planets revolve about the ſun as their center, it is manifeſt, that to a ſpectator at the ſun they would appear to move in the direction in which they really do move, and ſhine with full faces. But to a ſpectator on the earth which is in motion, they will ſometimes appear to move in a direction contrary to their real motion, and ſometimes appear ſtationary; and as the ſame face is not always turned towards the earth as towards the ſun, ſome part of the diſc which is towards the earth will not be illuminated. Theſe, with ſome other appearances and circumſtances which are obſerved to take place among the planets, we ſhall next proceed to explain; and as theſe are matters in which great accuracy is never requiſite, being of no great practical uſe, but rather ſubjects of curioſity, we ſhall conſider the motion of all the planets as performed in circles about the ſun in the center, and lying in the plane of the ecliptic.

313. To find the *poſition* of a planet when ſtationary. Let S be the ſun, E the earth, P the cotemporary poſition of the planet, $X\Upsilon$ the ſphere of the fixed ſtars to which we refer the motions of all the planets; let EF, $P\mathcal{Q}$ be two indefinitely ſmall arcs deſcribed in the ſame time, and let EP, $F\mathcal{Q}$ produced, meet at L; then it is manifeſt, that whilſt the earth was moving from E to F, the planet appeared ſtationary at L; and on account of the immenſe diſtance of the fixed ſtars, EPL, $F\mathcal{Q}L$ may be conſidered as parallel. Draw SE, SFw, SvP and $S\mathcal{Q}$; then as EP and $F\mathcal{Q}$ are parallel, the angle $\mathcal{Q}FS - PES = PwS - PES = ESF$, and $SPw - S\mathcal{Q}F = SvF - S\mathcal{Q}F = PS\mathcal{Q}$; that is, the cotemporary variations of the angles E and P are as $ESF : PS\mathcal{Q}$, or (becauſe the angular velocities are inverſely as the periodic times, or inverſely in the ſeſquiplicate ratio of the diſtances) as $SP^{\frac{3}{2}} : SE^{\frac{3}{2}}$, or as $a^{\frac{3}{2}} : 1^{\frac{3}{2}}$. But the ſines of the angles E and P being in the conſtant ratio of $a : 1$, the cotemporary variations of theſe angles will (as is well known) be as their tangents. Hence if x and y be the ſines of the angles E and P, we have $x : y :: a : 1$, and $\frac{x}{\sqrt{1-x^2}} : \frac{y}{\sqrt{1-y^2}}$ $:: a^{\frac{3}{2}} : 1$, whence $x^2 = \frac{a^3-a^2}{a^3-1} = \frac{a^2}{a^2+a+1}$, and $x = \frac{a}{\sqrt{a^2+a+1}}$ the ſine of the planet's elongation from the ſun when ſtationary. FIG. 68.

Ex. If P be the earth, and E Venus; and we take the mean diſtances of the earth and Venus to be 100000 and 72333, we find $x = 0{,}48264$ the ſine of 28°. 51′. 5″, the elongation of Venus when ſtationary, upon the ſuppoſition of circular orbits.

For excentric orbits, the points will depend upon the poſition of the apſides and place of the bodies at the time. We may however get a very near approximation thus. Find the time when the planet would be ſtationary if the orbits were circular, and compute for ſeveral days, about that time, the geocentric place of the planet, ſo that you get two days, on one of which the planet was direct and on the other retrograde, in which interval it muſt have been ſtationary, and the point of time when this happened may be determined by interpolation. The arc of retrogradation muſt manifeſtly be different in different parts of the orbit. M. de la LANDE has given us the following circumſtances reſpecting the ſtationary ſituations, and retrograde motions of the planets. The *firſt* ſtationary, means the ſtationary poſition after the planet has been direct; and the *ſecond* ſtationary, after it has been retrograde. The titles above ſhow the places of the planet and earth in their orbits when the planet is *firſt* ſtationary; all other elongations at the time they are ſtationary, arcs and durations of retrogradation, muſt neceſſarily be contained within theſe limits. If the time of retrogradation be ſubtracted from the time of a *ſynodic* revolution, the remainder gives the time in which the motion of the planet has been *direct*.

MERCURY.

	☿ in perihelion ⊖ in aphelion	☿ in aphelion ⊖ in perihelion
Elongation at the firſt ſtationary	15°. 23′. 34″	18°. 39′. 23″
——— ſecond ſtationary	20. 50. 55	14. 48. 39
Arc of retrogradation	15. 43. 58	9. 21. 56
Duration of retrogradation	21 *days* 12*h*.	23 *days* 12*h*.

VENUS.

	♀ in perihelion ⊖ in aphelion	♀ in aphelion ⊖ in perihelion
Elongation at the firſt ſtationary	29°. 6′. 42″	28°. 28′. 0″
——— ſecond ſtationary	29. 40. 42	27. 41. 0
Arc of retrogradation	17. 12. 15	14. 35. 58
Duration of retrogradation	43 *days* 12*h*.	40 *days* 21*h*.

MARS.

MARS.

	♂ in perihelion ☉ in aphelion	♂ in aphelion ☉ in perihelion
Elongation at the firſt ſtationary	4ˢ. 25°. 3′. 9″	4ˢ. 10°. 18′. 59″
——— ſecond ſtation.	4. 26. 36. 51	4. 8. 44. 20
Arc of retrogradation	0. 10. 6. 11	0. 19. 34. 38
Duration of retrogradation	60 *days* 18 *hours*	80 *days* 15 *hours*

JUPITER.

	♃ in perihelion ☉ in aphelion	♃ in aphelion ☉ in perihelion
Elongation at the firſt ſtationary	4ˢ. 0°. 7′. 47″	3ˢ. 24°. 2′. 35″
——— ſecond ſtation.	3. 26. 41. 49	3. 23. 35. 18
Arc of retrogradation	0. 9. 51. 30	0. 9. 59. 23
Duration of retrogradation	116 *days* 18 *hours*	122 *days* 12 *hours*

SATURN.

	♄ in perihelion ☉ in aphelion	♄ in aphelion ☉ in perihelion
Elongation at the firſt ſtationary	3ˢ. 20°. 19′. 38″	3ˢ. 17°. 51′. 5″
——— ſecond ſtation.	3. 20. 45. 50	3. 17. 24. 48
Arc of retrogradation	0. 6. 55. 44	0. 6. 40. 39
Duration of retrogradation	135 *days* 9 *hours*	138 *days* 18 *hours*

GEORGIAN.

	♅ in perihelion ☉ in aphelion	♅ in aphelion ☉ in perihelion
Elongation at the firſt ſtationary	3ˢ. 12°. 23′	3ˢ. 13°. 33′
——— ſecond ſtation.	3. 15. 5	3. 13. 47
Arc of retrogradation	0. 4. 13	0. 4. 3
Duration of retrogradation	151*d*. 12*h*.	149*d*. 18*h*.

314. To find the *time* when a planet is ſtationary, we muſt know the time of it's oppoſition, or inferior conjunction. Let m and n be the daily motions of the earth and planet, and v the angle PSE when the planet is ſtationary; then $m-n$, or $n-m$, is the daily variation of the angle at the ſun between the earth and planet, according as it is a ſuperior or inferior planet; hence $m-n$, or $n-m$, : v :: 1 day : $\frac{v}{m-n}$ or $\frac{v}{n-m}$ the time from oppoſition or conjunction to the ſtationary points both before and after. Hence the planet muſt be ſtationary twice every *ſynodic* * revolution.

Ex. Let P be the earth, E Venus; then by the Example to Art. 313. the angle $SPE = 28°.\ 51',5$, therefore $PSE = 13°$; alſo $n-m = 37'$; hence $37'$: $13°$ :: 1 day : 21 days the time between the inferior conjunction and the ſtationary poſitions.

315. If the elongation be obſerved when ſtationary, we may find the diſtance of the planet from the ſun, compared with the earth's diſtance, ſuppoſed to be unity. For (313) $x^2 = \frac{a^2}{a^2+a+1}$; hence $a^2 + \frac{x^2}{x^2-1} \times a = -\frac{x^2}{x^2-1} =$ (if $t =$ the tangent of the angle whoſe ſine is x) $a^2 - t^2 a = t^2$; conſequently $a = \frac{1}{2}t^2 + t\sqrt{1+\frac{t^2}{4}}$, upon the ſuppoſition of circular orbits.

FIG. 69. 316. A ſuperior planet is retrograde in oppoſition, and an inferior planet in it's inferior conjunction. For let E be the earth, P a ſuperior planet in oppoſition; then as the velocities are in the inverſe ſquare roots of the radii of the orbits, the ſuperior planet moves ſloweſt; hence if EF, $P\mathcal{Q}$ be two indefinitely ſmall cotemporary arcs, $P\mathcal{Q}$ is leſs than EF, and on account of the immenſe diſtance of the ſphere ΥZ of the fixed ſtars, $F\mathcal{Q}$ muſt cut EP in ſome point x between P and m, conſequently the planet has appeared to move retrograde from m to n. If P be the earth, and E an inferior planet in inferior conjunction, it will have appeared to have moved retrograde from v to w. Hence from this and the laſt Article, a ſuperior planet appears to move retrograde from it's ſtationary point before oppoſition to it's ſtationary point after; and an inferior planet, from it's ſtationary point before inferior conjunction to it's ſtationary point after.

FIG. 70. 317. If S be the ſun, E the earth, V Venus or Mercury, and EV a tangent to the orbit of the planet, then will the angle SEV be the greateſt elongation of the planet from the ſun; which angle, if the orbits were circles having the ſun

* A *Synodic* revolution is the time between two conjunctions or oppoſitions of a planet.

in

in their center, would be found by ſaying, $ES : SV ::$ rad. : ſin. SEV. But the orbits are not circular, in conſequence of which the angle EVS will not be a right angle, unleſs the greateſt elongation happens when the planet is at one of it's apſides. The angle SEV is alſo ſubject to an alteration from the variation of SE and SV. The *greateſt* angle SEV happens, when the planet is in it's *aphelion* and the earth in it's *perigee*; and the *leaſt* angle SEV, when the planet is in it's *perihelion* and the earth in it's *apogee*. M. de la LANDE has calculated theſe greateſt elongations, and finds them 47°. 48′ and 44°. 57′ for *Venus*, and 28°. 20′ and 17°. 36′ for *Mercury*. If we take the mean of the greateſt elongations of Venus, which is 46°. 22′,5, it gives the angle $VSE = 43°. 37′,5$; and as the difference of the daily *mean* motions of Venus and the earth about the ſun is 37′, we have 37′ : 43°. 37′,5 :: 1 day : 70,7 days, the time that would elapſe between the greateſt elongations and the inferior conjunction, if the motions had been uniform, which will not vary much from the true time.

318. To delineate the appearance of a planet at any time. Let S be the ſun, E the earth, V Venus, for example, aVb the plane of illumination perpendicular to SV, cVd the plane of viſion perpendicular to EV, and draw av perpendicular to cd; then ca is the breadth of the viſible illuminated part, which is projected into cv, the verſed ſine of cVa, or SVZ, for SVc is the complement of each. Now the circle terminating the illuminated part of the planet, being ſeen obliquely, appears to be an ellipſe; therefore if $cmdn$ repreſent the projected hemiſphere of Venus next to the earth, mn, cd, two diameters perpendicular to each other, and we take $cv =$ the verſed ſine of SVZ, and deſcribe the ellipſe mvn, then $mcnvm$ will repreſent the viſible enlightened part, as it appears at the earth; and from the property of the ellipſe, this area varies as cv. Hence, *the viſible enlightened part : the whole diſc :: the verſed ſine of SVZ : diameter*. FIG. 71.

Hence *Mercury* and *Venus* will have the ſame phaſes from their inferior to their ſuperior conjunction, as the moon has from the new to the full; and the ſame from the ſuperior to the inferior conjunction, as the moon has from the full to the new. *Mars* will appear gibbous in quadratures, as the angle SVZ will then differ conſiderably from two right angles, and conſequently the verſed ſine from the diameter. For *Jupiter*, *Saturn* and the *Georgian*, the angle SVZ never differs enough from two right angles to make them appear gibbous, ſo that they always appear to ſhine with a full face.

319. Let V be the moon; then as EV is very ſmall compared with VS, ES, theſe lines will be very nearly parallel, and the angle SVZ very nearly equal to SEV; hence, *the viſible enlightened part of the moon varies very nearly as the verſed ſine of it's elongation*.

320. Dr. HALLEY propoſed the following Problem: To find the poſition of *Venus* when brighteſt, ſuppoſing it's orbit, and that of the earth, to be circular, having the ſun in the center. Draw Sr perpendicular to EVZ, and put $a = SE$, $b = SV$, $x = EV$, $y = Vr$; then $b - y$ is the verſed ſine of the angle SVZ; and as the intenſity of light varies inverſely as the ſquare of it's diſtance, the quantity of light received at the earth varies as $\frac{b-y}{x^2} = \frac{b}{x^2} - \frac{y}{x^2}$; but by Euclid, B. II. P. 12. $a^2 = b^2 + x^2 + 2xy$, hence $y = \frac{a^2 - b^2 - x^2}{2x}$; ſubſtitute this for y, and we get the quantity of light to be as $\frac{b}{x^2} - \frac{a^2 - b^2 - x^2}{2x^3} = \frac{2bx - a^2 + b^2 + x^2}{2x^3}$ = a maximum; put the fluxion equal to nothing, and $x = \sqrt{3a^2 + b^2} - 2b$. Now if $a = 1$, $b = ,72333$ as in Dr. HALLEY's Tables, then $x = ,43036$; hence the angle $ESV = 22°.\ 21'$; but the angle ESV at the time of the planet's greateſt elongation is $43°.\ 40'$; hence Venus is brighteſt between it's inferior conjunction and it's greateſt elongation; alſo the angle $SEV = 39°.\ 44'$ the elongation of Venus from the ſun at the ſame time. The angle $SVZ = VSE + VES = 62°.\ 5'$, the verſed ſine of which is 0,53, radius being unity; hence (318) the viſible enlightened part : whole diſc :: 0,53 : 2; Venus therefore appears a little more than one fourth illuminated, and anſwers to the appearance of the moon when five days old. The diameter of Venus is about $39''$, and therefore the enlightened part is about $10'',25$. At this time, Venus is bright enough to caſt a ſhadow at night. This ſituation happens about 36 days before and after it's inferior conjunction; for the daily variation of the angle ESV is the *difference* of the daily motions of the earth and Venus about the ſun, which (taking their mean motions) is $37'$; an angle ESV therefore of $22°.\ 21'$ correſponds to about 36 days. It paſſes the meridian about $2h.\ 31'$ before or after the ſun, according as we take the ſituation after or before the inferior conjunction. If inſtead of ſuppoſing Venus and the earth at their mean diſtances, we ſuppoſe Venus in it's perihelion and the earth in it's apogee, the elongation of Venus when brighteſt would be $39°.\ 6'$; and if Venus were in it's aphelion and the earth in it's perigee, it would be $40°.\ 20'$. *Memoirs de Berlin*, 1750.

321. If we apply this to Mercury, $b = ,3171$, and $x = 1,00058$; hence the angle $ESV = 78°.\ 55'\frac{2}{3}$; but the ſame angle at the time of the planet's greateſt elongation is $67°.\ 13'\frac{1}{2}$. Hence Mercury is brighteſt between it's greateſt elongation and ſuperior conjunction. Alſo the angle $SEV = 22°.\ 18'\frac{3}{4}$ the elongation of Mercury at that time.

322. When

322. When Venus is brighteſt, and at the ſame time is at it's greateſt north latitude, it can then be ſeen with the naked eye at any time of the day; for when it's north latitude is the greateſt, it riſes higheſt above the horizon, and therefore is more eaſily ſeen. This happens (325) once in about eight years, Venus and the earth returning to the ſame parts of their orbits after that interval of time.

323. *Venus* is a *morning* ſtar from inferior to ſuperior conjunction, and an *evening* ſtar from ſuperior to inferior conjunction. For let S be the ſun, E the earth, $ACBD$ the orbit of Venus, arm, esn, two tangents to the earth, repreſenting the horizon at each place. Then the earth revolving about it's axis according to the order abc, when a ſpectator is at a, the part rCm of the orbit of Venus is above the horizon, but the ſun is not yet riſen; therefore Venus, in going from r through C to m, appears in the morning before ſun riſe. When the ſpectator is carried by the earth's rotation to c, the ſun is then ſet, but the part nDs of Venus' orbit is ſtill above the horizon; therefore Venus, in going from n through D to s, appears in the evening after ſun ſet. FIG. 72.

324. If two planets revolve in circular orbits, to find the time from conjunction to conjunction. Let $P =$ the periodic time of the earth, $p =$ that of the planet, ſuppoſe an inferior, $t =$ the time required. Then P : 1 day :: 360° : $\frac{360°}{P}$ the angle deſcribed by the earth in 1 day; for the ſame reaſon, $\frac{360°}{p}$ is the angle deſcribed by the planet in 1 day; hence $\frac{360°}{p} - \frac{360°}{P}$ is the daily angular velocity of the planet *from* the earth. Now if they ſet out from conjunction, they will return into conjunction again after the planet has gained 360°; hence $\frac{360°}{p} - \frac{360°}{P}$: 360° :: 1 day : $t = \frac{Pp}{P-p}$. For a ſuperior planet, $t = \frac{pP}{p-P}$. This will alſo give the time between two oppoſitions, or between any two ſimilar ſituations.

325. To find the time when a planet and the earth return to the ſame point of the Heavens. Find, from a Table of their mean motions, a number of years agreeing to a complete number of revolutions of the planet. Now *Mercury* in 13 years, (of which three are biſſextiles) and three days, make 54 revolutions and 2°. 55′ over; and the earth has made 13 revolutions and 2°. 49′ over. In this time therefore the earth and Mercury return to the ſame ſituation in the Heavens, very nearly. It will be 13 years and two days, if there be four biſſextiles. *Venus*, after a ſpace of eight years, is found within 1°. 32′ of the ſame place, and the earth within 4′. *Mars*, in 15 years wanting 18 days, has changed it's place 11ˢ. 11°. 26′, and the earth 11ˢ. 11°. 38′; if there

have

have been four biſſextiles, it will be 15 years wanting 19 days. But in 79 years and 4 days, ſuppoſing there are 20 biſſextiles, Mars returns to the ſame ſituation within 3°. 39′, and the earth within 3°. 48′. *Jupiter* in 83 years returns to the ſame point within 12′, and the earth within 6′. The period of 12 years 5 days approaches very near, for Jupiter has in that time made 4°. 47′ above one revolution, and the earth 5°. 1′ above 12 revolutions. *Saturn* in 59 years and two days returns to the ſame ſituation within 1°. 45′, and the earth within 1°. 41′. M. de la LANDE, who has given theſe returns of the planets and earth to the ſame point of the Heavens, has alſo added the following GRAND CONJUNCTIONS.

On May 22, 1702, *Jupiter* and *Saturn* were within 1°. 4′ of each other. *Miſcel. Berolin.* p. 217.

On February 11, 1524, *Venus*, *Mars*, *Jupiter* and *Saturn* were very near each other, and *Mercury* not above 16° from them, according to the Ephemeris of STOFFLER.

On November 11, 1544, *Mercury*, *Venus*, *Jupiter* and *Saturn* were within the ſpace of 10°.

On March 17, 1725, *Mercury*, *Venus*, *Mars* and *Jupiter* appeared within the ſame teleſcope. SOUCIET, *Obſ. Mathem.* T. 1. p. 103.

On December 23, 1769, *Venus*, *Mars* and *Jupiter* were within 1° of each other.

CHAP.

CHAP. XVIII.

ON THE MOON'S MOTION FROM OBSERVATION, AND IT'S PHÆNOMENA.

Art. 326. THE moon being the neareſt, and moſt remarkable body in our ſyſtem next to the ſun, and alſo uſeful for the diviſion of time, it is no wonder that the ancient Aſtronomers were attentive to diſcover it's motions; and it is a very fortunate circumſtance, that their obſervations have come down to us, as from thence it's mean motion can be more accurately ſettled, than it could have been by modern obſervations only; and it moreover gave occaſion to Dr. HALLEY, from the obſervations of ſome ancient eclipſes, to diſcover an acceleration in it's mean motion. The proper motion of the moon in it's orbit about the earth is from weſt to eaſt; and from comparing it's place with the fixed ſtars in one revolution, it is found to deſcribe an orbit inclined to the ecliptic; it's motion alſo appears not to be uniform; and the poſition of the orbit, and the line of it's apſides are obſerved to be ſubject to a continual change. Theſe circumſtances, as they are eſtabliſhed by obſervation, we come now to explain; the phyſical cauſes thereof will afterwards become the ſubject of our conſideration.

To determine the Place of the Moon's Nodes.

327. *Firſt Method.* Let AE be the ecliptic, A the firſt point of Aries, OL the moon's orbit, N the node, m the place of the moon in it's orbit when it paſſes the meridian on the day before it comes to the ecliptic, n the place when it paſſes the day after, and draw mv, nw perpendicular to EA. Find (124) it's latitudes mv, nw on theſe two days, and it's longitudes Av, Aw; then $mv+nw : mv :: vw :: vN$, which added to AN gives the longitude of the node. To find the time when the moon is in the node, we have $vw : vN$ :: the interval of time between the paſſages of the moon over the meridian : the interval from the time of the firſt paſſage over the meridian till it comes to the node; this interval therefore added to the time of that paſſage, gives the time of the paſſage through the node. FIG. 73.

328. *Second Method.* In a *central* eclipſe of the moon, the moon's place at the middle of the eclipſe is directly oppoſite to the ſun, and the moon muſt alſo then be in the node; calculate therefore the true place of the ſun, or which is more exact, find it's place by obſervation, and the oppoſite point will be the true place of the moon, and conſequently the place of it's node.

Ex. M. CASSINI, in his Aſtronomy, pag. 281, informs us, that on April 16, 1707, a central eclipſe was obſerved at Paris, the middle of which was determined to be at 13*h*. 48′ apparent time. Now the true place of the ſun calculated for that time was 0ˢ. 26°. 19′. 17″; hence the place of the moon's node was 6ˢ. 26°. 19′. 17″. The moon paſſed from north to ſouth latitude, and therefore this was the deſcending node.

FIG. 74. 329. *Third Method.* To find the place of the node by a partial eclipſe. Find, by obſervation, the magnitude *AB* of the eclipſe at the middle, and ſubtract it from the ſemidiameter *AD* of the earth's ſhadow, and we have *DB*, to which add *BC* the ſemidiameter of the moon, and we have *CD*. Now at the time of a lunar eclipſe, we may ſuppoſe the angle $CND = 5°. 17'$, from which it will never differ but a very little. Hence in the right angled triangle *DCN*, right angled at *C*, we have *DC* and the angle *DNC*, to find *DN*; and as the point *D* is oppoſite to the true place of the ſun, which is known by computation, the place *N* of the node will be known.

Ex. On March 26, 1717, the middle of an eclipſe was obſerved at Paris at 15*h*. 16′, and the digits eclipſed were $7\frac{17}{60}$ towards the north. Now the ſemidiameter of the moon was 15′. 46″, and that of the ſhadow 42′. 43″; hence 12 dig. :: $7\frac{17}{60}$ dig. :: 31′. 32″ the diameter of the moon : 19′. 8″ = *AB*; therefore $BD = 23'. 35''$, to which add $BC = 15'. 46''$ and we have $CD = 39'. 21''$, which is ſouth, becauſe the ſhadow upon the moon is towards the north. Hence in the right angled triangle *DCN*, we have $CD = 39'. 21''$, and the angle $N = 5°. 17'$, conſequently $DN = 7°. 8'. 26''$, which is the diſtance of the center of the earth's ſhadow from the aſcending node, becauſe the ſhadow of the earth is on the north ſide of the moon and the latitude is decreaſing. Now the true place of the ſun at that time was 0ˢ. 6°. 20′. 43″, and therefore the true place of the center *D* of the earth's ſhadow was 6ˢ. 6°. 20′. 43″, to which add $DN = 7°. 8'. 26''$ and we get the true place of the aſcending node of the moon to be in 6ˢ. 13°. 29′. 9″. M. de la LANDE makes the epoch of the aſcending node for 1780, to be 2ˢ. 0°. 3′. 2″.

On

On the Mean Motion of the Nodes.

330. To determine the mean motion of the nodes, find (327) the place of the nodes at different times, and it will give their motion in the interval. We muſt firſt compare the places at a ſmall interval, to get nearly their mean motion, and then at a greater interval to get it more accurately. Now on April 16, 1707, at 13*h*. 48′ at Paris, the aſcending node was in 0ˢ. 26°. 19′; and on March 26, 1717, at 15*h*. 16′, the place of the ſame node was in 6ˢ. 13°. 29′; alſo by an eclipſe obſerved at the ſame place on September 9, 1718, at 8*h*. 4′, the place of the aſcending node was in 5ˢ. 16°. 40′. From the two laſt obſervations it appears that the node is retrograde. Now the interval of theſe two obſervations was 531*d*. 0*h*. 16′. 48″, during which time the nodes moved retrograde through 26°. 49′, which gives the diurnal motion 3′. 2″. If we compare the firſt and laſt obſervations, they give the daily motion 3′. 10″.

331. But to determine the mean motion of the nodes with greater accuracy, we muſt compare together more diſtant obſervations. PTOLEMY, in his *Almageſt*, mentions three lunar eclipſes, that were obſerved at Babylon by the Chaldeans. The firſt was total on March 19, 720 years before J. C. the beginning was at 7*h*. 30′ in the evening, and the middle was at 9*h*. 30′. The ſecond was on March 8, 719 years before J. C. the middle of which happened at midnight, and the greateſt quantity eclipſed was 3 digits towards the ſouth. The third happened on Sepember 1, 719 years before J. C. the middle of which was at 8*h*. 30′ in the evening, and the moon was eclipſed a very little more than one half towards the north. Now it being uncertain whether the firſt eclipſe was central, M. CASSINI takes the ſecond; and the difference of the meridians of Babylon and Paris being 2*h*. 42′, it gives the middle of the eclipſe at Paris 9*h*. 18′ in the evening. And, by computation (329), we find the center of the earth's ſhadow to be 8°. 24′. 50″ from the node. The middle of the third eclipſe happened at Paris at 5*h*. 48′, and M. CASSINI takes the digits eclipſed to be $6\frac{1}{4}$, and computes (329) the diſtance of the center of the ſhadow from the node to be 8°. 15′. 28″. But we cannot tell from either of theſe obſervations, whether the latitude of the moon was aſcending or deſcending, and therefore we do not know at which node the eclipſes happened. To determine this, take the total eclipſe on March 19, the middle of which was at 9*h*. 30′ at Babylon, or 6*h*. 48′ at Paris, at which time the ſun's place, by computation, was 11ˢ. 21°. 27′, therefore the moon's place, was 5ˢ. 21°. 27′. Between this time and the eclipſe on September 1, there was very nearly 18 months, in which time the nodes had moved retrograde about 29°, which ſubtracted from the place of the moon in the obſervation on March 19, which we ſuppoſe to be

nearly the ſame as that of it's node, as the eclipſe was total, gives the place of the node on September 1, in $4^s.\ 23^\circ$, and the oppoſite node in $10^s.\ 23^\circ$. Now the true place of the ſun at the middle of this eclipſe was $5^s.\ 1^\circ.\ 7'$, and conſequently that of the moon $11^s.\ 1^\circ.\ 7'$. Hence the place of the moon in this eclipſe was about 8° *before* the place of the node, and the moon being eclipſed on the north ſide, this muſt have been the deſcending node. Hence if we ſubtract $8^\circ.\ 15'$, the diſtance of the node from the center of the ſhadow on September 1, 719, from $11^s.\ 1^\circ.\ 7'$ the place of the center of the ſhadow, we ſhall have $10^s.\ 22^\circ.\ 52'$ for the place of the deſcending node on September 1, 719 years before J. C. conſequently the true place of the aſcending node was $4^s.\ 22^\circ.\ 52'$. Now the place of the aſcending node on September 9, 1718, at $8h.\ 4'$ of the evening, was $5^s.\ 16^\circ.\ 40'$; and as the motion of the nodes is retrograde, the node in this latter caſe wants $23^\circ.\ 48'$ of being up to the place of the node in the former caſe; conſequently in this interval of time, which is 2437 years (of which 608 were biſſextiles) $19d.\ 2h.\ 16'$, the nodes made a certain number of revolutions and $336^\circ.\ 12'$ over. Now Art. 330. gives $3'.\ 10''$ for the mean diurnal motion of the nodes, and conſequently in the above time, the nodes muſt have made 131 complete revolutions; if therefore we divide $2437y.\ 19d.\ 2h.\ 16'$ by 131 revolutions $336^\circ.\ 12'$, it gives $6798d.\ 7h.$ for the time of a mean revolution of the nodes; hence if we divide $6798d.\ 7h.$ by $365d.$ it gives $19^\circ.\ 19'.\ 45''$ for the mean motion of the nodes in a common year of 365 days; and if we divide $19^\circ.\ 19'.\ 45''$ by 365, it gives $3'.\ 10''.\ 38'''$ for the mean daily motion of the nodes. This differs only $38'''$ from the motion determined from the obſervations in 1707 and 1718. The motion of the nodes is not uniform; certain equations therefore are neceſſary to be applied to the mean place in order to get the true place at any time. MAYER in his Tables makes the mean annual motion $19^\circ.\ 19'.\ 43'',1$.

If we examine the motion of the nodes from the eclipſes on March 8, and September 1, 719 years before J. C. it gives $3'.\ 10''.\ 20'''$ for their mean daily motion. We have no reaſon therefore to think, that the mean motion of the nodes is ſubject to any change.

On the Inclination of the Orbit of the Moon to the Ecliptic.

332. To determine the inclination of the orbit, obſerve the moon's right aſcenſion and declination when it is 90° from it's nodes, and thence compute it's latitude (124), which will be the inclination at that time. Repeat the obſervation for every diſtance of the ſun from the earth, and for every poſition of the ſun and moon in reſpect to the moon's nodes, and you will get the

the inclination at thoſe times. From theſe obſervations it appears, that the inclination of the orbit to the ecliptic is variable, and that the *leaſt* inclination is about 5°, which is found to happen when the nodes are in quadratures; and the *greateſt* is about 5°. 18′, which is obſerved to happen when the nodes are in ſyzygies. The inclination is found alſo to depend upon the ſun's diſtance from the earth.

On the Mean Motion of the Moon.

333. The mean motion of the moon is found from obſerving it's place at two different times, and you get the mean motion in that interval, ſuppoſing the moon to have had the ſame ſituation in reſpect to it's apſides at each obſervation; and if not, if there be a very great interval of the times, it will be ſufficiently exact. To determine this, we muſt compare together the moon's places, firſt at a ſmall interval of time from each other, in order to get very nearly the mean time of a revolution; and then at a greater interval, in order to get it more accurately. The moon's place may be determined directly from obſervation, or deduced from an eclipſe.

334. M. CASSINI, in his Aſtronomy, pag. 294, obſerves, that on September 9, 1718, the moon was eclipſed, the middle of which happened at 8*h*. 4′, when the ſun's true place was 5ˢ. 16°. 40′. This he compared with another eclipſe, the middle of which was obſerved at 8*h*. 32′ on Auguſt 29, 1719, when the ſun's place was 5ˢ. 5°. 47′. In this interval of 354*d*. 28′ the moon made 12 revolutions and 349°. 7′ over; divide therefore 354*d*. 28′ by 12 revolutions 349°. 7′, and it gives 27*d*. 7*h*. 6′ for the time of one revolution. This is ſufficiently accurate to compare eclipſes at a greater interval.

335. On March 26, 1717, the middle of a lunar eclipſe was obſerved at 15*h*. 16′ at Paris, when the ſun's place was 0ˢ. 6°. 21′. And on March 15, 1699, an eclipſe was obſerved, the middle of which was at 7*h*. 23′ at which time the ſun's place was 11ˢ. 25°. 30′. In this interval of 18 years (of which 4 were biſſextiles) 11*d*. 7*h*. 53′, the moon, beſides a certain number of revolutions, was advanced 10°. 51′. This interval of 6585*d*. 7*h*. 53′ divided by 27*d*. 7*h*. 6′ gives 241 revolutions and about $\frac{1}{4}$, which ſhows that the number of complete revolutions muſt have been 241. Hence if we divide 6585*d*. 7*h*. 53′ by 241 revolutions 10°. 51′, it gives 27*d*. 7*h*. 43′. 6″ for the time of one revolution. This will be ſufficiently accurate to give the time for the moſt diſtant eclipſes.

336. The

336. The moon was obſerved at Paris to be eclipſed on September 20, 1717, the middle of which was at 6*h*. 2′. Now PTOLEMY mentions that a total eclipſe of the moon was obſerved at Babylon on March 19, 720 years before J. C. the middle of which happened at 9*h*. 30′, at that place, which gives 6*h*. 48′ at Paris. The interval of theſe times was 2437 years (of which 609 were biſſextiles) 174 days wanting 46′; divide this by 27*d*. 7*h*. 43′. 6″ and it gives 32585 revolutions and a little above $\frac{1}{2}$. Now the difference of the two places of the ſun, and conſequently of the moon, at the times of obſervation, was 6ˢ. 6°. 12′. Therefore in the interval of 2437*y*. 174*d*. wanting 46′ the moon had made 32585 revolutions 6ˢ. 6°. 12′, which gives 27*d*. 7*h*. 43′. 5″ for the mean time of a revolution. This determination is very exact, as the moon was at each time very nearly at the ſame diſtance from it's apſide. Hence the mean *diurnal* motion is 13°. 10′. 35″, and the mean *hourly* motion 32′. 56″. 27‴ $\frac{1}{2}$. M. de la LANDE makes the mean *diurnal* motion 13°. 10′. 35″,02784394. This is the mean time of a revolution in reſpect to the equinoxes. The place of the moon at the middle of the eclipſe has here been taken the ſame as that of the ſun, which is not accurate, except for a central eclipſe; it is ſufficiently accurate, however, for this long interval. From the unequal angular motion of the moon about the earth, the hourly motion of the moon is ſubject to change from 29′. 55″ to 38′. 22″; the excentricity of the orbit produces a variation of 3′. 36″; the evection produces one of 42″, and the variation produces one of 40″. The corrections for all the inequalities of the moon's motion will be found in the Tables of the moon.

337. Hence to find the mean motion for any other time, ſay, the interval between the eclipſes 2437*y*. 174*d*. wanting 46′ : any other time :: 32585 revolutions 6ˢ. 6°. 12′ : the mean motion in that time. This is more exact than taking the mean diurnal motion 13°. 10′. 35″ and multiplying it by the time, as ſmall errors are thus multiplied and become conſiderable. M. de LAMBRE makes the ſecular motion to be 10ˢ. 7°. 53′. 12″, which M. de la LANDE uſes in his Tables. MAYER in his Tables makes it 10ˢ. 7°. 53′. 35″. In this motion of 100 years, 25 are ſuppoſed to be biſſextiles.

338. As the preceſſion of the equinoxes is 50″,25 in a year, or about 4″ in a month, the mean revolution of the moon in reſpect to the fixed ſtars muſt be greater than that in reſpect to the equinox by the time the moon is deſcribing 4″ with it's mean motion, which is about 7″. Hence the time of a ſidereal revolution of the moon is 27*d*. 7*h*. 43′. 12″.

339. M. de la LANDE has determined the revolution in reſpect to the equinoxes to be 27*d*. 7*h*. 43′. 4″,6795, which does not differ $\frac{1}{2}$″ from the above; and hence he makes the ſidereal revolution 27*d*. 7*h*. 43′. 11″,5259. Hence the mean ſynodic revolution (324) is 29*d*. 12*h*. 44′. 2″,8283. If we take

take unity to repreſent the mean motion of the moon in reſpect to the fixed ſtars, then will 0,0040218 53526 repreſent the motion of the node, found by comparing their mean motions; hence, as the nodes move retrograde, the ſidereal revolution of the moon, 27*d*. 7*h*. 43′. 11″,5259, : it's revolution in reſpect to it's nodes :: 1,0040218 53526 : 1, the moon approaching the node with the ſum of the velocities; hence the revolution of the moon in reſpect to the nodes is 27*d*. 5*h*. 5′. 35″,603. This is the determination of the mean revolutions to the beginning of this century.

To determine the Place of the Moon's Apogee, and the Equation of it's Orbit.

340. Compare the obſerved place of the moon at any time with the place obſerved at any time afterwards; take the mean motion correſponding to the interval of time, and add it to the moon's place at the firſt obſervation, and the difference between that ſum and the moon's place at the ſecond obſervation ſhows the effect of the equation of the orbit between theſe two ſituations of the moon. Repeat this for a great many intervals, and mark thoſe where the difference between the ſum before mentioned and the moon's true place is greateſt both in exceſs and defect. If the greateſt exceſs and defect be equal, it is a proof that at the time of the firſt obſervation, the moon was in it's apogee or perigee, and that it's true and mean places were the ſame. In this caſe each of theſe differences is the greateſt equation of the moon's orbit. If the greateſt exceſs and defect be not equal, half the ſum will meaſure the greateſt equation; and if from the greateſt equation we ſubtract the leaſt of the differences, we ſhall have the equation of the moon at the time of the firſt obſervation. M. CASSINI uſes the place of the moon as determined from it's eclipſes, ſelecting thoſe which were proper for this purpoſe; and although the apogee has moved in the interval, yet, as the true and mean place of the moon always coincide at the apogee, it will not affect the concluſion. *Elem. d'Aſtron.* pag. 297.

341. Hence to find the place of the apogee, let *AMPV* be the orbit of the moon, *A* the apogee, *P* the perigee, *C* the center of the orbit, *T* the earth in the focus, *F* the other focus, *M* the place of the moon at the time of the firſt obſervation; produce *TM* to *R*, take *MR* = *MF*, and join *RF*. From the greateſt equation find (231) the ratio of *AC* to *CT*; this being known, we have, *TF* : *TR* :: ſin. *TRF* : ſin *TFR*, or *AFR*; now *FRT* = ½*FMT* the equation of the moon at the firſt obſervation, upon the *ſimple elliptic* hypotheſis (227); hence we know *AFR*, from which ſubtract *FRT*, and we get *ATM* the moon's diſtance from it's apogee. FIG. 75.

342. Let

342. Let the firſt eclipſe, with which the others are to be compared, be a total one, the middle of which happened at Paris on December 10, 1685, at 10*h*. 38′. 10″ mean time. The true place of the ſun at that time, by calculation, was 8ˢ. 19°. 40′, and conſequently the moon's place was 2ˢ. 19°. 40′. Let the next eclipſe be the total one on May 16, 1696, the middle of which was 12*h*. 7′. 56″ mean time at Paris, and the moon's place was 7ˢ. 26°. 53′. 35″. Now in this interval of 10 years (of which 3 were biſſextiles) 157*d*. 1*h*. 29′. 46″, the mean motion of the moon, omitting the complete revolutions, was 5ˢ. 12°. 53′. 10″; this added to 2ˢ. 19°. 40′, the place at the firſt eclipſe, gives 8ˢ. 2°. 33′. 10″ for the mean place at the ſecond eclipſe, the difference between which and the true place 7ˢ. 26°. 53′. 35″ is 5°. 39′. 35″. The next eclipſe compared with the firſt was that on March 15, 1699, the middle of which was at 7*h*. 14′ mean time at Paris, at which time the moon's true place was 5ˢ. 25°. 28′. 41″. Now in this interval of 13 years (of which 3 were biſſextiles) 94*d*. 20*h*. 35′. 50″, the mean motion of the moon, omitting the revolutions, was 3ˢ. 1°. 24′. 47″; this added to 2ˢ. 19°. 40′, the place at the firſt eclipſe, gives 5ˢ. 21°. 4′. 47″ for the mean place at this third eclipſe, the difference between which and 5ˢ. 25°. 28′. 41″ the true place is 4°. 23′. 54″. Now in the former caſe, the true place was leſs than the mean place by 5ˢ. 39°. 35′, and in the latter caſe, the mean place is the leaſt by 4°. 23′. 54″. Theſe are the greateſt differences of all the eclipſes between 1685, and 1720. Now the ſum of theſe differences is 10°. 3′. 29″, and the half ſum is 5°. 1′. 44″,5 the greateſt equation of the moon's orbit deduced from theſe obſervations. And if from 5°. 1′. 44″,5 we take 4°. 23′. 54″, the leaſt difference, we have 37′. 50″,5 for the equation of the moon at the time of the firſt eclipſe; and this taken from 2ˢ. 19°. 40′, the true place of the moon at that time, gives 2ˢ. 19°. 2′. 10″ for the mean place of the moon on Dec. 10, 1685, at 10*h*. 38′. 10″ mean time at Paris. This therefore may be conſidered as an *epoch* of the mean place of the moon. This is the method uſed by M. Cassini. But the beſt method is, to obſerve accurately the place of the moon for a whole revolution as often as it can be done, and by comparing the true and mean motions, the greateſt difference will be double the equation. If two obſervations be found, where the difference of the true and mean motions is nothing, the moon muſt then have been in it's apogee and perigee. Mayer makes the mean excentricity 0,05503568, and correſponding greateſt equation 6°. 18′. 31″,6. It is 6°. 18′. 32″ in his laſt Tables publiſhed by Mr. Mason, under the direction of Dr. Maskelyne.

343. To determine the place of the apogee, from M. Cassini's obſervations, we have the greateſt equation = 5°. 1′. 44″,5, therefore (231), 57°. 17′. 48″,8 : 2°. 30′. 52″,25 :: $AC = 100000 : CT = 4388$ for the moon's excentricity.

Alſo

Alſo $TF = 8776$: $TR = 200000$ :: ſin. $TRF = 18'.\ 55'',25$: ſin. TFR, or AFR, $= 7°.\ 12'.\ 20''$, from which take $TRF = 18'.\ 55'',25$, and we have $ATM = 6°.\ 53'.\ 25''$ the diſtance of the moon from it's apogee; add this to $2^s.\ 19°.\ 40'$, the true place of the moon, and it gives $2^s.\ 26°.\ 33'.\ 25''$ for the place of the apogee on December 10, 1685, at $10h.\ 38'.\ 10''$ mean time at Paris. This therefore may be conſidered as an *epoch* of the place of the apogee.

344. If we compare the ſame eclipſe in 1685 with two others, one of which happened on July 7, 1675, and the other on April 14, 1642, we ſhall get the equation of the orbit $5°.\ 2'.\ 14''$, differing only $37''$ from the other determination. Alſo the place of the apogee at the eclipſe in 1685, comes out $2^s.\ 25°.\ 57'.\ 58''$, which is $35'.\ 27''$ leſs advanced than by the former caſe. If the moon's place be determined by obſervation at any time when it is not eclipſed, the ſame method may be applied.

345. It has been here ſuppoſed, that at the time of the eclipſes the moon was at it's mean diſtance, and of the great number of obſervations which were compared in order to get the greateſt difference of the true and mean places, it is ſuppoſed that thoſe which gave the greateſt differences were ſo circumſtanced. Alſo, no other inequalities have been ſuppoſed, but thoſe which aroſe from the excentricity of the moon's orbit. The way therefore to get at the greateſt accuracy is to make a great number of ſuch compariſons, and take the mean.

346. The place of the moon's apogee may alſo be thus found. Take a great many meaſures of the moon's diameter, when at, or very near, the full, with a micrometer, and reduce them to the meaſure at the ſame altitude, and note the times of obſervation; ſeek out amongſt them, thoſe which are the greateſt or the leaſt, and you have the time when the moon was in it's perigee or apogee. Or if you find two diameters equal to each other, very near together, the moon muſt have been in it's apogee or perigee in the middle point of time. Now at the apogee, the difference between the true and mean motion of the moon for every degree is about $5'$; and at the perigee, about $5'.\ 30''$. Hence the places of the moon being known at the time when the two diameters were found equal, and the mean motion between the times being known, the mean motion from one of the times to the middle point of time between will be known; therefore, as the difference of the true and mean motion is known, the true motion is known from one of the times to the half interval of time, and conſequently the true place of the moon at the half interval, or place of the apogee or perigee, will be known. But on account of the great difficulty of meaſuring accurately the diameter of the moon, this method cannot be depended upon to a great degree of accuracy.

It was from obſerving the diameter of the moon, that HORROX found the motion of the apogee was ſometimes in antecedentia, and ſometimes in conſequentia; and that the excentricity of the orbit muſt be variable, in order to account for the *ſecond* equation (349) obſerved by PTOLEMY. By this method, M. CASSINI found, from the eclipſe on December 10, 1685, at 10*h*. 38′. 10″ apparent time at Paris, the place of the apogee to be 2ˢ. 25°. 41′. 30″. From the mean of a great number of obſervations, he determined, at the above time, the place of the apogee to be 2ˢ. 24°. 32′, and the greateſt equation 4°. 58′. 44″. But the excentricity, and conſequently the greateſt equation, is ſubject to a variation; and the excentricity here determined is about the leaſt. According to MAYER, the mean excentricity is 0,05503568, and the correſponding greateſt equation 6°. 18′. 31″,6.

To determine the mean Motion of the Apogee.

347. Find it's place at different times, and compare the difference of the places with the interval of the time between. To do this, we muſt firſt compare obſervations at a ſmall diſtance from each other, leaſt we ſhould be deceived in a whole revolution; and then we can compare thoſe at a greater diſtance. Now we may either compute (343) the place of the apogee at ſeveral times, or we may find it from knowing the place once, according to the following method, given by M. CASSINI in his Aſtronomy, page 307. The place of the apſide has been determined for Dec. 10, 1685; and to find from thence it's place at any other time, obſerve the true place of the moon at that time, and find the mean motion correſponding to that interval, and add it to, or ſubtract it from, the place of the apogee on December 10, 1685, according as the time was after or before that, and you have the mean place of the moon at that time; the difference between which and the true place obſerved, is the equation of the orbit at that time; if the mean place be forwarder than the true, the moon is in the firſt ſix ſigns; if backwarder, in the laſt ſix. But the ſame equation may anſwer to two different mean anomalies; this therefore leaves an uncertainty in reſpect to the place of the apogee. Now from the mean place of the moon ſubtract each mean anomaly, and it gives the place of the apogee correſponding to each; conſequently you get the motion of the apogee correſponding to each place thus found; and to determine which is the true motion, repeat the operation for ſome other time compared with the place of the apogee on December 10, 1685, and you will get the motion correſponding to two places again. Then compare theſe two motions

motions with the other two, and thoſe two which agree, muſt be the true motion.

348. By thus comparing the place of the apogee on December 21, 1684, at 10*h*. 55′. 58″ apparent time, with the place determined on Dec. 10, 1685, M. CASSINI found the time of a revolution of the apſides to be either 8 years and nearly 9 months, or about 3 years. And by comparing the place of the apogee on Nov. 29, 1686, at 11*h*. 7′. 18″ apparent time, with the place on December 10, 1685, he found that the motion of the apſides, deduced from thence, came out, one between eight and nine years, but that the other motion did not agree with either of the former. The time of a revolution therefore muſt be about 8 years 9 months. The time being thus nearly determined, he computed the motion from more diſtant obſervations, and from a mean of the whole, he found the time of a revolution of the apſides to be 8 common years, 311*d*. 8*h*. and hence the mean annual motion is 1°. 10°. 39′. 52″, and daily motion 6′. 41″. 1‴. MAYER in his Tables makes the annual motion 1°. 10°. 39′. 50″. This is the mean motion in reſpect to the equinoxes. M. de la LANDE makes the daily motion, in reſpect to the equinoxes, 6′. 41″,069815. Hence he deduces the daily motion in reſpect to the fixed ſtars to be 6′. 40″,932238. If we take unity to repreſent the mean motion of the moon in reſpect to the fixed ſtars, then will the motion of it's apogee be repreſented by 0,00845226445, found by comparing their mean motions; hence, as the motion of the apogee is direct, the ſidereal revolution of the moon, 27*d*. 7*h*. 43′. 11″,4947, : it's revolution in reſpect to it's apogee :: 1 − 0,00845226445 : 1, the moon approaching the apogee with the difference of the velocities; hence the revolution of the moon in reſpect to it's apogee is 27*d*. 13*h*. 18′. 33″,95. The motion of the apogee is not uniform, as is implied in this method of determining it's mean motion, and therefore it will be ſubject to a ſmall error, unleſs the equation ſhould be the ſame at both obſervations; this error may be corrected, by reducing the true to the mean place at each obſervation. HORROX, from obſerving the diameters of the moon, found the apogee ſubject to an annual equation of 12°,5. Having thus explained the methods of determining the moon's mean motions, ſituation of it's apogee, and the equation of it's orbit, or firſt inequality, we proceed to ſhow how that, and ſome of the other principal inequalities were diſcovered.

349. The motion of the moon having been examined for one month, it was immediately diſcovered, that it was ſubject to an irregularity, which ſometimes amounted to 5° or 6°, but that this irregularity diſappeared about every 14 days. And by continuing the obſervations for different months, it alſo appeared, that the points where the inequalities were the greateſt, were not fixed, but that they moved forwards in the Heavens about

3° in a month, ſo that the motion of the moon in reſpect to it's apogee was about $\frac{1}{120}$ leſs than it's abſolute motion; thus it appeared that the apogee had a progreſſive motion. PTOLEMY determined this *firſt* inequality, or equation of the orbit, from three lunar eclipſes obſerved in the years 719 and 720, before J. C. at Babylon by the Chaldeans; from which he found it amounted to 5°. 1′ when at it's greateſt. But he ſoon diſcovered that this inequality would not account for all the irregularities of the moon. The diſtance of the moon from the ſun obſerved both by HIPPARCHUS and himſelf, ſometimes agreed with this inequality, and ſometimes it did not. He found that when the apſides of the moon's orbit were in quadratures, this *firſt* inequality would give the moon's place very well; but that when the apſides were in ſyzygies, he diſcovered that there was a further inequality of about $2°\frac{2}{3}$, which made the whole inequality about $7°\frac{2}{3}$. This *ſecond* inequality is called the *Evection*, and ariſes from a change of excentricity of the moon's orbit. The inequality of the moon was therefore found, by PTOLEMY, to vary from about 5° to $7°\frac{2}{3}$, and hence the mean quantity was 6°. 20′. MAYER makes it 6°. 18′. 31″,6. It is very extraordinary, that PTOLEMY ſhould have determined this to ſo great a degree of accuracy. This mean quantity is the greateſt equation of the orbit for the mean excentricity, and is called the *firſt* equation. The *Evection*, or variation of the equation of the orbit from the mean equation, is at it's maximum 1°. 20′. 28″,9 according to MAYER. Hence when the apſides are in ſyzygies, at which time the excentricity is found to be the greateſt, the greateſt equation is 7°. 39′. 0″,5; and when the apſides are in quadratures, at which time the excentricity is found to be the leaſt, it is only 4°. 58′. 2″,7. D'ARZACHEL, an Arab, who obſerved in Spain about the year 1080, from comparing the obſervations of PTOLEMY and thoſe of D'ALBATEGNIUS with his own, diſcovered that the apſides were ſometimes progreſſive and ſometimes regreſſive; and that the excentricity was ſubject to a change. KEPLER believed this to be the caſe. HORROX diſcovered the ſame from his own obſervations; he found that when the diſtance of the ſun from the apogee of the moon was about 45° and 225°, the apogee was more advanced by 25″ than when the diſtance was about 135° and 315°, in ſuch a manner that the mean motion was not uniform, but ſubject from thence to an equation of about 12°,5. He firſt made the moon revolve in an ellipſe about the earth in it's focus, and although ſome difficulties aroſe from this ſuppoſition, yet, he ſays, he durſt not give up the hypotheſis.

FIG. 76. 350. TYCHO explained theſe irregularities thus. Let the earth be at T the center of the circle $sqxy$, whoſe radius is 100000; Tr, the ſemidiameter of the circle $Tdet$, $=21741$ the circle of excentricity, in whoſe circumference the cen-

ter

ter of excentricity is ſuppoſed to move in conſequentia *Tde*, with a motion equal to double the diſtance of the moon from the ſun; *so* the radius of the circle *acbo* = 5800; and *om*, the radius of *mwzv*, = 2900. Let *sq* = 90°; and let the moon move from it's ſyzygies and apogee at *s* to quadratures at *q*, and conceive in the ſame time the center of excentricity to move from *T* through *d* to *e*, with twice the angular velocity of the moon from the ſun. Then, conſidering *r* as the mean place of the center, when the moon comes to *q*, the equation is the angle *eqr* = 1°. 15′, which is to be ſubtracted in the firſt quadrature at *q*, and added in the third quadrature at *y*; this will produce an inequality of 2°. 30′, and account for the *Evection*. But inſtead of ſuppoſing the moon to revolve in the circumference *sqxy*, let the center of the circle *oacb* revolve in conſequentia, and the moon revolve in antecedentia in the circumference *obca*, and be at *o* when the moon is in it's apogee, and to deſcend through *b* and arrive at *c* when the moon comes to it's perigee; this will produce an inequality of 3°. 19′, which is part of the equation of the center. Laſtly, let us ſuppoſe the moon to revolve in the circumference *zvmw* in conſequentia, whilſt the center *o* moves. When the center is at *o* let the moon be at *z*, and when the center has moved to *b* or *a*, let the moon be at *m*; this will produce an equation of 1°. 40′; which added to the laſt gives 4°. 59′. In this manner TYCHO repreſented the irregularities of the moon diſcovered by PTOLEMY, who explained the *Evection*, by making the center of excentricity deſcribe a circle *Tdet*, and the *equation of the center*, by one circle *obca*. HORROX explained the *ſecond* inequality thus. Let *E* be the earth, *C* the mean place of the center of the orbit, *EBCA* the correſponding line of the apſides, *EC* the mean excentricity of the orbit; and if we ſuppoſe the center of the orbit, inſtead of being at *C*, to deſcribe the circle *ADB*, and take the angle *ACD* double the diſtance of the apogee from the ſun, then *AED* will repreſent the equation of the apogee, and *ED* the excentricity. Sir I. NEWTON followed the ſame hypotheſis. FIG. 77.

351. But TYCHO being able to determine more accurately, from his obſervations, the true place of the moon, found that the place, computed from the above principles, would not agree with obſervations out of ſyzygies and quadratures, particularly in the octants, where the difference was the greateſt, and where he found it from 37′ to 40′. Thus TYCHO diſcovered a *third* irregularity, which he called the *Variation*. To explain this, he ſubſtituted another circle *ni*, and gave the center of the circle *obca* a libratory motion in the diameter *ni* perpendicular to *Ts*, correſponding to a motion in the circumference, which is double the diſtance of the moon from the ſun. Thus, with the center *s* and radius equal to the variation, deſcribe the circle *iyn*; take *hy* = double the diſtance of the moon from the ſun, draw *yp* perpendicular FIG. 76.

cular to *in*, and where it cuts *in* in *s* will be the place of the moon corrected for the *Variation*. For the different methods by which the inequalities of the moon's motion have been repreſented, ſee RICCIOLI *Almageſtum Novum*. Sir I. NEWTON makes this inequality vary from 33′. 14″ to 37′. 11″, it depending upon the ſun's diſtance from the earth. HORROX makes it 36′. 27″ in his Tables. MAYER makes it 36′. 59″,8.

352. TYCHO alſo diſcovered another, called the *annual* equation, becauſe it depends upon the diſtance of the earth from the ſun, the variation of which cauſes a variation of the effect of the ſun's action upon the earth and moon. CASSINI makes this equation 9′. 44″. Sir I. NEWTON makes it 11′. 50″. TYCHO obſerved, that the mean motion of the moon, in order to be uniform, required an equation of days, different from that which the motion of the ſun gave; but this did not agree with the equation which we now employ. KEPLER alſo employed an equation for this purpoſe, which, he ſaid, aroſe either from the motion of the moon, or the equation of time. HORROX, after employing the three equations already mentioned, corrected the apparent time at which he would calculate the true place of the moon by the equation of time, additive in the firſt ſix ſigns, which at the mean diſtance went as far as 13′. 24″, which is the ſame as if he had added 7′. 21″ to the mean longitude; at the ſame time, he neglected one part of the true equation, which would have been 7′. 42″ ſubtractive, in ſuch a manner that it would have added 4′. 14″ to the place of the moon; theſe two would have made the whole 11′. 35″, which agrees with the annual equation. FLAMSTEAD obſerved, that this equation of time was not the equation belonging to the ſolar ſyſtem; nevertheleſs he granted that this equation ought to be employed, which he ſays is peculiar to the moon, it being affected by the earth. Afterwards Dr. HALLEY obſerved that the moon moved faſteſt when the ſun was in it's apogee; and he fixed this equation at about 13′. MAYER makes it 11′. 8″,8.

353. It is very eaſy to conceive how this annual equation might be diſcovered by obſervation. By computing the moon's place for a great many times in the year, allowing for the equation of the orbit, the evection and variation, and comparing it with the obſerved place, it would appear that they agreed very well about the beginning of January and July, but that they differed conſiderably at the beginning of April and October. This would point out an equation. But beſides theſe four principal equations, the only ones deduced ſolely from obſervation, there are a great many others which are ſmaller, which are found by theory and corrected by obſervations. The theory of the moon muſt therefore be conſulted by thoſe, who would wiſh to have an intimate knowledge of the ſubject. We ſhall afterwards give ſo much of it, as is conſiſtent with the plan of this Work.

354. TYCHO

354. TYCHO found that the motion of the nodes and variation of the inclination of the orbit, were ſubject to an irregularity, and might be repreſented by the motion of the pole of the orbit in a circle *ECFG*, whoſe radius *GD* = 9′. 30″, half the difference of the greateſt and leaſt inclinations, the center *D* being 5°. 8′ from the pole *A* of the ecliptic, that being the mean inclination of the orbit, according to TYCHO, or mean diſtance of the poles of the ecliptic and moon's orbit. By more accurate obſervations, *GD*=8′. 48″, and the mean inclination 5°. 8′. 49″. Let the pole of the lunar orbit move in the circumference *GEC*, and be at *G* in ſyzygies and *C* in quadratures, and at *F* and *K* in octants, it's motion being twice the true diſtance of the ſun from the moon. Then when the pole is at any point *H*, *HA* is the inclination, and the angle *HAD* the equation of the node, the angle *ADH* being double the diſtance of the moon from the ſun. At *F* this equation is the greateſt, and = 1°. 46′, found from the triangle *DFA*. Hence MAYER gave a method of finding the equation of latitude, of which the following is the inveſtigation, given by M. de la LANDE in his Aſtronomy. FIG. 78.

355. Let *L* be the moon 90° from the true pole *E* of it's orbit, *D* being the mean pole, draw *LEM*, and *DM* perpendicular to it; then as the angle *DLM* is very ſmall, we may ſuppoſe *LD* = *LM*, and conſequently *EM* = *LD* − *LE*. Now as *DA* is very ſmall compared with *DL*, *LE* and *LD* will be very nearly circles of latitude, and therefore their difference *EM*, will be the equation of latitude, being the difference of the diſtances of the moon from the true and mean pole. Draw *DB* perpendicular to *AD*, and it muſt paſs through the nodes, therefore *LDB* is the moon's diſtance from the node, or the argument of latitude, and which is equal to *ADM*, *MDB* being the complement of each; alſo *ADE* is twice the diſtance of the moon from the ſun. Now $EM = ED \times \text{ſin. } EDM = ED \times \text{ſin. } \overline{ADE - ADM}$; that is, the *equation of the moon's latitude is* 8′. 48″ *multiplied by the ſine of double the diſtance of the moon from the ſun — the argument of latitude.* TYCHO, and after him FLAMSTEAD, HALLEY, NEWTON, &c. conſidered the equations of the nodes and inclination ſeparately.

Elements of the Theory of the Moon according to Observation.

Secular motion for 100 years, of which 25 are biſſextiles	KEPLER and HORROX	10ˢ. 7°. 48′. 51″
	NEWTON, FLAMSTEAD and HALLEY	10. 7. 50. 25
	CASSINI	10. 7. 49. 52
	MAYER, (ſecond Tables) . . .	10. 7. 53. 35
	M. de LAMBRE	10. 7. 53. 12
Secular motion of the Apogee	KEPLER	3ˢ. 19°. 14′. 16″
	HORROX	3. 19. 4. 16
	CASSINI	3. 19. 14. 16
	FLAMSTEAD, HALLEY and MAYER	3. 19. 11. 15
Secular motion of the Node	KEPLER and HORROX	4ˢ. 14°. 11′. 7″
	FLAMSTEAD and HALLEY . .	4. 14. 11. 15
	CASSINI	4. 14. 11. 5
	MAYER	4. 14. 11. 15
Epoch of the mean longitude of the moon for 1750	KEPLER	6ˢ. 8°. 18′. 54″
	HORROX	6. 8. 17. 54
	FLAMSTEAD	6. 8. 21. 24
	CASSINI	6. 8. 20. 0
	MAYER (ſecond Tables) . . .	6. 8. 22. 24
	MASON	6. 8. 22. 21
	M. de LAMBRE	6. 8. 22. 20
Epoch of the longitude of the Apogee for 1750	KEPLER	5ˢ. 21°. 30′. 36″
	HORROX	5. 20. 30. 36
	FLAMSTEAD and HALLEY . . .	5. 20. 58. 55
	CASSINI	5. 21. 1. 24
	MAYER	5. 20. 55. 54
	MASON	5. 20. 54. 56

Epoch

Epoch of the Longitude of the Node for 1750	KEPLER	9ˢ. 10°. 33′. 13″
	HORROX	9. 10. 15. 13
	FLAMSTEAD	9. 10. 14. 59
	HALLEY	9. 10. 13. 58
	CASSINI	9. 10. 18. 7
	MAYER	9. 10. 19. 8
	MASON	9. 10. 19. 59

Mean Equation of the Orbit	FLAMSTEAD	6°. 18′. 43″
	EULER	6. 18. 18
	D'ALEMBERT	6. 18. 43
	CLAIRAUT	6. 18. 1
	MAYER	6. 18. 32

Times of the Revolutions of the Moon, of it's Apogee and Node, as determined by M. de la LANDE.

Tropical revolution	27ᵈ.	7ʰ.	43′.	4″,6795
Sidereal revolution	27.	7.	43.	11, 5259
Synodic revolution	29.	12.	44.	2, 8283
Anomaliſtic revolution	27.	13.	18.	33, 9499
Revolution in reſpect to the node	27.	5.	5.	35, 603
Tropical revolution of the apogee	8ʸ. 311.	8.	34.	57, 6177
Sidereal revolution of the apogee	8. 312.	11.	11.	39, 4089
Tropical revolution of the node	18. 228.	4.	52.	52, 0296
Sidereal revolution of the node	18, 223.	7.	13.	17, 744
Diurnal motion of the moon in reſpect to the equinox		13°.	10′.	35″,02784394
Diurnal motion of the apogee		0.	6.	41, 069815195
Diurnal motion of the node		0.	3.	10, 638603696

The years here taken are the common years of 365 days.

According to PTOLEMY, the mean annual motion of the moon is 4ˢ. 9°. 22′. 46″, and the diurnal motion 13°. 10′. 34″. 58‴; the mean annual motion

of the nodes in antecedentia is 19°. 20′. 0″. 58‴, and the diurnal motion 3′. 10″. 41‴,25; the mean annual motion of the apogee is 40°. 39′. 35″,75, and the diurnal motion 6′. 41″. 2‴,25; and the time of a mean ſynodic revolution is 29*d*. 13*h*. 5′. 39″. If the reader will compare theſe, with our preſent Tables, he will be ſurpriſed at their accuracy; and if he conſider alſo that PTOLEMY diſcovered the two firſt irregularities, the lunar motions will be found to have been known to a very conſiderable degree of accuracy, near 2000 years ago.

On the Acceleration of the Moon's Motion.

356. Dr. HALLEY, by comparing the ancient eclipſes obſerved at Babylon, with thoſe obſerved by ALBATEGNIUS in the ninth century, and with thoſe obſerved in his own time, diſcovered the mean motion of the moon to be accelerated; and ſays, that he could have found out the quantity of acceleration, if he had had the longitude of Bagdat, Alexandria and Aleppo; becauſe in, or near theſe places, the obſervations were made; for it is neceſſary to know their longitudes, in order to reduce the times, to thoſe under the meridian in which the modern obſervations are made. In the *Phil. Tranſ.* 1749, Mr. DUNTHORNE has examined ſome ancient eclipſes obſerved under known meridians, and determined what the acceleration is. The eclipſes which he has choſen for this purpoſe are theſe.

357. An eclipſe of the ſun was obſerved at Alexandria, by THEON, in the year 364, on June 16; the beginning was in the afternoon at 3*h*. 18′, and the end at 5*h*. 15′. In the year 977, an eclipſe of the ſun was obſerved, at Grand Cairo, on December 13; the beginning was at 8*h*. 25′, and the end at 10*h*. 45′, apparent time, in the morning, and the digits eclipſed were 8; alſo the ſun's altitude at the beginning was 15°. 43′, and at the end 33°½. In the year 978, at the ſame place, the ſun was obſerved to be eclipſed on June 8; the beginning was at 2*h*. 31′, and the end at 4*h*. 50′, apparent time, in the afternoon. Mr. DUNTHORNE then computed the diſtance of the moon from the ſun in longitude, at the beginning of each eclipſe, from the above *data*; he alſo computed their diſtance at that time from the Tables of the ſun's and moon's motion, and found that at the beginning of the firſt eclipſe, the Tables gave the difference of the ſun's and moon's places leſs than that deduced from the obſervation by 4′. 16″; in the ſecond eclipſe it was greater by 7′. 36″; and in the third, greater by 8′. 45″; the computed places at the two laſt eclipſes being

ſo

ſo much *before* the obſerved places, but at the firſt eclipſe the computed place was ſo much *behind*. The agreement of the two laſt ſhows, that the Tables repreſent the mean motion of the moon's apogee very well for above 700 years, the moon having been very near it's perigee at the time of one of thoſe eclipſes, and near it's apogee at the time of the other. Now HIPPARCHUS mentions an eclipſe obſerved at Babylon, which happened on December 22, in the year 313 before J. C. when a ſmall part of the moon was eclipſed on the north eaſt, half an hour before the end of the night, and the moon ſet eclipſed; Mr. DUNTHORNE, from his Tables, makes the middle of it at $9h.$ $4'$ apparent time, and the duration $1h.$ $37'$; PTOLEMY makes it $1h.$ $30'$ nearly. Hence the beginning, according to Mr. DUNTHORNE's calculation, was about $8h.$ $15'$ after midnight. But, according to PTOLEMY, the ſun roſe at $7h.$ $12'$, and as the moon had then ſouth latitude, and was not quite come to the ſun's oppoſition, it's apparent ſetting muſt have been a little ſooner, that is, above an hour before the beginning of the eclipſe, according to his Tables; whereas the moon was ſeen eclipſed ſome time before it's ſetting, which proves that it's true place was then forwarder than the Tables make it, by $40'$ or $50'$. In the year 201 before J. C. on September 22, an eclipſe was obſerved at Alexandria, when the moon began to be eclipſed about half an hour before it's riſing, and ended about $3\frac{1}{2}h.$ in the morning. Now, by the Tables, the middle of the eclipſe was at $7h.$ $44'$ apparent time, and the duration, $3h.$ $4'$, which makes the beginning at $6h.$ $12'$, or about $10'$ after the moon roſe, and conſequently $40'$ after the time by obſervation; which makes the moon's true place forwarder than by the Tables, by about $20'$. In the year 721 before J. C. on March 19, an eclipſe was obſerved at Babylon, the middle of which, by the Tables, was at $10h.$ $26'$ apparent time, and the beginning was at $8h.$ $32'$; but the beginning, by obſervation, was at $6h.$ $46'$, or $1h.$ $46'$ ſooner than by the Tables; therefore the moon's true place preceded it's place by the Tables, by a little more than $50'$. Hence, as the ſame Tables repreſent the moon's place in the ancient eclipſes *behind* it's true place, and in the two eclipſes obſerved in 977 and 978 *before* it, it follows that it's mean motion in ancient times was ſlower, and in latter times quicker than the Tables give, and therefore it muſt have been accelerated. There muſt alſo have been a time when the Tables would give the true place. And although the ancient obſervations of the times of the eclipſes were not very accurate, yet they were ſufficiently ſo to prove, beyond all doubt, that the moon's motion is greater at this time, than it was at the times when the ancient eclipſes were obſerved.

358. As we have no *data* to determine directly what this acceleration is, and at what point of time the moon's place from the Tables would agree with it's true place, we muſt make a ſuppoſition for each, and then compute the errors of the Tables, and ſee how they agree with the above errors; and that ſuppoſition which, upon the whole, agrees beſt, muſt give the acceleration the moſt to be depended upon, and probably near the truth. Now whatever be the cauſe of this acceleration, it is very probable that it continues conſtant, or very nearly ſo, and therefore the quantity of acceleration will vary as the ſquare of the time. Upon this principle, if we ſuppoſe the Tables to give the true place of the moon at the year 700, and the acceleration to be 10″ for the firſt 100 years from that time, it will give reſults agreeing better with the obſervations than any other ſuppoſition. Theſe reſults may be thus computed. The quantity of acceleration at the beginning of the ſucceſſive centuries from 700 to 1700, will be, 10″, 40″, 90″, 160″, 250″, 360″, 490″, 640″, 810″, 1000″. Now as the whole acceleration in theſe 10 centuries is 1000″, the mean acceleration is 100″ in a year, and would be, in the above reſpective centuries, 100″, 200″, 300″, 400″, 500″, 600″, 700″, 800″, 900″, 1000″; ſubtract therefore the above actual accelerations from theſe mean ones which the Tables give, and we have 1′. 30″, 2′. 40″, 3′. 30″, 4′. 0″, 4′. 10″, 4′. 0″, 3′. 30″, 2′. 40″, 1′. 30″, 0′. 0″ for the error of the Tables at the beginning of every century from 700 to 1700, ſhowing how much the Tables give the place too forward. If we go downwards from 700, the motion will be diminiſhed at the ſame rate of 10″, 40″, 90″, &c. for every century; whereas our Tables give it increaſing at the rate of 100″, 200″, 300″, &c. therefore the errors of the Tables will be the ſums of theſe reſpective quantities, or 1′. 50″, 4′. 0″, 6′. 30″, 9′. 20″, 12′. 30″, 16′. 0″, 19′. 50″, 24′. 0″, 28′. 30″, 33′. 20″, 38′. 30″, 44′. 0″, 49′. 50″, 56′. 6″, ſhowing how much the Tables give the place too backward at the beginning of each century from the year 700 to 700 before J. C. Hence the following Table.

Years

Years before Chriſt.	Error of Tables.	Years after Chriſt.	Error of Tables.	Years after Chriſt.	Error of Tables.
700	−56′. 6″	0	−19′. 50″	900	+ 2′. 40″
600	−49. 50	100	−16. 0	1000	+ 3. 30
500	−44. 0	200	−12. 30	1100	+ 4. 0
400	−38. 30	300	− 9. 20	1200	+ 4. 10
300	−33. 20	400	− 6. 30	1300	+ 4. 0
200	−28. 30	500	− 4. 0	1400	+ 3. 30
100	−24. 0	600	− 1. 50	1500	+ 2. 40
* *	* * *	700	0. 0	1600	+ 1. 30
* *	* * *	800	+ 1. 30	1700	0. 0

If we compare the errors of the Tables, in the eclipſes above related, with the errors in this Table, they will be found to differ not more than might be expected from the uncertainty of the times of the eclipſes, and the different errors which the Tables may be ſubject to at different times. Theſe obſervations therefore make the ſecular variation 10″, and to vary as the ſquare of the time. M. de la LANDE, from the eclipſes in 977 and 978, makes it 9″,886. In MAYER's Tables it is 9″, beginning from 1700.

359. M. de la PLACE, in the *Mem. de l'Acad. Roy. des Scien.* for 1786, has ſhown, that this acceleration of the moon's motion ariſes from the action of the ſun upon the moon, combined with the variation of the excentricity of the earth's orbit. The excentricity of the earth's orbit is, at preſent, diminiſhing, and this ariſes from the action of the planets upon the earth. The major axis of the earth's orbit is not altered by this, but the excentricity is. The mean force of the ſun to dilate and contract the orbit of the moon depends on the ſquare of the excentricity of the earth's orbit. By the diminution of the excentricity, the moon's mean motion is accelerated, and this is a circumſtance which takes place at preſent. When the excentricity is come to it's *minimum*, the acceleration of the mean motion will ceaſe; after which the excentricity will increaſe and the moon's mean motion will be retarded. This therefore cauſes a ſecular equation of the moon's mean motion, the period of which is very long. If n be the number of centuries from 1700, M. de la PLACE has computed the ſecular equation to be $+11'',135n^2+0'',04398n^3$; this however cannot be true whatever be the value of n, becauſe the acceleration would then continually increaſe; but it may be extended back to the moſt ancient obſervations of the moon, and for 1000 or 1200 years to come, without any ſenſible error. M. de LAMBRE, from comparing the modern obſervations at about

about the diſtance of a century, found that the ſecular mean motion of the moon in the laſt Tables of MAYER was too great by 25″; and that the place of the moon, calculated by theſe Tables, ought to be corrected by the quantity $-25''n+2'',135n^2+0''04398n^3$. If the ancient obſervations of the moon be compared with the places calculated by MAYER's Tables with this correction, the errors will be comparatively very ſmall, and no greater than what might ariſe from the inaccuracy of their obſervations. M. de la LANDE, in his Tables of the moon, has thus corrected MAYER's Tables. Hence it appears, that the preſent acceleration of the moon is nothing more than an equation, the period of which is very long; it will be accelerated and retarded by the ſame quantity, and therefore if the mean motion be taken for the whole time of acceleration or retardation, it will be found never to vary

The mean motion of the nodes and apogee of the moon's orbit is ſubject to a ſecular equation. The ſecular equation of the nodes is $-2'',784n^2-0''010995n^3$, which being negative ſhows that it is to be applied contrary to their mean motion. This ſecular equation is $\frac{1}{4}$ of the ſecular equation of the mean motion. The ſecular equation of the apogee is $\frac{7}{4}$ of the ſecular equation of the mean motion, and is therefore $-19'',486n^2-0'',07697n^3$, where the negative ſign ſhows that it is to be applied contrary to it's mean motion. Hence all the irregularities of the moon are but ſo many equations, which return again in their regular order; and the ſame is ſhown to be true of the irregularities of *Jupiter* and *Saturn*; alſo as the major axes of their orbits remain undiſturbed, it is manifeſt that the ſyſtem can never be deſtroyed, all the irregularities being periodical, and confined to ſuch ſmall limits as to produce no inconvenience. Theſe are circumſtances which furniſh great matter for our attention; the ſtability of the ſyſtem ſhows the power and wiſdom of the Framer.

On the Diameter of the Moon.

360. The diameter of the moon may be meaſured, at the time of it's full, by a micrometer; or it may be meaſured by the time of it's paſſing over the vertical wire of a tranſit teleſcope; but this muſt be when the moon paſſes within an hour or two at the time of the full, before the viſible illumination is ſenſibly changed from a circle. To find the diameter by the time of it's paſſage over the meridian, let $d''=$ the horizontal diameter of the moon, $c=$ ſec. of it's declination, and $m=$ the length of a lunar day, or the time from the paſſage of the moon over the meridian on the day we calculate, to the paſſage over the meridian the next day. Then (108) cd'' is the moon's diameter in right aſcenſion; hence $360° : cd'' :: m :$ the time (t) of paſſing the meri-

meridian; therefore $d'' = 360° \times \frac{t}{cm}$. If we obſerve the time when the limb of the moon comes to the meridian, we can find the time when the center comes to it, by adding to, or ſubtracting from the time when the firſt or ſecond limb comes to the meridian, half the time of the paſſage of the moon over the meridian. The time in which the ſemidiameter of the moon paſſes the meridian, may be found by two Tables, in the Tables of the moon's motion.

361. ALBATEGNIUS made the diameter of the moon to vary from 29′. 30″ to 35′. 20″, and hence the mean 32′. 25″. COPERNICUS found it from 27′. 34″ to 35′. 38″, and therefore the mean 31′. 36″. KEPLER made the mean diameter 31′. 22″. M. de la HIRE made it from 29′. 30″ to 33′. 30″. M. CASSINI made it from 29′. 30″ to 33′. 38″. M. de la LANDE, from his own obſervations, found the mean diameter to be 31′. 26″; the extremes from 29′. 22″ when the moon is in apogee and conjunction, and 33′. 31″ when in perigee and oppoſition. The mean diameter here taken is the arithmetic mean between the greateſt and leaſt diameters; the diameter at the mean diſtance is 31′. 7″. Hence, according to the theory of MAYER, the horizontal diameter of the moon at any time is 31′. 7″ − 1′. 42″,3 cos. anom. + 5″,4 cos. 2 anom. + 13″,7 cos. 2 diſt. ☾ from ☉ − 20″,2 cos. (2 diſt. ☾ from ☉ − anom. ☾).

362. When the moon is at different altitudes above the horizon, it is at different diſtances from the ſpectator, and therefore there is a change of the apparent diameter. Let C be the center of the earth, A the place of a ſpectator on it's ſurface, Z his zenith, M the moon; then ſin. CAM or ZAM : ſin. ZCM :: CM : $AM = \frac{CM \times \text{ſin. } ZCM}{\text{ſin. } ZAM}$; but the apparent diameter is inverſely as it's diſtance; hence the apparent diameter varies as $\frac{\text{ſin. } ZAM}{\text{ſin. } ZCM}$, CM being ſuppoſed conſtant. Now in the horizon, $\frac{\text{ſin. } ZAM}{\text{ſin. } ZCM}$ may be conſidered as equal to unity; hence 1 : $\frac{\text{ſin. } ZAM}{\text{ſin. } ZCM}$, or ſin. ZCM : ſin. ZAM, or cos. true alt. (a) : cos. apparent alt. (A) :: the horizontal diameter : the diameter at the apparent altitude (A). Hence the horizontal diameter : it's increaſe :: cos. a : cos. A − cos. $a = 2$ ſin. $\overline{\frac{1}{2}a + \frac{1}{2}A} \times$ ſin. $\overline{\frac{1}{2}a - \frac{1}{2}A}$; therefore the increaſe of the ſemidiameter = hor. ſemidiam. $\times \frac{\text{ſin. } \overline{\frac{1}{2}a + \frac{1}{2}A} \times \text{ſin. } \overline{\frac{1}{2}a - \frac{1}{2}A}}{\text{cos. } a}$; from this we may eaſily conſtruct a Table of the increaſe of the ſemidiameter for any horizontal ſemidiameter; and then for any other horizontal ſemidiameter, the increaſe will vary in proportion.

FIG. 79.

364. Some

363. Some Aſtronomers have thought, that in finding the time of the tranſit of the moon over the meridian, we ought to take the apparent diameter inſtead of that ſeen from the center of the earth. But this, as M. de la LANDE has obſerved, muſt not be; for although the apparent diameter is increaſed by the moon being nearer to the ſpectator, yet the angular velocity about the point where the ſpectator is ſituated is increaſed in the ſame ratio, the angular velocity about any point, and the apparent diameter, being inverſely as the diſtance, and therefore the time of the tranſit is the ſame.

On the Phaſes of the Moon.

364. By Art. 319. the greateſt breadth of the viſible illuminated part of the moon's ſurface varies as the verſed ſine of the moon's elongation from the ſun, very nearly; and the circle terminating the light and dark part being ſeen obliquely will appear an ellipſe; hence the following delineation of the phaſes.

FIG. 80. Let E be the earth, S the ſun, M the moon; deſcribe the circle $abcd$, repreſenting that hemiſphere of the moon which is towards the earth projected upon a plane; ac, db two diameters perpendicular to each other; take $dv =$ the verſed ſine of elongation SEM, and deſcribe the ellipſe avc, and (318, 319) $adcva$ will repreſent the viſible enlightened part; which will be horned between conjunction and quadratures, biſected at quadratures, and gibbous between quadratures and oppoſition, the verſed ſine being leſs than radius in the firſt caſe, equal to it in the ſecond, and greater in the third. The viſible enlightened part varying as dv, we have, *the viſible enlightened part : whole :: verſed ſine of elongation : diameter.*

On the Libration of the Moon.

365. Many Aſtronomers have given maps of the face of the moon; but the moſt celebrated are thoſe of HEVELIUS in his *Selenographia*, in which he has repreſented the appearance of the moon in it's different ſtates from the new to the full, and from the full to the new; theſe figures MAYER prefers.

FIG. 81. Figure 81 repreſents the face of the moon in it's mean ſtate of libration, as ſhown by the beſt teleſcopes. LANGRENUS and RICCIOLUS denoted the ſpots upon the ſurface by the names of philoſophers, mathematicians, and other celebrated men, giving the names of the moſt celebrated characters to the largeſt ſpots; HEVELIUS marked them with the geographical names of places upon the earth. The former diſtinction is now generally followed, and is that which we have here given. The numbers in the figure repreſent, nearly, the order in which the ſpots are eclipſed, going from the eaſt to the weſt.

1. Grimaldus

1. Grimaldus
2. Galileus
3. Ariſtarchus
4. Keplerus
5. Gaſſendus
6. Schikardus
7. Harpalus
8. Heraclides
9. Lanſbergius
10. Reinoldus
11. Copernicus
12. Helicon
13. Capuanus
14. Bullialdus
15. Eratoſthenes
16. Timocharis
17. Plato
18. Archimedes
19. Inſulaſinus Medii
20. Pitatus
21. Tycho
22. Eudoxus
23. Ariſtotles
24. Manilius
25. Menelaus
26. Hermes
27. Poſſidonius
28. Dionyſius
29. Plinius
30. Theophilus
31. Fracaſtorius
32. Promontorium Acutum, Cenſorinus
33. Meſſala
34. Promontorium Somnii
35. Proclus
36. Cleomedes
37. Snellius
38. Petavius
39. Langrenus
40. Taruntius

A. Mare Humorum
B. Mare Nubium
C. Mare Imbrium
D. Mare Nectaris
E. Mare Tranquilitatis
F. Mare Serenitatis
G. Mare Fæcunditatis
H. Mare Criſium.

The ſpots upon the moon are cauſed by the mountains and vallies upon it's ſurface; for certain parts are found to project ſhadows oppoſite to the ſun; and when the ſun becomes vertical to any of them, they are obſerved to have no ſhadow; theſe therefore are mountains; other parts are always dark on that ſide next to the ſun, and illuminated on the oppoſite ſide; theſe therefore are cavities. Hence the appearance of the face of the moon continually varies, from it's altering it's ſituation in reſpect to the ſun. The tops of the mountains, on the dark part of the moon, are frequently ſeen enlightened at a diſtance from the confines of the illuminated part. The dark parts have, by ſome, been thought to be ſeas; and by others, to be only a great number of caverns and pits, the dark ſides of which, next to the ſun, would cauſe thoſe places to appear darker than others. The great irregularity of the line bounding the light and dark part, on every part of the ſurface, proves that there can be no very large tracts of water, as ſuch a regular ſurface would neceſſarily produce a line, terminating the bright part, perfectly free from all irregularity.

If there was much water upon it's ſurface, and an atmoſphere, as conjectured (377) by ſome Aſtronomers, the clouds and vapours might eaſily be diſcovered by the teleſcopes which we have now in uſe; but no ſuch phænomena have ever been obſerved.

366. Very nearly the ſame face of the moon is always turned towards the earth, it being ſubject only to a ſmall change within certain limits, thoſe ſpots which lie near to the edge appearing and diſappearing by turns; this is called it's *Libration*, and ariſes from four cauſes. 1. GALILEO, who firſt obſerved the ſpots of the moon after the invention of teleſcopes, diſcovered this circumſtance; he perceived a ſmall daily variation ariſing from the motion of the ſpectator about the center of the earth, which, from the riſing to the ſetting of the moon, would cauſe a little of the weſtern limb of the moon to diſappear, and bring into view a little of the eaſtern limb; this is called the *diurnal* libration. 2. He obſerved likewiſe, that the north and ſouth poles of the moon appeared and diſappeared by turns; this ariſes from the axis of the moon not being perpendicular to the plane of it's orbit, and is called a libration in *latitude*. 3. From the unequal angular motion of the moon about the earth, and the uniform motion of the moon about it's axis, a little of the eaſtern and weſtern parts muſt gradually appear and diſappear by turns, the period of which is a month, and this is called a libration in *longitude*; the cauſe of this libration was firſt aſſigned by RICCIOLUS, but he afterwards gave it up, as he made many obſervations which this ſuppoſition would not ſatisfy. HEVELIUS however found that it would ſolve all the phænomena of this libration. 4. Another cauſe of libration ariſes from the attraction of the earth upon the moon, in conſequence of it's ſpheroidical figure.

367. If the angular velocity of the moon about it's axis were equal to it's angular motion about the earth, the libration in *longitude* would not take place. For if E be the earth, $abcd$ the moon at v and w, and avc be perpendicular to $Ebvd$; then abc is that hemiſphere of the moon at v next to the earth. When the moon comes to w, if it did not revolve about it's axis, bwd would be parallel to bvd, and the ſame face would not be towards the earth. But if the moon, by revolving about it's axis in the direction $abcd$, had brought b into the line Ew, the ſame face would have been towards the earth; and the moon would have revolved about it's axis through the angle bwE, which is equal to the alternate angle wEc, the angle which the moon has deſcribed about the earth.

FIG. 82.

368. When the moon returns to the ſame point of it's orbit, the ſame face is obſerved to be towards the earth, and therefore (367) the time of the revolution in it's orbit is equal to the time about it's axis. But in the intermediate points it varies, ſometimes a little more to the eaſt, and ſometimes to the weſt, becomes viſible;

viſible; and this ariſes from it's angular motion about the earth being not uniform, whilſt the angular motion about it's axis is ſo. Hence the libration in *longitude* is nearly equal to the equation of the orbit, or about $7°\frac{1}{2}$ at it's maximum, and would be accurately ſo, if the axis of the moon were perpendicular to it's orbit. The ſame face will be towards the earth in apogee and perigee, for at thoſe points there is no equation of the orbit. If E be the earth, M the moon, pq it's axis, not perpendicular to the plane of the orbit ab; then at a the pole p will be viſible to the earth, and at b the pole q will be viſible; as the moon therefore revolves about the earth, the poles muſt appear and diſappear by turns, cauſing the libration in *latitude*. This is exactly ſimilar to the cauſe of the variety of our ſeaſons, from the earth's axis not being perpendicular to the plane of it's orbit. Hence nearly one half of the moon is never viſible at the earth. Alſo the time of it's rotation about it's axis being a month, the length of the lunar days and nights will be about a fortnight each, they being ſubject but to a very ſmall change, on account of the axis of the moon being nearly perpendicular to the ecliptic. FIG. 83.

369. HEVELIUS *(Selenographia*, pag. 245.) obſerved, that when the moon was at it's greateſt north latitude, the libration in latitude was the greateſt, the ſpots which are ſituated near to the northern limb being then neareſt to it; and as the moon departed from thence, the ſpots receded from that limb, and when the moon came to it's greateſt ſouth latitude, the ſpots ſituated near the ſouthern limb were then neareſt to it. This variation he found to be about $1'.45''$, the diameter of the moon being $30'$. Hence it follows, that when the moon is at it's greateſt latitude, a plane drawn through the earth and moon perpendicular to the plane of the moon's orbit, paſſes through the axis of the moon; conſequently the equator of the moon muſt interſect the ecliptic in a line parallel to the line of the nodes of the moon's orbit, and therefore, in the Heavens, the nodes of the moon's orbit and of it's equator coincide; and this will be further confirmed, when we treat on the ſituation of the moon's equator and axis.

370. It is a very extraordinary circumſtance, that the time of the moon's revolution about it's axis ſhould be equal to that in it's orbit. Sir I. NEWTON, from the altitude of the tides on the earth, has computed that the altitude of the tides on the moon's ſurface muſt be 93 feet, and therefore the diameter of the moon perpendicular to a line drawn from the earth to the moon, ought to be leſs than the diameter directed to the earth, by 186 feet; hence, ſays he, the ſame face muſt always be towards the earth, except a ſmall oſcillation; for if the longeſt diameter ſhould get a little out of that direction, it would be brought into it again, by the attraction of the earth. The ſuppoſition of D. de MAIRAN is, that that hemiſphere of the moon next the earth

is more denſe than the oppoſite one, and hence the ſame face would be kept towards the earth, upon the ſame principle as above. M. de la GRANGE, in the *Mem. de l'Acad. des Scien.* 1780, has examined this ſubject very fully, and ſhown, that from the attraction of the earth, that diameter of the moon's equator which is directed towards the earth, will be lengthened four times more than that which is perpendicular to it. If h be the ſemidiameter of the moon in parts of it's mean diſtance from the earth, m the quantity of matter in the moon expreſſed in parts of that of the earth, he has ſhown that the increaſe of the ſemidiameters will be $\frac{5h^3}{m}$ and $\frac{5h^3}{4m}$, the radius being unity.

371. Sir I. NEWTON propoſes the following method of repreſenting the libration of the moon in *latitude* and *longitude*. Take a common globe, and elevate the pole to the zenith, ſo that the equator may coincide with the horizon, and let the ecliptic repreſent the moon's orbit. Conceive the center of this globe to repreſent the place of the earth, and the ſurface of the globe the ſphere in which the moon revolves. Take two ſmall ſpheres, having each a meridian, and ſuſpend each by a ſtring from one of it's poles. Let one of theſe repreſent a fictitious moon carried uniformly round the earth, having it's equator coinciding with the horizon of the globe, and revolving uniformly about it's axis in the ſame time in which it revolves about the earth; then the ſame meridian of the moon will always paſs through the earth, and the moon would not be ſubject to any libration. Let the other ſphere, repreſenting the true moon, be carried in the ecliptic with it's proper angular motion about the earth, having it's axis and meridian parallel to thoſe of the other moon. Then as the true moon moves from the perigee to it's apogee, preceding the fictitious moon, the meridian will appear towards the left of it's diſc, and the ſpots will appear to move towards the eaſt, by as many degrees as there are between the longitudes of the true and fictitious moons, or by the equation of the orbit; when the true moon moves from apogee to perigee, the meridian of the true moon will appear towards the right of the diſc, and the ſpots will appear to move towards the weſt; thus repreſenting the libration in longitude. When the true moon moves from it's aſcending node to it's greateſt north latitude, the north pole of the moon will diſappear, and the ſouth pole, with the ſpots about it, will come into view; and as the moon leaves this northern limit, they will begin to diſappear, and when the moon has reached it's greateſt ſouthern latitude, the northern pole, with the ſpots about it, will be brought into view, and appear furtheſt upon it's diſc; thus repreſenting the libration in longitude. NICOLAI MERCATORIS, *Inſtitut. Aſtron.* pag. 286.

372. When the moon is about three days from the new, the dark part is very viſible, by the light reflected from the earth, which is moon light to the

Luna-

Lunarians, conſidering our earth as a moon to them; and in the moſt favourable ſtate, ſome of the principal ſpots may be ſeen. But when the moon gets into quadratures, it's great light then prevents the dark part from being viſible. According to Dr. SMITH, the ſtrength of moon light, at the full moon, is 90 thouſand times leſs than the light of the ſun; but from ſome experiments of M. BOUGUER, he concluded it to be 300 thouſand times leſs. The light of the moon, condenſed by the beſt mirrors, produces no ſenſible effect upon the thermometer. Our earth, in the courſe of a month, ſhows the ſame phaſes to the Lunarians, as the moon does to us; the earth is at the full at the time of the new moon, and at the new at the time of the full moon. The ſurface of the earth being about 13 times greater than that of the moon, it affords 13 times more light to the moon than the moon does to the earth.

On the Altitude of the Lunar Mountains.

373. The method uſed by HEVELIUS, and others ſince his time, to determine the height of a lunar mountain is this. Let SLM be a ray of light from the ſun, paſſing by the moon at L, and touching the top of the mountain at M; then the ſpace between L and M appears dark. With a micrometer, meaſure LM, and compare it with LC; then knowing LC, we know LM, and by Euc. B. 1. P. 47. $CM = \sqrt{CL^2 + LM^2}$ is known; from which ſubtract Cp, and we get the height pM of the mountain. But, as Dr. HERSCHEL obſerves in the *Phil. Tranſ.* 1781, this method will only do when the moon is in quadratures; he has therefore given the following general method. Let E be the earth, draw EMn, and Lo, perpendicular to the moon's radius RC, and Lr parallel to on, alſo ME' perpendicular to SM. Now ML would meaſure it's full length when ſeen from the earth in quadratures at E', but ſeen from E, it only meaſures the length of Lr. As the plane paſſing through SM, EM, is perpendicular to a line joining the cuſps, the circle RLV may be conceived to be a ſection of the moon perpendicular to that line. Now it is manifeſt, that the angle SLo or LCR, is very nearly equal to the elongation of the moon from the ſun; and the triangles LrM, LCo being ſimilar, $Lo : LC :: Lr : LM = \frac{LC \times Lr}{Lo} = Lr$ divided by the ſine of elongation, radius being unity. Hence we find Mp as before.

FIG. 84.

FIG. 85.

Ex. On June 1780, at 7 o'clock, Dr. HERSCHEL found the angle under which LM, or Lr, appeared to be 40″,625, for a mountain in the ſouth eaſt quadrant; and the ſun's diſtance from the moon was 125°. 8′, whoſe ſine is ,8104;

,8104; hence 40″,625 divided by ,8104 gives 50″,13, the angle under which *LM* would appear, if ſeen directly. Now the ſemidiameter of the moon was 16′. 2″,6, and taking it's length to be 1090 miles, we have, 16′. 2″,6 :50″,13 :: 1090 : *LM* = 56,73 miles; hence *Mp* = 1,47 miles.

374. Dr. HERSCHEL found the height of a great many more mountains, and thinks he has good reaſon to believe, that their altitudes are greatly overrated; and that, a few excepted, they generally do not exceed half a mile. He obſerves, that it ſhould be examined whether the mountain ſtands upon level ground, which is neceſſary that the meaſurement may be exact. A low tract of ground between the mountain and the ſun will give it higher, and elevated places between will make it lower, than it's true height above the common ſurface of the moon.

FIG. 86. 375. The line *Lr* was meaſured thus. 1. Set the immoveable hair of the micrometer parallel to *AB*, then moving the other hair parallel to it from *L* to *r*, it gives the meaſure under which *Lr* appears. Or 2. Obſerve ſome ſpot near to *L*, to which the line *rL* is directed; or take a view of the ſhadow of ſome neighbouring mountains; either of theſe will indicate a line perpendicular to a line joining the cuſps, ſufficiently near to ſet the micrometer by. The laſt method Dr. HERSCHEL thinks the beſt. But if the micrometer be furniſhed with an hair perpendicular to the moveable wire, and that hair be made to coincide with *Lr*, it at once gives the poſition of the micrometer.

376. On April 19, 1787, Dr. HERSCHEL diſcovered three volcanos in the dark part of the moon; two of them ſeemed to be almoſt extinct, but the third ſhowed an actual eruption of fire, or luminous matter, reſembling a ſmall piece of burning charcoal covered by a very thin coat of white aſhes; it had a degree of brightneſs about as ſtrong as that with which ſuch a coal would be ſeen to glow in faint day-light. The adjacent parts of the volcanic mountain ſeemed faintly illuminated by the irruption. A ſimilar irruption appeared on May 4, 1783. *Phil. Tranſ.* 1787. On March 7, 1794, a few minutes before 8 o'clock in the evening, Mr. WILKINS of Norwich, an eminent Architect, obſerved, with the naked eye, a very bright ſpot upon the dark part of the moon; it was there when he firſt looked at the moon; the whole time he ſaw it, it was a fixed, ſteady light, except the moment before it diſappeared, when it's brightneſs increaſed; he conjectures that he ſaw it about 5 minutes. The ſame phænomenon was obſerved by Mr. T. STRETTON, in St. John's Square, Clerkenwell, London. *Phil. Tranſ.* 1794. On April 13, 1793, and on February 5, 1794, M. PIAZZI, Aſtronomer Royal at Palermo, obſerved a bright ſpot upon the dark part of the moon, near Ariſtarchus. Several other Aſtronomers have obſerved the ſame phænomenon. See the *Memoirs de Berlin*, for 1788.

377. It

377. It has been a doubt amongſt Aſtronomers, whether the moon has any atmoſphere; ſome ſuſpecting that at an occultation of a fixed ſtar by the moon, the ſtar did not vaniſh inſtantly, but loſt it's light gradually; whilſt others could never obſerve any ſuch appearance. M. SCHROETER of Lilianthan, in the dutchy of Bremen, has endeavoured to eſtabliſh the exiſtence of an atmoſphere, from the following obſervations. 1. He obſerved the moon when two days and an half old, in the evening ſoon after ſun ſet, before the dark part was viſible; and continued to obſerve it till it became viſible. The two cuſps appeared tapering in a very ſharp, faint, prolongation, each exhibiting it's fartheſt extremity faintly illuminated by the ſolar rays, before any part of the dark hemiſphere was viſible. Soon after, the whole dark limb appeared illuminated. This prolongation of the cuſps beyond the ſemicircle, he thinks muſt ariſe from the refraction of the ſun's rays by the moon's atmoſphere. He computes alſo the height of the atmoſphere, which refracts light enough into it's dark hemiſphere to produce a twilight, more luminous than the light reflected from the earth when the moon is about 32° from the new, to be 1356 Paris feet; and that the greateſt height capable of refracting the ſolar rays is 5376 feet. 2. At an occultation of Jupiter's ſatellites, the third diſappeared, after having been about 1″ or 2″ of time indiſtinct; the fourth became indiſcernible near the limb; this was not obſerved of the other two. *Phil. Tranſ.* 1792. If there be no atmoſphere in the moon, the Heavens, to a Lunarian, muſt always appear dark like night, and the ſtars be conſtantly viſible; for it is owing to the reflection and refraction of the ſun's light by the atmoſphere, that the Heavens, in every part, appear bright in the day.

On the Phænomenon of the Harveſt Moon.

378. The full moon which happens at, or neareſt to, the autumnal equinox, is called the *Harveſt* moon; and at that time, there is a leſs difference between the times of it's riſing on two ſucceſſive nights, than at any other full moon in the year; and what we here propoſe, is to account for this phænomenon.

379. Let P be the north pole of the equator $\mathcal{Q}AU$, HAO the horizon, EAC the ecliptic, A the firſt point of Aries; then, in *north* latitudes, A is the aſcending node of the ecliptic upon the equator, AC being the order of the ſigns, and $A\mathcal{Q}$ that of the apparent diurnal motion of the heavenly bodies. When Aries riſes in north latitudes, the ecliptic makes the leaſt angle with the horizon; and as the moon's orbit makes but a ſmall angle with the ecliptic, let us firſt ſuppoſe EAC to repreſent the moon's orbit. Let A be the place of the moon at it's riſing on one night; now, in mean ſolar time, the earth FIG. 87.

makes

makes one revolution in 23*h*. 56′. 4″, and brings the ſame point *A* of the equator to the horizon again; but in that time, let the moon have moved in it's orbit from *A* to *c*, and draw the parallel of declination *tcns*; then it is manifeſt, that 3′. 56″ before the *ſame* hour the next night, the moon, in it's diurnal motion, has to deſcribe *cn* before it riſes. Now *cn* is manifeſtly the leaſt poſſible, when the angle *CAn* is the leaſt, *Ac* being given. Hence it riſes more nearly at the ſame hour, when it's orbit makes the leaſt angle with the horizon. Now at the autumnal equinox, when the ſun is in the firſt point of Libra, the moon, at that time at it's full, will be at the firſt point of Aries, and therefore it riſes with the leaſt difference of times, on two ſucceſſive nights; and it being at the time of it's full, it is more taken notice of; for the ſame thing happens every month when the moon comes to Aries.

Hitherto we have ſuppoſed the ecliptic to repreſent the moon's orbit, but as the orbit is inclined to it at an angle of 5°. 9′ at a mean, let *xAz* repreſent the moon's orbit when the aſcending node is at *A*, and *Ar* the arc deſcribed in a day; then the moon's orbit making the leaſt poſſible angle with the horizon in that poſition of the nodes, the arc *rn*, and conſequently the difference of the times of riſing, will be the leaſt poſſible. As the moon's nodes make a revolution in about 19 years, the leaſt poſſible difference can only happen once in that time. In the latitude of London the leaſt difference is about 17′.

380. The ecliptic makes the greateſt angle with the horizon when the firſt point of Libra riſes, conſequently when the moon is in that part of it's orbit, the difference of the times of it's riſing will be the greateſt; and if the deſcending node of it's orbit be there at the ſame time, it will make the difference the greateſt poſſible; and this difference is about 1*h*. 17′ in the latitude of London. This is the caſe with the vernal full moons. Thoſe ſigns which make the leaſt angle with the horizon when they riſe, make the greateſt angle when they ſet, and vice verſâ; hence when the difference of the times of riſing is the leaſt, the difference of the times of ſetting is the greateſt, and the contrary.

381. By increaſing the latitude, the angle *tAn*, and conſequently *rn*, is diminiſhed; and when the time of deſcribing *rn*, by the diurnal motion, is 3′. 56″, the moon will then riſe at the ſame ſolar hour. Let us ſuppoſe the latitude to be increaſed until the *rAn* vaniſhes, then the moon's orbit becomes coincident with the horizon, every day, for a moment of time, and conſequently the moon riſes at the ſame ſidereal hour, or 3′. 56″ ſooner, by ſolar time. Now take a globe, and elevate the north pole to this latitude, and marking the moon's orbit in this poſition upon it, turn the globe about, and it will appear, that at the inſtant after the above coincidence, one half of the moon's orbit

corresponding to Capricorn, Aquarius, Pisces, Aries, Taurus, Gemini, will rise; hence when the moon is going through that part of it's orbit, or for 13 or 14 days, it rises at the same sidereal hour. Now taking the angle $xAE = 5°.\ 9'$, and the angle $EA\mathfrak{Q} = 23°.\ 28'$, the angle $\mathfrak{Q}Ax$, or $\mathfrak{Q}AH$ when the moon's orbit coincides with the horizon, is $28°.\ 37'$; hence the latitude $\mathfrak{Q}Z$ is $61°.\ 23'$ where these circumstances take place. If the descending node be at A, then $\mathfrak{Q}Ax$, or $\mathfrak{Q}AH = 18°.\ 19'$, and the latitude is $71°.\ 41'$. In any other situation of the orbit, the latitude will be between these limits. When the angle $\mathfrak{Q}Ax$ is greater than the complement of latitude, the moon will rise every day sooner by sidereal time. As there is a complete revolution of the nodes in about 18 years 8 months, all the varieties of the rising and setting of the moon must happen within that time.

On the Horizontal Moon.

382. The phænomenon of the horizontal moon is this, that it appears larger in the horizon than in the meridian; whereas, from it's being nearer to us in the latter case than in the former, it subtends a greater angle. GASSENDUS thought that, as the moon was less bright in the horizon, we looked at it there with a greater pupil of the eye, and therefore it appeared larger. But this is contrary to the principles of Optics, the image of an object upon the retina not depending upon the pupil. This opinion was supported by a French *Abbé*, who supposed that the opening of the pupil made the chrystalline humour flatter, and the eye longer, and thereby increased the image. But there is no connection between the muscles of the iris and the other parts of the eye, to produce these effects. Des CARTES thought that the moon appeared largest in the horizon, because, when comparing it's distance with the intermediate objects, it appeared then furthest off; and as we judge it's distance greatest in that situation, we of course think it larger, supposing that it subtends the same angle. This opinion was supported by Dr. WALLIS in the *Phil. Trans.* N°. 187. Dr. BERKLEY accounts for it thus. Faintness suggests the idea of greater distance; the moon appearing most faint in the horizon, suggests the idea of greater distance, and, supposing the visual angle the same, that must suggest the idea of a greater tangible object. He does not suppose the *visible* extension to be greater, but that the idea of a greater *tangible* extension is suggested, by the alteration of the appearance of the visible extension. He says, 1. That which suggests the idea of greater magnitude, must be something perceived; for what is not perceived can produce no effect. 2. It must be something which is variable, because the moon does not always appear of

the ſame magnitude in the horizon. 3. It cannot lie in the intermediate objects, they remaining the ſame; alſo, when theſe objects are excluded from ſight, it makes no alteration. 4. It cannot be the viſible magnitude, becauſe that is leaſt in the horizon; the cauſe therefore muſtl ie in the viſible appearance, which proceeds from the greater paucity of rays coming to the eye, producing *faintneſs*. Mr. Rowning ſuppoſes, that the moon appears furtheſt from us in the horizon, becauſe the portion of the ſky which we ſee, appears, not an entire *hemiſphere*, but only a portion of one; and in conſequence of this, we judge the moon to be furtheſt from us in the horizon, and therefore to be then largeſt. Dr. Smith, in his *Optics*, gives the ſame reaſon. He makes the apparent diſtance in the horizon to be to that in the zenith as 10 to 3, and therefore the apparent diameters in that ratio. The methods by which he eſtimated the apparent diſtances, may be ſeen in Vol. I. pag. 65. The ſame circumſtance alſo takes place in the ſun, which appears much larger in the horizon than in the zenith. Alſo, if we take two ſtars near each other in the horizon, and two other ſtars near the zenith at the ſame angular diſtance from each other, the two former will appear at a much greater diſtance from each other, than the two latter. Upon this account, people are, in general, very much deceived in eſtimating the altitudes of the heavenly bodies above the horizon, judging them to be much greater than they are. Dr. Smith found, that when a body was about 23° above the horizon, it appeared to be half way between the zenith and horizon, and therefore at that real altitude it would be eſtimated to be 45° high. Upon the ſame principle, the lower part of a rainbow appears broader than the upper part. And this may be conſidered as an argument that the phænomenon cannot depend entirely upon the greater degree of faintneſs in the object when in the horizon, becauſe the lower part of the bow frequently appears brighter than the upper part, at the ſame time that it appears larger; alſo this cauſe could have no effect upon the diſtance of the ſtars; and as the difference of the apparent diſtance of two ſtars, whoſe angular diſtance is the ſame, in the horizon and zenith, ſeems to be fully ſufficient to account for the apparent variation of the moon's diameter in theſe ſituations, it may be doubtful, whether the faintneſs of the object enters into any part of the cauſe.

CHAP.

CHAP. XIX.

ON THE ROTATION OF THE SUN, MOON AND PLANETS.

Art. 383. THE time of rotation of the ſun, moon and planets, and the poſition of their axes, are determined from the ſpots which are obſerved upon their ſurfaces. The poſition of the ſame ſpot, obſerved at three different times, will give the poſition of the axis; for three points of any ſmall circle will determine it's ſituation, and hence we know the axis of the ſphere which is perpendicular to it. The time of rotation may be found, either from obſerving the arc of the ſmall circle deſcribed by a ſpot in any time, or by obſerving the return of a ſpot to the ſame poſition in reſpect to the earth.

On the Rotation of the Sun.

384. It is doubtful by whom the ſpots on the ſun were firſt diſcovered. SCHEINER, Profeſſor of Mathematics at Ingolſtadt, obſerved them in May, 1611, and publiſhed an account of them in 1612, in a Work entitled, *Roſa urſina*. He ſuppoſed them not to be ſpots upon the body of the ſun, but that they were bodies of irregular figures revolving about the ſun, very near to it. GALILEO, in the Preface to a Work entitled, *Iſtoria, Dimoſtrazioni, intorno alle Macchie Solari, Roma* 1613, ſays, that being at Rome in April 1611, he then ſhowed the ſpots of the ſun to ſeveral perſons, and that he had ſpoken of them, ſome months before, to his friends at Florence. He imagined them to adhere to the ſun. KEPLER, in his Ephemeris, ſays, that they were obſerved by the ſon of DAVID FABRICIUS, who publiſhed an account of them in 1611. In the Papers of HARRIOT, not yet printed, it is ſaid, that ſpots upon the ſun were obſerved on December 8, 1610. As teleſcopes were in uſe at that time, it is probable that each might make the diſcovery. Admitting theſe ſpots to adhere to the ſun's body, the reaſons for which we ſhall afterwards give, we proceed to ſhow, how the poſition of the axis of the ſun, and the time of it's rotation, may be found.

385. To determine the poſition of a ſpot upon the ſun's ſurface, find, by the method given in my *Practical Aſtronomy*, Art. 125. the difference between the right aſcenſions and declinations of the ſpot and ſun's center; from

which, find the latitude of the ſpot, and the difference between it's longitude and that of the ſun's center; this may be done thus Let ♈♎ be the ecliptic, ♈*C* the equator, *AB* the ſun, *S* the center of it's diſc, *v* a ſpot on it's ſurface; draw *St* parallel to ♈*C*, and *Sb*, *xva* ſecondaries to ♈*C*, and *vr* perpendicular to ♈♎; then *ab* is the obſerved difference of the right aſcenſions of the ſpot and the ſun's center, and *vx* the difference of their declinations. By Art. 13. *ab* × cos. *Sb* = *Sx*; hence in the right angled triangle *vSx*, we know *Sx* and *vx*, to find *vS*, and the angle *vSx*; alſo in the right angled triangle ♈*Sb*, we know ♈*b* the ſun's right aſcenſion, and *bS* it's declination, to find the angle ♈*Sb*, the difference between which and the right angle *bSx* gives ♈*Sx*, and as *vSx* is known, we get *vSr*; hence in the right angled triangle *vSx*, we know *vS* and the angle *vSr*, to find *vr* the latitude of the ſpot, and *rS* the difference of longitudes of the ſpot and ſun's center. Reduce it's geocentric latitude and longitude to the heliocentric latitude and longitude.

FIG. 89. To do which, let *EACD*, be the projection of the ſun's diſc, *ESC* the ecliptic, *S* the center of the diſc, *M* a ſpot on the ſurface; draw *ML* perpendicular to *EC*, and *ML*, *LS*, are the obſerved geocentric latitude of the ſpot, and difference of longitudes between that and the ſun's center; hence we know *SM*, which is the projection of the arc of a great circle between the point *S* on the ſun's ſurface to which the earth is vertical and the ſpot, into it's ſine. To find this arc, let *E* be the earth, *Ea* a tangent to the ſun, and draw *ab* perpendicular to *EeS*; then the angle *Sab* being equal to *SEa*, the apparent ſemidiameter of the ſun, the arc *ae* is the complement of the ſun's ſemidiameter. Hence if *d* be a ſpot upon the ſun, and *dc* be perpendicular to *Se*; then, *as* ba *the obſerved ſemidiameter* : *the ſine of the arc* ae :: *the obſerved angle under which* dc *appears* : *the ſine of the arc* de. Thus we find the arc correſponding to *SM*, or the angular diſtance of the ſpot upon the ſun's ſurface from the middle of the ſun's diſc. Now the angle *MSL* in the projection, is equal to that upon the ſurface of the ſun formed by the great circles; compute therefore this angle from the right angled plain triangle *MLS*. Let *p* be the pole of the ecliptic upon the ſurface of the ſun. Then the angle *pSL* being a right angle, we know the angle *pSM* on the ſun's ſurface, together with *SM* and *Sp*, *Sp* being = 90°; hence we find *pM*, and the angle *MpS*. Now as *S* is a point on the ſun's diſc to which the earth is vertical, *S* ſeen from the ſun's center has the ſame longitude as the earth, and is therefore known; hence if to that we add, or from it ſubtract, *MpS*, according as *L* is to the eaſt or weſt of *S*, we get the longitude of *M* ſeen from the ſun's center; and the difference of *PM* and *Pv*, or *vM*, is the heliocentric latitude of *M*.

FIG. 90.

FIG. 89.

FIG. 91. 386. To determine the pole *P* of the ſun's equator ♌*nRN*, let *ab* be the path deſcribed by a ſpot, and *M*, *N*, *O*, three obſerved poſitions of that

ſpot,

ſpot, the apparent motion of which is from eaſt to weſt, the ſun revolving about it's axis according to the order of the ſigns; then (385) we know *Mp*, *Np*, *Op*, and the angles *MpN*, *NpO*; for as we know the angles which *Mp*, *Np*, *Op* make with *pS*, the angles between theſe circles will be known, which is the difference of their longitudes. Join the points *M*, *N*, *O*, by three great circles, dotted in the Figure; then in each of the triangles *MpN*, *MpO*, *NpO*, we know two ſides and the included angle at *p*, to find the arcs of the great circles *MN*, *MO*, *NO*, denoted by the dotted lines. Now to find the arcs of the ſmall circle *ab* correſponding, take the ſines of half the arcs of the great circles, and the double will be the chords. Let *aMNOb* be the ſmall circle, *C* it's center, produce *MC* to *V*, and join *OV*; then knowing the chords *MN*, *NO*, *MO*, we know the angle *ONM*, the ſupplement of which is the angle *OVM*, the double of which is the angle *OCM* at the center, or the arc *ONM* of the ſmall circle. Let *OvM* be an arc of the great circle paſſing through *OM*, whoſe radius *OD* is equal to the radius of the ſphere; draw *DCw*, which muſt be perpendicular to *OM*; then the angle *OCw* ſhows the degrees contained in half the arc *ONM* of the ſmall circle, and the angle *ODw*, half the degrees in the great circle *OvM*; and ſin. *OCw*, or *OCD*, : ſin. *ODC* :: *OD* : *OC* :: the radius of the ſphere : radius of the ſmall circle parallel to the ſolar equator, :: radius : cos. of the diſtance of the ſmall circle *ab* from the ſolar equator; hence the diſtance of this ſmall circle from the pole *P* is known. Therefore in the triangle *POM*, we know all the ſides, to find the angle *PMO*; and in the triangle *pMO*, we know all the ſides, to find the angle *pMO*; hence we know the angle *PMp*, together with *PM*, *pM*; therefore we can find *Pp*, which meaſures the inclination of the ſun's axis to the ecliptic.

FIG. 92.

FIG. 91.

387. Let *N* be the aſcending node of the ſun's equator; to find the ſituation of which from the ſun's center, produce *pP* to *t*, then *Pt* paſſing through the poles of the ecliptic and equator muſt cut each 90° from the node *N*, therefore $Nt = 90$. Now to find the poſition of *t*, find, in the triangle *OPp*, the angle at *p*, which meaſures the arc *te*; find (385) alſo the angle *OpS*, or the arc *eS*; hence we know *tS*; but the longitude of *S* ſeen from the ſun's center, is oppoſite to the ſun's place in the ecliptic; find this therefore at the time of the obſervation at *O*, and we get the longitude of *t*, conſequently we get the place *N* of the node. The beſt time to determine the place of the node and the inclination of the equator, is about the beginning of June and Deccember, becauſe at thoſe times the earth being in the plane of the equator, the path of the ſpot is moſt inclined to the ecliptic, and it's latitude changes the faſteſt.

388. To

388. To find the time of the ſun's rotation, we have given the degrees of the arc MNO, and the time the ſpot is moving from M to O; hence the arc MO : 360° :: the time of deſcribing MO : the time of a revolution.

389. But there is a ſhorter and more elegant method of determining the place of the node and inclination of the axis, given by M. CAGNOLI, in his Trigonometry, from the variation of a triangle when two of it's ſides remain conſtant, and the third ſide varies by any finite quantity; this is the caſe with the triangles PpO, PpN, PpM, where Pp is conſtant, and $PO = PN = PM$. Now taking any two of theſe triangles, PpM, PpN, he proves that, ſin. $\frac{1}{2} \times \overline{pN - pM}$: tan. $\frac{1}{2}$ MpN :: ſin. $\frac{1}{2} \times \overline{Np + pM}$: cot. $\frac{1}{2} \times \overline{PMp + PNp}$, where all the terms are known, except the laſt, which therefore is known; in like manner, from the triangles PpN, PpO, we get $PNp + POp$; therefore if we put L', L'', L''' for the obſerved longitudes of the ſpot at M, N, O, and D', D'', D''' for PM, PN, PO; alſo $a = \frac{1}{2} \times \overline{PMp + PNp}$, $b = \frac{1}{2} \times \overline{PMp + POp}$, $c = \frac{1}{2} \times \overline{PNp + POp}$; then

$$\text{tan. } a = \frac{\text{ſin. } \frac{1}{2} \times \overline{D'' - D'} \times \text{cot. } \frac{1}{2} \times \overline{L'' - L'}}{\text{ſin. } \frac{1}{2} \times \overline{D'' + D'}}$$

$$\text{tan. } b = \frac{\text{ſin. } \frac{1}{2} \times \overline{D''' - D'} \times \text{cot. } \frac{1}{2} \times \overline{L''' - L'}}{\text{ſin. } \frac{1}{2} \times \overline{D''' + D'}}$$

$$\text{tan. } c = \frac{\text{ſin. } \frac{1}{2} \times \overline{D''' - D''} \times \text{cot. } \frac{1}{2} \times \overline{L''' - L''}}{\text{ſin. } \frac{1}{2} \times \overline{D''' + D''}}$$

Alſo tan. $\frac{1}{2}$ OpN : tan. $\frac{1}{2} \times \overline{PNp - POp}$ :: tan. $\overline{PpO + \frac{1}{2} OpN}$: tan. $\frac{1}{2} \times \overline{PNp + POp}$, where all the terms are known, except the third, of which one part OpN is $L''' - L''$; hence tan. $\overline{PpO + \frac{1}{2} \times \overline{L''' - L''}}$ = tan. $\frac{1}{2} \times \overline{L''' - L''} \times$ tan. $c \times$ cot. $\overline{a - b}$, which put = tan. x, and we have

$$PpO + \tfrac{1}{2} \times \overline{L''' - L''} = x$$

Add $\tfrac{1}{2} \times \overline{L''' + L''} = \tfrac{1}{2} \times \overline{L''' + L''}$

$\therefore PpO + L''' = x + \frac{1}{2} \times \overline{L''' + L''}$ the longitude of the pole P of the ſun, or of t, to which add 90°, and we get the longitude of the node N.

Now to find Pp, put P = the longitude of the pole P, then $PpO = P - L''' = s$, and $POp = b + c - a = d$; conſequently the tangent of half the difference of PM and Pp is $\frac{\text{tan. } \frac{1}{2} D''' \times \text{ſin. } \frac{1}{2} . \overline{s - d}}{\text{ſin. } \frac{1}{2} . s + d} = \text{tan. } y$, and the tangent of half the ſum is $\frac{\text{tan. } \frac{1}{2} D''' \times \text{cos. } \frac{1}{2} . \overline{s - d}}{\text{cos. } \frac{1}{2} . s + d} = \text{tan. } z$; hence $z + y = PM$

(PM

(*PM* being greater than *Pp*) and $z \sim y = Pp$ the inclination of the ſolar equator to the ecliptic. If s be greater than 180°, take $360° - s$ for s; and the ſame for d. But if d be leſs than 90°, then $Pp = 180° - \overline{z+y}$, and $PM = 180° - \overline{z \sim y}$.

Ex. According to the obſervations of M. de la LANDE, the three longitudes of a ſpot ſeen from the center of the ſun, and it's diſtance from the pole of the ecliptic in 1775, were as follows;

		Longitude	Diſtance from the Pole
June 14,	7ˢ.	8°. 34′. 21″ $= L'$	90°. 38′. 6″ $= D'$
18,	9.	5. 48. 51 $= L''$	97. 30. 8 $= D''$
21,	10.	19. 0. 14 $= L'''$	101. 35. 16 $= D'''$

$$\text{Hence } \tan. a = \frac{\text{ſin. } \frac{1}{2} \times \overline{D'' - D'} \times \text{cot. } \frac{1}{2} \times \overline{L'' - L'}}{\text{ſin. } \frac{1}{2} \times \overline{D'' + D'}} = 6°.\ 16'.\ 45''$$

$$\tan. b = \frac{\text{ſin. } \frac{1}{2} \times \overline{D''' - D'} \times \text{cot. } \frac{1}{2} \times \overline{L''' - L'}}{\text{ſin. } \frac{1}{2} \times \overline{D''' + D'}} = 4°.\ 34'.\ 10''$$

$$\tan. c = \frac{\text{ſin. } \frac{1}{2} \times \overline{D''' - D''} \times \text{cot. } \frac{1}{2} \times \overline{L''' - L''}}{\text{ſin. } \frac{1}{2} \times \overline{D''' + D''}} = 5°.\ 13'.\ 2''$$

Hence tan. $\frac{1}{2} \times \overline{L''' - L''}$ × tan. c × cot. $\overline{a-b}$ = tan. of 50°. 26′. 50″ = x; conſequently = $PpO + L''' = x + \frac{1}{2} \times \overline{L''' + L''} = 11^s.\ 17°.\ 51'.\ 20''$ the longitude P of the pole of the ſun; hence the longitude of the node N is $2^s.\ 17°.\ 51'.\ 20''$.

Now $P - L''' = 28°.\ 51'.\ 6''$, $b + c - a = 3°.\ 30'.\ 27''$; hence

$$\tan. \tfrac{1}{2} \times \overline{PM \sim Pp} = \tan. y = \frac{\tan. \frac{1}{2} D''' \times \text{ſin. } \frac{1}{2} \times \overline{s-d}}{\text{ſin. } \frac{1}{2} \times \overline{s+d}} = 43°.\ 59'.\ 0''$$

$$\tan. \tfrac{1}{2} \times \overline{PM + Pp} = \tan. z = \frac{\tan. \frac{1}{2} D''' \times \text{cos. } \frac{1}{2} . \overline{s-d}}{\text{cos. } \frac{1}{2} . \overline{s+d}} = 51°.\ 14'.\ 10''$$

Hence, (*PM* being greater than *Pp*,) we have $PM = z + y = 95°.\ 13'.\ 10''$ the diſtance of the ſpot from the north pole of the ſun; and $Pp = z - y = 7°.\ 15'.\ 10''$ the inclination of the ſolar equator to the ecliptic.

390. M. CASSINI, from his own obſervations, makes the inclination of the ſun's axis $7\frac{1}{2}°$, calling the inclination the diſtance from the perpendicular to the ecliptic; and the place of the node $2^s.\ 8°$. Le P. SCHEINER ſuppoſes the inclination to be 7°. M. de l'ISLE found it 6°. 35′, from one ſet only of obſervations. The place of the node was determined by M. CASSINI the Son, to be $2^s.\ 10°$. M. de l'ISLE found it $1^s.\ 26°$. Le P. SCHEINER, in 1626, fixed it at $2^s.\ 10°$. From the difficulty of determining the exact poſition of the ſpots,

ſpots, the place of the node and inclination, more particularly the former, are ſubject to conſiderable errors, and accuracy can only be depended upon, from the mean of a great number of obſervations. It does not appear that the place of the node, and the inclination, are ſubject to any change.

391. M. de la LANDE has given the following method of correcting the place of the node, and the inclination of the equator. He ſuppoſes the place of the node, and the inclination, to be nearly known; and from three obſerved latitudes and longitudes of a ſpot, he computes it's declination, which ought to be the ſame in each caſe, if the above quantities be rightly aſſumed; if the declinations come out different, he changes the aſſumed place of the node and inclination, according to the errors, until the declination comes out the ſame for each obſervation, and then concludes the quantities to be rightly aſſumed, ſo far as the obſervations are true. For example; He aſſumes the place of the node ☊ 8ˢ. 17°, and inclination 7°. 30′. Now in 1775, he found by obſervation on June 14, the latitude of a ſpot 0°. 38′ ſouth, longitude 7ˢ. 8°. 34′; on June 18, the latitude 7°. 30′, and longitude 9ˢ. 5°. 49′; and on June 21, the latitude 11°. 35′; and longitude 10ˢ. 19°; hence, (393) the correſponding declinations by calculation are 5°. 17′, 5°. 2′ and 4°. 57′. By making the inclination 7°. 20′, the firſt and laſt declinations become 5°. 11′ and 5°. 6′; therefore by diminiſhing the inclination 10′, the declinations of the ſpot at the firſt and laſt obſervations are brought nearer by 15′; hence 15′ : 10′ :: 5′ (the difference of 5°. 11′ and 5°. 6′) : 3′, which ſubtracted from 7°. 20′ gives 7°. 17′ for the inclination, which will give the firſt and laſt declination 5°. 9′. With this inclination 7°. 17′, the ſecond obſerved place gives 5°. 6′ for the declination, differing 3′ for the two other. His ſecond hypotheſis is, to change the place of the node in order to make the declinations at the firſt and third obſervations agree; he therefore ſuppoſes the place of the node to be 8ˢ. 22°. And by going through the calculations as before, he finds, that an inclination of 7°. 10′ will give 5°. 33′ for the declination at the firſt and third obſervations, and 5°. 47′ at the ſecond, differing 14′. Hence he arranges the two hypotheſes thus.

Node	Inclination	Decl. on June 14 and 21	Declination on June 18	Difference of Declins.
8ˢ. 17°. 0′	7°. 17′	5°. 9′	5°. 6′	3′ leſs
8. 22. 0	7. 10	5. 33	5. 47	14 more
Diff. 5. 0	0. 7	0. 24	0. 41	17 diff.

Here

Here a change of 5° of the node and 7′ in the inclination has made a difference of 17′ in the ſum of the errors. Hence to alter the place of the node and inclination to make both the differences 3′ and 14′ vaniſh, ſay, 17′ : 5° :: 3′ : 53′, which added to 8ˢ. 17°. gives 8ˢ. 17°. 53′; alſo 17′ : 7′ :: 3′ : 1′, ſubtract therefore 1′ from 7°. 17′ and it gives 7°. 16′ for the correſponding inclination. Laſtly, to find the correſponding declinations, ſay, 17′ : 24′ :: 3′ : 4′, add this 4′ to 5°. 9′ and it gives 5°. 13′ for the declination on June 14 and 21; and 17′ : 41′ :: 3′ : 7′, add this 7′ to the declination 5°. 6′ on June 18, and it gives 5°. 13′ for the declination at that time. Hence the place of the node 8ˢ. 17°. 53′, and inclination 7°. 16′, give 5°. 13′ for the declination of the ſpot at the three obſervations, and therefore we may conclude the place of the node and inclination to be truly aſcertained, as near as the obſervations can give it. It will be always proper to go through with all the calculations again, after you have thus deduced the place of the node and inclination, and ſee whether they give the declinations the ſame at each obſervation; if not, another correction muſt be made in the ſame manner; but this will not be found neceſſary, unleſs you have conſiderably altered the place of the node and inclination; in which caſe, the approximations may not be ſufficiently exact; and after all, the ſmall errors which the obſervations muſt be ſubject to, renders it unneceſſary to ſeek for a nearer agreement in the declinations than 3′ or 4′. This may be conſidered as a correction of the place of the node and inclination, as determined nearly by any other method.

392. When the earth is in the nodes of the ſun's equator, it being then in it's plane, the ſpots appear to deſcribe ſtraight lines; this happens about the beginning of June and December. As the earth recedes from the nodes, the path of a ſpot grows more and more elliptical, till the earth gets 90° from the nodes, which happens about the beginning of September and March, at which time the ellipſe has it's minor axis the greateſt, and is then to the major axis, as the ſine of the inclination of the ſolar equator to radius.

393. To find the right aſcenſion nv of the ſpot at O from the deſcending node n, and the declination Ov, we have, in the right angled triangle neO, ne the difference of the longitudes of n and O, with eO the latitude of O, to find On, and the angle One; and as we know vne, we ſhall know vnO; hence in the right angled triangle Ovn, we know nO and the angle Onv, to find vn the right aſcenſion of O meaſured from the node n, and Ov it's declination. FIG. 91.

394. If the latitude, longitude and declination of a ſpot be known, we may find it's right aſcenſion thus. By ſpher. trig. rad. × cos. nO = cos. ne × cos. Oe, and rad. × cos. nO = cos. nv × cos. Ov, hence cos. ne × cos. Oe = cos. nv × cos. Ov, conſequently the cos. of right aſcen. $nv = \frac{\text{cos. } ne \times \text{cos. } Oe}{\text{cos. } Ov} =$

$\frac{\text{cos.dist.from node} \times \text{cos hel lat.}}{\text{cos. hel. dec.}}$. If we therefore calculate the right aſcenſion of the ſame ſpot at two different times, we get it's motion in right aſcenſion in the interval of theſe times; hence that motion : 360° :: the interval of the times : the time of the rotation of the ſun in reſpect to the nodes, or, as it does not appear that the node has any ſenſible motion, it gives the true time of rotation. Or the time may be determined by the return of a ſpot to the ſame declination or right aſcenſion. Thus M. de la LANDE has found the time of rotation to be 25*d*. 10*h*. and the return of the ſpots to the ſame ſituation, to be 27*d*. 7*h*. 37′. 28″. M. CASSINI determined the time of rotation, from obſerving the time in which a ſpot returns to the ſame ſituation upon the diſc, or to the circle of latitude paſſing through the earth. Let *t* be that interval of time, and let *m* be equal to the *true* motion of the earth in that time, and *n* equal to it's *mean* motion; then 360° + *m* : 360° + *n* :: *t* : the time of return if the motion had been uniform, and this, from a great number of obſervations he determines to be 27*d*. 12*h*. 20′; now the mean motion of the earth in that time is 27°. 7′. 8″; hence 360° + 27°. 7′. 8″ : 360° :: 27*d*. 12*h*. 20′ : 25*d*. 14*h*. 8′ the time of rotation. *Elem. de 'Aſtron.* pag. 104. But this method is not capable of ſo much accuracy as the other.

395. There has been a great difference of opinions reſpecting the nature of the ſolar ſpots. SCHEINER ſuppoſed them to be ſolid bodies revolving about the ſun, very near to it; but as they are as long viſible as they are inviſible, this cannot be the caſe. Moreover, we have a phyſical argument againſt this hypotheſis, which is, that moſt of them do not revolve about the ſun in a plane paſſing through it's center, which they neceſſarily muſt, if they revolved, like the planets, about the ſun. GALILEO confuted SCHEINER's opinion, by obſerving that the ſpots were not permanent; that they varied their figure; that they increaſed and decreaſed, and ſometimes diſappeared. He compared them to ſmoak and clouds. HEVELIUS appears to have been of the ſame opinion; for in his *Cometographia*, page 360, ſpeaking of the ſolar ſpots, he ſays, *hæc materia nunc ea ipſa eſt evaporatio et exhalatio (quia aliunde minime oriri poteſt) quæ ex ipſo corpore ſolis, ut ſupra oſtenſum eſt, expiratur et exhalatur.* But the permanency of moſt of the ſpots is an argument againſt this hypotheſis. M. de la HIRE ſuppoſed them to be ſolid, opaque bodies, which ſwim upon the liquid matter of the ſun, and which are ſometimes entirely immerſed. M. de la LANDE ſuppoſes that the ſun is an opaque body, covered with a liquid fire, and that the ſpots ariſe from the opaque parts, like rocks, which, by the alternate flux and reflux of the liquid igneous matter of the ſun, are ſometimes raiſed above the ſurface. The ſpots are frequently dark in the middle, with an umbra about them; and M. de la

LANDE

LANDE ſuppoſes that that part of the rock which ſtands above the ſurface forms the dark part in the center, and thoſe parts which are but juſt covered by the igneous matter form the umbra. Dr. WILSON, Profeſſor of Aſtronomy at Glaſgow, oppoſes this hypotheſis of M. de la LANDE, by this argument. Generally ſpeaking, the umbra immediately contiguous to the dark central part, or nucleus, inſtead of being very dark, as it ought to be, from our ſeeing the immerſed parts of the opaque rock through a thin ſtratum of the igneous matter, is, on the contrary, very nearly of the ſame ſplendor as the external ſurface, and the umbra grows darker the further it recedes from the nucleus; this, it muſt be acknowledged, is a ſtrong argument againſt the hypotheſis of M. de la LANDE. Dr. WILSON further obſerves, that M. de la LANDE produces no optical arguments in ſupport of the rock ſtanding above the ſurface of the ſun. The opinion of Dr. WILSON is, that the ſpots are excavations in the luminous matter of the ſun, the bottom of which forms the umbra. They who wiſh to ſee the arguments by which this is ſupported, muſt conſult the *Phil. Tranſ.* 1774 and 1783. Dr. HALLEY conjectured that the ſpots are formed in the atmoſphere of the ſun. Dr. HERSCHEL ſuppoſes the ſun to be an opaque body, and that it has an atmoſphere; and if ſome of the fluids which enter into it's compoſition ſhould be of a ſhining brilliancy, whilſt others are merely tranſparent, any temporary cauſe which may remove the lucid fluid will permit us to ſee the body of the ſun through the tranſparent ones. See the *Phil. Tranſ.* 1795. Dr. HERSCHEL on April 19, 1779, ſaw a ſpot which meaſured 1′. 8″,06 in diameter, which is equal in length to more than 31 thouſand miles; this was viſible to the naked eye. Beſides the dark ſpots upon the ſun, there are alſo parts of the ſun, called *Faculæ*, *Luculi*, &c. which are brighter than the general ſurface; theſe always abound moſt in the neighbourhood of the ſpots themſelves, or where ſpots recently have been. Moſt of the ſpots appear within the compaſs of a zone lying 30° on each ſide of the equator; but on July 5, 1780, M. de la LANDE obſerved a ſpot 40° from the equator. Spots which have diſappeared have been obſerved to break out again. The ſpots appear ſo frequently, that Aſtronomers very ſeldom examine the ſun with their teleſcopes, but they ſee ſome; SCHEINER ſaw 50 at once. The following phænomena of the ſpots are deſcribed by SCHEINER and HEVELIUS.

I. Every ſpot which hath a nucleus, hath alſo an umbra ſurrounding it.

II. The boundary between the nucleus and umbra is always well defined.

III. The increaſe of a ſpot is gradual, the breadth of the nucleus and umbra dilating at the ſame time.

IV. The decreaſe of a ſpot is gradual, the breadth of the nucleus and umbra contracting at the ſame time.

 V. The

V. The exterior boundary of the umbra never conſiſts of ſharp angles, but is always curvilinear, however irregular the outline of the nucleus may be.

VI. The nucleus, when on the decreaſe, in many caſes changes it's figure, by the umbra encroaching irregularly upon it.

VII. It often happens, by theſe encroachments, that the nucleus is divided into two or more nuclei.

VIII. The nucleus vaniſhes ſooner than the umbra.

IX. Small umbræ are frequently ſeen without nuclei.

X. An umbra of any conſiderable ſize is ſeldom ſeen without a nucleus.

XI. When a ſpot, conſiſting of a nucleus and umbra, is about to diſappear, if it be not ſucceeded by a facula, or more fulgid appearance, the place it occupied, is, ſoon after, not diſtinguiſhable from any other part of the ſun's ſurface.

On the Rotation of the Moon.

FIG. 89. 396. The latitude and longitude of ſome one ſpot, as ſeen from the moon's center, muſt be determined (385) as for the ſun; but (referring to Fig. 89.) pS is not, as for the ſun, equal to 90°, but it is the moon's diſtance from the pole of the ecliptic, for the pole of the ecliptic will not lie in the circumference of the moon's diſc, as in the caſe of the ſun, except when the moon is in the ecliptic; for as the moon leaves the ecliptic, it is manifeſt that the pole of the ecliptic will approach upon the diſc, or recede behind the moon, by a quantity equal to the moon's latitude; at the time of obſervation therefore, pS will be known, by knowing the moon's latitude; alſo SM and the angle pSM are determined as for the ſun; hence we can compute pM the diſtance of the ſpot from the north pole of the ecliptic, and the angle SpM the difference between the longitude of the ſpot and that of the earth ſeen from the moon, therefore the longitude of the earth being known, the longitude of the ſpot ſeen from the moon's center will be known. We thus find the latitude and longitude of a ſpot at three different times, ſeen from the center of the moon, in reſpect to the ecliptic, or to a circle drawn through the center of the moon parallel to the ecliptic; and with theſe three obſervations, we can determine the ſituation of the lunar equator, in the ſame manner as for the ſun; but MAYER has given another method by approximation, by which he can employ more obſervations for one operation, and thereby increaſe the accuracy of the concluſion. Thoſe ſpots near the center are the beſt for this purpoſe, becauſe their change is moſt ſenſible; MAYER has therefore choſen that called Manilius, the obſervations upon which are contained in the following Table.

True

True Time at Nuremberg	$SM=$	$pSM=$	Apparent long. of ☾	Appar. lat. of ☾	$pM=$	$SpM=$	Longitude of Manilius at ☾'s center
1748. D. H. M.	° ′	° ′	° ′	° ′	° ′	° ′	° ′
April 11. 11. 1	17. 20	58. 11	6. 0. 35	4. 16 s.	76. 50	15. 4	0. 15. 39
—— 13. 9. 30	15. 8	58. 35	6. 27. 24	5. 27 -	76. 52	13. 14	1. 10. 39
May 11. 10. 56	15. 29	60. 40	7. 6. 19	5. 51 -	76. 48	13. 50	1. 20. 9
—— 16. 16. 11	13. 26	28. 45	9. 22. 14	2. 31 -	75. 45	6. 38	3. 28. 52
—— 17. 15. 56	14. 23	20. 49	10. 6. 33	1. 17 -	75. 18	5. 14	4. 11. 37
June 5. 9. 58	18. 2	62, 16	6. 2. 53	4. 56 -	76. 59	16. 20	0. 19. 13
—— 13. 14. 0	14. 18	25. 48	10. 0. 24	1. 41 -	75. 29	6. 23	4. 6. 47
—— 14. 12. 50	15. 12	16. 47	10. 14. 43	0. 25 -	75. 3	4. 30	4. 19. 13
July 2. 9. 23	18. 2	61. 56	5. 28. 25	4. 54 -	76. 55	16. 17	0. 14. 42
—— 4. 6. 49	17. 36	64. 29	6. 23. 11	5. 48 -	76. 57	16. 16	1. 9. 27
—— 5. 8. 4	17. 23	64. 49	7. 7. 18	6. 8 -	76. 48	16. 7	1. 23. 25
—— 6. 8. 34	16. 20	62. 37	7. 21. 34	5. 57 -	76. 49	14. 52	2. 6. 26
—— 7. 9. 4	15. 43	58. 10	8. 6. 15	5. 30 -	76. 26	13. 42	2. 19. 57
—— 8. 10. 4	15. 8	52. 0	8. 21. 33	4. 44 -	76. 7	12. 14	3. 3. 47
—— 9. 11. 15	14. 38	44. 26	9. 7. 12	3. 38 -	76. 2	10. 30	3. 17. 42
—— 10. 12. 5	14. 34	34. 40	9. 22. 50	2. 19 -	75. 46	8. 29	4. 1. 19
—— 11. 13. 15	15. 23	23. 24	10. 8. 37	0. 51 -	75. 4	6. 16	4. 14. 53
—— 12. 13. 5	16. 0	16. 57	10. 23. 34	0. 30 N.	75. 13	4. 46	4. 28. 20
—— 15. 13. 35	19. 38	2. 14	0. 6. 37	3. 41 -	74. 4	0. 47	6. 7. 24
Aug. 3. 7. 5	16. 10	60. 27	7. 29. 58	5. 46 s.	76. 31	14. 25	2. 14. 23
—— 14. 11. 34	20. 23	4. 16	1. 11. 2	4. 25 N.	74. 5	1. 33	7. 12. 35
Nov. 1. 5. 44	19. 27	15. 33	11. 24. 42	3. 4 -	74. 21	5. 19	6. 0. 1
—— 2. 6. 29	20. 26	11. 50	0. 9. 1	3. 46 -	73. 51	4. 16	6. 13. 17
Dec. 27. 4. 47	20. 54	7. 19	0. 14. 44	4. 21 -	73. 36	2. 43	6. 17. 27
1749.	* *	* *	* * *	* *	* *	* *	* * *
Jan. 28. 3. 59	8. 56	9. 59	2. 16. 0	3. 0 -	74. 22	3. 21	8. 19. 21
Feb. 25. 11. 43	17. 30	14. 53	2. 27. 53	2. 0 -	75. 6	4. 35	9. 2. 28
March 4. 11. 42	14. 46	54. 26	5. 22. 9	4. 42 s.	76. 53	12. 17	0. 4. 26

397. Let QDV repreſent the face of the moon next to the earth; C the center of the moon's diſc; QNX the lunar equator, P it's pole; DNW the ecliptic referred to the moon's ſurface, or rather a circle paſſing through it's center parallel to the ecliptic, and which extended to the heavens may be conſidered as coinciding with it, p it's pole, which is not, as in the ſun, in the outward circle QDV; M Manilius, through which draw the great circles pMB, PML; and let γ be the firſt point of Aries ſeen from the moon's center; then MB is the latitude of Manilius, which is a variable quantity, and known FIG. 93.

known from obſervation, and therefore we know pM it's complement; Pp is the diſtance of the two poles, or the inclination of the lunar equator to the ecliptic; ML is the *declination** of Manilius; and γN is the longitude of the node N of the lunar equator. Now when p falls between P and M, Mp is the leaſt; and when p is oppoſite to that ſituation, Mp is the greateſt; and half the difference gives Pp the diſtance of the poles, or the inclination of the lunar equator to the ecliptic. But as Mp is the complement of latitude of M, it is manifeſt that the above mentioned half difference is half the difference of the greateſt and leaſt complements of latitudes of M. Now by inſpection in the Table, the greateſt obſerved value of pM is 76°. 59′, and the leaſt value is 73°. 36′, half the difference of which is 1°. 41′,5, which is nearly the value of Pp, and would be accurately ſo, if we could be ſure that the above values of pM were the greateſt and leaſt poſſible. Alſo (369) the node N of the lunar equator coincides, or nearly ſo, with the node of the lunar orbit. Put $a = Pp$, $b = LM$, $g = \gamma B$, $h = pM$, $t =$ the diſtance of the node N of the lunar equator from the node of it's orbit, $k =$ the longitude of the aſcending node of the orbit; then $k + t = \gamma N$ the longitude of the node of the lunar equator; hence $g - k - t = NB$, or the angle NpB, and therefore $MpP = 90° - g + k + t$, becauſe the great circle paſſing through the poles of any two great circles muſt be 90° from their interſection. Now in the triangle MPp, cos. PM = cos. Pp × cos. pM + ſin. Pp × ſin. pM × cos. PpM, that is, cos. $\overline{90° - b}$ = cos. a × cos. h + ſin. a × ſin. h × cos. $\overline{90° - g + k + t}$, or ſin. b = cos. a × cos. h + ſin. a × ſin. h × ſin. $\overline{g - k - t}$. Now by plain trig. ſin. $\overline{g - k - t}$ = ſin. $\overline{g - k}$ × cos. t − ſin. t × cos. $\overline{g - k}$; but as t is very ſmall, we may aſſume the cos. $t = 1$; and as a is alſo very ſmall, cos. $a = 1$; hence, by ſubſtitution and tranſpoſition, ſin. b − cos. h = ſin. a × ſin. h × ſin. $\overline{g - k}$ − ſin. a × ſin. h × ſin. t × cos. $\overline{g - k}$. But as Pp is very ſmall, $b = 90° - h + x$, where x muſt be very ſmall, it never being more than Pp; hence ſin. b = cos. $\overline{h - x}$ = cos. h × cos. x + ſin. h × ſin. x = (as cos. $x = 1$ very nearly, and ſin. $x = x$) cos. $h + x$ × ſin. h, therefore ſin. b − cos. h. = x × ſin. $h = b - \overline{90° - h}$ × ſin. h. Subſtitute this quantity for ſin. b − cos. h in the above equation, divide by ſin. h, and for ſin. a ſubſtitute a, and we have $b - \overline{90° - h} = a$ × ſin. $\overline{g - k}$ − a × ſin. t × cos. $\overline{g - k}$. Now the quantities g, h, k are known from obſervation, to find a, b, t; to do which, we muſt form three equations from three different values of g, h and k, from whence we can find a, b, t.

* Writers upon this ſubject call this the *Lunar Latitude*, but this makes a confuſion of terms; I have choſen to call it *Declination*, it being the diſtance of the ſpot from the lunar equator.

For

For this purpose, MAYER has taken the observations on July 2, 10, and 15, in the Tables; hence

Times of observation	July 2, at $9^h.\ 23'$	July 10, at $12^h.\ 5'$	July 15, at $13^h.\ 35'$
$g =$	$0^s.\ 14^\circ.\ 42'$	$4^s.\ 1^\circ.\ 19'$	$6^s.\ 7^\circ.\ 24'$
$h =$	$0.\ 76.\ 55$	$0.\ 75.\ 46$	$0.\ 74.\ 4$
$k =$	$10.\ 9.\ 14$	$10.\ 8.\ 48$	$10.\ 8.\ 32$
sin. $\overline{g-k}$	$+0{,}9097$	$+0{,}1302$	$-0{,}8560$
cos. $\overline{g-k}$	$+0{,}4152$	$-0{,}9915$	$-0{,}5170$

These values substituted into the above equations give,

$$b - 13^\circ.\ 5' = +0{,}9097a - 0{,}4152a \times \text{sin. } t.$$
$$b - 14^\circ.\ 14' = +0{,}1302a + 0{,}9915a \times \text{sin. } t.$$
$$b - 15^\circ.\ 56' = -0{,}8560a + 0{,}5170a \times \text{sin. } t.$$

Subtract the first from the second, and the second from the third, and

$$-171' = -1{,}7657a + 0{,}9322a \times \text{sin. } t.$$
$$-102' = -0{,}9862a - 0{,}4745a \times \text{sin. } t.$$

Divide the first by 0,9322, and the second by $-0{,}4745$, and

$$-183',44 = -1{,}8941a + a \times \text{sin. } t.$$
$$214',47 = +2{,}0784a + a \times \text{sin. } t.$$

Subtract the first from the second, and we get $397{,}91 = 3{,}9725a$; hence $a = 100' = 1^\circ.\ 40'$; substitute this value of a into one of the other two equations, and we get $t = 3^\circ.\ 36'$; and these values of a and t substituted into one of the three first equations, give $b = 14^\circ.\ 33'$ the declination of Manilius. This value of t shows that the node of the lunar equator does not sensibly differ from the place of the node of the lunar orbit; and from any thing which appears here, they may accurately coincide, as there is no greater difference than what may arise from the inaccuracy to which the observations may be liable; for a very small error in the observed place of Manilius, will make a considerable error in the place of the node. This determination also gives the inclination of the moon's axis to the ecliptic $= 1^\circ.\ 40'$. Produce pP to meet the ecliptic and moon's equator in r and s; then $rs = 1^\circ.\ 40'$. Now the ascending node of the moon's orbit, and the descending node of it's equator, are those which go together. Let therefore Nv be the situation of the moon's orbit in respect to the

the ecliptic Nr, then $vr = 5°.\ 9'$ at the mean inclination of the lunar orbit; and as $rs = 1°.\ 40'$, we have vs, or the angle vNs, equal to $6°.\ 49'$ the inclination of the axis of the moon to the plane of it's orbit. To have all the accuracy poſſible, the three latitudes obſerved ſhould be very different, and NB about $90°$; if two of the obſervations be towards the extreme latitudes, and the other near the node, the inclination will be determined with the greateſt accuracy; and if two be near the node, and one near the greateſt latitude, the node will be beſt determined.

To apply more than three obſervations to one operation, MAYER, having calculated the 27 obſervations in the Table, formed 27 equations ſimilar to the three formed before; then he added nine of them together, and thus formed the following equations.

$$9b - 118°.\ 8' = +8{,}4987a - 0{,}7932a \times \text{ſin. } t.$$
$$9b - 140.\ 17 = -6{,}1404a + 1{,}7443a \times \text{ſin. } t.$$
$$9b - 127.\ 32 = +2{,}7977a + 7{,}9649a \times \text{ſin. } t.$$

In the forming of theſe equations, nine equations were taken for the firſt, ſo as to make the poſſitive coefficient of a as great as poſſible; nine for the ſecond, to make the negative coefficient the greateſt; and the third was formed from the other nine. By this means, when we exterminate all but a, it's coefficient will be the greateſt, and will give the moſt accurate value of a. Proceeding therefore as before, we get $a = 89',9 = 1°.\ 30'$ very nearly, differing $10'$ from the other determination, which cannot be conſidered ſo accurate as this; $b = 14°.\ 33'$, the ſame as before; $t = -3°.\ 45'$, giving the longitude of the node of the lunar equator about as much leſs as the other gave it greater. This value of a gives the inclination of the moon's axis to the plane of it's orbit $= 6°.\ 39'$. And as the longitude of the node of the moon's orbit at the beginning of 1748, was $10^s.\ 18°.\ 56'$, that of it's equator was $10^s.\ 15°.\ 11'$.

In the year 1763, M. de la LANDE, in the month of October, repeated theſe obſervations, and found the inclination to be $1°.\ 43'$, and the declination of Manilius $14°.\ 35'$; he thinks this determination is more to be depended upon than that from the obſervations of MAYER. He alſo found the diſtance of the nodes of the moon's orbit and equator to be about $2°$, at a time when the diſtance of the node of the lunar orbit was $60°$ from the place where it was in 1748. We may therefore, with CASSINI, conclude, that *the nodes of the lunar equator agree with the mean place of the nodes of the lunar orbit, and conſequently their mean motions are the ſame.*

398. The values of γB and γN being known, we know NB the longitude of M; and it's latitude MB being alſo known, together with the angle BNL,

we

we can (393) find the right aſcenſion *NL* of Manilius. Hence compute the right aſcenſion at any intervals of time, and it appears that the right aſcenſion increaſes uniformly, therefore the rotation of the moon about it's axis is uniform, and conſequently is performed (355) in 27*d*. 7*h*. 43′. 11″,5.

399. As *L* is a fixed point upon the moon's ſurface, if the right aſcenſion of any other point eſtimated from *L* be found, and alſo it's declination, the ſituation of that point will be known. Thus we might lay down the figure of the lunar diſc.

On the Rotation of the Planets.

400. The *Georgian* is at ſo great a diſtance, that Aſtronomers, with their beſt teleſcopes, have not been able to diſcover whether it has any revolution about it's axis.

401. *Saturn* was ſuſpected by CASSINI and FATO, in 1683, to have a revolution about it's axis; for they one day ſaw a bright ſtreak, which diſappeared the next, when another came into view near the edge of it's diſc; theſe ſtreaks are called *Belts*. In 1719, when the ring diſappeared, CASSINI ſaw it's ſhadow upon the body of the planet, and a belt on each ſide parallel to the ſhadow. When the ring was viſible, he perceived their curvature was ſuch as agreed with the elevation of the eye above the plane of the ring. He conſidered them as ſimilar to our clouds floating in the atmoſphere; and having a curvature ſimilar to the exterior circumference of the ring, he concluded that they ought to be nearly at the ſame diſtance from the planet, and conſequently the atmoſphere of Saturn extends to the ring. Dr. HERSCHEL found that the arrangement of the belts always followed the direction of the ring; thus, as the ring opened, the belts began to ſhow an incurvature anſwering to it. And during his obſervations on June 19, 20 and 21, 1780, he ſaw the ſame ſpot in three different ſituations. He conjectured therefore, that Saturn revolved about an axis perpendicular to the plane of it's ring. Another argument in defence of this is, that the planet is an oblate ſpheroid, having the diameter in the direction of the ring to the diameter perpendicular to it as about 11 : 10, according to Dr. HERSCHEL; the meaſures were taken with a wire micrometer prefixed to his 20 feet reflector. The truth of his conjecture he has now verified, having determined that Saturn revolves about it's axis in 10*h*. 16′. 0″,4. *Phil. Tranſ.* 1794. The rotation is according to the order of the ſigns.

402. *Jupiter* is obſerved to have belts, and alſo ſpots, by which the time of it's rotation can be very accurately aſcertained. M. CASSINI found the time of rotation to be 9*h*. 56′, from a remarkable ſpot which he obſerved in 1665.

In October 1691, he obſerved two bright ſpots almoſt as broad as the belts; and at the end of the month he ſaw two more, and found them to revolve in 9*h*. 51′; he alſo obſerved ſome other ſpots near Jupiter's equator, which revolved in 9*h*. 50′; and, in general, he found that the nearer the ſpots were to the equator, the quicker they revolved. It is probable therefore that theſe ſpots are not upon Jupiter's ſurface, but in it's atmoſphere; and for this reaſon alſo, that ſeveral ſpots which appeared round at firſt, grew oblong by degrees in a direction parallel to the belts, and divided themſelves into two or three ſpots. M. MARALDI, from a great many obſervations of the ſpot obſerved by CASSINI in 1665, found the time of rotation to be 9*h*. 56′; and concluded that the ſpots had a dependence upon the contiguous belt, as the ſpot had never appeared without the belt, though the belt had without the ſpot. It continued to appear and diſappear till 1694, and was not ſeen any more till 1708; hence he concluded, that the ſpot was ſome effuſion from the belt upon a fixed place of Jupiter's body, for it always appeared in the ſame place. Dr. HERSCHEL found the time of rotation of different ſpots to vary; and that the time of revolution of the ſame ſpot diminiſhed; for the ſpot obſerved in 1778 revolved as follows. From February 25, to March 2, in 9*h*. 55′. 20″; from March 2 to the 14th, in 9*h*. 54′. 58″; from April 7 to the 12th, in 9*h*. 51′ 35″. Alſo, from a ſpot obſerved in 1779, it's rotation was, from April 14 to the 19th, in 9*h*. 51′. 45″; from April 19 to the 23d, in 9*h*. 50′. 48″. This, he obſerves, is agreeable to the theory of equinoctial winds, as it may be ſome time before the ſpot can acquire the velocity of the wind; and if Jupiter's ſpots ſhould be obſerved in different parts of it's year to be accelerated and retarded, it would amount almoſt to a demonſtration of it's monſoons, and their periodical changes. M. SCHROETER makes the time of rotation 9*h*. 55′. 36″,6; he obſerved the ſame variations as Dr. HERSCHEL. The rotation is according to the order of the ſigns. This planet is obſerved to be flat at it's poles. Dr. POUND meaſured the polar and equatorial diameters, and found them as 12 : 13. Mr. SHORT made them as 13 : 14. Dr. BRADLEY made them as 12,5 : 13,5. Sir I. NEWTON makes the ratio $9\frac{1}{3} : 10\frac{1}{3}$ by theory. The belts of Jupiter are generally parallel to it's equator, which is very nearly parallel to the ecliptic; they are ſubject to great variations, both in reſpect to their number and figure; ſometimes eight have been ſeen at once, and at other times only one; ſometimes they continue for three months without any variation, and ſometimes a new belt has been formed in an hour or two. From their being ſubject to ſuch changes, it is very probable, that they do not adhere to the body of Jupiter, but exiſt in it's atmoſphere.

403. GALILEO diſcovered the phaſes of *Mars*; after which, ſome Italians, in 1636, had an imperfect view of a ſpot. But in 1666, Dr. HOOK and M. CASSINI diſcovered ſome well defined ſpots; and the latter determined the time of the rotation to be 24*h*. 40′. Soon after, M. MARALDI obſerved ſome ſpots, and determined the time of rotation to be 24*h*. 39′. He alſo obſerved a very bright part near the ſouthern pole, appearing like a polar zone; this, he ſays, has been obſerved for 60 years; it is not of equal brightneſs, more than one half of it being brighter than the reſt; and that part which is leaſt bright, is ſubject to great changes, and has ſometimes diſappeared. Something like this has been ſeen about the north pole. The rotation is made according to the order of the ſigns. Dr. HERSCHEL makes the time of a ſidereal revolution to be 24*h*. 39′. 21″,67, without the probability of a greater error than 2″,34. He propoſes to find the time of a ſidereal revolution, in order to diſcover, by future obſervations, whether there is any alteration in the time of the revolution of the earth, or of the planets, about their axes; for a change of either would thus be diſcovered. He choſe Mars, becauſe it's ſpots are permanent. See the *Phil. Tranſ.* 1781. From further obſervations upon Mars, which he publiſhed in the *Phil. Tranſ.* 1784, he makes it's axis to be inclined to the ecliptic 59°. 42′, and 61°. 18′ to it's orbit; and the north pole to be directed to 17°. 47′ of Piſces upon the ecliptic, and 19°. 28′ on it's orbit. He makes the ratio of the diameters of Mars to be as 16 : 15. Dr. MASKELYNE has carefully obſerved Mars at the time of oppoſition, but could not perceive any difference in it's diameters. Dr. HERSCHEL obſerves, that Mars has a conſiderable atmoſphere.

404. GALILEO firſt diſcovered the phaſes of *Venus* in 1611, and ſent the diſcovery to WILLIAM de' MEDICI, to communicate it to KEPLER. He ſent it in this cypher, *Hæc immaturæ a me fruſtra leguntur, o, y,* which put in order, is, *Cynthiæ figuras æmulatur mater amorum,* that is, *Venus emulates the phaſes of the moon.* He afterwards wrote a letter to him, giving an account of the diſcovery, and explaining the cypher. In 1666, M. CASSINI, at a time when Venus was dichotomiſed, diſcovered a bright ſpot upon it at the ſtraight edge, like ſome of the bright ſpots upon the moon's ſurface; and by obſerving it's motion, which was upon the edge, he found the ſidereal time of rotation to be 23*h*. 16′. In the year 1726, BLANCHINI made ſome obſervations upon the ſpots of Venus, and aſſerted the time of rotation to be $24\frac{1}{3}$ days; that the north pole anſwered to the 20th degree of Aquarius, and was elevated 15° or 20° above it's orbit; and that the axis continued parallel to itſelf. The ſmall angle which the axis of Venus makes with it's orbit, is a ſingular circumſtance; and muſt cauſe a very great variety in the ſeaſons. M. CASSINI, the Son, has vin-

dicated his Father, and ſhown, from BLANCHINI's obſervations being interrupted, that he might eaſily miſtake different ſpots for the ſame; and he concludes, that if we ſuppoſe the periodic time to be 23*h*. 20′, it agrees equally with their obſervations; but if we take it $24\frac{1}{3}$ days, it will not at all agree with his Father's obſervations. M. SCHROETER has endeavoured to ſhow that Venus has an atmoſphere, from obſerving that the illuminated limb, when horned, exceeds a ſemicircle; this he ſuppoſes to ariſe from the refraction of the ſun's rays through the atmoſphere of Venus at the cuſps, by which they appear prolonged. The cuſps appeared ſometimes to run 15°. 19′ into the dark hemiſphere; from which he computes that the height of the atmoſphere to refract ſuch a quantity of light muſt be 15156 Paris feet. But this muſt depend on the nature and denſity of the atmoſphere, of which we are ignorant. *Phil. Tranſ.* 1792. He makes the time of rotation to be 23*h*. 21′, and concludes, from his obſervations, that there are conſiderable mountains upon this planet. *Phil. Tranſ.* 1795. Dr. HERSCHEL agrees with M. SCHROETER, that Venus has a conſiderable atmoſphere; but he has not made any obſervations, by which he can determine, either the time of rotation, or the poſition of the axis. *Phil. Tranſ.* 1793.

405. The phaſes of *Mercury* are eaſily diſtinguiſhed to be like thoſe of *Venus*; but no ſpots have yet been diſcovered by which we can aſcertain whether it has any rotation.

406. There is reaſon to believe that the ſatellites of *Jupiter* and *Saturn* revolve about their axes; for the ſatellites of the former appear at different times to be of very different magnitudes and brightneſs. The fifth ſatellite of Saturn was obſerved by M. CASSINI for ſeveral years as it went through the eaſtern part of it's orbit to appear leſs and leſs, till it became inviſible; and in the weſtern part to increaſe again. Theſe phænomena can hardly be accounted for, but by ſuppoſing ſome parts of the ſurfaces to be unfit to reflect light, and therefore when ſuch parts are turned towards the earth, they appear to grow leſs, or to diſappear. As the ſame appearances of this ſatellite returned again when it came to the ſame part of it's orbit, it affords an argument that the time of the rotation about it's axis is equal to the time of it's revolution about it's primary, a circumſtance ſimilar to the caſe of the moon and earth. See Dr. HERSCHEL's account of this in the *Phil. Tranſ.* 1792. The appearance of this ſatellite of Saturn is not always the ſame, and therefore it is probable that the dark parts are not permanent.

CHAP.

CHAP. XX.

ON THE SATELLITES.

Art. 407. ON January 8, 1610, GALILEO diſcovered the four ſatellites of *Jupiter*, and called them *Medicea Sidera*, or *Medicean Stars*, in honor of the family of the MEDICI, his patrons. This was a diſcovery, very important in it's conſequences, as it furniſhed a ready method of finding the longitudes of places, by means of their eclipſes; the eclipſes led M. ROEMER to the diſcovery of the progreſſive motion of light; and hence Dr. BRADLEY was enabled to ſolve an apparent motion in the fixed ſtars, which could not otherwiſe have been accounted for.

408. The ſatellites of *Jupiter* in going from the weſt to the eaſt are eclipſed by the ſhadow of Jupiter, and as they go from eaſt to weſt are obſerved to paſs over it's diſc; hence they revolve about Jupiter, and in the ſame direction as Jupiter revolves about the ſun. The three firſt ſatellites are always eclipſed, when they are in oppoſition to the ſun, and the lengths of the eclipſes are found to be different at different times; but ſometimes the fourth ſatellite paſſes through oppoſition without being eclipſed. Hence it appears, that the planes of the orbits do not coincide with the plane of Jupiter's orbit, for in that caſe, they would always paſs through the center of Jupiter's ſhadow, and there would always be an eclipſe, and of the ſame, or very nearly the ſame duration, at every oppoſition to the ſun. As the planes of the orbits which they deſcribe ſometimes paſs through the eye, they will then appear to deſcribe ſtraight lines paſſing through the center of Jupiter; but at all other times they will appear to deſcribe ellipſes, of which Jupiter is the center.

On the Periodic Times, and Diſtances of Jupiter's Satellites.

409. To get the times of their mean *ſynodic* revolutions, or of their revolutions in reſpect to the ſun, obſerve, when Jupiter is in oppoſition, the paſſage of a ſatellite over the body of Jupiter, and note the time when it appears to be exactly in conjunction with the center of Jupiter, and that will be the time of conjunction with the ſun. After a conſiderable interval of time, repeat the ſame obſervation, Jupiter being in oppoſition, and divide the interval of time by

by the number of conjunctions with the ſun in that interval, and you get the time of a *ſynodic* revolution of the ſatellite. This is the revolution which we have occaſion principally to conſider, it being that on which the eclipſes depend. But owing to the equation of Jupiter's orbit, this will not give the *mean* time of a ſynodic revolution, unleſs Jupiter was at the ſame point of it's orbit at both obſervations; if not, we muſt proceed thus.

FIG. 94. 410. Let *AIPR* be the orbit of Jupiter, *S* the ſun in one focus, and *F* the other focus; and as the excentricity of the orbit is ſmall, the motion may be conſidered (227) as uniform about *F*. Let Jupiter be in it's aphelion at *A* in oppoſition to the earth at ♈, and *L* a ſatellite in conjunction; and let *I* be the place of Jupiter at it's next oppoſition with the earth at *D*, and the ſatellite in conjunction at *G*. Then if the ſatellite had been at *O*, it would have been in conjunction with *F*, or in mean conjunction; therefore it wants the angle *FIS* of being come to the mean conjunction, which angle is (227) the equation of the orbit according to the *ſimple elliptic hypotheſis*, which may be here uſed, as the excentricity of the orbit is but ſmall; the angle *FIS* therefore meaſures the difference between the mean *ſynodic* revolutions in reſpect to *F*, and the ſynodic revolutions in reſpect to the ſun *S*. If therefore *n* be the number of revolutions which the ſatellite has made in reſpect to the ſun, $n \times 360° - SIF =$ the revolutions in reſpect to *F*; hence $n \times 360° - SIF : 360°$ :: the time between the two oppoſitions : the time of a mean ſynodic revolution about the ſun.

411. As the ſatellite is at *O* at the mean conjunction, and at *G* when in conjunction with the ſun, it is manifeſt, that if the angle *FIS* continued the ſame, the time of a revolution in reſpect to *S* would be equal to the time in reſpect to *F*, or to the time of a mean ſynodic revolution; hence the difference between the times of any two ſucceſſive revolutions in reſpect to *S* and *F* reſpectively is as the variation of the angle *FIS*, or variation of the equation of the orbit. When Jupiter is at *A* the equation vaniſhes, and the times of the two conjunctions at *F* and *S* coincide. When Jupiter comes to *I*, the mean conjunction at *O* happens after the true conjunction at *G*, by the time of deſcribing the angle *SIF*, the equation of Jupiter's orbit. This is the *firſt* inequality, and has for it's argument a number called *A*, which is the mean anomaly of Jupiter, calculated to hundredths of a degree. By this inequality of the intervals of the conjunctions, the returns of the eclipſes are affected.

412. But as it may not often happen that there will be a conjunction of the ſatellite exactly at the time when Jupiter is in oppoſition, the time of a mean revolution may be found, out of oppoſition, thus. Let *H* be the earth when the ſatellite is at *Z* in conjunction with Jupiter at *R*; and let *V* be another poſition of the earth when the ſatellite is at *C* in conjunction with Jupiter at *I*;

and

and produce *RH*, *IV* to meet in *M*; then the motion of Jupiter about the earth in this interval is the ſame as if the earth had been fixed at *M*. Now the difference between the true and mean motions of Jupiter is $RFI - RMI = FIM + FRM$, which ſhows how much the number of mean revolutions in reſpect to *F* exceeds the ſame number of apparent revolutions in reſpect to the earth; hence $n \times 360^\circ - MIF - MRF : 360^\circ ::$ the time between the obſervations : the time of a mean ſynodic revolution of the ſatellite. If *C* and *Z* lie on the other ſide of *O* and ♈, the angles *MIF*, *MRF* muſt be added to $n \times 360^\circ$; and if one lie on one ſide and the other on the other, one muſt be added and the other ſubtracted, according to the circumſtances.

413. As it is difficult, from the great brightneſs of Jupiter, to determine accurately the time when the ſatellite is in conjunction with the center of Jupiter as it paſſes over it's diſc, the time of conjunction is determined by obſerving it's entrance upon the diſc, and it's going off; but as this cannot be determined with ſo much accuracy as the time of immerſion into the ſhadow of Jupiter, and emerſion from it, the time of conjunction can be moſt accurately determined from the eclipſes.

414. Let *I* be the center of Jupiter's ſhadow *FG*, *Nmt* the orbit of a ſatellite, *N* the node of the ſatellite upon the orbit of Jupiter; draw *Iv* perpendicular to *IN*, and *Ic* to *Nt*; and when the ſatellite comes to *v* it is in conjunction* with the ſun. Now both the immerſion at *m* and emerſion at *t* of of the ſecond, third, and fourth ſatellites may ſometimes be obſerved, the middle point of time between which gives the time of the middle of the eccliſpe at *c*, and by calculating *cv*, from knowing the angle *N* and *NI*, we get the time of conjunction at *v*. If both the immerſion and emerſion cannot be obſerved, take the time of either, and after a very long interval of time, when an eclipſe happens as nearly as poſſible in the ſame ſituation in reſpect to the node, take the time of the ſame phænomenon, and from the interval of theſe times you will get the time of a revolution. By theſe different methods, M. CASSINI found the times of the mean *ſynodic* revolutions of the four ſatellites to be as follows; FIG. 95.

Firſt	Second	Third	Fourth
$1^d. 18^h. 28'. 36''$	$3^d. 13^h. 17'. 54''$	$7^d. 3^h. 59'. 36''$	$16^d. 18^h. 5'. 7''$

* A Satellite is ſaid to be in conjunction, both when it is between the Sun and Jupiter, and when it is oppoſite to the Sun; the latter may be called *Superior*, and the former *Inferior*.

415. Hence

415. Hence it appears, that 247 revolutions of the firſt ſatellite are performed in 437*d*. 3*h*. 44′; 123 revolutions of the ſecond, in 437*d*. 3*h*. 41′; 61 revolutions of the third, in 437*d*. 3*h*. 35′, and 26 revolutions of the fourth, in 435*d*. 14*h*. 13′. Therefore after an interval of 437 days, the three firſt ſatellites return to their relative ſituation within nine minutes.

416. In the return of the ſatellites to their mean conjunction, they deſcribe a revolution in their orbits together with the mean angle a° deſcribed by Jupiter in that time; therefore to get the *periodic* time of each, we muſt ſay, $360^\circ + a^\circ$: 360° :: time of a ſynodic revolution : the time of a periodic revolution; hence the *periodic* times of each are;

Firſt	Second	Third	Fourth
$1^d.\ 18^h.\ 27'.\ 33''$	$3^d.\ 13^h.\ 13'.\ 42''$	$7^d.\ 3^h.\ 42'.\ 33''$	$16^d.\ 16^h.\ 32'.\ 8''$

417. The diſtances of the ſatellites from the center of Jupiter may be found at the time of their greateſt elongations, by meaſuring, with a micrometer, at that time, their diſtances from the center of Jupiter, and alſo the diameter of Jupiter, by which you get their diſtances in terms of the diameter. Or it may be done thus. When a ſatellite paſſes over the middle of the diſc of Jupiter, obſerve the whole time of it's paſſage, and then, the time of a revolution : the time of it's paſſage over the diſc :: 360° : the arc of it's orbit correſponding to the time of it's paſſage over the diſc; hence, the ſine of half that arc : radius :: the ſemidiameter of Jupiter : the diſtance of the ſatellite. Thus M. Cassini determined their diſtances in terms of the ſemidiameter of Jupiter to be, of the *firſt* 5,67, of the *ſecond* 9, of the *third* 14,38, and of the *fourth* 25,3.

418. Or having determined the periodic times and the diſtance of one ſatellite, the diſtances of the other may be found from the proportion of the ſquares of the periodic times being as the cubes of their diſtance. Dr. Pound, with a teleſcope 15 feet long, found, at the mean diſtance of Jupiter from the earth, the greateſt diſtance of the *fourth* ſatellite to be 8′. 16″; and by a teleſcope 123 feet long he found the greateſt diſtance of the *third* to be 4′. 42″; hence the greateſt diſtance of the *ſecond* appears to be 2′. 56″. 47‴, and of the *firſt* 1′. 51″. 6‴. Now the diameter of Jupiter, at it's mean diſtance, was determined, by Sir I. Newton, to be $37''\frac{1}{4}$; hence the diſtances of the ſatellites, in terms of the ſemidiameter of Jupiter, come out 5,965; 9,494; 15,141, and 26,63 reſpectively. *Prin. Math. Lib. ter. Phæn.*

419. Hence

418. Hence by knowing the greateſt elongations of the ſatellites in minutes and ſeconds, we get their diſtances from the center of Jupiter compared with the mean diſtance of Jupiter from the earth, by ſaying, the ſine of the greateſt elongation of the ſatellite : radius :: the diſtance of the ſatellite from Jupiter : the mean diſtance of Jupiter from the earth.

On the Equations of Jupiter's Satellites.

419. The conjunction of the ſatellites with the ſun, and their eclipſes, cannot (411) return at equal intervals of time, on account of the unequal motion of Jupiter, which conſtitutes the greateſt inequality; becauſe theſe intervals are equal to a revolution in their orbits increaſed by the time of deſcribing an angle equal to that which Jupiter has deſcribed in theſe intervals, which angle is variable. The true conjunctions compared with the mean may therefore vary by twice the greateſt equation of Jupiter's orbit, or by 11°. 8′. 2″ according to M. WARGENTIN; becauſe Jupiter in one part of it's orbit will be 5°. 34′. 1″ behind it's mean place, and in another part 5°. 34′. 1″ before it. To find this inequality, or equation, in time, ſay, 360° : 5°. 34′. 1″ :: a ſynodic revolution : the equation anſwering to the greateſt equation of Jupiter's orbit, which is found to be 39′. 22″, 1*h*. 19′. 13″, 2*h*. 39′. 42″, and 6*h*. 12′. 59″ for the firſt, ſecond, third and fourth ſatellite reſpectively. This equation depending on Jupiter's anomaly, has (411) for it's argument *A* the mean anomaly of Jupiter. But as the excentricity, and conſequently the greateſt equation of Jupiter's orbit, is ſubject to a change, this equation muſt alſo be variable M. CASSINI firſt employed this equation in calculating the eclipſes.

420. Another equation ariſes from the progreſſive motion of light. When the earth is at *T* and Jupiter in oppoſition at *A*, the eclipſe begins ſooner by 16′. 15″ than when the earth is at *N*, and Jupiter at *A*, light taking that time to move over the diameter of the earth's orbit*. If therefore we ſuppoſe Jupiter to revolve about the ſun in a circle at it's mean diſtance, FIG. 94.

* M. CASSINI firſt ſuſpected that light was progreſſive, from obſerving that the immerſions of the *firſt* ſatellite, as they are obſerved from the conjunction of Jupiter to it's oppoſition, took place ſooner and ſooner in reſpect to the computed time; and that the emerſions, as they are obſerved from oppoſition to conjunction, took place later and later. But he perceived that if he admitted this for the firſt ſatellite, it muſt be admitted for the three others, which did not appear to him to require this equation; he therefore gave up the idea. M. ROEMER did not think that M. CASSINI's objection to the progreſſive motion of light was well founded; he therefore adopted the idea, and eſtabliſhed the fact. Dr. HALLEY obſerved, that it was neceſſary to allow for the motion of light in the other ſatellites.

and v and w be the places of the earth when at it's mean diſtance from Jupiter, whilſt the earth is in the part vNw of it's orbit, the light from the ſatellite comes *later* to the earth, than when at it's mean diſtance; and when the earth is in the part wTv, the light comes *ſooner*; conſequently the eclipſe happens *later* in the *former* caſe, and *ſooner* in the *latter*, than it would, if the earth were at it's mean diſtance. This difference conſtitutes the *firſt* and greateſt equation of light; it is nothing when Jupiter is at it's mean diſtance from the earth, and is at it's maximum when Jupiter is in conjunction and oppoſition, at which time it's quantity is half $16'.\ 15''$, or $8'.\ 7'',5$. This equation is ſubtractive in vNw, and additive in wTv; and has for it's argument, the elongation of Jupiter from the ſun. But Jupiter does not move in a circular orbit; and if A be the apogee and P the perigee, the difference between AS and PS is ſuch that light moves through PS in $4'.\ 5''$ ſooner than it does through AS. Now this equation beginning when Jupiter is at it's mean diſtance, the half of $4'.\ 5''$, or $2'.\ 2'',5$, is the greateſt equation ariſing from this cauſe, the excentricity of the orbit. Hence the argument for this equation is the anomaly of Jupiter. This equation is *additive* when Jupiter is at a *leſs* than it's mean diſtance, and *ſubtractive*, when at a *greater*. This is the *ſecond* equation of light. Theſe three equations, that is, the equation of Jupiter's orbit (419) and the two equations of light, are manifeſtly common to all the ſatellites, the apparent times at which the eclipſes of all the ſatellites happen, being equally affected by them. But beſides theſe equations, there are others which belong to each, the manner of determining which has generally been, to compare a great number of obſervations with the calculations, after taking into conſideration the preceding equations, and the difference between ſuch computations and the obſervations muſt give the equation required. Such an equation however may be the reſult of ſeveral inequalities, in which caſe it muſt be ſeparated into ſeveral equations; and by trying one ſet of equations after another, and by increaſing ſome and diminiſhing others, or adding new ones, Aſtronomers have made their Tables agree very well with obſervations. Equations thus introduced, are called *Empyric*. And this is the only way, where there is not proper *data* to compute their value from theory, or to ſeparate them by. The uncertainty of the quantity of matter in the ſatellites, renders the theory, in eſtimating the effects of the diſturbing forces upon each other, ſubject to the ſame degree of uncertainty.

421. M. BAILLY, in his *Eſſai ſur la Theorie des Satellites de Jupiter*, has ſhown, that the inequalities of the *firſt* ſatellite ariſe from the attraction of the *ſecond*, which produces an equation of about $3'.\ 30''$ in time, or of $29'.\ 30''$ on the orbit, as was found by M. WARGENTIN. In the year 1719, Dr. BRADLEY found that in the years 1682, 1695, 1718, the eclipſe of the firſt ſatellite laſted about

2*h*. 20′; but at the other node in 1677 and 1689, the duration was only 2*h*. 14′; this appeared to indicate, that the motion of the ſatellite was not uniform, and conſequently that the orbit might be excentric; he nevertheleſs ſuſpected that it aroſe from the attraction of the ſecond, as the reader may ſee in the *Phil. Tranſ.* 1726. M. WARGENTIN's Tables, which agree very well with obſervations, contain this equation. M. BAILLY and M. de la GRANGE examined this matter very fully, and found that all the irregularities of the firſt ſatellite aroſe from the attraction of the ſecond, and produced an effect of about 3′. 30″ in time. This equation is as the ſine of the diſtance from the point where it is nothing.

422. The *ſecond* ſatellite is ſubject to the greateſt irregularities. It appears from obſervation, that the equation amounts to about 16′½ in time, of which the period is 437 days, which indicates that it is produced by the attractions of the firſt and third, for in that time the three firſt ſatellites return to the ſame ſituation in reſpect to Jupiter. M. BAILLY ſuſpected an excentricity of the orbit, and a motion of it's apſides; but this he ſpeaks of as a circumſtance very doubtful.

423. The *third* ſatellite has it's motion diſturbed by the firſt, ſecond and fourth; the whole effect of theſe, according to M. BAILLY, produces an equation of 16′. 11″ of a degree. M. WARGENTIN makes it, from obſervation, to be 16′. 46″ in the Tables publiſhed in 1759; but in the laſt edition of his Tables, he has employed three equations; one about 2′½ of time, of which the period is 437 days, which he determined from obſervation; the other two are 4′½ and 2′½ of time, and which he determined alſo from obſervation; the periods of theſe are about 12½ and 14 years. Perhaps, ſays he, the variation of the excentricity of the orbit is ſubject to ſome change, which may produce the two laſt equations. He afterwards doubted, whether it would not be better to ſubſtitute one equation inſtead of theſe two. M. de la LANDE ſays, that the third equation may be ſuppreſſed, and the computations will then not deviate much from obſervation. M. MARALDI ſuſpected that this ſatellite had an equation of it's center, and that the annual motion of it's apſide was 1°. 30′. M. BAILLY having calculated a great number of obſervations, and compared them with his Theory, after allowing for all the other equations, found it neceſſary to aſſume 10′ for the equation of it's center; he alſo found it neceſſary to give a motion to the apſides of about 2° in a year; but this motion appeared to him to be rather too ſmall to ſatisfy the obſervations. According to his Theory, the motion of the apſides is 2°. 12′. 3″, from the diſturbing force of the ſun, and the action of the other ſatellites, without taking into conſideration the figure of Jupiter, which will alſo cauſe a motion of the apſides. He joined to the equation of the center, five other equations; the

firſt of 25″ from the action of the firſt ſatellite; the ſecond of 4′. 10″ from the action of the ſecond; the third of 1′. 19″ from the action of the ſecond, on account of the excentricity of the orbit; and laſtly, two others of 17″ and 59″ from the action of the fourth. Theſe equations, M. BAILLY ſays, may in certain caſes go as far as 16′. 11″, which is very nearly 16′. 46″, the value of the total equation which had been before determined by obſervation.

424. Dr. BRADLEY found by obſervation, that the orbit of the *fourth* ſatellite was elliptical, and made the greateſt equation 0°. 48′. Before this was publiſhed, M. MARALDI had obſerved, that M. CASSINI's Tables erred nearly two hours, and always the ſame way, when Jupiter returned to the ſame point of it's orbit; and that the error was nothing, when Jupiter was at it's mean diſtance. This might evidently ariſe from the excentricity of the orbit; for as Jupiter revolved about the ſun and carried the orbit of it's ſatellite with it, in one revolution of Jupiter, the apſides of the orbit of the ſatellite would have had every poſition in reſpect to the ſun; ſo that the ſatellite would ſometimes come into oppoſition to the ſun when it was in it's lower apſide, where it's motion was greateſt, and therefore the eclipſe would happen ſooner than if it's motion was uniform; ſometimes the eclipſe would happen when the ſatellite was in it's higher apſide, and then it's motion being ſloweſt, the eclipſe would happen later; ſometimes the eclipſe would happen when the ſatellite was at it's mean diſtance, and then the true motion being equal to it's mean, the time of the eclipſe would happen at the time by computation according to it's mean motion. From a compariſon of the true and mean place of the ſatellite in it's orbit, M. MARALDI found the equation of the center to be 55′. 56″. According to the Tables of M. WARGENTIN, this equation amounts to 1*h*. 0′. 30″. The attractions of the other ſatellites do not ſenſibly affect it's motion; but M. BAILLY found two or three ſmall inequalities ariſing from the action of the ſun; he fixed the equation of the center at 50′. 20″, and the motion of the apſides 45′. 18″ in a year. In the year 1717, Dr. BRADLEY found the place of the apſide to be 11ˢ. 8°; but the obſervations in 1671, 1676 and 1677, require the place in 1677 to be 10ˢ. 14°; hence he fixed the motion at about 36′ in a year; and found this to agree very well with obſervations. M. MARALDI made the motion of the apſides 44′. 15″ in a year; and the place of the apſide 10ˢ. 29°. 22′ for the beginning of 1700; and the mean longitude at that time 7ˢ. 17°. 18′. 2″. Upon this hypotheſis, he computed 152 obſervations, of which not above 30 differed more than $5\frac{1}{2}'$ minutes from obſervations, amongſt which, 4 only differed 10′, and only 3 differed 13′. This was nearer than could have been expected, conſidering that the diſturbing force of *Saturn* was not conſidered. The motion of the apſides ariſes partly from the attraction of the ſun, and partly from the figure of the body of Jupiter. But

it

it being uncertain whether Jupiter be homogeneous, or what is the accurate ratio of it's diameters, the part which ariſes from the figure of the planet muſt be very uncertain. M. de la PLACE found an equation of 1′. 54″ of a degree, which depends on the action of the ſun and on the diſtance of Jupiter from it's aphelion; this is ſimilar to the annual equation of the moon; and another of about 28″, which anſwers to the evection of the moon.

425. M. MARALDI found the excentricity of the orbit, in the manner deſcribed in Article 340. In the conjunction on April 6, 1708, he found the place of the ſatellite on it's orbit to be 5ˢ. 27°. 55′. 26″, and on March 3, 1753, to be 3ˢ. 15°. 51′. 7″; hence the true motion was 9ˢ. 17°. 55′. 41″; but the mean motion in the ſame time was 9ˢ. 19°. 13′. 5″, or 1°. 17′. 24″ greater. Between the obſervation in 1708, and one on Auguſt 4, 1759, he found the true motion greateſt by 34′. 28″; hence half the ſum of 1°.17′. 24″ and 34′. 28″, or 55′. 56″, is the greateſt equation of the orbit.

426. The reduction of the orbit of a ſatellite to the orbit of Jupiter, furniſhes another equation. Let I be the center of the ſhadow of Jupiter, Nt the orbit of the ſatellite, draw Iv perpendicular to NI the plane of Jupiter's orbit, and Ic perpendicular to Nt, and take $Na = NI$. The point a is here called the conjunction of the ſatellite, that point upon the orbit having (268) the ſame longitude as the point I, or Jupiter; at c is the middle of the eclipſe, and ac is called the *Reduction*; when the ſatellite is at v it is in conjunction in reſpect to the orbit of Jupiter. The reduction is *ſubtractive* when the argument of latitude is between 0° and 90°, and between 180° and 270°; and *additive* for the other two quadrants. FIG. 95.

M. de la LANDE, in the laſt edition of his Aſtronomy, has given new Tables of Jupiter's ſatellites, computed by M. de LAMBRE, from the theory of their mutual attractions, given by M. de la PLACE, in the *Mem. de l'Acad.* 1784, 1788; the theory gave the form of the equations; the values of the coefficients were determined from obſervation. He alſo introduced the effect ariſing from the diſturbing force of Jupiter. In theſe Tables there are no empyric equations, and M. de la LANDE ſays they give the times of the eclipſes to a degree of accuracy, beyond what could be expected.

On the Eclipſes of Jupiter's Satellites.

427. Let S be the ſun, EF the orbit of the earth, I Jupiter, abc the orbit of one of it's ſatellites. When the earth is at E before the oppoſition of Jupiter, the ſpectator will ſee the immerſion at a; but if it be the firſt ſatellite, upon account of it's nearneſs to Jupiter the emerſion is never viſible, the ſatellite FIG. 96.

tellite being then always behind the body of Jupiter; the other three ſatellites *may* have both their immerſions and emerſions viſible; but this will depend upon the poſition of the earth. When the earth comes to *F* after oppoſition, we ſhall then ſee the emerſion of the firſt, but can never ſee the immerſion, and *may* ſee both the emerſion and immerſion of the other three. Draw *EIr*; then *sr*, the diſtance of the center of the ſhadow from the center of Jupiter referred to the orbit of the ſatellite, is meaſured at Jupiter by *sr*, or the angle $sIr = EIS$ the annual parallax. The ſatellite may be hidden behind the body at *r* without being eclipſed, which is called an *Occultation*. When the earth is at *E*, the conjunction of the ſatellite happens *later* at the earth than at the ſun; but when the earth is at *F*, it happens *ſooner*.

428. The diameter of the ſhadow of Jupiter at the diſtance of any of the ſatellites, is beſt found by obſerving the time of an eclipſe when it happens at the node, at which time the ſatellite paſſes through the center of the ſhadow; for the time of a ſynodic revolution : the time the ſatellite is paſſing through the center of the ſhadow :: 360° : the diameter of the ſhadow in degrees. But when the firſt and ſecond ſatellites are in the nodes, the immerſion and emerſion cannot both be ſeen. Aſtronomers therefore compare the immerſions ſome days *before* the oppoſition of Jupiter with the emerſions ſome days *after*, and then knowing how many ſynodic revolutions have been made, they get the time of the tranſit through the ſhadow, and thence the degrees correſponding. But on account of the excentricity of ſome of the orbits, the time of the central tranſit muſt vary: for example, the ſecond ſatellite is ſometimes found to be 2*h*. 50′ in paſſing through the center of the ſhadow, and ſometimes 2*h*. 54′; this indicates an excentricity.

429. The duration of the eclipſes being very unequal, ſhows that the orbits are inclined to the orbit of Jupiter; ſometimes the fourth ſatellite paſſes through oppoſition without ſuffering an eclipſe. The duration of the eclipſes muſt therefore depend upon the ſituation of the nodes in reſpect to the ſun, juſt the ſame as in a lunar eclipſe; when the line of the nodes paſſes through the ſun, the ſatellite will paſs through the center of the ſhadow; but as Jupiter revolves about the ſun, the line of the nodes will be carried out of conjunction with the ſun, and the time of the eclipſe will be ſhortened, as the ſatellite will then deſcribe only a chord of a ſection of the ſhadow inſtead of the diameter.

FIG. 97. 430. Let *S* be the ſun, *I* Jupiter, *Nbnv* the plane of Jupiter's orbit, *Ncan* the orbit of one of it's ſatellites, *Nn* the line of the nodes; draw *Ia*, *Ib* perpendicular to *Nn*, and *ab* perpendicular to the plane *Nbnv*; and let *c* be the point in oppoſition to the ſun, and draw *cd* perpendicular to *Nbnv*. Now the angle *aIb* is the inclination of the orbit of the ſatellite, whoſe ſine we will call *s*, to radius unity, and put $r = Ia$; then $1 : s :: r : ab = sr$; and if $v =$ the ſine

ſine of Nc the diſtance of the node from oppoſition, $1 : sr :: v : cd = vsr$ the latitude of the ſatellite at the time of oppoſition. Let $AFBG$ be a ſection of the ſhadow of Jupiter where the ſatellite paſſes through, $NAIB$ the plane of the orbit of Jupiter, Nmt the orbit of the ſatellite, and draw Ic perpendicular to Nt; then $Ic = vsr$; put $R = IA$, $d = mc$; then $\sqrt{R^2 - d^2} = vsr$; hence, $s = \frac{\sqrt{R^2 - d^2}}{vr}$. But R, r, and d may be taken in time, that is, d may repreſent the half duration of the eclipſe; call that time d'; and R may repreſent half the greateſt duration; call this R'. And to find the time the ſatellite is in paſſing through a ſpace equal to r, put t = an arc of 57°. 17′. 45″, which is equal in length to radius; hence, 360° : 57°. 17′. 45″ :: the time of a ſynodic revolution : the time r' of deſcribing a ſpace equal to r; hence, $s = \frac{\sqrt{R'^2 - d'^2}}{vr'}$. If therefore the ſemiduration be given, and the place of the node, the inclination of the orbit will be known; and if the inclination be given, we have $d' = \sqrt{R'^2 - v^2 s^2 r'^2}$ the half duration. This will be a little affected by the diſturbing forces of the ſatellites, and the excentricity of the orbits. M. BAILLY eſtimates what this diſturbing force is; but as it depends upon the quantity of matter in the ſatellites, which cannot be determined to a great degree of accuracy, any correction of that kind muſt be ſubject to a proportional degree of error.

Ex. On November 19, 1761, at 6 o'clock, the inclination of the orbit of the fourth ſatellite was 2°. 36′, and the diſtance of the node from Jupiter 46°. 43′; alſo, the greateſt duration was 2h. 23′, according to M. de la LANDE. Hence, $r' = 2h.\ 23'$, $s = ,04536$, $v = ,72797$; therefore $d' = 1h.\ 6'.\ 6''$ the half duration. When $Ic = IA$, or $vsr' = R'$, the ſatellite will not enter the ſhadow, but juſt touch it; hence, $v = \frac{R'}{sr'}$. Now by the Table, page 268, it appears that R' may be repreſented by 2°. 8′. 2″, r' being repreſented by 57°. 17′. 45″. Hence, $v = ,8209$ the ſine of 55°. 11′, within which diſtance muſt the node be from conjunction, in order that there may be an eclipſe.

431. Draw Iv perpendicular to BN; then in the right angled triangle Icv, if we know Ic and the angle vIc (the complement of cIN), we ſhall know cv the diſtance from the middle of the eclipſe to the conjunction of the ſatellite. The ſuppoſition that mt is a ſtraight line, produces no error of any conſequence.

432. Hitherto we have ſuppoſed the ſection of the ſhadow of Jupiter to be a circle; but as Jupiter is a ſpheroid, and not a ſphere, and the plane of it's

it's equator very nearly coincides with it's orbit, we ſhould conſider the ſection of the ſhadow as an ellipſe and not a circle, the major axis of which is nearly coincident with the orbit. M. de la LANDE therefore propoſes the following correction. Let $AFBG$ be the ſection, ſuppoſed to be a circle, $AxBz$ the elliptical ſection of the ſhadow, and draw nm parallel, and nc', mc perpendicular to Ix. Let nc' be half the duration; then, upon ſuppoſition that the ſection was circular, the ſame half duration would be repreſented by mc; ſo that the diſtance Ic before computed : the true diſtance Ic' :: (by the property of the ellipſe) $IF : Ix$; hence, $Ic' = \frac{Ix}{IF} \times Ic = \frac{Ix}{IF} \times \sqrt{R^2 - d^2}$; conſequently $\frac{Ix}{IF} \times \sqrt{R^2 - d^2} = vsr$, therefore $s = \frac{Ix}{IF} \times \frac{\sqrt{R'^2 - d'^2}}{vr'}$, R, r and d being expreſſed by R', r' and d' in time. M. de la LANDE puts $Ix : IF :: 13 : 14$, and therefore $s = \frac{13}{14} \times \frac{\sqrt{R'^2 - d'^2}}{vr'}$. To find the inclination of the orbit of the fourth ſatellite upon this ſuppoſition, M. WARGENTIN ſuppoſed the limit of the diſtance of the node from conjunction to be 55°. 11′. 10″; and upon ſuppoſition of a circular ſection, he found the inclination to be 2°. 36′; hence, by diminiſhing the ſine of the inclination in the ratio of 14 : 13, he found the true inclination of the orbit to be 2°. 24′. 51″.

FIG. 98.

433. The orbit of the ſecond ſatellite is found to change it's inclination, the period of which change is 30 years. M. MARALDI found the leaſt inclination at the beginning of the years 1672, 1702, 1732 and 1762 to be 2°. 48′; and at the beginning of the years 1687, 1717, 1747 and 1772 he found the greateſt inclination to be 3°. 48′. The inclination of the orbit of the firſt ſatellite, upon which he made the motion of the node of the ſecond depend, is 3°. 18′, calculated for a circular ſection, which is a mean between the greateſt and leaſt inclinations of the orbit of the ſecond. This determination of M. MARALDI, combined with the libration of the node, made his calculation of the eclipſes agree very well with obſervations; for of 122 which he calculated, only 12 differed more than 1 minute. According to the new Tables of M. WARGENTIN, the leaſt inclination is 2°. 46′ and the greateſt 3°. 46′, upon ſuppoſition that the ſection of the ſhadow is a circle.

434. This variation of the inclination of the orbit of the ſecond ſatellite ariſes from the libration of it's nodes. M. MARALDI, by an obſervation on Oct. 18, 1714, found the place of the node to be 10°. 21°. 21′. 45″; and by an obſervation on September 11, 1751, he found that place of the node to be 10°. 0°. 54′. 9″, the difference of which is 20°. 27′. 36″ for the whole libration of the node, ſuppoſing that theſe were the extreme points; hence, it's half, 10°. 13′. 48″

13′. 48″ ſhows the libration from the mean place, which therefore is 10°. 11°. 8′. M. WARGENTIN makes it 10°. 12°. 15′. M. de la LANDE firſt pointed out this libration of the nodes, and the conſequent change of the inclinations of the orbits. In conſequence of this, M. BAILLY propoſed to explain this motion of the nodes and variation of the inclination, in the following manner, ſimilar to that by which M. de la LANDE explained the changes of the inclinations of the orbits of the planets.

435. Let AC be the orbit of Jupiter, CB the orbit of the ſatellite which is diſturbed by the motion of another ſatellite moving in the orbit BA, ſo that we may ſuppoſe the orbit BC firſt to have been in the ſituation AB'; the angle B is the mutual inclination of the two orbits, which is ſuppoſed to be conſtant; let AB be the movement of the node of the orbit CB which is diſturbed, upon the other orbit AB, in any given time; then AC is the motion of the node upon the orbit of Jupiter. By Trigonometry, tan. $AC = \frac{\text{tan. } B \times \text{ſin. } AB}{\text{cos. } AB \times \text{cos. } A \times \text{tan. } B + \text{ſin. } A}$, and cos. $C = -\text{cos. } B \times \overline{\text{cos. } A - \text{ſin. } A \times \text{tan. } B \times \text{cos. } AB}$. Now to determine when AC becomes a maximum, put $y = \text{tan. } AC$, $x = \text{ſin. } AB$, $a = \text{tan. } B$, $b = \text{cos. } A$, $m = \text{ſin. } BAC$; then $y = \frac{ax}{ab\sqrt{1-x^2}+m}$ = a maximum, whoſe fluxion being put = o and reduced gives $x = \sqrt{1-\frac{a^2b^2}{m^2}} = \sqrt{1-\frac{\text{tan. } B^2}{\text{tan. } A^2}}$, the ſine of AB, when AC is a maximum, where AB is greater than 90°, for from the tan. of AC, it appears that AC increaſes till AB is greater than 90°. The motion of the node of the ſecond ſatellite upon the orbit of the firſt is found by obſervation to be about 12° in a year, and therefore it completes it's revolution in 30 years; hence at the end of 30 years, the node of BC upon the orbit of Jupiter will return to the ſame ſituation, and to the ſame inclination. Hence the node C has a movement of libration about A; if b be the utmoſt limit of the node of BC from A on one ſide, and a on the other, the node will librate between a and b.

436. The two inclinations A and C are not equal at the limits a and b; for as cos. $C = \text{cos. } B \times \overline{\text{cos. } A - \text{ſin. } A \times \text{tan. } B \times \text{cos. } AB}$, therefore when the inclinations become equal, cos. $C = \text{cos. } B \times \overline{\text{cos. } C - \text{ſin. } C \times \text{tan. } B \times \text{cos. } AB}$, hence cos. $AB = \frac{\text{cos. } C \times \overline{\text{cos. } B - 1}}{\text{ſin. } C \times \text{tan. } B}$, which being negative, ſhows that AB is greater than 90°. Alſo ſin. $AB = \sqrt{1-\frac{\overline{\text{cos. } B - 1}^2}{\text{tan. } B^2 \times \text{tan. } C^2}}$; let us aſſume this $= \sqrt{1-\frac{\text{tan. } B^2}{\text{tan. } A^2}}$, which is the ſine of AB when AC is a maximum, and

(ſuppoſing $A = C$) we deduce $1 = \cos. B^2$, which is abſurd, conſequently the inclinations are not equal when IC is a maximum. Alſo, as $\sqrt{1 - \frac{\overline{\cos. B - 1}^2}{\tan. B^2 \times \tan. C^2}}$ is greater than $\sqrt{1 - \frac{\tan. B^2}{\tan. C^2}}$, and both greater than 90°, the two inclinations become equal before the node comes to it's limits.

437. From an eclipſe of the third ſatellite on January 25, 1763, the half duration of which was 43′, M. MARALDI found the inclination 3°. 25′. 41″, ſuppoſing the ſemidiameter of the ſhadow to be 1*h*. 47′. 10″, and to be circular. In 1745, it was found to be greater by $7\frac{1}{2}'$; but from 1763 it has appeared to decreaſe, for in 1769 it was found to be 3°. 23′. 33″. M. de la GRANGE judged the period of it's augmentation, to be 195 years; M. BAILLY made it 200 years. M. WARGENTIN made the leaſt inclination to be in 1697, and the greateſt in 1782. M. MARALDI made the period 132 years, finding the greateſt inclination in the years 1633, 1765 to be 3°. 25′. 57″; and the leaſt inclination 3°. 2′ in the year 1697. Upon this he computed the inclination for every intermediate time, with the libration of the node ariſing from the attraction of the firſt ſatellite. But ſome of his computations make the duration of the eclipſe err 6′, which renders his period very uncertain. M. de la LANDE has found the inclination of the third ſatellite by Art. 435. The annual motion of the node B of the third upon the orbit AB of the firſt was found to be 2°. 43′. 38″,2, and therefore it was 27°. 16′. 22″ between the obſervations made in 1763 and 1773, a period of 10 years; let $AB =$ 27°. 16′. 22″, the angle $A = 3°. 14'$, and the angle $B = 12'$; hence the angle $C = 3°. 24'. 44''$, the inclination of the orbit of the third ſatellite in 1773. Alſo $AC = 1°. 32'. 24''$, the libration in that interval.

438. The inclination of the orbit of the fourth ſatellite is 2°. 36′ according to M. MARALDI, with very little, if any, variation. Dr. BRADLEY made it 2°. 42′. M. WARGENTIN, in 1781, found an increaſe of 1′ or 2′ in the five laſt years, and he eſtimated it at 2°. 38′. M. de la LANDE makes it 2°. 36′ in a circular, and 2°. 24′. 51″ in an elliptical ſhadow. The motion of the nodes of this ſatellite, which is 4′. 19″ in a year according to M. WARGENTIN, ought to produce a change in the inclination, and M. BAILLY thought that in 1720 the inclination was a little diminiſhed, the nodes of the firſt and fourth ſatellites then coinciding. M. MARALDI could not reconcile the ſemiduration of the eclipſes with any variation of inclination, or motion of the node; yet in ſuppoſing the inclination to be conſtantly 2°. 36′, and the ſemidiameter of the ſhadow to be 2°. 8′. 2″, and the place of the node in 1745 to be 4*s*. 16°. 11′, with an annual progreſſive motion of 5′. 33″, his computations have agreed very well with obſervation.

439. M. de la PLACE

439. M. de la PLACE has endeavoured to ſhow that the nodes of the fourth ſatellite have a retrograde motion in a plane which paſſes between Jupiter's equator and orbit, inclined to the former at about half a degree. The plane of of the orbit of the fourth preſerves a conſtant inclination of 14′ or 15′, and a retrograde motion of the node of about 35′ in a year upon this plane. This theory will ſatisfy all the obſervations, and explain why the inclination is conſtant, and the motion of the nodes direct. This reſults from the action of the ſun and of the other ſatellites, and from the flatneſs of Jupiter.

440. The inclination of the fourth ſatellite being conſiderable, it may be found by finding the minor axis of the ellipſe which it appears to deſcribe when Jupiter is 90° from the node, which is done by obſerving it's apparent diſtance from Jupiter in it's conjunctions, which is the ſemi-minor axis, and the ſemi-major axis being the greateſt elongation, the latter is to the former as radius to the ſine of the inclination.

On the Nodes of the Orbits of Jupiter's Satellites.

441. The place of the node may be determined at the time of the greateſt duration of an eclipſe, for at that time the plane of the orbit of the ſatellite muſt paſs through the ſun, and therefore the place of Jupiter at that time gives the place of the node. Or the place of the node may be found by obſerving two eclipſes of the ſame duration on each ſide of the node, in which caſe the place of the node will biſect the two ſituations of Jupiter. This method ſuppoſes that Jupiter has moved uniformly in the intermediate time, and that the nodes of the ſatellite remained fixed. On March 12, 1687, FLAMSTEAD obſerved the duration of an eclipſe of the third ſatellite to be 2*h*. 33′, Jupiter's heliocentric longitude at that time being 8ˢ. 11°. 58′. On December 6, 1702, the duration was exactly the ſame, and the heliocentric longitude of Jupiter was 0ˢ. 15°. 21′; half the difference of theſe longitudes added to the firſt gives 10ˢ. 13°. 29′ for the place of the node nearly. Or the place of the node may be found when the ſatellite paſſes in a right line over the diſc of Jupiter, which may be obſerved by it's ſhadow upon Jupiter. This we may determine from the belts, as the motion of the ſatellites is very nearly in their direction.

442. In the year 1693, M. CASSINI, in his Aſtronomy, places the nodes of all the ſatellites in 10ˢ. 14°. 30′. Dr. BRADLEY thought the place of the nodes of them all in 1718 to be 10ˢ. 11°,5. From obſervations ſince, it appears that the nodes do not all coincide. The node of the *firſt* ſatellite is found to be in 10ˢ. 14°. 30′, and obſervations ſhow that it has no ſenſible motion.

443. According to M. WARGENTIN in his firſt Tables, the place of the node of the *ſecond* was 10ˢ. 11°. 48′, fixed; but in his new Tables, he gives it a progreſſive motion upon the orbit of Jupiter of 1°. 42′, in reſpect to the aphelion of Jupiter, in 100 years. M. BAILLY gives the node a libration of 9°. 21′. M. MARALDI makes it 8°. 42½′. M. de la GRANGE makes it 11°. 27′. The mean place of the aſcending node is 10ˢ. 13°. 52′ according to M. MARALDI. The nodes of the third and fourth ſatellite have a like libration about the nodes of the firſt, whilſt the nodes of the firſt have a libratory motion about a point as the mean place.

444. The mean place of the node of the *third* ſatellite is conſtantly in 10ˢ. 14°. 24′ according to M. WARGENTIN. M. MARALDI ſuppoſes it to have a motion of about 3′ in a year; and as we have ſeen (437) that the inclination is ſubject to a change, it may be neceſſary that the nodes ſhould have a motion to account for it.

445. From the theory of attraction, Dr. BRADLEY thought that the nodes of the *fourth* ſatellite ought to be retrograde; the motion would be retrograde from the attraction of the ſun only, but the attraction of the other ſatellites may make it direct; and obſervations ſhow that it is direct. According to M. MARALDI, it's place, in 1745, was 4ˢ. 16°. 11′, with an annual motion of 5′. 33″. M. BAILLY finds it 5′. 15″. M. WARGENTIN placed the node in 1760 in 10ˢ. 16°. 39′, and gave the node an annual motion of 3′. 18″ in reſpect to the aphelion of Jupiter, which gives 4′. 15″ in reſpect to the equinoxes.

446. M. BAILLY, from his Theory, deduces the following concluſions reſpecting the motion of the nodes.—1. The node of the firſt has a libratory motion about it's mean place of 18′, of which the period is 30 or 32 years.—2. The node of the ſecond librates about the ſame point, about 9°. 37′, with a period of 30 or 32 years.—3. The node of the third has a libratory motion about the ſame point, of about 3°. 53′, of which the period is about 200 years.—4. The node of the fourth librates about the ſame point, about 12° or 13°, with a period of 4 or 500 years.—5. This point, or the mean place of the node of the firſt ſatellite, about which the nodes of the other ſatellites librate, has a retrograde motion upon the orbit of Jupiter of 33′. 30″ in a year, from the diſturbing force of the ſun. M. BAILLY, from his Theory of the ſatellites, has computed a ſet of Tables of the motions of each.

447. The aſcending nodes of the orbits of all the ſatellites we may conſider in 10ˢ. 15°, in all caſes where great accuracy is not required. When Jupiter therefore is in 10ˢ. 15° and 4ˢ. 15°, the planes of the orbits paſs through the ſun, and to a ſpectator, there ſituated, the ſatellites would appear to deſcribe ſtraight lines, as *AB*, in the direction of the belts; in any other ſituation of Jupiter, they would appear to deſcribe an ellipſe *AmBn*. When Jupiter comes

to

to 1ˢ. 15° and 7ˢ. 15° the minor axis becomes the greateſt, and in that ſituation, the major axis : minor :: rad. : ſine of the inclination of the orbit of the planet; in any other ſituation of Jupiter, the minor axis is as the ſine of the diſtance of Jupiter from the node. As Jupiter paſſes from 10ˢ. 15° to 4ˢ. 15° the *furtheſt* ſemicircle of the orbit of the ſatellite appears moſt to the north, or, as we may expreſs it, higheſt, and therefore it will be repreſented by *AmB*; but as the planet paſſes from 4ˢ. 15° to 10ˢ. 15° the *neareſt* ſemicircle will appear higheſt, and therefore it will be repreſented by *AmB*. Hence we may judge of the ſituation of the ſatellites in reſpect to the poſition of the belts, or to the line *AB*; a circumſtance which we take into conſideration in the configuration of the ſatellites; we will explain this by the example there given. The longitude of Jupiter is 9ˢ. 23°. 5′, conſequently *AmB* repreſents the *neareſt* ſemicircle; hence *m* is the inferior conjunction and *n* the ſuperior, and therefore *n* has the ſame longitude ſeen from Jupiter, as Jupiter has from the ſun; that is, the longitude of the point *n* of the orbit of the ſatellite is 9ˢ. 23°, omitting the minutes; hence the longitude of *A* is 6ˢ. 23°, of *B* 0ˢ. 23°, and of *m* 3ˢ. 23°. Deſcribe the circle *Am′Bn′*; now the longitude of the firſt ſatellite is 2ˢ. 21°; hence ſet off $Ba = 2^s.\ 21° - 0^s.\ 23° = 1^s.\ 28°$, and draw *ars* perpendicular to *AB*, and *s* will be the place of the ſatellite, and *sr* the apparent elevation *above* the line of the belts; in like manner the ſituations of the others may be found. If four figures of a conſiderable ſize, and of the proper proportions, be thus deſcribed about *I*, and in each, *AB* : *mn* :: rad. : ſin. of the inclination of the orbit × ſine of the diſtance of Jupiter from the nodes, and the ellipſe be accurately drawn, and the orbits divided into every 5°, the ſituations of the ſatellites will, from inſpection, appear ſufficiently accurate. The ſpectator has here been ſuppoſed to be at the ſun; if he be at the earth, the appearance will be very nearly the ſame; however, when Jupiter comes near to the nodes, it may be conſidered at what time the planes of the orbits paſs through the earth inſtead of the ſun.

On the Magnitudes of Jupiter's Satellites.

448. The ſatellites appear ſo ſmall in the field of view of a teleſcope, that they cannot be meaſured by a micrometer; their magnitudes have therefore been determined from obſerving the times they are entering into the ſhadow of Jupiter, in a central eclipſe; but this muſt always give their diameters too ſmall, as we cannot tell the inſtant the ſatellite touches the ſhadow; a certain quantity of light muſt be loſt before the eclipſe appears to begin, and it muſt become inviſible before it be wholly immerſed in the ſhadow. Their magnitudes have

have alſo been found from meaſuring the diameters of their ſhadows upon the diſc of Jupiter; or by obſerving how long they are entering upon the diſc of Jupiter when they paſs centrally over it. By the obſervations of Mr. LYNN *(Phil. Tranſ.)* Mr. WHISTON found that the firſt entered into the ſhadow in 1′. 10″, the ſecond in 2′. 20″, the third in 3′. 40″ and the fourth in 5′. 30″, when they entered perpendicularly; hence their apparent diameters ſeen from the center of Jupiter become known. From this he deduced the magnitude of the third to be very nearly as big as the earth; the firſt nearly as big; the ſecond a little leſs than the firſt; the fourth the leaſt of all, and a little greater than the moon. M. WARGENTIN compared the ſhadows of the ſatellites upon the diſc of Jupiter, from which he found the third and fourth to be five or ſix times greater than the firſt, and the ſecond to be half as large as the firſt. M. MARALDI having examined and calculated three obſervations of M. CASSINI made in 1695, found that the firſt ſatellite entered upon the diſc of Jupiter in 7′, the ſecond in 9′. 40″, the third in 12′. 6″; and their continuance upon the diſc was 2*h*. 27′, 3*h*. 4′. 20″ and 3*h*. 43′. 38″ reſpectively; in reſpect to the fourth, he concluded from the Tables that it ought to be 15′ in entering upon the diſc, and 5*h*. in it's continuance upon it; hence he deduced the diameter of the third to be $\frac{1}{18}$ of that of Jupiter, and of the three others $\frac{1}{20}$. The difference of theſe concluſions ſhows that no great dependance can be placed upon them. The diſappearance of a ſatellite will be later the better the teleſcope is, and it will appear ſooner. M. de FOUCHY obſerved, that the diſappearance and re-appearance of the ſatellites depended on the diſtance of Jupiter from the ſun and earth. M. de BARROS obſerved, that different ſtates of the atmoſphere, different altitudes, and their diſtance from Jupiter, would influence the times of their appearance and diſappearance. All theſe circumſtances, ſo far as they cannot be conſidered, muſt tend to render the meaſures of their diameters very uncertain. Mr. WHISTON obſerves, that the compariſon of the obſervations ſhows that the quantities are ſometimes conſiderably larger than at others. *Longitude diſcovered by Jupiter's Planets.* page 5.

The

The following Table contains the diameters of the three firſt ſatellites as ſeen from Jupiter, according to CASSINI, WHISTON and BAILLY; the fourth as determined by M. de la LANDE.

Satellites.	CASSINI	WHISTON	BAILLY
I	59′. 4″	60′. 58″	60′. 20″
II	38. 1	28. 25	29. 42
III	24. 59	53. 40	22. 28
IV	13′. 32″	11′. 19″	9′. 39″

449. If their diameters could be aſcertained to any great degree of certainty, their quantities of matter would ſtill be very uncertain, becauſe their denſities are not known. Aſtronomers have endeavoured therefore to find out their quantities of matter, from obſerving the quantities of the effects produced by their actions upon each other. From ſuppoſing the maſſes of the firſt and third equal, M. de la GRANGE found, from the inequalities which they produce in the ſecond, that their maſſes were 0,00006869, that of Jupiter being =1; M. BAILLY found it 0,0000638 from the ſame ſuppoſition.

The maſs of the ſecond, from the inequalities which it produces in the firſt, of which it is principally the cauſe, is found by M. BAILLY to be 0,0000211; it is 0,00002417 according to M. de la GRANGE.

The maſs of the third, from it's effect upon the movement of the node of the ſecond in conjunction with the firſt, is, according to M. BAILLY, 0,00007624; but from it's effect upon the inequalities of the motion of the ſecond, ſuppoſing it equal to the firſt, it is 0,0000638; it is 0,0000687 according to M. de la GRANGE.

The maſs of the fourth, from the ſmall effect which it has upon the third, is not eaſily to be determined; M. BAILLY made it 0,00005.

Theſe maſſes, M. BAILLY obſerves, repreſent very well the motions of the ſatellites, of their nodes, and the variation of their inclinations; we may therefore conclude, that they are pretty accurately eſtabliſhed; and at the ſame time it proves that the variation of gravity, according to the inverſe ſquare of the diſtance, will explain all the phænomena of the ſatellites. In reſpect to the motion of the apſides, that depends upon the figure of Jupiter, it's denſity, and how the denſity may vary from the center to the ſurface, but as this is unknown,

unknown, the theory cannot be here applied. M. BAILLY has, however, pointed out the method by which we may, from the obſerved motion of the apſides, deduce the law of the variation of the denſity.

On the Conſtruction of the Epochs of the Mean Conjunctions of Jupiter's Satellites.

450. The epoch of the mean conjunction is the moment when the ſatellite arrives the firſt time every year at the mean place of Jupiter, reckoned upon the orbit of the ſatellite, diminiſhed by the ſum of the maxima of all the equations (the equations being expreſſed in time), in order to render the equations all additive; the maxima of the equations being added to the equations themſelves, in order to make up for that ſubtraction. For example, the firſt equation of light, at it's maximum, is $8'.\ 7'',5$, the time therefore is, in the epoch, diminiſhed by this quantity. Now let us ſuppoſe that at the time we are making any computation, this equation of light is $\pm 4'$, then the equation, as we ſhall find it, is $8'.\ 7'',5 \pm 4' = 12'.\ 7'',5$ or $4'.\ 7'',5$, both additive, and this is manifeſtly the ſame as if $8'.\ 7'',5$ had not been ſubtracted at firſt, and the equation $\pm 4'$ applied, what was at firſt ſubtracted being now added; and as we add the maximum, the quantity by which it is diminiſhed can never render it negative. It is the ſame with all the other equations, except that depending on the excentricity of Jupiter's orbit, which being variable, does not admit of this method. All the epochs are thus put down; and the computations are rendered more eaſy and ſimple by making the equations additive.

451. To find the epoch for any year, we will take and explain that Example which is given by M. de la LANDE in the laſt edition of his Aſtronomy, Vol. III. pag. 185. On January 2, 1764, the firſt ſatellite was eclipſed, the emerſion of which was at $10h.\ 27'.\ 44''$ mean time, at Paris. Now to make the equation of time always additive, we muſt ſubtract $14'.\ 42''$ which is the greateſt equation ſubtractive, and we have $10h.\ 13'.\ 2''$ for the time of the emerſion, according to the conſtruction of M. WARGENTIN's Tables.

The diſtance from the node was $60°.\ 17'$, the ſemidiameter of the ſhadow $1h.\ 7'.\ 55''$, and the inclination of the orbit $3°.\ 18'\frac{2}{3}$; hence (430) the ſemiduration of the eclipſe was $1h.\ 4'.\ 51''$; therefore the time of the middle of the eclipſe was $9h.\ 8'.\ 11''$. From this we muſt deduce the time of the mean conjunction, by applying all the equations for that time.

The mean anomaly of Jupiter was about $7^s.\ 8°,5$, and the equation of it's orbit was $4°.\ 51'.\ 30''$ additive, which converted into time (419) according to the motion of the ſatellite, gives the equation $34'.\ 39''$ to be ſubtracted from the middle of the eclipſe, and hence their remains $8h.\ 33'.\ 32''$.

From

From Art. 420, the maximum of the *firſt* part of the equation of light is 8′. 7″,5; but at the time of the eclipſe the equation was found to be 7′. 0″,5 additive; hence we have to ſubtract only 1′. 7″, which gives the time 8*h*. 32′. 25″.

The maximum (420) of the *ſecond* part of the equation of light is 2′. 2″,5; but the equation was 59″,5 additive at the time; therefore we muſt ſubtract 1′. 3″, which gives 8*h*. 31′. 22″.

The maximum (421) of the equation, marked in the Tables *C*, is 3′. 30″; but that equation at the time of the eclipſe was 0′. 27″ additive; hence we have to ſubtract 3′. 3″, which gives 8*h*. 28′. 19″.

The ſmall equations which come from the inequalities of Jupiter amounted at the ſame time to 15″ ſubtractive; and the maximum being 1′, we muſt ſubtract 1′. 15″, which gives 8*h*. 27′. 4″.

Laſtly, we muſt ſubtract 17″ for the reduction (426), and we have 8*h*. 26′. 47″ on January 2; but it being biſſextile, we muſt ſubtract one day, which gives January 1, 1764, 8*h*. 26′. 47″ for the epoch of the mean conjunction for that year, by M. WARGENTIN's conſtruction of the Tables.

452. Having ſhown how the epochs for any year are eſtabliſhed, we have only to ſhow how they are carried on for any number of years. According to M. WARGENTIN, the ſynodic revolution of the firſt ſatellite is 1*d*. 18*h*. 28′. 35″,947909; of the ſecond, 3*d*. 13*h*. 17′. 53″,74893; of the third, 7*d*. 3*h*. 59′. 35″,86754; and of the fourth, 16*d*. 18*h*. 5′. 7″,09174. Let us take for our example the firſt ſatellite. If we multiply 1*d*. 18*h*. 28′. 35″,947909 by 207 it gives 366*d*. 8*h*. 40′. 1″,21716, which is a common year of 365 days, and 1*d*. 8*h*. 40′. 1″,21716 over. Therefore at the beginning of the next year the ſatellite will be forwarder than it was at the beginning of the preceding, by 1*d*. 8*h*. 40′. 1″,21716. If we add again 1*d*. 8*h*. 40′. 1″,21716, it gives 2*d*. 17*h*. 20′. 2″,43432, which being more than a revolution, by ſubtracting a revolution from it, we get 0*d*. 22*h*. 51′. 26″,48641, which is the quantity by which the ſatellite will be forwarder at the beginning of the ſecond year. If to this we again add the ſame quantity, it gives 2*d*. 7*h*. 31′. 27″,70357, which being more than a revolution, by ſubtracting a revolution from it, we get 0*d*. 13*h*. 2′. 51″,75566, the quantity by which the ſatellite will be forwarder at the beginning of the third year. But as the fourth year is ſuppoſed to be biſſextile, the epoch will take place on the firſt of January, therefore this year conſiſts of 366 days, and conſequently contains 207 revolutions and 0*d*. 8*h*. 40′. 1″,21716; this therefore added to 0*d*. 13*h*. 2′. 51″,75566 gives 0*d*. 21*h*. 42′. 52″,97282 the quantity by which the ſatellite is forwarder at the beginning of the fourth year. But as this year begins on the firſt of January inſtead of December 31, in the mean motions for days, in January and February,

we muſt take the day of the month one leſs than it is, as will be further explained in the conſtruction of the Tables. Hence it appears, that if we begin at the epoch of any leap-year, and add to it 1*d*. 8*h*. 40′. 1″,21716, 0*d*. 22*h*. 51′. 26″,48641, 0*d*. 13*h*. 2′. 51″,75566, and 0*d*. 21*h*. 42′. 52″,97282, the ſums, rejecting a whole revolution when neceſſary, will be the epochs for the firſt, ſecond, third and fourth years after. Thus we may continue the epochs as far as we pleaſe. In like manner we proceed with the arguments *A*, *B*, *C*, &c. of the equations, rejecting a revolution when the ſum exceeds it.

To find the Configuration of Jupiter's Satellites at any Time.

453. 1. Find by Tables I, II, III, IV, V, the mean place of each ſatellite for the given time, which will be ſufficiently near to the true place, except for the fourth ſatellite, for which we muſt apply the equation of the center; for that purpoſe we muſt, with it's mean motion, take out the place of it's apſide, which ſubtracted from the mean longitude gives it's mean anomaly, to which find the equation in Table IX, and apply it to the mean longitude, and it gives the true longitude.

2. From the place of each ſatellite thus found, ſubtract the geocentric longitude of Jupiter,* and in Table VI with that difference, find the correſponding numbers, which repreſent the apparent diſtance of each from the center of Jupiter in terms of it's ſemidiameter. When the argument of this Table is *leſs* than ſix ſigns, the ſatellite will be to the *eaſt* of Jupiter; when *greater*, to the *weſt*. This is Dr. HALLEY's method.

As the principal deſign of finding the configurations of the ſatellites is to diſtinguiſh one from another, the equation of light is commonly of but little importance, and may be neglected. Or if that accuracy be deſired, compute the configuration to any hour mean time, and then add the equation of light to it, and you have the configuration as they appear at that time. The operation is rendered ſhorter by calculating to an hour; and it will be ſufficient if the places be calculated to minutes of a degree.

454. To find the equation of light, from the ſun's longitude ſubtract the heliocentric longitude of Jupiter, and with the remainder enter Table VII, and take out the firſt part of the equation; and with the anomaly of Jupiter enter Table VIII, and take out the ſecond part. The anomaly is found by ſubtracting the place of the aphelion from the longitude.

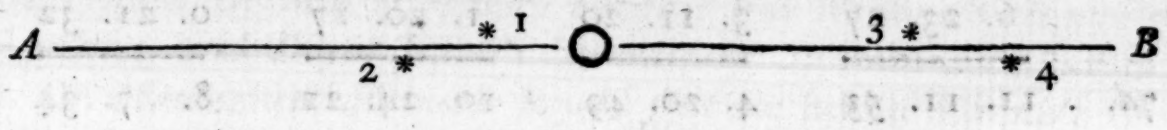
* The method of finding this will be ſhown in the Introduction to the Tables in the ſecond Volume.

Ex.

Ex. To calculate the configuration of the ſatellites on April 6, 1795, at four o'clock in the morning, mean time, by the civil account; or on the *5d.* 16*h.* aſtronomical time.

	I.	II.	III.	IV.	Aps. IV.
1795,	1ˢ. 25°. 8′	0ˢ. 15°. 8′	2ˢ. 25°. 11′	8ˢ. 26°. 53′	1ˢ. 10°. 49′
March	10. 14. 1	4. 3. 44	6. 28. 35	4. 21. 24	0. 0. 11
5 days	9. 27. 27	4. 26. 52	8. 11. 35	3. 17. 51	0. 0. 1
16 hours	4. 15. 40	2. 7. 35	1. 3. 33	0. 14. 23	
	2. 22. 16	11. 23. 19	7. 8. 54	5. 20. 31	1. 11. 1
24 Geo.	10. 3. 50	10. 3. 50	10. 3. 50	−39	5. 20. 22
	4. 18. 26	1. 19. 29	9. 4. 4	5. 19. 52	4. 9. 21
				10. 3. 50	
				7. 16. 2	
	Eaſt 3,91	*Eaſt* 7,14	*Weſt* 14,95	*Weſt* 18,98	

Hence, and by Article 447, this Configuration.



The line *AB* ſhows the direction of the belts.

Sun's longitude . .	0ˢ. 17°. 24′	
Jupiter's hel. long. .	9. 23. 3	
Difference	2. 24. 21	. Equation of light 8′. 59″
Jupiter's aphelion .	6. 11. 4	
Jupiter's anomaly . .	3. 11. 59	. Equation of light 1. 37
	Total equation of light	10. 36

This configuration therefore which is calculated for four o'clock mean time, is ſuch as will appear at 10′. 36″ after four; but it will not ſenſibly differ from the appearance at four. To have computed the configurations as they appear at four o'clock, we muſt have computed for 3*h.* 49′. 24″ mean time.

455. The configuration at the ſame hour, for a month, may be very readily determined, without repeating the whole operation, in this manner. The difference from day to day ariſes from the addition of the daily mean motions, and from the variation of the geocentric place of Jupiter, the mean daily variation of which for a month will be ſufficiently accurate for our purpoſe. As the geocentric place of Jupiter is *ſubtracted* from the mean place of the ſatellite, if the geocentric motion be *direct*, *ſubtract* it's mean daily variation from the mean daily motion of the ſatellite; but if the geocentric motion be *retrograde*, you muſt *add*, and you will have the whole daily motion to be applied to the calculation for any one day in order to get the ſituation of the ſatellites for the next day; and thus you may continue the proceſs for a month, at the end of which, it may be proper to reſume the firſt calculation, and then proceed for that month in like manner.

To take our example, I find Jupiter's geocentric motion is *direct*, at the mean rate of about 2′ in a day for the month; ſubtract therefore 2′ from the daily motions of the ſatellites, and we have 6ˢ. 23°. 27′, 3ˢ. 11°. 20′, 1ˢ. 20°. 17′ and 0ˢ. 21°. 32′, for the relative daily motions of the firſt, ſecond, third and fourth ſatellites in reſpect to Jupiter; hence

	I.	II.	III.	IV.
	4ˢ. 18°. 26′	1ˢ. 9°. 29′	9ˢ. 4°. 4′	7ˢ. 16°. 2′
	6. 23. 27	3. 11. 20	1. 20. 17	0. 21. 32
7*d*.	11. 11. 53	4. 20. 49	10. 24. 21	8. 7. 34
	6. 23. 27	3. 11. 20	1. 20. 17	0. 21. 32
8*d*.	6. 5. 20	8. 2. 9	0. 14. 38	8. 29. 6
	6. 23. 27	3. 11. 20	1. 20. 17	0. 21. 32
9*d*.	0. 28. 47	11. 13. 29	2. 4. 55	9. 20. 38

Hence by Table VI, we get the following configurations;

7*d*.	*Weſt* 1,84	*Eaſt* 5,94	*Weſt* 8,75	*Weſt* 24,38
8*d*.	*Weſt* 0,54	*Weſt* 8,31	*Eaſt* 3,79	*Weſt* 26,38
9*d*.	*Eaſt* 2,82	*Weſt* 2,67	*Eaſt* 13,58	*Weſt* 24,71

As there is the ſame motion of the ſatellites to be added every time, it will be beſt to put them down upon a ſtrip of paper, and by laying it under, the addition may be made from it without the trouble of writing the motions down every time. In this manner we may lay down the configurations with great expedition, and with more accuracy than by the mechanical contrivances of FLAMSTEAD and CASSINI.

The

The mean time at which theſe configurations are ſhown, may be reduced to apparent time, by applying the equation of time; thus the configuration on the ſixth day at 10′. 36″ after four o'clock mean time, is 8′. 11″ after four apparent time; to have calculated therefore for four apparent time, we muſt have calculated for 3*h*. 51′. 49″.

456. The principle upon which the ſixth Table is conſtructed is this. Let *E* be the earth, *I* Jupiter, *a* the place of a ſatellite in it's orbit; join *EI*, and produce it to the heavens at *m*, and produce *Ia* to *n*, and draw *ac* perpendicular to *Ib*. Now *m* is the geocentric place of Jupiter in the heavens, and *n* the true place of the ſatellite; if therefore from ♈ *n* the longitude of the ſatellite we ſubtract ♈ *m* the geocentric longitude of Jupiter, we get *mn*, or the angle *bIa*, whoſe ſine *ac* repreſents the apparent diſtance of the ſatellite from Jupiter; if therefore *ac* be expreſſed in terms of the ſemidiameter of Jupiter, we ſhall get the apparent diſtance in ſemidiameters, and in this manner the Tables are conſtructed. FIG. 101.

When a ſatellite is *approaching* Jupiter, the figure is put *between* Jupiter and the point; when it is *receding*, the figure is put on the other ſide.

A TABLE

Of the apparent Diſtances of Jupiter's Satellites from it's limb at the time of an Eclipſe, for every tenth Day of Jupiter's Diſtance from Oppoſition or Conjunction; by the Rev. Dr. MASKELYNE, Aſtronomer Royal; from the Britiſh Mariner's Guide.

Diſtance of Jupiter from *oppoſition* to the ☉.	Diſtance of the Satellites from Jupiter's limb at the Eclipſes, in ſemidiameters of Jupiter and decimal parts.				Diſtance of Jupiter from *conjunction* with ☉.	Diſtance of the Satellites from Jupiter's limb at the Eclipſes, in ſemidiameters of Jupiter and decimal parts.			
Days.	I.	II.	III.	IV.	Days.	I.	II.	III.	IV.
10	0 ,20	0,33	0,50	0,85	10	0,15	0,25	0,35	0,55
20	0,40	0,66	1,05	1,66	20	0,30	0,45	0,70	1,25
30	0,60	0,95	1,50	2,65	30	0,40	0,67	1,05	1,70
40	0,75	1,20	1,90	3,35	40	0,55	0,90	1,40	2,50
50	0,90	1,40	2,25	3,95	50	0,70	1,00	1,80	3,20
60	1,00	1,60	2,50	4,40	60	0,80	1,25	2,00	3,50
70	1,05	1,70	2,66	4,70	70	0,90	1,40	2,25	3,95
80	1,10	1,75	2,75	4,85	80	1,00	1,55	2,45	4,33
90	1,10	1,75	2,75	4,85	90	1,05	1,66	2,60	4,60
100	1,10	1,70	2,70	4,80	100	1,10	1,75	2,70	4,80

The

The diſtances of the ſatellites from Jupiter's limb in this Table, are to be meaſured either in a line with Jupiter's equator, or longer axis, or in a line parallel thereto; or, which is the ſame thing, to it's belts; for the ſatellites generally appear a little to the north or ſouth of this line.

In this Table the apparent diſtances are thoſe of the regular eclipſes, that is, at the immerſions before oppoſition and emerſions after; but at the emerſions which are viſible before oppoſition, and immerſions after, the diſtances from Jupiter's limb will be leſs than in the Table, by a quantity which is in the ſame proportion to Jupiter's diameter, as the duration of the eclipſe is to the longeſt duration when in the nodes.

Various Circumſtances reſpecting the Phænomena of the Satellites.

FIG. 102. 457. To find when an immerſion and emerſion are viſible, let s be the center of the ſhadow AB, r the center of the diſc of Jupiter CD, the radius sn being expreſſed in minutes of the orbit of the ſatellite, and rn expreſſed in the ſame meaſure. Let rs be a portion of the orbit of the ſatellite equal to the annual parallax, expreſſed in minutes, which may be taken from the *Nautical Almanac*; and let AB and CD be repreſented as ſeen from the earth; let wcn be the path of the ſatellite, then the immerſion at w being viſible, the emerſion at n will alſo be juſt viſible, or rather it is the limit. In the triangle rns, we know all the ſides, to find the perpendicular nm on rs, which, as the orbit wn is very nearly parallel to sr, is very nearly equal to sc; but (434) $sc = vsr$; or if we repreſent the radius r by unity, $sc = vs$; make therefore $mn = vs$, and we get v, the ſine of the diſtance of Jupiter from the node. We have here ſuppoſed the earth in the plane of the orbit of Jupiter; but as the earth is not in that plane, it will make Jupiter appear a little higher or lower in the ſhadow, by the latitude of the earth ſeen from Jupiter; this when greateſt is about $15'$, and varies as the ſine of the diſtance of the earth from the node of Jupiter. If rs repreſent the orbit of Jupiter, in the firſt ſix months of the year, the center of the ſhadow will lie at t to the ſouth of s. Hence knowing sr and st, we can find the angle srt, and rt; conſequently we know the three ſides of nrt, to find the angle nrt; hence we know nrs; therefore in the triangle nrs, we know rn, rs and the angle nrs; to find nm. The latitude of the earth ſeen from Jupiter is nearly equal to one ſeventh part of the equation of the earth's orbit (*Mem. Acad.* 1765), at leaſt when Jupiter is about quadratures. This is the method which is given by M. de la LANDE, to determine when an emerſion will be viſible before oppoſition, and an immerſion, after oppoſition.

458. M. de

458. M. de la PLACE, in the *Mem. de l'Acad.* 1784, in his theory of the motions of the ſatellites, has deduced ſome very extraordinary concluſions, which are confirmed by obſervations. If p, q and r repreſent the mean motions of the firſt, ſecond and third ſatellites, he has ſhown, that $p-q=2q-2r$, or $p-3q+2r=0$; and if x, y and z repreſent their mean longitudes, he has proved that $x-3y+2z=180°$. The Tables therefore muſt always ſatisfy theſe conditions. The laſt equation ſhows that the three ſatellites can never be eclipſed at the ſame time. But it may be obſerved, that the firſt equation is a conſequence of the ſecond; for the correſponding mean longitudes will always be repreſented by $x+p$, $y+q$, $z+r$, and hence $z+p-3\times\overline{y+q}+2\times\overline{z+r}=180°$, from which ſubtract $x-3y+2z=180°$, and we get $p-3q+2r=0$. M. de la PLACE makes the annual motion of the apſide of the third to be 3°, and of the fourth to be 37′. M. BAILLY makes the former 2°, in his Tables, and the epoch for 1700, in 11ˢ. 13°; the latter he makes 45′. 18″. In the Tables which we have here added, the epochs of the apſide of the fourth are thoſe given by M. BAILLY, in his *Eſſai ſur la Theorie des Satellites de Jupiter*.

459. The firſt ſatellite is moſt proper for finding the longitude, the Tables of that being the moſt correct; it is alſo the beſt on account of the greater velocity of the ſatellite, the inſtant of it's appearing or diſappearing being, on that account, more certain. It is better to compare an eclipſe obſerved under one meridian with an eclipſe obſerved under another, rather than with one computed, becauſe of the imperfection of the Tables. The obſervers ſhould alſo be furniſhed with the ſame kind of teleſcopes, as the time when a ſatellite becomes viſible at an emerſion, or inviſible at an immerſion, depends upon the quantity of light which the teleſcope receives, and it's magnifying power; it depends alſo upon the proximity of the ſatellite to Jupiter, and it's altitude above the horizon. M. BAILLY has given us ſome Rules to correct the difference ariſing from theſe circumſtances; theſe we ſhall, in brief, here explain.

460. As the ſatellite enters the ſhadow of Jupiter, it's light diminiſhes by degrees, until the ſatellite becomes inviſible; and it is of great importance to aſcertain how much of the ſatellite is immerſed in the ſhadow at the time it diſappears. M. FOUCHY firſt obſerved that this would depend upon the diſtance of the earth from Jupiter. Let PR be the ſhadow of Jupiter, LM the orbit of the ſatellite, and let v be the center of the ſatellite $mnrt$ at the time it becomes inviſible, then mnr is the part not yet immerſed, and which is called the *inviſible* ſegment; let $O\mathcal{Q}$ be perpendicular to LM, and join Oc, and draw $Ovsn$. Now if we know sn, ſubtract it from vn, and we get vs; hence we know $Os-vs$, or Ov; and knowing $O\mathcal{Q}$, we know $\mathcal{Q}v$, which reduce into time; and as $O\mathcal{Q}$ and Oc are known, we can find $\mathcal{Q}c$, and therefore we FIG. 103.

we know the time of deſcribing $\mathfrak{Q}c$; hence having found the time of deſcribing $\mathfrak{Q}c$ and $\mathfrak{Q}v$, we know the time of deſcribing cv, which ſubtracted from the time at which the center was at v gives the time when it was at c, or the time of the immerſion of the center, called the true time, which ought to be the ſame to all obſervers. This quantity vc is M. FOUCHY's equation, and when applied to the obſerved time ſhould give the ſame time for all obſervations. We have here ſuppoſed sn to be known; the idea how to find this was firſt ſuggeſted by M. FOUCHY, and afterwards improved upon by M. BAILLY; his method we ſhall here explain.

461. M. BAILLY, by diaphragms with a circular hole in the middle, diminiſhed gradually the field of view of his teleſcope until the ſatellite diſappeared; hence the aperture in the diaphragm at the time the ſatellite becomes inviſible, is to the whole aperture, as the quantity of light received from the ſatellite at the time it diſappears, to the quantity of light in the whole aperture; let the whole aperture $= 1$, the aperture of the diaphragm $= a$; then the light of the ſatellite when not eclipſed being repreſented by unity, the light when it diſappears at an immerſion will be $= a$; conſequently $a : 1 ::$ ſegment mnr : whole ſurface $mnrt$; hence we know the ſegment mnr, and conſequently it's verſed ſine ns, for on account of the ſmallneſs of the arc msr, we may conſider it as a ſtraight line. In a teleſcope whoſe aperture was 24 lines, M. BAILLY found the fourth ſatellite, when at it's greateſt elongation, to vaniſh at an aperture of 5.5 lines; hence $a = \overline{\frac{5.5}{24}}^2 = 0{,}0525$, which is equal to the ſegment mnr, the circle $mnrt$ being unity; hence the verſed ſine $ns = 0{,}4303$. For the third ſatellite, he found $a = 0{,}0156$; and for the firſt and ſecond $a = 0{,}0646$. Theſe determinations were made, when the diſtance of Jupiter from the ſun was 5,2207, and the diſtance of the earth from Jupiter 4,8456, the earth's diſtance from the ſun being unity; alſo the altitude of Jupiter above the horizon was $15°$, and the ſatellites were at their greateſt elongations. To reduce the inviſible ſegments to any other ſituations of the earth and Jupiter, and any other altitude, he takes the light received at Jupiter to vary inverſely as the ſquare of it's diſtance from the ſun, and the light received at the earth from the ſatellites to vary inverſely as the ſquare of the diſtance of the earth from Jupiter; and for the variation of the quantity of light at different altitudes, he takes that which is given by M. BOUGUER in his Optics. To find the allowance to be made for the different diſtances of the ſatellites from Jupiter, he proceeds thus.

On

On July 17 and 23, 1771, the following obſervations were made, and the inviſible ſegments determined as above explained, by taking into conſideration the diſtances of Jupiter from the ſun and the earth, and the altitude of Jupiter.

	H. M.	Diſt. of Sat. in ſemid. ♃	Segment
July 17,	9. 58	1,36	0,2485
	10. 17	1,62	0,1677
	10. 43	1,96	0,1361
23,	11. 5	1,49	0,1862
	11. 20	1,32	0,2357
	11. 28	1,21	0,2910
	11. 35	1,11	0,3201
	11. 39	1,6	0,3521

The law which the variation of theſe ſegments follow is nearly as $\frac{b}{x^2}+\frac{c}{x}$, x being the diſtance of the ſatellite from the center of Jupiter in ſemidiameters of Jupiter; M. BAILLY therefore aſſumes $\frac{b}{x^2}+\frac{c}{x}=y$, y being the ſegment; and by taking two values of y and the correſponding values of x, we get two equations, from which we can determine b and c; the two values of y which he aſſumed are 0,1862 and 0,3521, and taking the correſponding values of x, he found $b=0{,}3397$, and $c=0{,}0495$; hence $\frac{0{,}3397}{x^2}+\frac{0{,}0495}{x}=y$; by applying this to other obſervations, he found the errors much ſmaller than could be expected. By proceeding thus for the ſecond and third ſatellites, he found for the

Satellites

I . . . $\frac{0{,}3397}{x^2}+\frac{0{,}0495}{x}=y$

II . . . $\frac{0{,}3933}{x^2}+\frac{0{,}0375}{x}=y$

III . . . $\frac{0{,}0756}{x^2}+\frac{0{,}2157}{x}=y$

IV . . . $\frac{0{,}192}{x^2}+\frac{0{,}053}{x}=y$

The fourth was determined by M. de la LANDE, M. BAILLY not having ſufficient obſervations upon the ſatellite, to determine the law of variation.

462. M. BAILLY, in the laſt place, conſiders the effects of different teleſcopes. The greater the quantity of light which a teleſcope receives, or the greater the aperture, the leſs will be the inviſible ſegment, and that in the inverſe ratio of the aperture, for in this caſe, the ſame quantity of light comes to the eye. Hence by taking into conſideration all the circumſtances, he reduced the obſervations, and found, in general, a very near agreement after the reduction, compared with the agreement between the obſervations themſelves. The calculation requires that we ſhould know the diameters of the ſatellites; theſe he deduced in the following manner.

463. On June 30, 1771, he obſerved the immerſion of the firſt ſatellite. At 24 minutes in time before the immerſion, with an aperture of 10,5 lines he loſt ſight of the ſatellite, the whole aperture being 24 lines. With an aperture of 13 lines, he made his obſervation of the immerſion; and taking off the diaphragm, he then obſerved it 4′. 54″ longer. Now an aperture of 10,5 lines gives the inviſible ſegment $BFG = 0{,}1914$; but here we muſt take into conſideration, the proximity of the ſatellite to Jupiter. At the time when the ſatellite diſappeared with an aperture of 10,5 lines, it was 1,57 diſtant from Jupiter, and at the immerſion it was 1,25; hence if we put theſe for x, we ſhall get the correſponding ſegments 0,2571 and 0,1695; hence 0,1695 : 0,2571 :: 0,1914 : 0,2903 the inviſible ſegment BFG which correſponds to the whole aperture at the diſtance 1,25, deduced from that which was obſerved at the diſtance 1,75. But with an aperture of 13 lines, the ſegment BDE muſt be greater in proportion as the aperture is leſs, or in the ratio of $13^2 : 24^2$; hence $BDE = 0{,}9895$. Now the verſed ſine $BK = 0{,}664$, and $BH = 1{,}99852$, therefore $KH = 1{,}33452$, the ſpace paſſed over in 4′. 54″; here the ſatellite entered obliquely into the ſhadow; but if it had entered perpendicularly, it would have taken only 4′. 51″ to have paſſed over the ſame part; hence 1,33452 : 2 (the diameter) :: 4′. 51″ : 7′. 16″ the diameter in time, which anſwers to 1°. 1′. 36″ the diameter of the ſatellite ſeen from Jupiter. If the reader wiſh for any further ſatisfaction upon this ſubject, he may conſult the *Mem. de l'Acad. Roy. des Scien.* 1771; or the *Phil. Tranſ.* Vol. LXIII.

FIG. 104.

464. Dr. MASKELYNE obſerves, that the method here propoſed of correcting the immerſion and emerſion of a ſatellite, muſt be ſubject to a certain degree of inaccuracy from hence, that when you reduce the aperture of the teleſcope ſo as to make the ſatellite diſappear, you alſo diminiſh the quantity of light from Jupiter in the ſame proportion, on which account the ſatellite will be viſible with a leſs quantity of light than it would be if Jupiter con-

tinued

tinued of the ſame brightneſs, and therefore the inviſible ſegment will have a leſs ratio to the whole ſurface, than the quantity of light in the aperture when the ſatellite is rendered inviſible has to the quantity of light in the whole aperture. A correction therefore for this circumſtance ought to be applied. We may alſo further obſerve, that beſides the circumſtances which are here taken notice of, the twilight, the clearneſs of the air, the proximity of Jupiter to the moon, and the eye of the obſerver, all combine to affect the time at which the ſatellite becomes inviſible.

465. When Jupiter is ſo far diſtant from conjunction with the ſun, as to be about 8° above the horizon when the ſun is 8° below, an eclipſe of the ſatellites will be viſible at any place; this may be determined near enough by a celeſtial globe *(Nautical Almanac)*. *Before* the oppoſitions of Jupiter to the ſun, the immerſions and emerſions happen to the *weſt* of Jupiter; *after* oppoſition, they happen to the *eaſt*. If an aſtronomical teleſcope be uſed, which reverſes objects, the appearance will be contrary. The ſatellites in the configurations in the *Nautical Almanac* are put down on their proper ſides of Jupiter; ſuch a teleſcope therefore reverſes their ſituation in reſpect to Jupiter, making thoſe on the eaſt appear to the weſt, and thoſe on the weſt appear to the eaſt. The immerſions ſignify the inſtant of the diſappearance of the ſatellite by entering the ſhadow of Jupiter; and the emerſions ſignify the inſtant at which they firſt appear at the coming out of the ſame. For directions to the obſerver, I refer the Reader to my *Practical Aſtronomy*, page 186.

466. M. Cassini ſuſpected that the ſatellites had a rotation about their axes, as ſometimes in their paſſage over Jupiter's diſc they were viſible, and at other times not; he conjectured therefore that they had ſpots upon one ſide and not on the other, and that they were rendered viſible in their paſſage when the ſpots were next the earth. At different times alſo they appear of different magnitudes and of different brightneſs. The fourth appears generally the ſmalleſt, but ſometimes the greateſt; and the diameter of it's ſhadow on Jupiter appears ſometimes greater than the ſatellite. The third alſo appears of a variable magnitude, and the like happens to the other two. M. Maraldi alſo concluded from his own obſervations, that they had a rotation. M. Pound alſo obſerved that they appeared more luminous at one time than another, and therefore he concluded that they revolved about their axes.

THE FOLLOWING TABLE EXHIBITS THE ELEMENTS OF THE SATELLITES, AS GIVEN BY M. DE LA LANDE, FROM THE BEST OBSERVATIONS.

ELEMENTS.	I.	II.	III.	IV.
Periodic revolution	$1^d.18^h.27'.33'',476$	$3^d.13^h.13'.41'',929$	$7^d.3^h.42'.32'',879$	$16^d.16^h.32'.8'',491$
Synodic revolution	1. 18. 28. 36	3. 13. 17. 54	7. 3. 59. 36	16. 18. 5. 7
Diſt. in ſemid. 4 by CASSINI . .	5,67	9,00	14,38	25,30
——— Sir I. NEWTON	5,965	9,494	15,141	26,63
Mean diſt. in minutes at mean diſt. 4	1'.51''	2'. 57''	4'. 42''	8'. 16''
Semid. of ſhadow in deg. of the orbit	9°. 35. 37	6°. 1. 33	3°. 43. 58	2°. 8. 2
——— time . . .	1^h. 7. 55	1^h. 25. 40	1^h. 47. 0	2^h. 23. 0
——— that of 4 = 1 .	0,9941	0,9967	0,9857	0,9913
Half duration of an eclipſe 90° from node when the inclination is leaſt	1^h. 3'. 45''	1^h. 16'. 5''	1^h. 3'. 40''	0^h. 0'. 0''
——— greateſt -	1. 3. 45	1. 6. 49	0. 38. 22	0. 0. 0
For circ. ſhadow greateſt inclination	3°. 18. 38	3°. 46'. 0''	3°. 25. 57	2°. 36. 0
For circ. ſhadow mean ———	3. 18. 38	3. 16. 0	3. 13. 58	2. 36. 0
For circ. ſhadow leaſt ———	3. 18. 38	2. 46. 0	3. 2. 0	2. 36. 0
For ellip. ſhadow greateſt ———	3. 4. 27	3. 29. 42	3. 11. 14	2. 24. 51
For ellip. ſhadow leaſt ———	3. 4. 27	2. 34. 0	2. 49. 0	2. 24. 51
Epoch of conj. 1760 for mer. Greenwich	$0^d.10^h.35'.0''$	$1.14^h.49'.36''$	$2^d.5^h.32'.29''$	$1^d.7^h.20'.50''$
Mean place of the node in 1760 .	$10^s.14°.30'$	$10^s.13°.45'$	$10^s.14°.24'$	$10^s.16°.39'$
——— annual motion of the node . .	0'. 0''	2'. 3''	0'. 0''	4'. 19''
——— long. for 1700 for mer. Greenwich	$2^s.12°.33.26$	$2^s.12°.55.28$	$5^s.13°.0.48$	$7^s.17°.20.38$
——— diurnal motion	6. 23. 29. 20,3798	3. 11. 22. 29,14275	1. 20. 19. 3,5389	0. 21. 34. 16,0008
——— ſecular motion	7. 25. 31. 13	3. 23. 10. 39	1. 22. 9. 19	6. 29. 50. 29

On the Conſtruction of the Epochs in the Tables.

467. The epoch of a ſatellite for any year is found from a conjunction of the ſatellite; this will be beſt explained by an example. By Article 451, it was found that the time of a mean conjunction of the firſt ſatellite in 1764, was January 1, 8*h*. 26′. 47″, as computed for the conſtruction of the Tables; but this is not the true time of the mean conjunction, but it is that time diminiſhed by the ſum of the maxima of all the equations, except that ariſing from the equation of Jupiter's orbit, which ſum is 29′. 22″; add this therefore to the above time, and it gives January 1, 8*h*. 56′. 9″ for the true time of the mean conjunction, or the time when the mean place of the ſatellite upon it's orbit was the ſame as the mean place of Jupiter in it's orbit; but, by computation, the mean place of Jupiter at that time was 2ˢ. 8°. 56′. 41″, this therefore is the mean place of the ſatellite at the ſame time; but the mean motion of the ſatellite in 1*d*. 8*h*. 56′. 9″ was 9ˢ. 9°. 14′. 10″; ſubtract this therefore from 2ˢ. 8°. 56′. 41″, and we have 4ˢ. 29°. 42′. 31″ for the mean place of the ſatellite at the beginning of 1764, or the epoch for that year, at Paris. The conſtruction of the Tables here explained, has been for the mean diſtance of Jupiter from the earth, that is, to repreſent the ſatellites as ſeen from that diſtance, becauſe we applied the equations of light as explained in Article 420, which reduces the time at the place where the earth is at the time of obſervation, to the time at which the ſame phænomenon of the ſatellite (it's conjunction) would have appeared if the earth had been at it's mean diſtance from Jupiter. As Greenwich is 9′. 20″ eaſt of the Obſervatory at Paris, if 1°. 19′. 8″, the mean motion of the ſatellite for that time, be added to 4ˢ. 29°. 42′. 31″, it gives 5ˢ. 1°. 1′. 39″ for the epoch for Greenwich, the year at Greenwich beginning 9′. 20″ later than at Paris. But the epochs in theſe Tables are for the *leaſt* diſtance of Jupiter from the earth, and conſequently they are found from the epochs at the *mean* diſtance, by adding to them the mean motions of the ſatellites for 10′. 10″, that being the time (420) which light takes in paſſing over a ſpace equal to the difference between the leaſt and mean diſtances. For as any ſituations of the ſatellites appear 10′. 10″ ſooner at the leaſt than at the mean diſtance of Jupiter from the earth, at any point of time they muſt appear forwarder in their orbits at the former than at the latter diſtance by their motions in that time. Now the mean motion of the ſatellite in 10′. 10″ is 1°. 26′. 12″; hence the epoch for 1764, for the leaſt diſtance of Jupiter, is 5ˢ. 2°. 27′. 51″; the Tables which are here given, were conſtructed from other obſervations. Having determined the epoch for any one year, the epochs for the following years are found by con-

continually adding to it, the mean motions for a year, as explained in Article 452; and the epochs for the preceding years are found by ſubtraction; thus we continue the Tables as far as we pleaſe.

468. The firſt Table contains the epochs of the ſatellites at the beginning of the year, that is, on December 31 of the preceding year by the civil account, at 12 o'clock at noon, mean time; except on leap-year, in which the place is put down for January 1, at 12 o'clock at noon, mean time.

Table the ſecond contains the mean motions for months, ſhowing at the end of each month, how much forwarder the ſatellites are than they were at the beginning of the year. If therefore to the place at the beginning of the year, you add the mean motion for any month, it gives the mean place for the end of that month, or for the beginning of the next. The month of February is here ſuppoſed to contain 28 days. Now in leap-year, the epoch being for the firſt of January at noon, mean time, when we add the motion for January, it gives the place on February 1, at noon; and adding the motion for February, it gives the place for the laſt day at noon, becauſe from January 1, to February 29, in leap-year is the ſame as from December 31 to February 28, in the common years; hence the mean motions for the other months added to the epochs, will give the mean places as well in leap-year as in the common years.

The third Table contains the mean motions for days, as far as 31, that being ſufficient, as we have the mean motions for months. But in leap-year, in the months of January and February, we muſt take the motion for one day leſs than the day of the month, becauſe (as above explained) the epoch is for the firſt of January, and the motion for a month being added gives the place on the firſt of February; the places therefore being thus obtained after one day in each month has paſſed, the motion from that time to any other day muſt be one day leſs than the number of days of the month.

The fourth Table contains the mean motions for hours; and the fifth Table contains the mean motions for minutes and ſeconds.

The ſixth Table contains the apparent diſtances of the ſatellites from the center of Jupiter in terms of it's ſemidiameters, according to their ſituations in their orbits, and the geocentric place of Jupiter.

The ſeventh Table contains the firſt equation of light; the eighth contains the ſecond equation of light. Theſe Tables are conſtructed to the neareſt diſtance of Jupiter from the earth; and therefore at all other times, the ſatellites will appear to come later to the places found from the Tables than the time to which they were computed, by the equations in the Tables.

Table the ninth contains the equation of the center of the fourth ſatellite.

469. Theſe

469. Theſe Tables give only the *mean* places of the ſatellites, except for the fourth ſatellite, whoſe place may be corrected by the equation of the orbit. This accuracy is ſufficient for the purpoſe for which the Tables are here given, they being principally intended to find the configurations of the ſatellites. In the Tables for computing the eclipſes, the epochs for each year are thoſe of the firſt mean conjunction of the ſatellite after the commencement of the year; the conſtruction of theſe Tables, and their uſes, will be explained in the ſecond Volume.

470. If it be required to find the apparent poſitions of the ſatellites at any given apparent time, that time muſt be converted into mean time (the Tables being conſtructed to mean time) by applying the equation of time; and then from that mean time, the equation of llght muſt be ſubtracted, and the computation made for that time; and from the places thus found, we muſt ſubtract the geocentric place of Jupiter, and proceed as already explained.

EPOCHS

EPOCHS OF THE MEAN MOTIONS OF JUPITER's SATELLITES.

TABLE I.

YEARS, NEW STILE.	I. SATELLITE	II. SATELLITE	III. SATELLITE	IV. SATELLITE	IV.SAT.APS.
	S. D. M. S.	S. D. M. S.	S. D. M. S.	S. D. M. S.	S. D. M.
1790	0. 4. 13. 37	10. 4. 57. 2	0. 5. 9. 5	3. 28. 1. 10	1. 7. 4
1791	3. 27. 42. 36	7. 16. 44. 19	0. 11. 5. 36	2. 11. 28. 30	1. 7. 49
B. 1792	2. 14. 40. 55	8. 9. 54. 5	2. 7. 21. 12	1. 16. 31. 6	1. 8. 34
1793	6. 8. 9. 53	5. 21. 41. 22	2. 13. 17. 44	11 29. 58. 26	1. 9. 19
1794	10. 1. 38. 52	3. 3. 28. 39	2. 19. 14. 15	10. 13. 25. 46	1. 10. 4
1795	1. 25. 7. 50	0. 15. 7. 56	2. 25. 10. 47	8. 26. 53. 6	1. 10. 49
B. 1796	0. 12. 6. 9	1. 8 17. 43	4. 21. 26. 23	8. 1. 54. 42	1. 11. 35
1797	4. 5. 35. 8	10. 20. 5. 0	4. 27. 22. 54	6. 15. 22. 2	1. 12. 20
1798	7. 29. 4. 7	8. 1. 52. 17	5. 3. 19. 26	4. 28. 49. 22	1. 13. 5
1799	11. 22. 33. 5	5. 13. 39. 34	5. 9. 15. 58	3. 12. 17. 42	1. 13. 50
C. 1800	3. 16. 2. 4	2. 25. 26. 51	5. 15. 12. 30	1. 25. 45. 2	1. 14. 35
1801	7. 9. 31. 3	0. 7. 14. 8	5. 21. 9. 2	0. 9. 12. 22	1. 15. 20
1802	11. 3. 0. 1	9. 19. 1. 25	5. 27. 5. 34	10. 22. 39. 42	1. 16. 6
1803	2. 26. 29. 0	7. 0. 48. 42	6. 3. 2. 6	9. 6. 7. 2	1. 16. 51
B. 1804	1. 13. 27. 19	7. 23. 58. 29	7. 29. 17. 41	8. 11. 8. 38	1. 17. 36
1805	5. 6. 56. 18	5. 5. 45. 46	8. 5. 14. 13	6. 24. 35. 58	1. 18. 21
1806	9. 0. 25. 16	2. 17. 33. 5	8. 11. 10. 45	5. 8. 3. 18	1. 19. 6
1807	0. 23. 54. 16	11. 29. 20. 20	8. 17. 7. 16	3. 21. 30. 38	1. 19. 51
B. 1808	11. 10. 52. 34	0. 22. 30. 6	10. 13. 22. 52	2. 26. 32. 14	1. 20. 37
1809	3. 4. 21. 32	10. 4. 17. 23	10. 19. 19. 24	1. 9. 59. 34	1. 21. 22

THE FIRST TABLE CONTINUED.

YEARS, NEW STILE.	I. SATELLITE				II. SATELLITE				III. SATELLITE				IV. SATELLITE				IV. S. APS.		
	S.	D.	M.	S.	S.	D.	M.	S.	S.	D.	M.	S.	S.	D.	M.	S.	D.	M.	S.
1810	6.	27.	50.	31	7.	16.	4.	40	10.	25.	15.	55	11.	23.	26.	54	1.	22.	7
1811	10.	21.	19.	30	4.	27.	51.	57	11.	1.	12.	27	10.	6.	54.	14	1.	22.	52
B. 1812	9.	8.	17.	49	5.	21.	1.	44	0.	27.	28.	3	9.	11.	55.	50	1.	23.	37
1813	1.	1.	46.	47	3.	2.	49.	1	1.	3.	24.	34	7.	25.	23.	10	1.	24.	22
1814	4.	25.	15.	46	0.	14.	36.	18	1.	9.	21.	6	6.	8.	50.	30	1.	25.	8
1815	8.	18.	44.	45	9.	26.	23.	35	1.	15.	17.	38	4.	22.	17.	50	1.	25.	53
B. 1816	7.	5.	43.	4	10.	19.	33.	21	3.	11.	33.	14	3.	27.	19.	26	1.	26.	38
1817	10.	29.	12.	2	8.	1.	20.	38	3.	17.	29.	45	2	10.	46.	46	1.	27.	23
1818	2.	22.	41.	1	5.	13.	7.	55	3.	23.	26.	17	0.	24.	14.	6	1.	28.	8
1819	6.	16.	10.	0	2.	24.	55.	13	3.	29.	22.	49	11.	7.	41.	26	1.	28.	53
B. 1820	5.	3.	8.	18	3.	18.	4.	59	5.	25.	38.	24	10.	12.	43.	2	1.	29.	39

THE MEAN MOTION OF JUPITER's SATELLITES FOR *MONTHS*.

TABLE II.

MONTHS	I. SATELLITE	II. SATELLITE	III. SATELLITE	IV. SATELLITE	IV. APS
31 January	6ˢ. 8°. 9′. 32″	8ˢ. 22°. 37′. 3″	3ˢ. 29°. 50′. 50″	10ˢ. 8°. 42′. 16″	3′. 50″
28 February	4. 5. 51. 2	7. 11. 6. 38	2. 28. 44. 29	6. 12. 41. 44	7. 18
31 March	10. 14. 0. 34	4. 3. 43. 40	6. 28. 35. 18	4. 21. 24. 0	11. 7
30 April	9. 28. 40. 46	9. 14. 58. 14	9. 8. 7. 4	2. 8. 32. 0	14. 56
31 May	4. 6. 50. 17	6. 7. 35. 16	1. 7. 57. 53	0. 17. 14. 16	18. 46
30 June	3. 21. 30. 29	11. 18. 49. 50	3. 17. 29. 40	10. 4. 22. 16	22. 22
31 July	9. 29. 40. 1	8. 11. 26. 52	7. 17. 20. 29	8. 13. 4. 32	26. 12
31 Auguſt	4. 7. 49. 32	5. 4. 3. 55	11. 17. 11. 19	6. 21. 46. 48	30. 2
30 September	3. 22. 29. 44	10. 15. 18. 28	1. 26. 43. 5	4. 8. 54. 48	33. 45
31 October	10. 0. 39. 15	7. 7. 55. 31	5. 26. 33. 55	2. 17. 37. 4	37. 35
30 November	9. 15. 19. 27	0. 19. 10. 4	8. 6. 5. 41	0. 4. 45. 4	41. 17
31 December	3. 23. 28. 59	9. 11. 47. 7	0. 5. 56. 31	10. 13. 27. 20	45. 7

THE MEAN MOTION OF JUPITER's SATELLITES FOR *DAYS*.

TABLE III.

DAYS	I.	II.	III.	IV.	IV. Aps.
	S. D. M. S.	S. D. M. S.	S. D. M. S.	S. D. M. S.	M. S.
1	6. 23. 29. 20	3. 11. 22. 29	1. 20. 19. 4	0. 21. 34. 16	0. 7
2	1. 16. 58. 41	6. 22. 44. 58	3. 10. 38. 7	1. 13. 8. 32	0. 15
3	8. 10. 28. 1	10. 4. 7. 27	5. 0. 57. 11	2. 4. 42. 48	0. 22
4	3. 3. 57. 22	1. 15. 29. 57	6. 21. 16. 14	2. 26. 17. 4	0. 30
5	9. 27. 26. 42	4. 26. 52. 26	8. 11. 35. 18	3. 17. 51. 20	0. 37
6	4. 20. 56. 2	8. 8. 14. 55	10. 1. 54. 21	4. 9. 25. 36	0. 44
7	11. 14. 25. 22	11. 19. 37. 24	11. 22. 13. 25	5. 0. 59. 52	0. 52
8	6. 7. 54. 43	3. 0. 59. 53	1. 12. 32. 28	5. 22. 34. 8	0. 59
9	1. 1. 24. 3	6. 12. 22. 22	3. 2. 51. 32	6. 14. 8. 24	1. 7
10	7. 24. 53. 23	9. 23. 44. 51	4. 23. 10. 35	7. 5. 42. 40	1. 14
11	2. 18. 22. 44	1. 5. 7. 21	6. 13. 29. 39	7. 27. 16. 56	1. 22
12	9. 11. 52. 4	4. 16. 29. 50	8. 3. 48. 42	8. 18. 51. 12	1. 29
13	4. 5. 21. 25	7. 27. 52. 19	9. 24. 7. 46	9. 10. 25. 28	1. 36
14	10. 28. 50. 45	11. 9. 14. 48	11. 14. 26. 50	10. 1. 59. 44	1. 44
15	5. 22. 20. 6	3. 20. 37. 17	1. 4. 45. 53	10. 23. 34. 0	1. 51
16	0. 15. 49. 26	6. 1. 59. 46	2. 25. 4. 57	11. 15. 8. 16	1. 59
17	7. 9. 18. 46	9. 13. 22. 15	4. 15. 23. 0	0. 6. 42. 32	2. 6
18	2. 2. 48. 6	0. 24. 44. 45	6. 5. 43. 4	0. 28. 16. 48	2. 13
19	8. 26. 17. 27	4. 6. 7. 14	7. 26. 2. 7	1. 19. 51. 4	2. 21
20	3. 19. 46. 47	7. 17. 29. 43	9. 16. 21. 11	2. 11. 25. 20	2. 28
21	10. 13. 16. 8	10. 28. 52. 12	11. 6. 40. 14	3. 2. 59. 36	2. 36
22	5. 6. 45. 29	2. 10. 14. 41	10. 26. 59. 18	3. 24. 33. 52	2. 43
23	0. 0. 14. 49	5. 21. 37. 10	2. 17. 18. 21	4. 16. 8. 8	2. 51
24	6. 23. 44. 9	9. 2. 59. 39	4. 7. 37. 25	5. 7. 42. 34	2. 58
25	1. 17. 13. 29	0. 14. 22. 9	5. 27. 56. 28	5. 29. 16. 50	3. 5
26	8. 10. 42. 50	3. 25. 44. 38	7. 18. 15. 32	6. 20. 50. 56	3. 13
27	3. 4. 12. 10	7. 7. 7. 7	9. 8. 34. 36	7. 12. 25. 12	3. 20
28	9. 27. 41. 30	10. 18. 29. 36	10. 28. 53. 39	8. 3. 59. 28	3. 28
29	4. 21. 10. 51	1. 29. 52. 5	0. 19. 12. 43	8. 25. 33. 44	3. 35
30	11. 14. 40. 11	5. 11. 14. 34	2. 9. 31. 46	9. 17. 8. 0	3. 42
31	6. 8. 9. 32	8. 22. 37. 3	3. 29. 50. 50	10. 8. 42. 16	3. 50

In leap-year, for *January* and *February* take one day leſs, for reaſons already given.

THE MEAN MOTIONS OF JUPITER's SATELLITES FOR *HOURS*.

TABLE IV.

HOURS	I.				II.				III.				IV.			
	S.	D.	M.	S.	S.	D.	M.	S.	S.	D.	M.	S.	S.	D.	M.	S
1	0.	8.	28.	43	0.	4.	13.	26	0.	2.	5.	48	0.	0.	53.	56
2	0.	16.	57.	27	0.	8.	26.	52	0.	4.	11.	35	0.	1.	47.	51
3	0.	25.	26.	10	0.	12.	40.	19	0.	6.	17.	23	0.	2.	41.	47
4	1.	3.	54.	53	0.	16.	53.	45	0.	8.	23.	11	0.	3.	35.	43
5	1.	12.	23.	36	0.	21.	7.	11	0.	10.	28.	58	0.	4.	29.	38
6	1.	20.	52.	20	0.	25.	20.	37	0.	12.	34.	46	0.	5.	23.	34
7	1.	29.	21.	3	0.	29.	34.	3	0.	14.	40.	33	0.	6.	17.	30
8	2.	7.	49.	46	1.	3.	47.	30	0.	16.	46.	21	0.	7.	11.	25
9	2.	16.	18.	30	1.	8.	0.	56	0.	18.	52.	9	0.	8.	5.	21
10	2.	24.	47.	13	1.	12.	14.	22	0.	20.	57.	56	0.	8.	59.	17
11	3.	3.	15.	56	1.	16.	27.	48	0.	23.	3.	44	0.	9.	53.	13
12	3.	11.	44.	40	1.	20.	41.	15	0.	25.	9.	32	0.	10.	47.	8
13	3.	20.	13.	24	1.	24.	54.	41	0.	27.	15.	19	0.	11.	41.	4
14	3.	28.	42.	7	1.	29.	8.	7	0.	29.	21.	7	0.	12.	35.	0
15	4.	7.	10.	51	2.	3.	21.	33	1.	1.	26.	55	0.	13.	28.	55
16	4.	15.	39.	34	2.	7.	34.	59	1.	3.	32.	42	0.	14.	22.	51
17	4.	24.	8.	17	2.	11.	48.	26	1.	5.	38.	30	0.	15.	16.	47
18	5.	2.	37.	0	2.	16.	1.	52	1.	7.	44.	18	0.	16.	10.	42
19	5.	11.	5.	43	2.	20.	15.	18	1.	9.	50.	5	0.	17.	4.	38
20	5.	19.	34.	27	2.	24.	28.	44	1.	11.	55.	53	0.	17.	58.	34
21	5.	28.	3.	10	2.	28.	42.	11	1.	14.	1.	41	0.	18.	52.	29
22	6.	6.	31.	53	3.	2.	55.	37	1.	16.	7.	28	0.	19	46.	25
23	6.	15.	0.	37	3.	7.	9.	3	1.	18.	13.	16	0.	20.	40.	21
24	6.	23.	29.	20	3.	11.	22.	29	1.	20.	19.	4	0.	21.	34.	16

THE MEAN MOTIONS OF JUPITER's SATELLITES FOR *MINUTES* AND *SECONDS*.

TABLE V.

M. S.	D. M. S. M. S. T.	D. M. S. M. S. T.	D. M. S. M. S. T.	D. M. S. M. S. T.
	I.	II.	III.	IV.
1	0. 8. 29	0. 4. 13	0. 2. 6	0. 0. 54
2	0. 16. 57	0. 8. 27	0. 4. 12	0. 1. 48
3	0. 25. 26	0. 12. 40	0. 6. 17	0. 2. 42
4	0. 33. 55	0. 16. 54	0. 8. 23	0. 3. 36
5	0. 42. 24	0. 21. 7	0. 10. 29	0. 4. 30
6	0. 50. 52	0. 25. 21	0. 12. 35	0. 5. 24
7	0. 59. 21	0. 29. 34	0. 14. 41	0. 6. 18
8	1. 7. 50	0. 33. 47	0. 16. 46	0. 7. 11
9	1. 16. 18	0. 38. 1	0. 18. 52	0. 8. 5
10	1. 24. 47	0. 42. 14	0. 20. 58	0. 8. 59
11	1. 33. 16	0. 46. 28	0. 23. 4	0. 9. 53
12	1. 41. 45	0. 50. 41	0. 25. 10	0. 10. 47
13	1. 50. 13	0. 54. 55	0. 27. 15	0. 11. 41
14	1. 58. 42	0. 59. 8	0. 29. 21	0. 12. 35
15	2. 7. 11	1. 3. 22	0. 31. 27	0. 13. 39
16	2. 15. 40	1. 7. 35	0. 33. 33	0. 14. 23
17	2. 24. 8	1. 11. 48	0. 35. 39	0. 15. 17
18	2. 32. 37	1. 16. 2	0. 37. 44	0. 16. 11
19	2. 41. 6	1. 20. 15	0. 39. 50	0. 17. 5
20	2. 49. 34	1. 24. 29	0. 41. 56	0. 17. 59
21	2. 58. 3	1. 28. 42	0. 44. 2	0. 18. 52
22	3. 6. 32	1. 32. 56	0. 46. 7	0. 19. 46
23	3. 15. 1	1. 37. 9	0. 48. 13	0. 20. 40
24	3. 23. 29	1. 41. 22	0. 50. 19	0. 21. 34
25	3. 31. 58	1. 45. 36	0. 52. 25	0. 22. 28
26	3. 40. 27	1. 49. 49	0. 54. 31	0. 23. 22
27	3. 48. 55	1. 54. 3	0. 56. 37	0. 24. 16
28	3. 57. 24	1. 58. 16	0. 58. 42	0. 25. 10
29	4. 5. 53	2. 2. 30	1. 0. 48	0. 26. 4
30	4. 14. 22	2. 6. 43	1. 2. 54	0. 26. 58

THE MEAN MOTIONS OF JUPITER's SATELLITES FOR *MINUTES* AND *SECONDS*.

THE FIFTH TABLE CONTINUED.

M. S.	D. M. S. M. S. T.	D. M. S. M. S. T.	D. M. S. M. S. T.	D. M. S. M. S. T.
	I.	II.	III.	IV.
31	4. 22. 50	2. 10. 57	1. 5. 0	0. 27. 52
32	4. 31. 19	2. 15. 10	1. 7. 6	0. 28. 46
33	4. 39. 48	2. 19. 23	1. 9. 11	0. 29. 40
34	4. 48. 17	2. 23. 37	1. 11. 17	0. 30. 33
35	4. 56. 45	2. 27. 50	1. 13. 23	0. 31. 27
36	5. 5. 14	2. 32. 4	1. 15. 29	0. 32. 21
37	5. 13. 43	2. 36. 17	1. 17. 35	0. 33. 15
38	5. 22. 11	2. 40. 31	1. 19. 40	0. 34. 9
39	5. 30. 40	2. 44. 44	1. 21. 46	0. 35. 3
40	5. 39. 9	2. 48. 57	1. 23. 52	0. 35. 57
41	5. 47. 48	2. 53. 11	1. 25. 58	0. 36. 51
42	5. 56. 6	2. 57. 24	1. 28. 4	0. 37. 45
43	6. 4. 35	3. 1. 38	1. 30. 9	0. 38. 39
44	6. 13. 4	3. 5. 51	1. 32. 15	0. 39. 33
45	6. 21. 32	3. 10. 5	1. 34. 21	0. 40. 27
46	6. 30. 1	3. 14. 18	1. 36. 27	0. 41. 21
47	6. 38. 30	3. 18. 31	1. 38. 33	0. 42. 15
48	6. 46. 59	3. 22. 45	1. 40. 38	0. 43. 9
49	6. 55. 27	3. 26. 58	1. 42. 44	0. 44. 2
50	7. 3. 56	3. 31. 12	1. 44. 50	0. 44. 56
51	7. 12. 25	3. 35. 25	1. 46. 56	0. 45. 50
52	7. 20. 54	3. 39. 39	1. 49. 2	0. 46. 44
53	7. 29. 22	3. 43. 52	1. 51. 7	0. 47. 38
54	7. 37. 51	3. 48. 6	1. 53. 13	0. 48. 32
55	7. 46. 20	3. 52. 19	1. 55. 19	0. 49. 26
56	7. 54. 48	3. 56. 32	1. 57. 25	0. 50. 20
57	8. 3. 17	4. 0. 46	1. 59. 31	0. 51. 14
58	8. 11. 46	4. 4. 59	2. 1. 36	0. 52. 8
59	8. 20. 15	4. 9. 12	2. 3. 42	0. 53. 4
60	8. 28. 43	4. 13. 26	2. 5. 48	0. 53. 56

APPARENT DISTANCES OF THE SATELLITES FROM THE CENTER OF JUPITER, IN SEMI-DIAMETERS OF JUPITER.

TABLE VI.

	DISTANCE OF THE SATELLITES FROM THE GEOCENTRIC PLACE OF JUPITER.												
Sig.	O. *East.*			VI. *West.*	I. *East.*			VII. *West.*	II. *East.*			VIII. *West.*	
Sat.	I.	I I	III	IV.	I.	II.	III.	IV.	I.	II.	III.	IV.	
Deg	Semid.	Semid	Semid	Semid.	Semid.	Semid.	Semid.	Semid.	Semid.	Semid.	Semid.	Semid.	Deg.
0	0,0	0,0	0,0	0,0	2,95	4,70	7,50	13,19	5,12	8,14	12,99	22,85	30
1	0,10	0,16	0,26	0,46	3,04	4,84	7,73	13,59	5,17	8,22	13,12	23,07	29
2	0,21	0,33	0,52	0,92	3,13	4,98	7,95	13,98	5,22	8,30	13,24	23,29	28
3	0,31	0,49	0,78	1,38	3,22	5,12	8,17	14,37	5,27	8,38	13,36	23,51	27
4	0,41	0,66	1,05	1,84	3,30	5,26	8,39	14,75	5,31	8,45	13,48	23,71	26
5	0,51	0,82	1,31	2,30	3,39	5,39	8,60	15,13	5,36	8,52	13,59	23,91	25
6	0,62	0,98	1,57	2,76	3,47	5,53	8,82	15,51	5,40	8,59	13,70	24,10	24
7	0,72	1,14	1,83	3,22	3,56	5,66	9,03	15,88	5,44	8,66	13,81	24,28	23
8	0,82	1,31	2,09	3,67	3,64	5,79	9,24	16,24	5,48	8,72	13,91	24,46	22
9	0,92	1,47	2,34	4,13	3,72	5,92	9,44	16,60	5,52	8,78	14,00	24,63	21
10	1,03	1,63	2,60	4,58	3,80	6,04	9,64	16,96	5,55	8,84	14,10	24,79	20
11	1,13	1,79	2,86	5,03	3,87	6,17	9,84	17,31	5,59	8,89	14,18	24.95	19
12	1,23	1,95	3,11	5,48	3,95	6,29	10,04	17,65	5,62	8,94	14,27	25,09	18
13	1,33	2,12	3,37	5,93	4,03	6,41	10,23	17,99	5,65	8,99	14,34	25,23	17
14	1,43	2,27	3,63	6,38	4,10	6,53	10,42	18,32	5,68	9,04	14,42	25,36	16
15	1,53	2,43	3,88	6,83	4,18	6,65	10,61	18,65	5,71	9,08	14,49	25,48	15
16	1,63	2,59	4,13	7,27	4,25	6,76	10,79	18,98	5,73	9,12	14,56	25,60	14
17	1,73	2,75	4,38	7,71	4,32	6,88	10,98	19,30	5,76	9,16	14,62	25,71	13
18	1,83	2,91	4,63	8,15	4,39	6,99	11,15	19,61	5,78	9,20	14,67	25,81	12
19	1,92	3,06	4,88	8,59	4,46	7,09	11,32	19,91	5,80	9,23	14,72	25,90	11
20	2,02	3,22	5,13	9,02	4,53	7,20	11,49	20,21	5,82	9,26	14,77	25,98	10
21	2,12	3,37	5,37	9,45	4,59	7,31	11,66	20,50	5,83	9,29	14,81	26,06	9
22	2.22	3,52	5,62	9,88	4,66	7,41	11,82	20,79	5,85	9,31	14,85	26,13	8
23	2,31	3,67	5,86	10,31	4,72	7,51	11,98	21,07	5,87	9,33	14,89	26,19	7
24	2,40	3,82	6,11	10,73	4,78	7,61	12,14	21,34	5,88	9,35	14,92	26,24	6
25	2,50	3,97	6,34	11,15	4,84	7,70	12,29	21,61	5,89	9,37	14,94	26,28	5
26	2,59	4,12	6,57	11,57	4,90	7,80	12,44	21,87	5,89	9,38	14,96	26,32	4
27	2,68	4,27	6,81	11,98	4,96	7,89	12,58	22 13	5,90	9,39	14,98	26,35	3
28	2,77	4,41	7,04	12,39	5,01	7,97	12,72	22,37	5,91	9,40	14,99	26,37	2
29	2,86	4,56	7,27	12,79	5,07	8,06	12,86	22,61	5,91	9,40	15,00	26,38	1
30	2,95	4,70	7,50	13,19	5,12	8,14	12,99	22,85	5,91	9,40	15,00	26,38	0
Sig.	XI. *West.*			V. *East.*	X. *West.*			IV. *East.*	IX. *West.*			III. *East.*	

THE *FIRST* EQUATION OF LIGHT.

TABLE VII.

ARGUMENT. DISTANCE OF JUPITER FROM THE SUN.							
Deg.	Sig. O.	Sig. I.	Sig. II.	Sig. III.	Sig. IV.	Sig. V.	Deg.
0	16′. 15″	15′. 10″	12′. 12″	8′. 7″	4′. 3″	1′. 5″	30
1	16. 15	15. 6	12. 5	7. 59	3. 56	1. 1	29
2	16. 15	15. 1	11. 57	7. 51	3. 49	0. 57	28
3	16. 14	14. 56	11. 49	7. 42	3. 42	0. 53	27
4	16. 14	14. 52	11. 41	7. 34	3. 35	0. 49	26
5	16. 13	14. 47	11. 33	7. 25	3. 28	0. 45	25
6	16. 12	14. 42	11. 25	7. 16	3. 21	0. 42	24
7	16. 11	14. 37	11. 17	7. 8	3. 14	0. 38	23
8	16. 10	14. 32	11. 9	6. 59	3. 7	0. 36	22
9	16. 9	14. 27	11. 2	6. 51	3. 0	0. 33	21
10	16. 7	14. 21	10. 55	6. 42	2. 53	0. 30	20
11	16. 5	14. 15	10. 47	6. 34	2. 47	0. 27	19
12	16. 4	14. 9	10. 38	6. 26	2. 41	0. 24	18
13	16. 2	14. 3	10. 30	6. 18	2. 35	0. 22	17
14	16. 0	13. 58	10. 22	6. 10	2. 29	0. 19	16
15	15. 58	13. 52	10. 14	6, 1	2. 23	0. 17	15
16	15. 56	13. 46	10. 6	5. 53	2. 18	0. 15	14
17	15. 54	13. 40	9. 57	5. 45	2. 12	0. 13	13
18	15. 51	13. 34	9. 48	5. 37	2. 6	0. 11	12
19	15. 48	13. 28	9. 40	5. 29	2. 0	0. 10	11
20	15. 45	13. 22	9. 32	5. 21	1. 54	0. 8	10
21	15. 42	13. 15	9. 24	5. 13	1. 48	0. 6	9
22	15. 39	13. 8	9. 16	5. 6	1. 43	0. 5	8
23	15. 36	13. 1	9. 8	4. 58	1. 38	0. 4	7
24	15. 33	12. 54	8. 59	4. 50	1. 33	0. 3	6
25	15. 30	12. 47	8. 50	4. 42	1. 28	0. 2	5
26	15. 26	12. 40	8. 41	4. 34	1. 23	0. 1	4
27	15. 22	12. 33	8. 32	4. 26	1. 19	0. 1	3
28	15. 18	12. 26	8. 23	4. 18	1. 14	0. 0	2
29	15. 14	12. 19	8. 15	4. 11	1. 9	0. 0	1
30	15. 10	12. 12	8. 7	4. 3	1. 5	0. 0	0
	Sig. XI.	Sig. X.	Sig. IX.	Sig. VIII.	Sig. VII.	Sig. VI.	

THE *SECOND* EQUATION OF LIGHT.

TABLE VIII.

ARGUMENT. ANOMALY OF JUPITER.							
Deg.	Sig. O.	Sig. I.	Sig. II.	Sig. III.	Sig. IV.	Sig. V.	Deg.
0	4′. 4″	3′. 47″	3′. 3″	2′. 2″	1′. 1″	0′. 16″	30
1	4. 4	3. 46	3. 1	2. 0	0. 59	0. 15	29
2	4. 4	3. 45	2. 59	1. 58	0. 57	0. 14	28
3	4. 4	3. 44	2. 57	1. 56	0. 55	0. 13	27
4	4. 4	3. 43	2. 55	1. 53	0. 54	0 12	26
5	4. 3	3. 42	2. 53	1. 51	0. 52	0. 12	25
6	4. 3	3. 41	2. 51	1. 49	0. 51	0. 11	24
7	4. 3	3. 39	2. 49	1. 47	0. 49	0. 10	23
8	4. 3	3. 38	2. 47	1. 45	0. 47	0. 9	22
9	4. 2	3. 37	2. 45	1. 43	0. 46	0. 8	21
10	4. 2	3. 35	2. 44	1. 41	0. 44	0. 7	20
11	4. 2	3. 34	2. 42	1. 39	0. 43	0. 6	19
12	4. 1	3. 32	2. 40	1. 37	0. 41	0. 6	18
13	4. 0	3. 31	2. 38	1. 35	0. 40	0. 5	17
14	4. 0	3. 30	2. 36	1. 33	0. 38	0. 5	16
15	4. 0	3. 28	2. 34	1. 30	0. 36	0. 4	15
16	3. 59	3. 27	2. 32	1. 28	0. 35	0. 4	14
17	3. 59	3. 25	2. 29	1. 26	0. 33	0. 3	13
18	3. 58	3. 24	2. 27	1. 24	0. 32	0. 3	12
19	3. 57	3. 22	2. 25	1. 22	0. 30	0. 2	11
20	3. 56	3. 21	2. 23	1. 20	0. 29	0. 2	10
21	3 55	3. 20	2. 21	1. 18	0. 27	0. 1	9
22	3. 55	3. 17	2. 19	1. 16	0. 26	0. 1	8
23	3. 54	3. 15	2. 17	1. 14	0. 25	0. 1	7
24	3. 53	3. 13	2. 15	1. 12	0. 23	0. 1	6
25	3. 52	3. 12	2. 13	1. 10	0. 22	0. 0	5
26	3. 51	3. 10	2. 10	1. 8	0: 21	0. 0	4
27	3. 50	3. 8	2. 8	1. 7	0. 20	0. 0	3
28	3. 49	3. 6	2. 6	1. 5	0. 19	0. 0	2
29	3. 48	3. 5	2. 4	1. 3	0. 17	0. 0	1
30	3. 47	3. 3	2. 2	1. 1	0. 16	0. 0	0
	Sig. XI.	Sig. X.	Sig. IX.	Sig. VIII	Sig. VII.	Sig. VI.	

EQUATION OF THE CENTER OF THE *FOURTH* SATELLITE.

TABLE IX.

ARGUMENT. MEAN ANOMALY.				
Deg.	+VI – O.	+VII – I.	+VIII – II.	Deg.
0	0′. 0″	24′. 57″	43′. 22″	30
1	0. 52	25. 42	43. 48	29
2	1. 44	26. 26	44. 14	28
3	2. 37	27. 10	44. 39	27
4	3. 29	27. 54	45. 3	26
5	4. 21	28. 37	45. 26	25
6	5. 13	29. 20	45. 48	24
7	6. 5	30. 2	46. 9	23
8	6. 57	30. 44	46. 30	22
9	7. 48	31. 25	46. 50	21
10	8. 39	32. 6	47. 9	20
11	9. 31	32. 46	47. 26	19
12	10. 22	33. 25	47. 42	18
13	11. 13	34. 4	47. 58	17
14	12. 4	34. 43	48. 13	16
15	12. 55	35. 21	48. 28	15
16	13. 45	35. 57	48. 42	14
17	14. 36	36. 33	48. 56	13
18	15. 26	37. 8	49. 7	12
19	16. 15	37. 43	49. 18	11
20	17. 4	38. 18	49. 28	10
21	17. 53	38. 52	49. 37	9
22	18. 41	39. 25	49. 46	8
23	19. 29	39. 57	49. 54	7
24	20. 17	40. 28	50. 1	6
25	21. 5	40. 59	50. 6	5
26	21. 52	41. 29	50. 9	4
27	22. 39	41. 58	50. 14	3
28	23. 26	42. 27	50. 17	2
29	24. 12	42. 55	50. 19	1
30	24. 57	43. 22	50. 20	0
	+XI – V.	+X – IV.	+IX – III.	

On the Satellites of Saturn.

471. In the year 1655, HUYGENS diſcovered the fourth ſatellite of *Saturn*; and publiſhed a Table of it's mean motion in 1659. In 1671, M. CASSINI diſcovered the fifth, and the third in 1672; and in 1684, the firſt and ſecond; and afterwards publiſhed Tables of their motions. He called them *Sidera Lodoicea*, in honour of LOUIS le GRAND, in whoſe reign, and obſervatory, they were firſt diſcovered. Dr. HALLEY found by his own obſervations in 1682, that HUYGENS's Tables had conſiderably run out, they being about 15° in 20 years too forward, and therefore he compoſed new Tables from more correct elements. He alſo reformed M. CASSINI's Tables of the mean motions; and about the year 1720, publiſhed them a ſecond time, corrected from Mr. POUND's obſervations. He obſerves, that the four innermoſt ſatellites deſcribe orbits very nearly in the plane of the ring, which he ſays is, as to ſenſe, parallel to our equator; and that the orbit of the fifth is a little inclined to them. The following Table contains the periodic times of the five ſatellites, and their diſtances in ſemidiameters of the ring, as determined by Mr. POUND, by a micrometer fitted to the teleſcope given by HUYGENS to the Royal Society. Mr. POUND firſt meaſured the diſtance of the fourth, and then deduced the reſt from the proportion between the ſquares of the periodic times and cubes of their diſtances, and theſe are found to agree with obſervations.

Satellites.	Periodic Times by POUND.	Diſt. in ſemid. of Ring by POUND.	Diſt. in ſemid. of Saturn by POUND.	Diſt. in ſemid. of Ring by CASSINI.	Diſt. at the mean diſt. of Saturn.
I	$1^d.\ 21^h.\ 18'.\ 27''$	2,097	4,893	$1\ \frac{14}{15}$	$0'.\ 43'',5$
II	2. 17. 41. 22	2,686	6,286	$2\ \frac{1}{2}$	0. 56
III	4. 12. 25. 12	3,752	8,754	$3\ \frac{1}{2}$	1. 18
IV	15. 22. 41. 12	8,698	20,295	8	3. 0
V	79. 7. 49. 0	25,348	59,154	23	8. 42,5

The laſt column is from CASSINI; but Dr. HERSCHEL makes the diſtance of the fifth to be $8'.\ 31'',97$, which is probably more exact. In this and the two next Tables, the ſatellites are numbered from Saturn as they were before the diſcovery of the other two.

On

On June 9, 1749, at 10*h*. Mr. POUND found the diſtance of the fourth ſatellite to be 3′. 7″ with a teleſcope of 123 feet and an excellent micrometer fixed to it; and the ſatellite was at that time very near it's greateſt eaſtern digreſſion. Hence at the mean diſtance of the earth from Saturn that diſtance becomes 2′. 58″,21; Sir I. NEWTON makes it 3′. 4″.

472. The periodic times are found as for the ſatellites of Jupiter (409). To determine theſe, M. CASSINI choſe the time when the ſemi-minor axes of the ellipſes which they deſcribe were the greateſt, as Saturn was then 90° from their node, becauſe the place of the ſatellite in it's orbit is then the ſame as upon the orbit of Saturn; whereas in every other caſe it would be neceſſary to apply the reduction (426) in order to get the place in it's orbit.

473. As it is difficult to ſee Saturn and the ſatellites at the ſame time in the field of view of a teleſcope, their diſtances have ſometimes been meaſured by obſerving the time of the paſſage of the body of Saturn over a wire adjuſted as an hour circle in the field of the teleſcope, and the interval between the times when Saturn and the ſatellite paſſed. From comparing the periodic times and diſtances, M. CASSINI obſerved that KEPLER'S Rule (218) agreed very well with obſervations.

474. By comparing the ſatellites with the ring in different points of their orbits, and the greateſt minor axes of the ellipſes which they appear to deſcribe compared with the major axes, the planes of the orbits of the firſt four are found to be very nearly in the plane of the ring, and therefore are inclined to the orbit of Saturn about 30°; but the orbit of the fifth, according to M. CASSINI the Son, makes an angle with the ring of about 15°.

475. M. CASSINI places the node of the ring, and conſequently thoſe of the four firſt ſatellites, in 5ˢ. 22° upon the orbit of Saturn, and 5ˢ. 21° upon the ecliptic. M. HUYGENS had determined it to be in 5ˢ. 20°. 30′. M. MARALDI in 1716 determined the longitude of the node of the ring upon the orbit of Saturn to be 5ˢ. 19°. 48′. 30″; and upon the ecliptic to be 5ˢ. 16°. 20′. The node of the fifth ſatellite is placed by M. CASSINI in 5ˢ. 5° upon the orbit of Saturn. M. de la LANDE makes it 5ˢ. 0°. 27′. From the obſervation of M. BERNARD at Marſeilles in 1787, it appears that the node of this ſatellite is retrograde.

476. Dr. HALLEY diſcovered that the orbit of the fourth ſatellite was excentric. For having found it's mean motion, he diſcovered that it's place by obſervation was at one time 3° forwarder than by his calculations, and at other obſervations it was 2°. 30′ behind; this indicated an excentricity; and he placed the line of the apſides in 10ˢ. 22°. *Phil. Tranſ.* N°. 145.

TABLES OF THEIR REVOLUTIONS AND MEAN MOTIONS, ACCORDING TO M. DE LA LANDE.

Satel.	Diurnal Motion	Motion in 365 days
I	6°. 10°. 41′. 53″	4°. 4°. 44′. 42″
II	4. 11. 32. 6	4. 10. 15. 19
III	2. 19. 41. 25	9. 16. 57. 5
IV	0. 22. 34. 38	10. 20. 39. 37
V	0. 4. 32. 17	7. 6. 23. 37

Satel.	Periodic Revolution	Synodic Revolution
I	1^{d}. 21^{h}. 18′. 26″,222	1^{d}. 21^{h}. 18′. 54″,778
II	2. 17. 44. 51,177	2. 17. 45. 51,013
III	4. 12. 25. 11,100	4. 12. 27. 55,239
IV	15. 22. 41. 16,022	15. 23. 15. 23,153
V	79. 7. 53. 42,772	79. 22. 3. 12,883

477. M. CASSINI obſerved that the fifth ſatellite diſappeared regularly for about half it's revolution, when it was to the eaſt of Saturn; from which he concluded, that it revolved about it's axis; he afterwards however doubted of this. But Sir I. NEWTON in his *Principia*, Lib. III. Prop. 17, concludes from hence, that it revolves about it's axis, and in the ſame time that it revolves about Saturn; and that the variable appearance ariſes from ſome parts of the ſatellite not reflecting ſo much light as others. Dr. HERSCHEL has confirmed this, by tracing regularly the periodical change of light through more than 10 revolutions, and finding it, in all appearances, to be cotemporary with the return of the ſatellite to the ſame ſituation in it's orbit. This is further confirmed by ſome obſervations of M. BERNARD at Marſeilles in 1787; and is a remarkable inſtance of analogy among the ſecondary planets.

478. Theſe

478. Theſe are all the ſatellites which were known to revolve about Saturn till the year 1789, when Dr. HERSCHEL, in a Paper in the *Phil. Tranſ.* for that year, announced the diſcovery of a *ſixth* ſatellite, interior to all the others, and promiſed a further account in another paper. But in the intermediate time he diſcovered a *ſeventh* ſatellite, interior to the ſixth; and in a Paper upon Saturn and it's ring, in the *Phil. Tranſ.* 1790, he has given an account of the diſcovery, with ſome of the elements of their motions. He afterwards added Tables of their motions.

479. After his obſervations upon the ring, he ſays, he cannot quit the ſubject without mentioning his own ſurmiſes, and that of ſeveral other Aſtronomers, of a ſuppoſed roughneſs of the ring, or inequality in the planes and inclinations of it's flat ſides. This ſuppoſition aroſe, from ſeeing luminous points on it's boundaries projecting like the moon's mountains; or from ſeeing one arm brighter or longer than another; or even from ſeeing one arm when the other was inviſible. Dr. HERSCHEL was of this opinion, till he ſaw one of theſe points move off the edge of the ring in the form of a ſatellite. With his 20 feet teleſcope he ſuſpected that he ſaw a ſixth ſatellite; and on Auguſt 19, 1787, marked it down as probably being one; and having finiſhed his teleſcope of 40 feet focal length, he ſaw ſix of it's ſatellites the moment he directed his teleſcope to the planet. This happened on Auguſt 28, 1789. The retrograde motion of Saturn was then nearly $4'.\ 30''$ in a day, which made it very eaſy to aſcertain, whether the ſtars he took to be ſatellites were really ſo; and in about two hours and an half after, he found that the planet had viſibly carried them all away from their places. He continued his obſervations, and on September 17, he diſcovered the ſeventh ſatellite. Theſe two ſatellites lie within the orbits of the other five. Their diſtances from the center of Saturn are $36'',7889$, and $28'',6689$; and their periodic times are $1d.\ 8h.\ 53'.\ 8'',9$ and $22h.\ 37'.\ 22'',9$. The orbits of theſe ſatellites lie ſo near to the plane of the ring, that the difference cannot be perceived.

480. As ſoon as he had made obſervations ſufficient to conſtruct Tables of their mean motions, he calculated their places backwards, and found that his ſuſpicions of the exiſtence of theſe ſatellites, in the ſhape of protuberant points on the arms of the ring, were confirmed, and this ſerved to correct the Tables. He has alſo conſtructed Tables of the motions of the other five ſatellites; the epochs he deduced from his own obſervations, which differ conſiderably from thoſe given by M. de la LANDE, in the *Connoiſſance des Temps*, for 1791; but he aſſumed the mean motions the ſame as there given. The following Tables of the epochs and mean motions are given by Dr. HERSCHEL in the *Phil. Tranſ.* for 1790. The ſatellites are here numbered in their order from Saturn.

EPOCHS

EPOCHS OF THE MEAN LONGITUDES OF SATURN's SATELLITES.

* *	VII.	VI.	V.	IV.	III.	II.	I.
Years.	Deg. dec.	Deg. dec.	Deg. dec.	Deg. dec.	Deg. dec.	Deg. dec.	Deg. dec.
1787	335,91	149,16	87,21	272,18	176,46	269,31	307,07
1788	196,84	132,41	93,86	173,95	131,91	307,48	65,02
1789	53,23	93,09	20,82	304,19	256,66	82,92	161,00
1790	269,63	53,77	307,78	74,43	21,41	218,36	256,98
1791	126,02	14,45	234,74	204,68	146,16	353,81	352 97

THE MOTION OF THE SATELLITES ABOUT SATURN IN *MONTHS.*

* * *	VII.	VI.	V.	IV.	III.	II.	I.
Months.	Deg. dec.	Deg. dec.	Deg. dec.	Deg. dec.	Deg. dec.	Deg. dec.	Deg. dec.
January	000,00	000,00	000,00	000,00	000,00	000,00	000,00
February	140,68	339,89	310,40	117,58	151,64	224,54	320,81
March	267,75	252,05	21,73	200,56	91,18	20,91	215,73
April	48,43	231,95	332,13	318,14	242,81	245,45	176,54
May	184.57	189,26	202,84	304,19	203,75	207,27	115,39
June	325,25	169,16	153,24	61,77	355,39	71,81	76,20
July	101,39	126,47	23,94	47,82	316,33	33,63	15,05
Auguſt	242,07	106,37	334,34	165,40	107,96	258,17	335,86
September	22,75	86,26	284,74	282,98	259,60	122,72	296,67
October	158,89	43,58	155,45	269,03	220,54	84,54	235,52
November	299,57	23,47	105,85	26,61	12,17	309,08	196,33
December	75,71	340,78	336,56	12,66	333,11	270,90	135,17

In the months *January* and *February* of a biſſextile year, ſubtract 1 from the number of days given.

THE MOTION OF SATURN's SATELLITES IN *DAYS*.

DAYS	VII.	VI.	V.	IV.	III.	II.	I.
	Deg. dec.	Deg. dec.	Deg. dec.	Deg. dec.	Deg. dec.	Deg. dec.	Deg. dec.
1	4,54	22,58	79,69	131,53	190,70	262,73	21,96
2	9,08	45,15	159,38	263,07	21,40	165,45	43,92
3	13,61	67,73	239,07	34,60	212,09	68,18	65,88
4	18,15	90,31	318,76	166,14	42,79	330,91	87,85
5	22,69	112,89	38,45	297,67	233,49	233,64	109,81
6	27,23	135,46	118,14	69,21	64,19	136,36	131,77
7	31,77	158,04	197,83	200,74	254,89	39,09	153,73
8	36,30	180,62	277,52	332,28	85,58	301,82	175,69
9	40,84	203,19	357,21	103,81	276,28	204,55	197,65
10	45,38	225,77	76,90	235,35	106,98	107,27	219,62
11	49,92	248,35	156,59	6,88	297,68	10,00	241,58
12	54,46	270,93	236,28	138,42	128,38	272,73	263,54
13	58,99	293,50	315,97	269,95	319,07	175,45	285,50
14	63,53	316,08	35,66	41,49	149,77	78,18	307,46
15	68,07	338,66	115,35	173,02	340,47	340,91	329,42
16	72,61	1,24	195,04	304,56	171,17	243,64	351,39
17	77,15	23,81	274,74	76,09	1,87	146,36	13,35
18	81,69	46,39	354,43	207,63	192,56	49,09	35,31
19	86,22	68,97	74,12	339,16	23,26	311,82	57,27
20	90,76	91,54	153,81	110,70	213,96	214,54	79,23
21	95,30	114,12	233,50	242,23	44,66	117,27	101,19
22	99,84	136,70	313,19	13,77	235,35	20,00	123,16
23	104,38	159,28	32,88	145,30	66,05	282,73	145,12
24	108,91	181,85	112,57	276,84	256,75	185,45	167,08
25	113,45	204,43	192,26	48,37	87,45	88,18	189,04
26	117,99	227,01	271,95	179,91	278,15	350,91	211,00
27	122,53	249,58	351,64	311,44	108,84	253,64	232,96
28	127,07	272,16	71,33	82,98	299,54	156,36	254,92
29	131,60	294,74	151,02	214,51	130,24	59,09	276,89
30	136,14	317,32	230,71	346,05	320,94	321,82	298,85
31	140,68	339,89	310,40	117,58	151,64	224,54	320,81

THE MOTION OF SATURN's SATELLITES IN *HOURS*.

HOURS	VII.	VI.	V.	IV.	III.	II.	I.
	Deg. dec.	Deg. dec.	Deg. dec.	Deg. dec.	Deg. dec.	Deg. dec.	Deg. dec.
1	0,19	0,94	3,32	5,48	7,95	10,95	15,92
2	0,38	1,88	6,64	10,96	15,89	21,89	31,83
3	0,57	2,82	9,96	16,44	23,84	32,84	47,75
4	0,76	3,76	13,28	21,92	31,78	43,79	63,66
5	0,95	4,70	16,60	27,40	39,73	54,73	79,58
6	1,13	5,64	19,92	32,88	47,67	65,68	95,49
7	1,32	6,58	23,24	38,36	55,62	76,63	111,41
8	1,51	7,53	26,56	43,84	63,57	87,58	127,32
9	1,70	8,47	29,88	49,33	71,51	98,52	143,24
10	1,89	9,41	33,20	54,81	79,46	109,47	159,15
11	2,08	10,35	36,52	60,29	87,40	120,42	175,07
12	2,27	11,29	39,84	65,77	95,35	131,36	190,98
13	2,46	12,23	43,17	71,25	103,29	142,31	206,90
14	2,65	13,17	46,49	76,73	111,24	153,26	222,81
15	2,84	14,11	49,81	82,21	119,19	164,20	238,73
16	3,03	15,05	53,13	87,69	127,13	175,15	254,64
17	3,21	15,99	56,45	93,17	135,08	186,10	270,56
18	3,40	16,93	59,77	98,65	143,02	197,05	286,47
19	3,59	17,87	63,09	104,13	150,97	207,99	302,39
20	3,78	18,81	66,41	109,61	158,91	218,94	318,30
21	3,97	19,75	69,73	115,09	166,86	229,89	334,22
22	4,16	20,70	73,05	120,57	174,81	240,83	350,13
23	4,35	21,64	76,37	126,05	182,75	251,78	6,05
24	4,54	22,58	79,69	131,53	190,70	262,73	21,96

THE MOTION OF SATURN's SATELLITES IN *MINUTES*.

MINUTES	VII.	VI.	V.	IV.	III.	II.	I.
	Deg. dec.	Deg. dec.	Deg. dec.	Deg. dec.	Deg. dec.	Deg. dec.	Deg. dec.
1	0,00	0,02	0,06	0,09	0,13	0,18	0,27
2	0,01	0,03	0,11	0,18	0,26	0,36	0,53
3	0,01	0,05	0,17	0,27	0,40	0,55	0,80
4	0,01	0,06	0,22	0,37	0,53	0,73	1,06
5	0,02	0,08	0,28	0,46	0,66	0,91	1,33
6	0,02	0,09	0,33	0,55	0,79	1,09	1,59
7	0,02	0,11	0,39	0,64	0,93	1,28	1,86
8	0,03	0,13	0,44	0,73	1,06	1,46	2,12
9	0,03	0,14	0,50	0,82	1,19	1,64	2,39
10	0,03	0,16	0,55	0,91	1,32	1,82	2,65
11	0,04	0,17	0,61	1,00	1,46	2,01	2,92
12	0,04	0,19	0,66	1,10	1,59	2,19	3,18
13	0,04	0,20	0,72	1,19	1,72	2,37	3,45
14	0,05	0,22	0,77	1,28	1,85	2,55	3,71
15	0,05	0,24	0,83	1,37	1,99	2,74	3,98
16	0,05	0,25	0,89	1,46	2,12	2,92	4,24
17	0,06	0,27	0,94	1,55	2,25	3,10	4,51
18	0,06	0,28	1,00	1,64	2,38	3,28	4,78
19	0,06	0,30	1,05	1,73	2,52	3,47	5,04
20	0,07	0,31	1,11	1,83	2,65	3,65	5,31
21	0,07	0,33	1,16	1,92	2,78	3,83	5,57
22	0,07	0,34	1,22	2,01	2,91	4,01	5,84
23	0,08	0,36	1,27	2,10	3,05	4,20	6,10
24	0,08	0,38	1,33	2,19	3,18	4,38	6,37
25	0,08	0,39	1,38	2,28	3,31	4,56	6,63
26	0,09	0,41	1,44	2,37	3,44	4,74	6,90
27	0,09	0,42	1,49	2,47	3,57	4,93	7,16
28	0,09	0,44	1,55	2,56	3,71	5,11	7,43
29	0,10	0,45	1,60	2,65	3,84	5,29	7,69
30	0,10	0,47	1,66	2,74	3,97	5,47	7,96

THE MOTION OF SATURN's SATELLITES IN *MINUTES*.

MINUTES	VII.	VI.	V.	IV.	III.	II.	I.
	Deg. dec.	Deg. dec.	Deg. dec.	Deg. dec.	Deg. dec.	Deg. dec.	Deg. dec.
31	0,10	0,49	1,72	2,83	4,10	5,66	8,22
32	0,11	0,50	1,77	2,92	4,24	5,84	8,49
33	0,11	0,52	1,83	3,01	4,37	6,02	8,75
34	0,11	0,53	1,88	3,10	4,50	6,20	9,02
35	0,12	0,55	1,94	3,20	4,63	6,39	9,29
36	0,12	0,56	1,99	3,29	4,77	6,57	9,55
37	0,12	0,58	2,05	3,38	4,90	6,75	9,82
38	0,13	0,60	2,10	3,47	5,03	6,93	10,08
39	0,13	0,61	2,16	3,56	5,16	7,12	10,35
40	0,13	0,63	2,21	3,65	5,30	7,30	10,61
41	0,14	0,64	2,27	3,74	5,43	7,48	10,88
42	0,14	0,66	2,32	3,83	5,56	7,66	11,14
43	0,14	0,67	2,38	3,93	5,69	7,85	11,41
44	0,15	0,69	2,43	4,02	5,83	8,03	11,67
45	0,15	0,71	2,49	4,11	5,96	8,21	11,94
46	0,15	0,72	2,55	4,20	6,09	8,39	12,20
47	0,16	0,74	2,60	4,29	6,22	8,58	12,47
48	0,16	0,75	2,66	4,38	6,36	8,76	12,73
49	0,16	0,77	2,71	4,47	6,49	8,94	13,00
50	0,17	0,78	2,77	4,57	6,62	9,12	13,27
51	0,17	0,80	2,82	4,66	6,75	9,30	13,53
52	0,17	0,82	2,88	4,75	6,88	9,49	13,80
53	0,17	0,83	2,93	4,84	7,02	9,67	14,06
54	0,18	0,85	2,99	4,93	7,15	9,85	14,33
55	0,18	0.86	3,04	5,02	7,28	10,03	14,59
56	0,18	0,88	3,10	5,11	7,41	10,22	14,86
57	0,19	0,89	3,15	5,20	7,55	10,40	15,12
58	0,19	0,91	3,21	5,30	7,68	10,58	15,39
59	0,19	0,93	3,27	5,39	7,81	10,76	15,65
60	0,20	0,94	3,32	5,48	7,94	10,95	15,92

For the motion in *Seconds*, for Deg. dec. read Min. dec.

On theſe Tables, Dr. HERSCHEL makes the following obſervations. "I have not attempted to extend them farther than a few years backwards or forwards, as I am not in poſſeſſion of any obſervations that could authorize me to undertake ſuch a work. On the contrary, I am well convinced, that no Tables will give us the ſituation of the ſatellites accurately, till we have at leaſt eſtabliſhed the dimenſions of their elliptical orbits, and the motion as well as the ſituation of their aphelia. The epochs for 1789, therefore, muſt be looked upon not as *mean* ones, but ſuch as reſpect the orbits of theſe ſatellites in their ſituation during the time of the following obſervations; and the two preceding, and two following years, muſt be already a little affected with thoſe errors which are the neceſſary conſequence of our not knowing the required elements."

On the Configuration of Saturn's Satellites.

481. The following Tables, which were conſtructed and calculated by the Rev. Dr. MASKELYNE, *Aſtronomer Royal*, furniſh a very ſhort and eaſy method of finding the poſition of the ſatellites at any time.

TABLE I.

The Latitude and Reduction of Saturn's Satellites.

ARGUMENT.— { Geocentric Longitude of ♄ — ☊ of the Satellite's orbit. Or the Distance of the Apogee from ☊ + 3 Signs.

S.	O. *North.* VI. *South.*				I. *North.* VII. *South.*				II. *North.* VIII. *South.*				S.
	Satellites I. II. III. IV. V. VI.		Satellite VII.		Satellites I. II. III. IV. V. VI.		Satellite VII.		Satellites I. II. III. IV. V. VI.		Satellite VII.		
D.	Lat.	Red.	Lat.	Red.	Lat.	Red.	Lat.	Red.	Lat.	Red.	Lat.	Red.	D.
	D. M.	D. M.	D. M	D. M.	D. M.	D. M.	D. M.	D. M.	D. M.	D. M.	D. M.	D. M.	
0	0. 0	0. 0	0. 0	0. 0	14. 29	3. 26	7. 26	0. 51	25. 39	3. 41	12. 57	0. 52	30
1	0. 30	0. 8	0. 15	0. 2	14. 55	3. 31	7. 40	0. 52	25. 56	3. 37	13. 5	0. 51	29
2	1. 0	0. 16	0. 31	0. 4	15. 22	3. 35	7. 53	0. 53	26. 12	3. 33	13. 13	0. 50	28
3	1. 30	0. 24	0. 46	0. 6	15. 48	3. 39	8. 6	0. 54	26. 27	3. 28	13. 20	0. 49	27
4	2. 0	0. 32	1. 2	0. 8	16. 14	3. 42	8. 19	0. 55	26. 42	3. 23	13. 27	0. 47	26
5	2. 30	0. 40	1. 17	0. 10	16. 40	3. 46	8. 32	0. 55	26. 57	3. 18	13. 34	0. 46	25
6	3. 0	0. 48	1. 33	0. 12	17. 5	3. 49	8. 45	0. 56	27. 11	3. 12	13. 41	0. 45	24
7	3. 30	0. 56	1. 48	0. 14	17. 31	3. 52	8. 58	0. 57	27. 24	3. 7	13. 47	0. 44	23
8	3. 59	1. 4	2. 4	0. 16	17. 56	3. 55	9. 10	0. 57	27. 37	3. 1	13. 53	0. 42	22
9	4. 29	1. 12	2. 19	0. 18	18. 21	3. 58	9. 22	0. 58	27. 49	2. 55	13. 59	0. 41	21
10	4. 59	1. 19	2. 34	0. 20	18. 45	4. 0	9. 35	0. 58	28. 2	2. 48	14. 4	0. 39	20
11	5. 28	1. 27	2. 50	0. 22	19. 9	4. 2	9. 47	0. 59	28. 13	2. 41	14. 10	0. 37	19
12	5. 58	1. 34	3. 5	0. 24	19. 33	4. 3	9. 58	0. 59	28. 24	2. 34	14. 15	0. 35	18
13	6. 27	1. 42	3. 20	0. 26	19. 56	4. 5	10. 10	0. 59	28. 34	2. 27	14. 20	0. 34	17
14	6. 57	1. 49	3. 35	0. 27	20. 20	4. 6	10. 21	0. 59	28. 44	2. 19	14. 24	0. 32	16
15	7. 26	1. 56	3. 50	0. 29	20. 43	4. 6	10. 33	0. 59	28. 53	2. 12	14. 28	0. 30	15
16	7. 55	2. 3	4. 5	0. 31	21. 5	4. 7	10. 44	0. 59	29. 1	2. 4	14. 32	0. 28	14
17	8. 24	2. 10	4. 20	0. 33	21. 27	4. 7	10. 55	0. 59	29. 9	1. 56	14. 36	0. 27	13
18	8. 53	2. 17	4. 35	0. 34	21. 49	4. 7	11. 6	0. 59	29. 17	1. 47	14. 40	0. 25	12
19	9. 22	2. 24	4. 50	0. 36	22. 10	4. 6	11. 16	0. 59	29. 24	1. 39	14. 43	0. 23	11
20	9. 51	2. 30	5. 5	0. 38	22. 31	4. 6	11. 26	0. 59	29. 30	1. 30	14. 46	0. 21	10
21	10. 19	2. 37	5. 19	0. 39	22. 52	4. 5	11. 36	0. 58	29. 35	1. 22	14. 49	0. 19	9
22	10. 48	2. 43	5. 34	0. 41	23. 12	4. 3	11. 46	0. 58	29. 41	1. 13	14. 51	0. 17	8
23	11. 16	2. 49	5. 48	0. 43	23. 32	4. 2	11. 56	0. 57	29. 45	1. 4	14. 53	0. 15	7
24	11. 44	2. 55	6. 2	0. 44	23. 51	4. 0	12. 5	0. 57	29. 49	0. 55	14. 55	0. 13	6
25	12. 12	3. 1	6. 17	0. 45	24. 11	3. 58	12. 14	0. 56	29. 52	0. 46	14. 56	0. 11	5
26	12. 40	3. 6	6. 31	0. 46	24. 29	3. 55	12. 23	0. 56	29. 55	0. 37	14. 57	0. 9	4
27	13. 7	3. 11	6. 45	0. 48	24. 47	3. 52	12. 32	0. 55	29. 57	0. 28	14. 58	0. 7	3
28	13. 34	3. 16	6. 59	0. 49	25. 5	3. 49	12. 41	0. 54	29. 59	0. 19	14. 59	0. 4	2
29	14. 2	3. 21	7. 12	0. 50	25. 22	3. 45	12. 49	0. 53	30. 0	0. 10	15. 0	0. 2	1
30	14. 29	3. 26	7. 26	0. 51	25. 39	3. 41	12. 57	0. 52	30. 0	0. 0	15. 0	0. 0	0
S.	XI. *South.* + V. *North.* +				X. *South.* + IV. *North.* +				IX. *South.* + III. *North.* +				S.

TABLE II.

The apparent Distances of Saturn's first, second, third, fourth, fifth *and* sixth *Satellites from it's Center, in lines parallel to the line of the Anſæ of the Ring; and of the* seventh *Satellite in lines parallel to the longer axis of it's apparent orbit, in ſemidiameters of the Ring, and hundredths of the ſame.*

ARGUMENT.—The Distance of the Satellite from the Apogee.

S	O. *East.*						VI. *West.*	I. *East.*						VII. *West.*	II. *East.*						VIII. *West.*	S
	Distance of the Satellites.							Distance of the Satellites.							Distance of the Satellites.							
D	I.	II.	III.	IV.	V.	VI.	VII.	I.	II.	III.	IV.	V.	VI.	VII.	I.	II.	III.	IV.	V.	VI.	VII.	D
	Semid.	Semid.	Semid.	Semid.	Semid.	Semid.	Semid.	Semid.	Semid.	Semid.	Semid.	Semid.	Semid.	Semid.	Semid.	Semid.	Semid.	Semid.	Semid.	Semid.	Semid.	
0	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,72	0,93	1,05	1,34	1,87	4,35	12,67	1,24	1,59	1,82	2,33	3,25	7,53	21,95	30
1	0,02	0,03	0,04	0,05	0,06	0,15	0,44	0,74	0,96	1,08	1,38	1,93	4,48	13,05	1,25	1,61	1,84	2,36	3,28	7,61	22,17	29
2	0,05	0,06	0,07	0,09	0,13	0,30	0,88	0,76	0,98	1,11	1,42	1,99	4,61	13,42	1,26	1,62	1,85	2,38	3,31	7,68	22,38	28
3	0,08	0,10	0,11	0,14	0,20	0,45	1,32	0,78	1,01	1,14	1,46	2,05	4,74	13,79	1,27	1,64	1,87	2,40	3,34	7,75	22,58	27
4	0,10	0,13	0,14	0,19	0,26	0,60	1,76	0,80	1,03	1,17	1,50	2,10	4,86	14,16	1,28	1,65	1,89	2,42	3,37	7,82	22,78	26
5	0,12	0,16	0,18	0,23	0,32	0,75	2,20	0,82	1,06	1,20	1,54	2,16	4,99	14,53	1,28	1,67	1,91	2,44	3,40	7,89	22,97	25
6	0,15	0,19	0,22	0,28	0,39	0,91	2,64	0,84	1,08	1,23	1,58	2,21	5,11	14,89	1,30	1,68	1,92	2,46	3,43	7,95	23,16	24
7	0,18	0,22	0,25	0,33	0,46	1,06	3,08	0,86	1,11	1,26	1,62	2,26	5,24	15,25	1,31	1,70	1,94	2,48	3,46	8,01	23,33	23
8	0,20	0,26	0,29	0,38	0,52	1,21	3,52	0,88	1,13	1,29	1,65	2,31	5,36	15,60	1,32	1,71	1,95	2,50	3,48	8,07	23,50	22
9	0,22	0,29	0,32	0,42	0,58	1,36	3,96	0,90	1,16	1,32	1,69	2,36	5,48	15,94	1,33	1,72	1,96	2,52	3,51	8,13	23,66	21
10	0,25	0,32	0,36	0,47	0,65	1,51	4,40	0,92	1,18	1,35	1,73	2,41	5,59	16,28	1,34	1,73	1,97	2.53	3,53	8,18	23,82	20
11	0,28	0,35	0,40	0,52	0,72	1,66	4,83	0,94	1,21	1,38	1,77	2,46	5,71	16,62	1,35	1,74	1,98	2,55	3,55	8,23	23,97	19
12	0,30	0,38	0,44	0,56	0,78	1,81	5,26	0,96	1,23	1,40	1,80	2,51	5,82	16,96	1,36	1,75	1,98	2,56	3,57	8,27	24,11	18
13	0,33	0,42	0,48	0,61	0,85	1,96	5,70	0,98	1,26	1,43	1,84	2,56	5,93	17,28	1,37	1,76	2,00	2,58	3,59	8,32	24,24	17
14	0,35	0,45	0,51	0,65	0,91	2,11	6,13	1,00	1,28	1,46	1,87	2,61	6,04	17,60	1,37	1,77	2,02	2,59	3,60	8,36	24,37	16
15	0,38	0,48	0 55	0,70	0,97	2,26	6,56	1,02	1,30	1,49	1,91	2,66	6,15	17,92	1,38	1,78	2,03	2,60	3,62	8,40	24 48	15
16	0,40	0,51	0,58	0,74	1,03	2,40	6,98	1,03	1,32	1,51	1,94	2,70	6,26	18,23	1,39	1,79	2,04	2,61	3,64	8,44	24,59	14
17	0,42	0,54	0,62	0,79	1,10	2,55	7,41	1,05	1,35	1,54	1,97	2,75	6,36	18,53	1,40	1,80	2,05	2,62	3,66	8,48	24,70	13
18	0,44	0,57	0,65	0,83	1,16	2,69	7,83	1,06	1,37	1,56	2,00	2,79	6,46	18,83	1,40	1,80	2,05	2,63	3,67	8,51	24,80	12
19	0,47	0,60	0,69	0,88	1,22	2,83	8,25	1,08	1,39	1,59	2,03	2,83	6,56	19,13	1,40	1,81	2,06	2,64	3,68	8,54	24,88	11
20	0,49	0,63	0,72	0,92	1,28	2,97	8,66	1,09	1,41	1,61	2,06	2,87	6,66	19,42	1,41	1,81	2,07	2,65	3 69	8,56	24,96	10
21	0,51	0,66	0,76	0,97	1,34	3,12	9,08	1,11	1,43	1,64	2,09	2,91	6,76	19,70	1,41	1,82	2,08	2,66	3,70	8,59	25,03	9
22	0,53	0,69	0,79	1,01	1,40	3,26	9,50	1,13	1,45	1,66	2,12	2,95	6,85	19,97	1,41	1,82	2,08	2,66	3,71	8,61	25,10	8
23	0,56	0,72	0,83	1,06	1,46	3,40	9,90	1,15	1,47	1,68	2,15	2,99	6,95	20,24	1,42	1,83	2,08	2,67	3,72	8,63	25,16	7
24	0,58	0,75	0,86	1,10	1,52	3,54	10,30	1,16	1,49	1,70	2,18	3,03	7,04	20,51	1,42	1,83	2,09	2,67	3,73	8,65	25,21	6
25	0,61	0,78	0,89	1,14	1,58	3,68	10,71	1,18	1,51	1,72	2,21	3,07	7,13	20,76	1,43	1,84	2,09	2,68	3,74	8,67	25.25	5
26	0,63	0,81	0,92	1,18	1,64	3,82	11,11	1,19	1,53	1,74	2,23	3,11	7,21	21,01	1,43	1,84	2,09	2,68	3,74	8,68	25,29	4
27	0,65	0,84	0,96	1,22	1,70	3,96	11,50	1,20	1,55	1,76	2,26	3,15	7,30	21,26	1,43	1,84	2,10	2,69	3,74	8,69	25,32	3
28	0,67	0,87	0,99	1,26	1,76	4,10	11,89	1,21	1,56	1,78	2,28	3,18	7,38	21,50	1,43	1,84	2,10	2,69	3,75	8,69	25,33	2
29	0,69	0,90	1,02	1,30	1,82	4,23	12,28	1,23	1,58	1,80	2,31	3,22	7,46	21,73	1,43	1,84	2,10	2,69	3,75	8,70	25,35	1
30	0,72	0,93	1,05	1,34	1,87	4,35	12,67	1,24	1,59	1,82	2,33	3,25	7,53	21,95	1,43	1,84	2,10	2,69	3,75	8,70	25,35	0
S	XI. *West.*					V. *East.*		X. *West.*				IV. *East.*			IX. *West.*				III. *East.*			S

TABLE III.

The apparent Distances of Saturn's first, second, third, fourth, fifth *and* sixth *Satellites from the line of the Anſæ of the Ring; and of the* seventh *Satellite from the longer axis of it's apparent orbit, in ſemidiameters of the Ring, and hundredths of the ſame.*

ARGUMENT.—Diſtance of the Apogee from ☌ ± diſtance of the Satellite from the Apogee.

S	O. *North.*					VI. *South.*		I. *North.*					VII. *South.*		II. *North.*					VIII. *South.*		S
	Diſtance of the Satellites.							Diſtance of the Satellites.							Diſtance of the Satellites.							
D	I.	II.	III.	IV.	V.	VI.	VII.	I.	II.	III.	IV.	V.	VI.	VII.	I.	II.	III.	IV.	V.	VI.	VII.	D
	Semid.	semid.	semid.	semid.	semid.	semid.	semid.	semid.	semid.	semid.	semid.	semid.	Semid	semid.	semid.	semid.	semid.	semid	semid.	semid.	semid	
0	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,18	0,23	0,26	0,33	0,47	1,09	1,64	0,31	0,40	0,45	0,58	0,81	1,88	2,84	30
1	0,00	0,01	0,01	0,01	0,01	0,04	0,06	0,18	0,24	0,27	0,34	0,48	1,12	1,69	0,31	0,40	0,46	0,59	0,82	1,90	2,87	29
2	0,01	0,01	0,02	0,02	0,03	0,07	0,11	0,19	0,24	0,28	0,35	0,50	1,15	1,74	0,31	0,40	0,46	0,59	0,83	1,92	2,90	28
3	0,02	0,02	0,03	0,03	0,05	0,11	0,17	0,19	0,25	0,28	0,36	0,51	1,18	1,79	0,32	0,41	0,47	0,60	0,83	1,94	2,93	27
4	0,02	0,03	0,03	0,05	0,06	0,15	0,23	0,20	0,26	0,29	0,37	0,52	1,21	1,83	0,32	0,41	0,47	0,60	0,84	1,95	2,95	26
5	0,03	0,04	0,04	0,06	0,08	0,19	0,28	0,20	0,26	0,30	0,38	0,54	1,25	1,88	0,32	0,42	0,48	0,61	0,85	1,97	2,98	25
6	0,04	0,05	0,05	0,07	0,10	0,23	0,34	0,21	0,27	0,31	0,39	0,55	1,28	1,93	0,32	0,42	0,48	0,61	0,86	1,99	3,00	24
7	0,04	0,05	0,06	0,08	0,11	0,26	0,40	0,21	0,28	0,31	0,40	0,56	1,31	1,98	0,33	0,42	0,48	0,62	0,86	2,00	3,02	23
8	0,05	0,06	0,07	0,09	0,13	0,30	0,46	0,22	0,28	0,32	0,41	0,58	1,34	2,02	0,33	0,43	0,49	0,62	0,87	2,02	3,04	22
9	0,05	0,07	0,08	0,10	0,14	0,34	0,51	0,22	0,29	0,33	0,42	0,59	1,37	2,07	0,33	0,43	0,49	0,63	0,88	2,03	3,06	21
10	0,06	0,08	0,09	0,12	0,16	0,38	0,57	0,23	0,29	0,34	0,43	0,60	1,40	2,11	0,33	0,43	0,49	0,63	0,88	2,04	3,08	20
11	0,07	0,09	0,10	0,13	0,18	0,41	0,63	0,23	0,30	0,34	0,44	0,61	1,43	2,15	0,34	0,43	0,49	0,64	0,89	2,06	3,10	19
12	0,07	0,09	0,11	0,14	0,19	0,45	0,68	0,24	0,31	0,35	0,45	0,63	1,45	2,19	0,34	0,44	0,49	0,64	0,89	2,07	3,12	18
13	0,08	0,10	0,12	0,15	0,21	0,49	0,74	0,24	0,31	0,36	0,46	0,64	1,48	2,24	0,34	0,44	0,50	0,64	0,90	2,08	3,14	17
14	0,09	0,11	0,13	0,16	0,23	0,53	0,79	0,25	0,32	0,36	0,47	0,65	1,51	2,28	0,34	0,44	0,50	0,65	0,90	2,09	3,15	16
15	0,09	0,12	0,14	0,17	0,24	0,56	0,85	0,25	0,32	0,37	0,48	0,66	1,54	2,32	0,34	0,44	0,51	0,65	0,90	2,10	3,17	15
16	0,10	0,13	0,14	0,18	0,26	0,60	0,90	0,26	0,33	0,38	0,48	0,67	1,56	2,36	0,35	0,45	0,51	0,65	0,91	2,11	3,18	14
17	0,10	0,13	0,15	0,20	0,27	0,64	0,96	0,26	0,34	0,38	0,49	0,69	1,59	2,40	0,35	0,45	0,51	0,65	0,91	2,12	3,20	13
18	0,11	0,14	0,16	0,21	0,29	0,67	1,01	0,26	0,34	0,39	0,50	0,70	1,61	2,44	0,35	0,45	0,51	0,66	0,92	2,13	3,21	12
19	0,12	0,15	0,17	0,22	0,30	0,71	1,07	0,27	0,35	0,40	0,51	0,71	1,64	2,48	0,35	0,45	0,51	0,66	0,92	2,13	3,22	11
20	0,12	0,16	0,18	0,23	0,32	0,74	1,12	0,27	0,35	0,40	0,51	0,72	1,66	2,51	0,35	0,45	0,52	0,66	0,92	2,14	3,23	10
21	0,13	0,16	0,19	0,24	0,33	0,78	1,18	0,28	0,36	0,41	0,52	0,73	1,69	2,55	0,35	0,45	0,52	0,66	0,92	2,15	3,24	9
22	0,13	0,17	0,20	0,25	0,35	0,81	1,23	0,28	0,36	0,41	0,53	0,74	1,71	2,58	0,35	0,45	0,52	0,66	0,93	2,15	3,25	8
23	0,14	0,18	0,21	0,26	0,36	0,85	1,29	0,29	0,37	0,42	0,54	0,75	1,74	2,62	0,35	0,46	0,52	0,67	0,93	2,16	3,26	7
24	0,14	0,19	0,21	0,27	0,38	0,88	1,34	0,29	0,37	0,42	0,54	0,76	1,76	2,65	0,35	0,46	0,52	0,67	0,93	2,16	3,26	6
25	0,15	0,19	0,22	0,28	0,39	0,92	1,39	0,29	0,38	0,43	0,55	0,77	1,78	2,69	0,36	0,46	0,52	0,67	0,93	2,17	3,27	5
26	0,16	0,20	0,23	0,29	0,41	0,95	1,44	0,30	0,38	0,43	0,56	0,78	1,80	2,72	0,36	0,46	0,52	0,67	0,93	2,17	3,27	4
27	0,16	0,21	0,24	0,30	0,42	0,99	1,49	0,30	0,39	0,44	0,56	0,79	1,82	2,75	0,36	0,46	0,52	0,67	0,93	2,17	3,28	3
28	0,17	0,22	0,25	0,31	0,44	1,02	1,54	0,30	0,39	0,44	0,57	0,79	1,84	2,78	0,36	0,46	0,52	0,67	0,94	2,17	3,28	2
29	0,17	0,22	0,25	0,32	0,45	1,06	1,59	0,31	0,39	0,45	0,58	0,80	1,86	2,81	0,36	0,46	0,52	0,67	0,94	2,17	3,28	1
30	0,18	0,23	0,26	0,33	0,47	1,09	1,64	0,31	0,40	0,45	0,58	0,81	1,88	2,84	0,36	0,46	0,52	0,67	0,94	2,17	3,28	0
S	XI. *South.*					V. *North.*		X. *South*					IV. *North.*		IX. *South.*					III. *North.*		S

The Uſe of the Three foregoing Tables for finding the apparent Poſitions of Saturn's Satellites *at any Time, with reſpect to Saturn and the Line of the Anſæ of the Ring.*

482. From Saturn's geocentric longitude, taken out of the Nautical Almanac, ſubtract the place of the ſatellite's aſcending node (at preſent 5ˢ. 20°. 54′ for all but the *ſeventh*, and for that 5ˢ. 6°. 4′), and there will remain the argument of latitude; with which take the reduction out of Table I, and ſubtract it from the argument of latitude in the firſt or third quadrant, but add it to the ſame in the ſecond or fourth quadrant; that is, according to the ſign put at the top or bottom of the Table in the column of reduction, and you will have the diſtance of the apogee point of the ſatellite's orbit from the node. Apply the reduction alſo, and with the ſame ſign as before, to Saturn's geocentric longitude, and you will have the longitude of the apogee point on the ſatellite's orbit. Subtract the longitude of the apogee thus found from the ſatellite's longitude, and there will remain the diſtance of the ſatellite from the apogee; with which in Table II. find the apparent diſtances of the ſatellites from Saturn's center, meaſured in lines parallel to the longer axis of the ring; except of the ſeventh ſatellite, which will be meaſured in lines parallel to the longer axis of it's apparent elliptical orbit; and if the argument be under ſix ſigns, the ſatellite will be eaſt of Saturn's center; but if the argument be greater than ſix ſigns, the ſatellite will be weſt of Saturn's center; as is marked by the ſide of the ſigns of the argument in the Table.

Add and ſubtract the diſtance of the ſatellite from the apogee to and from the diſtance of the apogee from the node, and you will have two arguments, with which enter Table III. ſeparately, and take out the correſpondent latitudes, with their proper titles north or ſouth, ſtanding by the ſigns of the arguments; of which, if both of the ſame kind, the ſum with the common title; but if of different kinds, the difference with the title of the greater, will be the latitude of the ſatellite, as ſeen from the earth, meaſured by it's apparent diſtance from the line of the anſæ of the ring; except in the ſeventh ſatellite, which is meaſured by it's diſtance from the longer axis of it's apparent ellipſes; in ſemidiameters of the ring, and hundredths of the ſame.

Add three ſigns to the diſtance of the apogee from the node, with which take out the latitude from Table I. with it's proper title, north or ſouth, adjoining to the ſign of the argument, which will be the apparent inclination of the line of the anſæ of Saturn's ring, or of the longer axis of the ſeventh

ſatellite's

ſatellite's orbit, to Saturn's orbit; and the line of the anſæ, or longer axis of the ſeventh ſatellite's orbit, will aſcend from weſt to eaſt northward above Saturn's apparent orbit in the heavens, or deſcend from weſt to eaſt ſouthward below it, according as the title adjoining to the ſign of the argument in the Table is north or ſouth. Change the title of the inclination of the line of the anſæ of the ring to Saturn's orbit to the contrary; and if the inclination of the line of the anſæ of the ring with the title thus changed, and the inclination of the longer axis of the ſeventh ſatellite's orbit, be of the ſame kind, their ſum with the common title; but if of different kinds, their difference, with the title of the greater, will be the inclination of the longer axis of the ſeventh ſatellite's orbit to the line of the anſæ of the ring. And the longer axis of the ſeventh ſatellite's orbit will aſcend from weſt to eaſt northward above, or deſcend from weſt to eaſt ſouthward below the line of the anſæ, according as the reſulting title is north or ſouth.

EXAMPLE

EXAMPLE.

To compute the apparent Positions of Saturn's Satellites, on September 25, 1791, at 11h. mean time.

Geoc. long. of ♄	0°. 16°,47	Geoc. long. of ♄	0°. 16°,47	Geoc. long. of ♄	0°. 16°,47	Geoc. long. of ♄	0°. 16°,47
☌ of Ring .	− 5. 20,9	☌ of 7th Sat. orb.	− 5. 6,1	Redue. . . .	− 3,07	Reduc. . . .	− 0,97
Arg. Lat. . .	6. 25,57	Arg. Lat. . .	7. 10,37	Long. apog. of all the Satellites except 7th	0. 13,40	Long. apog. of 7th Satellite	0. 15,50
Table I. Red. .	− 3,07	Table I. Red. .	− 0,97				
Dist. apog. à ☌	6. 22,50	Dist. apog. à ☌	7. 9,40				
D° . .	+3°=9. 22,50	D° . .	+3°=10. 9,40				
Table I. Inclin. of line of anſæ to ♄'s orbit	27°. 30′,5 S.	Incl. of longer axis of 7th Sat. orb. to ♄ orb.	11°. 32′ S.				

Hence, 27°. 30′ N. − 11°. 32′ S. = 15°. 58′ N.; or the longer axis of the ſeventh ſatellite's orbit riſes ſo much above or north of the line of the anſæ to the eaſt.

	I.	II.	III.	IV.	V.	VI.	VII.
	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.
1791 - - -	11. 22,97	11. 23,81	4. 26,16	6. 24,68	7. 24,74.	0. 14,45	4. 6,02
September - -	9. 26,67	4. 2,72	8. 19,60	9. 12,98	9. 14,74	2. 26,26	0. 22,75
25D. - -	6. 9,04	2. 28,18	2. 27,45	1. 18,37	6. 12,26	6. 24,48	3. 23,45
11H. - -	5. 25,07	4. 0,42	2. 27,40	2. 0,29	1. 6,52	6. 10,35	0. 2,08
Long. of ſatellite -	9. 23,75.	10. 25,13	7. 10,61	7. 26,32	0. 28,26	10. 15,49	8. 24,30
Long. of apogee -	−0. 13,40	− 0. 13,40	−0. 13,40	−0. 13,40	0. 13,40	− 0. 13,40	−0. 15,50
Diſt. ſat. à apog. = Arg. II.	±9. 10,35	±10. 11,73	±6. 27,21	±7. 12,92	±0. 14,86	±10. 2,09	±8. 8,80
Arg. lat. - -	6. 25,57	6. 25,57	6. 25,57	6. 25,57	6. 25,57	6. 25,57	7. 10,37
Arg. of Tab. III.	4. 5,92	5. 7,30	1. 22,78	2. 8,49	7. 10,43	4. 27,66	3. 19,17
	9. 15,22	8. 13,84	11. 28,36	11. 12,65	6. 10,71	8. 3,48	11. 1,57
Diſt. ſat. à ♄ E. or W.	1,41 W.	1,06 W.	0,97 W.	1,84 W.	0,96 E.	7,37 W	23,63 W.
Numbers from Tab. III.	0,29 N.	0,18 N.	0,42 S.	0,62 N.	0,60 S.	1,16 N.	3,10 N.
	0,34 S.	0,44 S.	0,01 S.	0,20 S.	0,17 S.	2,16 S.	1,56 S.
Diſt. ſat. à ♄ N. or S.	0,05 S.	0,26 S.	0,43 S.	0,42 N.	0,77 S.	1,00 N.	1,54 N.

The epochs of the ſatellites which have been here given are for the meridian of *Slough*, which is about 2′. 25″ weſt of *Greenwich*.

On the Satellites of the Georgian.

483. On January 11, 1787, as Dr. HERSCHEL was obſerving the *Georgian*, he perceived, near it's diſc, ſome very ſmall ſtars, whoſe places he noted. The next evening, upon examining them, he found that two of them were miſſing. Suſpecting therefore that they might be ſatellites which had diſappeared in conſequence of having changed their ſituation, he continued his obſervations, and in the courſe of a month diſcovered them to be ſatellites, as he had at firſt conjectured. Of this diſcovery he gave an account in the *Phil. Tranſ.* 1787.

484. In the *Phil. Tranſ.* 1788, he publiſhed a further account of this diſcovery, containing their periodic times, diſtances, and poſitions of their orbits, ſo far as he was then able to aſcertain them. The moſt convenient method of determining the periodic time of a ſatellite is either from it's eclipſes, or from taking it's poſition in ſeveral ſucceſſive oppoſitions of the planet; but no eclipſes have yet happened ſince the diſcovery of theſe ſatellites, and it would be waiting a long time to put in practice the other method. Dr. HERSCHEL therefore took their ſituations whenever he could aſcertain them with ſome degree of preciſion, and then reduced them by computation to ſuch ſituations as were neceſſary for his purpoſe. In computing the periodic times, he has taken the ſynodic revolution, as the poſitions of their orbits, at the times when their ſituations were taken, were not ſufficiently known to get a very accurate ſidereal revolution. The mean of ſeveral reſults gave the ſynodic revolution of the firſt ſatellite $8d.\ 17h.\ 1'.\ 19'',3$, and of the ſecond $13d.\ 11h.\ 5'.\ 1'',5$. The reſults, he obſerves, of which theſe are a mean, do not much differ among themſelves, and therefore the mean is probably tolerably accurate. The epochs from which their ſituations may at any time be computed are, for the firſt, October 19, 1787, at $19h.\ 11'.\ 28''$, and for the ſecond, at $17h.\ 22'.\ 40''$, at which times they were $76°.\ 43'$ north following the planet.

485. The next thing to be determined in the elements of theſe ſatellites was their diſtances from the planet; to obtain which, he found one diſtance by obſervation, and then the other from the periodic times (218). Now in attempting to diſcover the diſtance of the ſecond, the orbit was ſeemingly elliptical. On March 18, 1787, at $8h\ 2'.\ 50''$, he found the elongation to be $46'',46$, this being the greateſt of all the meaſures he had taken. Hence at the mean diſtance of the Georgian from the earth, this elongation will be $44'',23$. Admitting therefore, for the preſent, ſays Dr. HERSCHEL, that the ſatellite moves in a circular orbit, we may take $44'',23$ for the true diſtance without much error; hence, as the ſquares of the periodic times are as the

cubes of the diſtances, the diſtance of the firſt ſatellite comes out 33″,09. The ſynodic revolutions were here uſed inſtead of the ſidereal, which will make but a ſmall error.

486. The laſt thing to be done was to determine the inclination of the orbits, and places of their nodes. And here a difficulty preſented itſelf which could not be got over at the time of his firſt obſervation; for it could not then be determined which part of the orbit was inclined *to* the earth, and which *from* it. On the two different ſuppoſitions therefore Dr. HERSCHEL has computed the inclinations of the orbits, and the places of the nodes, and found them as follows. The orbit of the ſecond ſatellite is inclined to the ecliptic 99°. 43′. 53″,3, or 81°. 6′. 4″,4; it's aſcending node upon the ecliptic is in 5ˢ. 18°, or 8ˢ. 6°; and when the planet comes to the aſcending node of this ſatellite, which will happen about the year 1799, or 1818, the northern half of the orbit will be turned towards the eaſt, or weſt, at the time of it's meridian paſſage. M. de LAMBRE makes the aſcending node in 5ˢ. 21°, or 8ˢ. 9°, from Dr. HERSCHEL's obſervations. The ſituation of the orbit of the firſt ſatellite does not materially differ from that of the ſecond. The light of the ſatellites is extremely faint; the ſecond is the brighteſt, but the difference is ſmall. The ſatellites are probably not leſs than thoſe of *Jupiter*. There will be eclipſes of theſe ſatellites about the year 1799, or 1818, when they will appear to aſcend through the ſhadow of the planet, in a direction almoſt perpendicular to the ecliptic.

CHAP.

CHAP. XXI.

ON THE RING OF SATURN.

Art. 487. GALILEO was the firſt perſon who obſerved any thing extraordinary in *Saturn*. The planet appeared to him like a large globe between two ſmall ones. In the year 1610 he announced this diſcovery. He continued his obſervations till 1612, when he was ſurpriſed to find only the middle globe; but ſometime after he again diſcovered the globes on each ſide, which, in proceſs of time, appeared to change their form; ſometimes appearing round, ſometimes oblong like an acorn, ſometimes ſemicircular, then with horns towards the globe in the middle, and growing by degrees ſo long and wide as to encompaſs it, as it were with an oval ring. Upon this HUYGENS ſet about improving the art of grinding object glaſſes; and made teleſcopes which magnified two or three times more than any which had been before made, with which he diſcovered very clearly the ring of Saturn; and having obſerved it for ſome time, he publiſhed the diſcovery in 1656. He made the ſpace between the globe and the ring equal to, or rather bigger than the breadth of the ring; and the greateſt diameter of the ring to that of the globe as 9 to 4. But Mr. POUND, with a micrometer applied to HUYGENS's teleſcope of 123 feet long, determined the ratio to be as 7 to 3. Mr. WHISTON, in his Memoirs of the Life of Dr. CLARK, relates, that the Doctor's Father once ſaw a fixed ſtar between the ring and the body of Saturn. In the year 1675, M. CASSINI ſaw the ring, and obſerved upon it a dark elliptical line dividing it as it were into two rings, the inner of which appeared brighter than the outer. He alſo obſerved a dark belt upon the planet, parallel to the major axis of the ring. Mr. HADLEY obſerved that the outer part of the ring ſeemed narrower than the inner part; and that the dark line was fainter towards it's upper edge; he alſo ſaw two belts, and obſerved the ſhadow of the ring upon Saturn. In October 1714, when the plane of the ring very nearly paſſed through the earth, and was approaching to it, M. MARALDI obſerved, that while the arms were decreaſing both in length and breadth, the eaſtern arm appeared a little larger than the other for three or four nights, and yet it vaniſhed firſt, for after two nights interruption by clouds, he ſaw the weſtern arm alone. This inequality of the ring made him ſuſpect that it was not bounded by exactly parallel planes, and that it

turned about it's axis. But the beſt deſcription of this ſingular phænomenon is that given by Dr. HERSCHEL in the *Phil. Tranſ.* 1790, who, by his extraordinary teleſcopes, has diſcovered many circumſtances which had eſcaped all other obſervers. We ſhall here give the ſubſtance of his account. Figure 105, is a view of Saturn and it's ring, as they appeared on June 20, 1778.

FIG. 105.

488. The black diſc, or belt, upon the ring of Saturn is not in the middle of it's breadth; nor is the ring ſubdivided by many ſuch lines, as has been repreſented by ſome Aſtronomers; but there is one * ſingle, dark, conſiderably broad line, belt, or zone, as in this Figure, which he has conſtantly found on the north ſide of the ring. As this dark belt is ſubject to no change whatever, it is probably owing to ſome permanent conſtruction of the ſurface of the ring. This conſtruction cannot be owing to the ſhadow of a chain of mountains, ſince it is viſible all round on the ring; for at the ends of the ring there could be no ſhade; and the ſame argument will hold againſt any ſuppoſed caverns. It is moreover pretty evident, that this dark zone is contained between two concentric circles, as all the phænomena anſwer to the projection of ſuch a zone. The nature of the ring is undoubtedly no leſs ſolid than the planet itſelf; and it is obſerved to caſt a ſtrong ſhadow upon the planet. The light of the ring is alſo generally brighter than that of the planet; for the ring appears ſufficiently bright, when the teleſcope affords ſcarcely light enough for Saturn. Dr. HERSCHEL next takes notice of the extreme thinneſs of the ring. He frequently ſaw the firſt, ſecond, third, fourth and fifth ſatellites paſs before and behind the ring in ſuch a manner, that they ſerved as an excellent micrometer to meaſure it's thickneſs by. It may be proper to mention a few inſtances, as they ſerve alſo to ſolve ſome phænomena obſerved by other Aſtronomers, without having been accounted for in any manner that could be admitted conſiſtently with other known facts. July 18, 1789, at $19^h.\ 41'.\ 9''$ ſidereal time, the third ſatellite ſeemed to hang upon the following arm, declining a little towards the north, and was ſeen gradually to advance upon it towards the body of Saturn; but the ring was not ſo thick as the lucid point. July 23, at $19^h.\ 41'.\ 8''$, the fourth ſatellite was a very little preceding the ring, but the ring appeared to be leſs than half the thickneſs of the ſatellite. July 27, at $20^h.\ 15'.\ 12''$, the fourth ſatellite was about the middle, upon the following arm of the ring, and towards the ſouth; and the ſecond at the farther end, towards the north; but the arm was thinner than either. Auguſt 29, at $22^h.\ 12'.\ 25''$, the fifth ſatellite was upon the ring, near the end of the preceding arm, and the thickneſs of the arm ſeemed to be about $\frac{1}{3}$ or $\frac{1}{4}$ of the diameter

* In a Paper in the *Phil. Tranſ.* 1792, Dr. HERSCHEL obſerves that, "ſince the year 1774 to the preſent time, I can find only four obſervations where any other black diviſion of the ring is mentioned than the one which I have conſtantly obſerved; theſe were all in June, 1780."

diameter of the ſatellite, which, in the ſituation it then was, he took to be leſs than one ſecond in diameter. At the ſame time, the firſt appeared at a little diſtance following the fifth, in the ſhape of a bead upon a thread, projecting on both ſides of the ſame arm; hence the arm is thinner than the firſt, which is conſiderably ſmaller than the ſecond, and a little leſs than the third. October 16, he followed the firſt and ſecond ſatellites up to the very diſc of the planet; and the ring, which was extremely faint, did not obſtruct his ſeeing them gradually approach the diſc. Theſe obſervations are ſufficient to ſhow the extreme thinneſs of the ring. But Dr. HERSCHEL further obſerves, that there may be a refraction through an atmoſphere of the ring, by which the ſatellites may be lifted up and depreſſed, ſo as to become viſible on both ſides of the ring, even though the ring ſhould be equal in thickneſs to the ſmalleſt ſatellite, which may amount to 1000 miles. From a ſeries of obſervations upon luminous points of the ring, he has diſcovered that it has a rotation about it's axis, the time of which is $10^h. 32'. 15'',4$.

489. The ring is inviſible* when it's plane paſſes through the ſun, the earth, or between them; in the firſt caſe, the ſun ſhines only upon it's edge, which is too thin to reflect ſufficient light to render it viſible; in the ſecond caſe, the edge only being oppoſed to us, it is not viſible for the ſame reaſon; in the third caſe, the dark ſide of the ring is expoſed to us, and therefore the edge being the only luminous part which is towards the earth, it is inviſible on the ſame account as before. Obſervers have differed 10 or 12 days in the time of it's becoming inviſible, owing to the difference of the teleſcopes, and of the ſtate of the atmoſphere. Dr. HERSCHEL obſerves, that the ring was ſeen in his teleſcope when we were turned towards the unenlightened ſide; ſo that he either ſaw the light reflected from the edge, or elſe the reflection of the light of Saturn upon the dark ſide of the ring, as we ſometimes ſee the dark part of the moon. He cannot however ſay which of the two might be the caſe; eſpecially as there are very ſtrong reaſons to think, that the edge of the ring is of ſuch a nature as not to reflect much light. M. de la LANDE thinks that the ring is juſt viſible with the beſt teleſcopes in common uſe, when the ſun is elevated $3'$ above it's plane, or 3 days before it's plane paſſes through the ſun; and when the earth is elevated $2'. 30''$ above the plane, or one day from the earth's paſſing it.

490. In a Paper in the *Phil. Tranſ.* 1790, Dr. HERSCHEL ventured to hint at a ſuſpicion that the ring was divided; this conjecture was ſtrengthened by

* The diſappearance of the Ring is only with the teleſcopes in common uſe among Aſtronomers; for Dr. HERSCHEL, with his large teleſcopes, has been able to ſee it in every ſituation. He thinks the edge of the Ring is not flat, but ſpherical or ſpheroidical.

by future obſervations, after he had had an opportunity of ſeeing both ſides of the ring. His reaſons are theſe: 1. The black diviſion upon the ſouthern ſide of the ring, is in the ſame place, of the ſame breadth, and at the ſame diſtance from the outer edge, that it always appeared upon the northern ſide. 2. With his ſeven feet reflector and an excellent ſpeculum, he ſaw the diviſion on the ring, and the open ſpace between the ring and the body, equally dark, and of the ſame colour with the heavens about the planet. 3. The black diviſion is equally broad on each ſide of the ring. From theſe obſervations, Dr. HERSCHEL thinks himſelf authorized to ſay that Saturn has two concentric rings, ſituated in one plane, which is probably not much inclined to the equator of the planet. The dimenſions of the two rings are in the following proportions, as nearly as they could be aſcertained.

	Parts.
Inſide diameter of the ſmaller ring	5900
Outſide diameter	7510
Inſide diameter of the larger ring	7740
Outſide diameter	8300
Breadth of the inner ring	805
Breadth of the outer ring	280
Breadth of the ſpace between the rings	115

In the *Mem. de l'Acad.* at Paris 1787, M. de la PLACE ſuppoſes that the ring may have many diviſions; but Dr. HERSCHEL remarks, that no obſervations will juſtify this ſuppoſition.

491. From the mean of a great many meaſures of the diameter of the larger ring, Dr. HERSCHEL makes it 46″,677 at the mean diſtance of Saturn. Hence, it's diameter : the diameter of the earth :: 25,8914 : 1. From the above proportions therefore, the diameter of this ring muſt be 204883 miles; and the diſtance of the two rings 2839 miles.

492. The ring being a circle, appears elliptical from it's oblique poſition; and it appears moſt open when Saturn is 90° from the nodes of the ring upon the orbit of Saturn, or when Saturn's longitude is about 2ˢ. 17°, and 8ˢ. 17°. In ſuch a ſituation, the minor axis is extremely nearly equal to half the major, when the obſervations are reduced to the ſun; conſequently the plane of the ring makes an angle of about 30° with the orbit of Saturn; it will alſo be ſhown that it continues parallel to itſelf. The ſituation of the nodes of the ring, and all it's other phænomena, may be determined as follows.

493. Let

493. Let S be the ſun, AB the orbit of the earth, $ntrzv$ the orbit of Saturn, vSn the line of the nodes; transfer theſe circles to the ſphere of the fixed ſtars, and let them be repreſented by NZ, NX; and let RV be the plane of the ring extended to the ſame ſphere; then F is the place of the node of the ring upon Saturn's orbit, and V the node upon the ecliptic; draw FtS, and t is the place of Saturn when the plane of the ring paſſes through the ſun; alſo let r be the place of Saturn when the plane paſſes through the earth at c, and draw crH; let z be any other poſition of Saturn, and e the correſponding place of the earth, and draw Szd, ezm to the abovementioned ſphere; join mV by a great circle, and let fall the perpendiculars dp, mw upon VR, and mG, HI and FT upon ZN. FIG. 106.

494. The place F of the node upon the orbit of Saturn may be immediately found, from obſerving the heliocentric place of Saturn in it's orbit when the plane paſſes through the ſun, for that place is the place of the node. Now as the ring is inviſible a few days before, and as many after it paſſes through the ſun, to get the time when it paſſes, obſerve the time when it diſappears and the time when it becomes viſible, and the middle point of time is the time when the plane paſſed through the ſun; and the place of Saturn at that time is the place of the node of the ring. According to M. de la LANDE, on January 8, 1774, the plane of the ring paſſed through the ſun, at which time the longitude of Saturn upon it's orbit was $5^s.\ 20^\circ.\ 38'$, which therefore was the place of the node F of the ring. Now the node N at that time was in $3^s.\ 21^\circ.\ 43'$; hence $FN = 58^\circ.\ 55'$, the diſtance of the node of the ring upon the orbit of Saturn from the node of Saturn; and to find the diſtance VN upon the ecliptic, we have in the right angled triangle FTN, $FN = 58^\circ.\ 55'$ and the angle $FNT = 2^\circ.\ 29'.\ 50''$; hence we find $TN = 58^\circ.\ 53'.\ 33''$, $TF = 2^\circ.\ 8'.\ 19''$, and the angle $NFT = 88^\circ.\ 42'.\ 36''$; from which ſubtract $NFV = 30^\circ$ (492) the angle which the plane of the ring makes with the orbit of Saturn, and we have $VFT = 58^\circ.\ 42'.\ 36''$; hence in the right angled triangle VFT, we have $FT = 2^\circ.\ 8'.\ 19''$, and $VFT = 58^\circ.\ 42'.\ 36''$, therefore $VT = 3^\circ.\ 30'.\ 49''$, which ſubtracted from TN, leaves $VN = 55^\circ.\ 22'.\ 44''$ the diſtance of the node of the ring on the ecliptic from Saturn's node; and the angle $FVT = 31^\circ.\ 21'.\ 19''$ the inclination of the ring to the ecliptic. M. MARALDI made it $31^\circ.\ 20'$; HEINSIUS made it $31^\circ.\ 23'.\ 17''$. Alſo as the longitude of N was $3^s.\ 21^\circ.\ 43'$, we have the longitude of the node V of the ring upon the ecliptic $5^s.\ 17^\circ.\ 5'.\ 44''$. MARALDI found it $5^s.\ 16^\circ.\ 17'$ in 1715; and if from that time to 1774 we allow $49'$ for the preceſſion of the equinoxes, it makes the place in 1774 to be $5^s.\ 17^\circ.\ 6'$, within $16''$ of what it was found by obſervation. Hence it appears that the plane of the ring is fixed. The place of the node of the ring upon the orbit of Saturn was, according to HUYGENS,

in the middle of the laſt century $5^s.\ 20°.\ 30'$; CASSINI made it $5^s.\ 22°$. This is the aſcending node.

495. The place of the node may alſo be found from the time when the plane of the ring paſſes through the earth, obſerving at that time, the geocentric latitude HI, and the longitude, from which we get IN, knowing the place of the node N. Hence in the right angled triangle NIv, we know IN and the angle N, to find Nv, Iv and the angle NvI; therefore in the triangle HvF, we know Hv, HvF and HFv, to find vF, which taken from vN leaves FN. Or if we ſuppoſe the angle HVI known, then knowing HI, we find IV, therefore if we know the longitude of I, we ſhall know that of V. Now according to M. de la LANDE, the ring paſſed through the earth on April 3, 1774, at which time the geocentric place I of Saturn was $5^s.\ 21°.\ 7'.\ 38''$, and latitude HI was $2°.\ 27'$; if therefore we ſuppoſe the angle HVI to be $31°\ 21'.\ 19''$, we have $IV = 4°.\ 1'.\ 35''$, which ſubtracted from $5^s.\ 21°.\ 7'.\ 38''$, leaves the longitude of the node V on the ecliptic $5^s.\ 17°.\ 6'.\ 3''$, which is within $19''$ of what it was found (494) from the paſſage of the plane through the ſun. When the earth and Saturn are moving in oppoſite directions, the place of the node may be more accurately determined by this method, than by the paſſage of the plane through the ſun; becauſe the diſappearance takes place quicker, and therefore you can determine the time more accurately

496. In determining the nodes of the ring, we ſuppoſed the inclination known, whereas the inclination is found from knowing the place of the nodes, by obſerving the ring when Saturn is 90° from the nodes. But by conſtantly obſerving the opening of the ring about the time when it is the greateſt, we ſhall get very nearly it's inclination, and a ſmall error in that will make but a very little alteration in the place of the node.

497. When the plane of the ring paſſes through the earth, we have $\frac{\text{tan. } IH}{\text{ſin. } IV}$ $=$ tan. $IVH =$ tan. $31°.\ 20' = 0{,}6088$. As this muſt take place when I is within a few degrees of the node V, it will be very eaſy to compute when the paſſage happens, by aſſuming ſome time, and taking at that time the geocentric latitude and longitude of Saturn from the *Nautical Almanac*, and thence finding IV, and dividing the tangent of the latitude HI by the ſine of IV; aſſume two times as near as you can conjecture to the time required, and the two reſults will direct you to find, by proportion, a time very near to that required; thus you will very eaſily get the time.

Ex. On May 3, 1789, the geocentric longitude of Saturn was $11^s.\ 20°.\ 23'$, and latitude $1°.\ 54'.\ 20''$; and taking the place of the deſcending node to be $11^s.\ 17°$

$11^s.\ 17^\circ.\ 18'$, we have $IV = 3^\circ.\ 5'$; hence $\frac{\text{tan. } 1^\circ.\ 54'.\ 20''}{\text{fin. } 3^\circ.\ 5'} = 0{,}619$. On May 4, we find the value to be 0,602; hence the ring paſſed through the earth on May 3, agreeing with Dr. MASKELYNE's computation (501).

498. To determine the appearance of the ring when the earth is at any point e, and Saturn at z, it is manifeſt, that e is at the ſame angular diſtance from the plane of the ring at Saturn that m is, the angles at z being vertical; but the angular diſtance of m from the plane of the ring at z is meaſured by mw, which therefore meaſures the elevation of the eye at e above the plane. Now to find mw, let there be given mG the geocentric latitude of Saturn, and it's geocentric longitude, or the point G on the ecliptic; then, as the point V is known, we ſhall know GV; hence we can find mV, and the angle mVG, which ſubtracted from wVG gives wVm; therefore in the triangle mwV, we can find mw the elevation of the eye above the plane of the ring. Hence rad. : ſin. mw :: major axis : minor axis of the ring.

Ex. On July 12, 1784, Mr. BUGGE obſerved the geocentric longitude of Saturn to be $9^s.\ 20^\circ.\ 34'.\ 48''$, and latitude $3'.\ 35''$ N. he alſo determined the aſcending node of Saturn's orbit to be $3^s.\ 21^\circ.\ 50'.\ 8''$; hence $GV = 4^s.\ 3^\circ.\ 29'.\ 48''$, and as $Gm = 3'.\ 35''$, we have $mV = 123^\circ.\ 29'.\ 48''$, and the angle $mVG = 4'.\ 18''$; and if we take $wVG = 31^\circ.\ 21'.\ 19''$, we have $mVw = 31^\circ.\ 17'.\ 1''$; hence $mw = 25^\circ.\ 38'.\ 37''$; conſequently the major axis : minor axis :: rad. : ſin. $25^\circ.\ 38'.\ 37''$ :: 100 : 43.

499. The arc dp meaſures the elevation of the ſun above the plane of the ring; hence, knowing the heliocentric longitude of Saturn on it's orbit, or of the point d, and the longitude of F the node of the ring upon it's orbit, we know dF; and we know alſo the angle dFp which the plane of the ring makes with the orbit; hence in the right angled triangle Fpd, we find dp the elevation required.

500. In the ſame manner as we have determined the inclination of the ring and poſition of the nodes, the inclination of the orbit of the ſeventh ſatellite and place of it's node may be determined, by meaſuring the minor axis of the orbit which it appears to deſcribe at the time when it is greateſt, and comparing it with the major axis, or twice the greateſt elongation. The ſemi-minor axis is determined by meaſuring the diſtance of the ſatellite from the planet, when that diſtance is the leaſt in the whole revolution.

501. In the *Nautical Almanac* for 1791, Dr. MASKELYNE has computed the diſappearances and re-appearances of the ring in 1789 and 1790; aſſuming

the place of the deſcending node on the ecliptic to be 11ˢ. 17°. 18′, the aſcending node of Saturn 3ˢ. 21°. 51′; inclination of it's orbit 2°. 30′. 20″; inclination of the ring to the ecliptic 31°. 20′, and place of the aſcending node of the ring on Saturn's orbit 5ˢ. 20°. 52′, all according to M. de la LANDE; and ſuppoſing, with him, that the ring is juſt viſible with the beſt teleſcopes in common uſe when the ſun is elevated 3′ above it's plane, or three days before the plane paſſes through the ſun; and when the earth is elevated 2′. 30″ above the plane, or one day from the earth's paſſing it.

May 3, 1789, the ring paſſes through the earth, the earth paſſing from the northern ſide which is enlightened, to it's ſouthern ſide which is dark.

Auguſt 26, the ring re-paſſes to the northern or enlightened ſide.

October 11, the ring paſſes through the ſun; when it will change it's enlightened ſide, from the northern to the ſouthern one; conſequently the dark ſide will then be towards the earth.

January 29, 1790, the ring paſſes through the earth, and the earth paſſing from the northern or dark, to the ſouthern, or enlightened ſide of the ring, the ring will become viſible, and continue ſo till 1803.

The phænomena may happen five days ſooner or later than here ſet down, if the Tables ſhould err 10′ in the geocentric longitude of Saturn.

By obſervation with an achromatic teleſcope of five feet focal length, Dr. MASKELYNE concluded that the ring re-paſſed through the earth on Auguſt 28, at 11 hour.

The

The following TABLES, taken from the *Recueil de Tables Aſtronomiques, Berlin*, 1776, are calculated to ſhow the apparent figure of the Ring, and of the orbits of the ſatellites, as ſeen either from the ſun or the earth.

For the Ring, and Six firſt Satellites.							For the Seventh Satellite.						
ARG. Long. ♄ +13°. 43′. 30″							ARG. Long. ♄ + 24°. 50′.						
Deg.	O. VI.		I. VII.		II. VIII		Deg.	O. VI.		I. VII.		II. VIII		Deg.
	−	+	−	+	−	+		−	+	−	+	−	+	
0	0,000		0,260		0,451		30	0,000		0,129		0,224		30
3	0,027		0,284		0,464		27	0,014		0,141		0,230		27
6	0,054		0,306		0,476		24	0,027		0,152		0,236		24
9	0,081		0,328		0,486		21	0,041		0,163		0,242		21
12	0,108		0,348		0,495		18	0,054		0,174		0,246		18
15	0,135		0,368		0,503		15	0,064		0,183		0,250		15
18	0,161		0,384		0,509		12	0,080		0,192		0,253		12
21	0,187		0,405		0,514		9	0,093		0,201		0,256		9
24	0,212		0,421		0,518		6	0,105		0,209		0,257		6
27	0,236		0,437		0,520		3	0,117		0,217		0,258		3
30	0,260		0,451		0,521		0	0,129		0,224		0,259		0
	+	−	+	−	+	−		+	−	+	−	+	−	
	XI.	V.	X.	IV.	IX.	III.		XI.	V.	X.	IV.	IX.	III.	

To the quantity taken from the Tables, apply the latitude of Saturn expreſſed in minutes divided by 4000, with the ſign − when the latitude is north, and + when it is ſouth; but for the ſeventh ſatellite, we muſt multiply this laſt quantity by $\frac{10}{9}$; and the reſult gives the minor axis of the ring, or of the orbits, the major axis being unity.

Ex.

Ex. On April 22, 1767, the geocentric latitude of Saturn was 1°. 10′ ſouth, and longitude 5ˢ. 16°. 55′; hence

For the Ring, and Six firſt Satellites.		For the Seventh Satellite.	
2ˢ. 16°. 55′		2ˢ. 16°. 55′	
13. 43		24. 50	
3. 0. 38 . . .	−0,521	3. 11. 45 . . .	−0,253
$\frac{70}{4000}$	+0,017	$\frac{70}{4000} \times \frac{10}{9}$ . .	+0,019
Minor axis . .	−0,504	Minor axis . .	−0,234

The ſign + ſhows that that half of the ring, or of the orbits, which is moſt diſtant, is more *north* than the center of Saturn, and the ſign − ſhows it to be more *ſouth*.

The *geocentric* latitude and longitude being here taken, we get the appearance as ſeen from the earth; the *heliocentric* latitude and longitude being aſſumed, gives the appearance at the ſun.

CHAP.

CHAP. XXII.

ON THE ABERRATION OF LIGHT.

Art. 502. IN the year 1725, Mr. MOLYNEUX, aſſiſted by Dr. BRADLEY, fitted up a zenith ſector at Kew, in order to diſcover whether the fixed ſtars had any ſenſible parallax*, that is, whether a ſtar s would appear to have changed it's place whilſt the earth moved from one extremity A of the diameter of it's orbit to the other extremity C; or which is the ſame, to determine whether the diameter AC of the earth's orbit ſubtends any ſenſible angle AsC at the ſtar s. The very important diſcovery ariſing from their obſervations is ſo clearly and fully related by Dr. BRADLEY in a Letter to Dr. HALLEY, that I cannot do better than to give it to the reader in his own words. *Phil. Tranſ.* N°. 406. FIG. 107.

503. "Mr. MOLYNEUX's apparatus was completed and fitted for obſerving about the end of November 1725, and on the third day of December following, the bright ſtar in the head of *Draco* (marked γ by BAYER) was for the firſt time obſerved as it paſſed near the zenith, and it's ſituation carefully taken with the inſtrument. The like obſervations were made on the 5th, 11th and 12th of the ſame month; and there appearing no material difference in the place of the ſtar, a farther repetition of them at this ſeaſon ſeemed needleſs, it being a part of the year wherein no ſenſible alteration of parallax in this ſtar could ſoon be expected. It was chiefly therefore curioſity that tempted me (being then at Kew where the inſtrument was fixed) to prepare for obſerving the ſtar on December 17, when having adjuſted the inſtrument as uſual, I perceived that it paſſed a little more ſouthwardly this day than when it was obſerved before. Not ſuſpecting any other cauſe of this appearance, we firſt concluded that it was owing to the uncertainty of the obſervations, and that either this or the foregoing were not ſo exact as we had before ſuppoſed; for which reaſon we purpoſed to repeat the obſervation again, in order to determine

* Dr. HOOK was the inventor of this method, and in the year 1669 he put it in practice at Greſham College, with a teleſcope 36 feet long. His firſt obſervation was July 6, at which time he found the bright ſtar in the head of *Draco*, marked γ by BAYER, paſſed about 2′. 12″ northward from the zenith; on July 9, it paſſed at the ſame diſtance; on Auguſt 6, it paſſed 2′. 6″ northward from the zenith; on October 21, it paſſed 1′. 48″ or 50″ north from the zenith, according to his obſervations. See his *Cutlerian* Lectures.

mine from whence this difference proceeded; and upon doing it on December 20, I found that the ſtar paſſed ſtill more ſouthwardly than in the former obſervations. This ſenſible alteration the more ſurprized us, in that it was the contrary way from what it would have been, had it proceeded from an annual parallax of the ſtar: but being now pretty well ſatisfied that it could not be entirely owing to the want of exactneſs in the obſervations, and having no notion of any thing elſe that could cauſe ſuch an apparent motion as this in the ſtar, we began to think that ſome change in the materials, &c. of the inſtrument itſelf might have occaſioned it. Under theſe apprehenſions we remained ſome time, but being at length fully convinced by ſeveral trials of the great exactneſs of the inſtrument, and finding by the gradual increaſe of the ſtar's diſtance from the pole, that there muſt be ſome regular cauſe that produced it, we took care to examine nicely at the time of each obſervation how much it was; and about the beginning of March 1726, the ſtar was found to be 20″ more ſouthwardly than at the time of the firſt obſervation. It now indeed ſeemed to have arrived at it's utmoſt limit ſouthward, becauſe in ſeveral trials made about this time no ſenſible difference was obſerved in it's ſituation. By the middle of April it appeared to be returning back again towards the north; and about the beginning of June it paſſed at the ſame diſtance from the zenith as it had done in December when it was firſt obſerved.

From the quick alteration of this ſtar's declination about this time (it increaſing a ſecond in three days) it was concluded that it would now proceed northward, as it before had gone ſouthward of it's preſent ſituation; and it happened as was conjectured, for the ſtar continued to move northward till September following, when it again became ſtationary, being then near 20″ more northwardly than in June, and no leſs than 39″ more northwardly than it was in March. From September the ſtar returned towards the ſouth, till it arrived in December to the ſame ſituation it was in at that time twelve months, allowing for the difference of declination on account of the preceſſion of the equinox.

This was a ſufficient proof that the inſtrument had not been the cauſe of this apparent motion of the ſtar, and to find one adequate to ſuch an effect ſeemed a difficulty. A nutation of the earth's axis was one of the firſt things that offered itſelf upon this occaſion, but it was ſoon found to be inſufficient; for though it might have accounted for the change of declination in γ Draconis, yet it would not at the ſame time agree with the phænomena in other ſtars, particularly in a ſmall one almoſt oppoſite in right aſcenſion to γ Draconis, at about the ſame diſtance from the north pole of the equator; for though this ſtar ſeemed to move the ſame way as a nutation of the earth's axis would have made it, yet it changing it's declination but about half as much as γ Draconis

in

in the ſame time, (as appeared upon comparing the obſervations of both made upon the ſame days at different ſeaſons of the year,) this plainly proved that the apparent motion of the ſtars was not occaſioned by a real nutation, ſince if that had been the cauſe, the alteration in both ſtars would have been nearly equal.

The great regularity of the obſervations left no room to doubt but that there was ſome regular cauſe that produced this unexpected motion, which did not depend on the uncertainty or variety of the ſeaſons of the year. Upon comparing the obſervations with each other, it was diſcovered that in both the fore-mentioned ſtars the apparent difference of declination from the maxima was always nearly proportional to the verſed ſine of the ſun's diſtance from the equinoctial points. This was an inducement to think that the cauſe, whatever it was, had ſome relation to the ſun's ſituation with reſpect to thoſe points. But not being able to frame any hypotheſis at that time ſufficient to ſolve all the phænomena, and being very deſirous to ſearch a little farther into this matter, I began to think of erecting an inſtrument for myſelf at Wanſted, that having it always at hand I might with the more eaſe and certainty enquire into the laws of this new motion. The conſideration likewiſe of being able by another inſtrument to confirm the truth of the obſervations hitherto made with Mr. MOLYNEUX's was no ſmall inducement to me; but the chief of all was the opportunity I ſhould thereby have of trying in what manner other ſtars were affected by the ſame cauſe, whatever it was. For Mr. MOLYNEUX's inſtrument being originally deſigned for obſerving γ Draconis (in order, as I ſaid before, to try whether it had any ſenſible parallax) was ſo contrived as to be capable of but little alteration in it's direction, not above ſeven or eight minutes of a degree; and there being few ſtars within half that diſtance from the zenith of Kew bright enough to be well obſerved, he could not with his inſtrument thoroughly examine how this cauſe affected ſtars differently ſituated with reſpect to the equinoctial and ſolſtitial points of the ecliptic.

Theſe conſiderations determined me; and by the contrivance and direction of the very ingenious perſon Mr. GRAHAM, my inſtrument was fixed up Auguſt 19, 1727. As I had no convenient place where I could make uſe of ſo long a teleſcope as Mr. MOLYNEUX's, I contented myſelf with one of but little more than half the length of his (viz. of about $12\frac{1}{2}$ feet, his being $24\frac{1}{4}$) judging from the experience which I had already had, that this radius would be long enough to adjuſt the inſtrument to a ſufficient degree of exactneſs, and I have had no reaſon ſince to change my opinion; for from all the trials I have yet made, I am very well ſatisfied that when it is carefully rectified, it's ſituation may be ſecurely depended upon to half a ſecond. As the place where my inſtrument was to be hung in ſome meaſure determined it's radius, ſo did

it alſo the length of the arch or limb on which the diviſions were made to adjuſt it; for the arch could not conveniently be extended farther than to reach to about 6°. 15′ on each ſide my zenith. This indeed was ſufficient, ſince it gave me an opportunity of making choice of ſeveral ſtars very different both in magnitude and ſituation, there being more than two hundred inſerted in the Britiſh Catalogue that may be obſerved with it. I needed not to have extended the limb ſo far, but that I was willing to take in Capella, the only ſtar of the firſt magnitude that comes ſo near my zenith.

My inſtrument being fixed, I immediately began to obſerve ſuch ſtars as I judged moſt proper to give me light into the cauſe of the motion already mentioned. There was variety enough of ſmall ones, and not leſs than twelve that I could obſerve through all the ſeaſons of the year, they being bright enough to be ſeen in the day time when neareſt the ſun. I had not been long obſerving before I perceived that the notion we had before entertained of the ſtars being fartheſt north and ſouth when the ſun was about the equinoxes, was only true of thoſe that were near the ſolſtitial colure; and after I had continued my obſervations a few months, I diſcovered what I then apprehended to be a general law, obſerved by all the ſtars, viz. that each of them became ſtationary or was fartheſt north or ſouth when they paſſed over my zenith at ſix o'clock either in the morning or evening. I perceived likewiſe that whatever ſituation the ſtars were in with reſpect to the cardinal points of the ecliptic, the apparent motion of every one tended the ſame way when they paſſed my inſtrument about the ſame hour of the day or night; for they all moved ſouthward while they paſſed in the day, and northward in the night; ſo that each was fartheſt north when it came about ſix o'clock in the evening, and fartheſt ſouth when it came about ſix in the morning.

Though I have ſince diſcovered that the maxima in moſt of theſe ſtars do not happen exactly when they come to my inſtrument at thoſe hours, yet not being able at that time to prove the contrary, and ſuppoſing that they did, I endeavoured to find out what proportion the greateſt alterations of declination in different ſtars bore to each other; it being very evident that they did not all change their declinations equally. I have before taken notice that it appeared from Mr. MOLYNEUX's obſervations that γ Draconis altered it's declination about twice as much as the fore-mentioned ſmall ſtar almoſt oppoſite to it; but examining the matter more particularly, I found that the greateſt alteration of declination in theſe ſtars was as the ſine of the latitude of each reſpectively. This made me ſuſpect that there might be the like proportion between the maxima of other ſtars; but finding that the obſervations of ſome of them would not perfectly correſpond with ſuch an hypotheſis, and not knowing whether the ſmall difference I met with might not be owing to the uncer-

uncertainty and error of the obſervations, I deferred the farther examination into the truth of this hypotheſis till I ſhould be furniſhed with a ſeries of obſervations made in all parts of the year; which might enable me not only to determine what errors the obſervations are liable to, or how far they may ſafely be depended upon; but to judge whether there had been any ſenſible change in the parts of the inſtrument itſelf.

Upon theſe conſiderations I laid aſide all thoughts at that time about the cauſe of the fore-mentioned phænomena, hoping that I ſhould the eaſier diſcover it when I was better provided with proper means to determine more preciſely what they were.

When the year was completed I began to examine and compare my obſervations, and having pretty well ſatisfied myſelf as to the general laws of the phænomena, I then endeavoured to find out the cauſe of them. I was already convinced that the apparent motion of the ſtars was not owing to a nutation of the earth's axis. The next thing that offered itſelf, was an alteration in the direction of the plumb-line with which the inſtrument was conſtantly rectified; but this upon trial proved inſufficient. Then I conſidered what refraction might do, but here alſo nothing ſatisfactory occurred. At laſt I conjectured that all the phænomena hitherto mentioned proceeded from the progreſſive motion of light and the earth's annual motion in it's orbit For I perceived if light was propagated in time, the apparent place of a fixed object would not be the ſame when the eye is at reſt, as when it is moving in any other direction than that of the line paſſing through the eye and object; and that when the eye is moving in different directions, the apparent place of the object would be different."

This is Dr. BRADLEY's account of this very important diſcovery; we ſhall therefore proceed to ſhow that his principle will ſolve all the phænomena.

504. The ſituation of any object in the heavens is determined by the poſition of the axis of the teleſcope annexed to the inſtrument with which we meaſure; for ſuch a poſition is given to the teleſcope, that the rays of light from the object may deſcend down the axis, and in that ſituation the index ſhows the angular diſtance required. Now if light be progreſſive, when a ray from any object deſcends down the axis, the poſition of the teleſcope muſt be different from what it would have been if light had been inſtantaneous, and therefore the place which is meaſured in the heavens will be different from the true place. For let S' be a fixed ſtar, vF the direction of the earth's motion, $S'F$ the direction of a particle of light, entering the axis ac of a teleſcope at a, and moving through aF while the earth moves from c to F; then if the teleſcope keep parallel to itſelf, the light will deſcend in the axis. For let the axis nm, FIG. 108.

Fw continue parallel to *ac*; then, conſidering each motion * as uniform, the ſpaces deſcribed in the ſame time will continue in the ſame proportion; but *cF* : *aF* :: *cn* : *av*, and by ſuppoſition *cF*, *aF* are deſcribed in the ſame time, therefore *cn*, *av* are deſcribed in the ſame time; hence when the teleſcope comes into the ſituation *nm*, the particle of light will be in the axis at *v*; and this being true for every inſtant, in this poſition of the teleſcope the ray will deſcend down the axis, and conſequently the place meaſured by the teleſcope at *F* is *s'*, and the angle *S'Fs'* is the *aberration*, or *the difference between the true place of the ſtar and the place meaſured by the inſtrument*. Hence if we take any line *FS* : *Ft* :: velocity of light : the velocity of the earth, and join *St*, and complete the parallelogram *FtSs*, the aberration will be equal to *FSt*. Alſo *S* repreſents the true place of the ſtar, and *s* the place meaſured by the inſtrument.

505. As the place meaſured by the inſtrument differs from the true place, let us next conſider how the progreſſive motion of light may affect the place of the ſtar ſeen by the naked eye. If a ray of light fall upon the eye in motion, it's relative motion in reſpect to the eye will be the ſame as if you were to impreſs equal motions in the ſame direction upon each at the inſtant of contact; for equal motions in the ſame direction impreſſed upon two bodies will not affect their relative motions, and therefore the effect of one upon the other will not be altered Let *vF* be a tangent to the earth's orbit at *F* which will repreſent the direction of the earth's motion at *F*, *S'* the ſtar, join *S'F* and produce it to *G*, and take *FG* : *Fn* :: the velocity of light : the velocity of the earth in it's orbit, and complete the parallelogram *nFGH*, and draw the diagonal *FH*. Now as *FG*, *nF* repreſent the motions of light and of the earth in it's orbit, conceive a motion *Fn* equal and oppoſite to *nF* to be impreſſed upon the eye at *F* and upon the ray of light, then the eye will be at reſt, and the ray of light, by the two motions *FG*, *Fn*, will deſcribe the diagonal *FH*; this therefore is the relative motion of the ray of light in reſpect to the eye itſelf. Hence the object appears in the direction *HF*, and conſequently it's apparent place differs from it's true place by the angle *GFH* = *FSt*. It appears therefore, that the apparent place of the object to the naked eye is the ſame as the place meaſured by the inſtrument. We may therefore call the place meaſured by the inſtrument, the apparent place. Many writers have given the explanation in this article, as the proof of the aberration, not conſidering that the aberration is the difference between the true place and that meaſured by the inſtrument, or the inſtrumental error; indeed, in this caſe, the apparent place to the naked eye coincides with the place meaſured by the inſtrument,

* The motion of the ſpectator ariſing from the rotation of the earth about it's axis is not here taken into the calculation, it being ſo ſmall as not to produce any ſenſible effect.

ſtrument, and therefore no error has been produced by conſidering it in that point of view; but it introduces a wrong idea of the ſubject; the correction which we apply, or the aberration, is the correction of the place meaſured by the inſtrument, and therefore the inveſtigation ought to proceed upon this principle; how much does the meaſured place differ from the true place?

506. By Trigonometry, ſin. FSt : ſin. FtS :: Ft : FS :: velocity of the earth : velocity of light, hence ſine of aberration = ſin. $FtS \times \frac{\text{vel. of earth}}{\text{vel. of light}}$; therefore if we conſider the velocity of the earth and of light to be conſtant, the ſine of aberration, or the aberration itſelf as it never exceeds 20″, varies as ſin. FtS, and therefore is greateſt when that angle is a right angle; if therefore s be put for the ſine of FtS, we have 1 (rad.) : s :: 20″ : $s \times 20''$ the aberration. Hence when Ft coincides with FS', or the earth be moving directly to or from a ſtar, there is no aberration.

507. As (by obſervation) the angle $FSt = 20''$ when $FtS = 90°$, we have, *the velocity of the earth : velocity of light :: ſin.* 20″ : *radius* :: 1 : 10314.

508. The aberration $S's'$ lies from the true place of the ſtar in a direction parallel to the direction of the earth's motion, and towards the ſame part.

509. Whilſt the earth makes one revolution in it's orbit, the curve, parallel to the ecliptic, deſcribed by the apparent place of a fixed ſtar, is a circle. For let $AFB\mathcal{Q}$ be the earth's orbit, K the focus in which the ſun is, H the other focus; on the major axis AB deſcribe a circle; draw a tangent ΥFZ to the point F, and $K\Upsilon$, HZ perpendicular to it; then, by Conics, the points Υ and Z will be always in the circumference of the circle. Let S' be the true place of the ſtar, any where out of the plane of the ecliptic, which therefore muſt be conceived to be elevated above the plane $AFB\mathcal{Q}$, and take tF : FS as the velocity of the earth : the velocity of light, and complete the parallelogram $FtSs$, and s will (504) be the apparent place of the ſtar. Draw FL perpendicular to AB, and let $WsVx$ be the curve deſcribed by the point s, and WSV be parallel to FL. Now (as will be proved when we come to the phyſical cauſes of the planets motions) the velocity of the earth varies as $\frac{1}{K\Upsilon}$, or as HZ; but tF, or Ss, repreſents the velocity of the earth; hence Ss varies as HZ. Alſo as Ss, SV are parallel to $F\Upsilon$, FL, the angle sSV = the angle ΥFL which is = the angle ZHL, becauſe the angle LFZ added to each makes two right angles, for in the quadrilateral figure $LFZH$ the angles L and Z are right ones. Hence as Ss varies as HZ, and the angle $sSV = ZHA$, the figures deſcribed by the points s and Z muſt be ſimilar; but Z deſcribes a circle in the time of one revolution of the earth in it's orbit, hence s muſt deſcribe a circle about S in the ſame time. And as Ss is always parallel to tF which lies in the

FIG. 109.

plane of the ecliptic, the circle *WsVx* is parallel to the ecliptic. Alſo, as *S* and *H* are two points ſimilarly ſituated in *WV* and *AB*, it appears that the true place of the ſtar divides that diameter which, although in a different plane, we may conſider as perpendicular to the major axis of the earth's orbit, in the ſame ratio as the focus divides the major axis. But as the earth's orbit is very nearly a circle, we may conſider *S* in the center of the circle without any ſenſible error.

510. As we may, for the purpoſes which we ſhall here want to conſider, conceive the earth's orbit *AFBQ* to be a circle, if from the center *C* we draw *Cs'* parallel to *Ss*, or *ϒF*, *s'* will be the point in that circle correſponding to *s* in the circle *WsVx*, and as *FCs'*=90°, the apparent place of the ſtar in the circle of aberration is always 90° *before* the place of the earth in it's orbit, and conſequently the apparent angular velocity of the ſtar and earth about their reſpective centers are always equal. It is further ſuppoſed, that the ſtar *S'* is at an indefinitely great diſtance; for the ſituation of the ſtar is not ſuppoſed to be altered from the motion of the earth, and conſidering *FS* as always parallel to itſelf, it will always be directed to *S'* as a fixed point in the heavens. Hence alſo, as the apparent place of the ſun is oppoſite to that of the earth, the apparent place of the ſtar in the circle of aberration is 90° *behind* that of the ſun.

511. As a ſmall part of the heavens may be conceived to be a plane perpendicular to a line joining the ſtar and eye, it follows from the principles of orthographic projection, that the circle parallel to the ecliptic deſcribed by the apparent place of the ſtar projected upon this plane will be an ellipſe; the apparent path of the ſtar in the heavens will therefore be an ellipſe, and the major axis will be to the minor as radius to the ſine of the ſtar's latitude. For let *CE* be the plane of the ecliptic, *P* it's pole, *PE* a ſecondary to it, *PC* perpendicular to *EC*, *C* the place of the eye, and let *ab* be parallel to *CE*, then it will be that diameter of the circle *anbm* of aberration which is ſeen moſt obliquely, and conſequently that diameter which is projected into the minor axis of the ellipſe; let *mn* be perpendicular to *ab*, and it will be ſeen directly, being perpendicular to a line drawn from it to the eye, and therefore will be the major axis; draw *Ca*, *Cbd*, and *ad* is the projection of *ab*; and as *ad* may be conſidered as a ſtraight line, we have *mn*, or *ab*, the major axis : *ad* the minor :: rad. : ſin. *abd*, or *ECd* the ſtar's latitude. As the angle *bad* is the complement of *abd*, or of the ſtar's latitude, the circle is projected upon a plane making an angle with it equal to the complement of the ſtar's latitude.

FIG. 110.

512. As the minor axis *da* coincides with a ſecondary to the ecliptic, it muſt be perpendicular to it, and the major axis *mn* is parallel to it, it's poſition not being altered by projection. Hence in the pole of the ecliptic, the

ſine

ſine of the ſtar's latitude being radius, the ellipſe becomes a circle; and in the plane of the ecliptic, the ſine of the ſtar's latitude being = o, the minor axis vaniſhes, and the ellipſe becomes a ſtraight line, or rather a very ſmall part of a circular arc.

513. To find the aberration in *latitude* and *longitude*. Let *ABCD* be the earth's orbit ſuppoſed to be a circle with the ſun in the center at *x*, and conceive *P* to be in a line drawn from *x* perpendicular to *ABCD*, and to repreſent the pole of the ecliptic; let *S* be the true place of the ſtar, and conceive *apcq* to be the circle of aberration parallel to the ecliptic, and *abcd* the ellipſe into which it is projected; let γT be an arc of the ecliptic, and draw the ſecondary *PSG* to it, and (511) it will coincide with the minor axis *bd* into which the diameter *pq* is projected; draw *GCxA*, and it (511) is parallel to *pq*, and *BxD* perpendicular to it muſt be parallel to the major axis *ac*; then when the earth is at *A*, the ſtar is in conjunction, and in oppoſition when the earth is at *C*. Now as the place of the ſtar in the circle of aberration is (510) always 90° before the earth in it's orbit, when the earth is at *A*, *B*, *C*, *D* the apparent places of the ſtar in the circle will be at *a*, *p*, *c*, *q*, or in the ellipſe at *a*, *b*, *c*, *d*; and to find the place of the ſtar in the circle when the earth is at any point *E*, take the angle $pSs = ExB$, and *s* will be the correſponding place of the ſtar in the circle; and to find the projected place in the ellipſe, draw *sv* perpendicular to *Sc*, and *vt* perpendicular to *Sc* in the plane of the ellipſe, and *t* will be the apparent place of the ſtar in the ellipſe; join *st* and it will be perpendicular to *vt*, becauſe the projection of the circle into the ellipſe is in lines perpendicular to the ellipſe; draw the ſecondary *PvtK*, which will, as to ſenſe, coincide with *vt*, unleſs the ſtar be very near to the pole of the ecliptic; therefore the rules here given will be ſufficiently accurate, except in that caſe. Now as *cvS* is parallel to the ecliptic, *S* and *v* muſt have the ſame latitude, hence *vt* is the aberration in latitude; and as *G* is the true, and *K* the apparent place of the ſtar in the ecliptic, *GK* is the aberration in longitude. To find theſe quantities, put *m* and *n* for the ſine and coſine of the angle *sSc*, or *CxE* the earth's diſtance from ſyzygies, radius being unity; and as (511) the angle *svt* = the complement of the ſtar's latitude, the angle *vst* = the ſtar's latitude, for the ſine and coſine of which put *v* and *w*, and put $r = Sa$ or *Ss*; then in the right angled triangle *Ssv*, $1 : m :: r : sv = rm$; hence in the triangle *vts*, $1 : v :: rm : tv = rvm$ the aberration in *latitude*. Alſo in the triangle *Ssv*, $1 : n :: r : vS = rn$, hence $w\ (13) : 1 :: rn : GK = \frac{rn}{w}$ the aberration in *longitude*. When the earth is in ſyzygies $m = o$, therefore there is no aberration in latitude; and, as *n* is then greateſt, there is the greateſt aberration in longitude; if the earth be at *A*, or the ſtar in conjunction, the apparent place of the ſtar is

FIG. 111.

is at a, and reduced to the ecliptic at H, therefore GH is the aberration, which diminiſhes the longitude of the ſtar, the order of the ſigns being $\curlyvee G\curlywedge$; but when the earth is at C, or the ſtar in oppoſition, the apparent place r reduced to the ecliptic is at F, and the aberration GF increaſes the longitude; hence the longitude is the greateſt when the ſtar is in oppoſition, and leaſt when in conjunction. When the earth is in quadratures at D or B, then $n = 0$, and m is greateſt, therefore there is no aberration in longitude, but the greateſt in latitude; when the earth is at D, the apparent place of the ſtar is at d and the latitude is there increaſed; but when the earth is at B the apparent place of the ſtar is at b and the latitude is diminiſhed; hence the latitude is leaſt in quadratures before oppoſition, and greateſt in quadratures after. From the mean of a great number of obſervations, Dr. BRADLEY determined the value of r to be 20″.

Ex. 1. What is the greateſt aberration in latitude and longitude of γ *Urſæ minoris*, whoſe latitude is 75°. 13′? Here $m = 1$, $v = ,9669$ the ſine of 75°. 13′; hence $20'' \times ,9669 = 19'',34$ the greateſt aberration in *latitude*. For the greateſt aberration in longitude, $n = 1$, $w = ,2551$; hence $\frac{20''}{,2551} = 78'',4$ the greateſt aberration in *longitude*.

Ex. 2. What is the aberration in latitude and longitude of the ſame ſtar, when the earth is 30° from ſyzygies? Here $m = ,5$; hence $19'',34 \times ,5 = 9'',67$ the aberration in *latitude*. If the earth be 30° beyond conjunction or before oppoſition, the latitude is diminiſhed; but if it be 30° after oppoſition or before conjunction the latitude is increaſed. Alſo $n = ,866$; hence $78'',4 \times ,866 = 67'',89$ the aberration in *longitude*. If the earth be 30° from conjunction, the longitude is diminiſhed; but if it be 30° from oppoſition, it is increaſed.

Ex. 3. For the *Sun*, $m = 0$ and $n = 1$, $w = 1$; hence it has no aberration in latitude, and the aberration in longitude $= r = 20''$ conſtantly. This quantity 20″ of aberration of the ſun anſwers to it's mean motion in 8′. 7″. 30‴, which is therefore the time the light is moving from the ſun to us at it's mean diſtance. Hence the ſun always appears 20″ backwarder than it's true place.

The following TABLE will render the calculations ſhorter.

The

The Argument for the Longitude is, *Long. Sun – Long. Star.* The Argument for the Latitude is, *Long. Sun – Long. Star – 3 Signs.*

Deg.	O. VI. −	O. VI. +	I. VII. −	I. VII. +	II. VIII. −	II. VIII. +	Deg.
0		20″, 0		17″, 32		10″, 0	30
1		20, 0		17, 14		9, 70	29
2		19, 99		16, 96		9, 39	28
3		19, 97		16, 77		9, 8	27
4		19, 95		16, 58		8, 77	26
5		19, 92		16, 38		8, 45	25
6		19, 89		16, 18		8, 13	24
7		19, 85		15, 97		7, 81	23
8		19, 81		15, 76		7, 49	22
9		19, 75		15, 54		7, 17	21
10		19, 70		15, 32		6, 84	20
11		19, 63		15, 9		6, 51	19
12		19, 56		14, 86		6, 18	18
13		19, 49		14, 63		5, 85	17
14		19, 41		14, 39		5, 51	16
15		19, 32		14, 14		5, 18	15
16		19, 23		13, 89		4, 84	14
17		19, 13		13, 64		4, 50	13
18		19, 2		13, 38		4, 16	12
19		18, 91		13, 12		3, 81	11
20		18, 80		12, 86		3, 47	10
21		18, 67		12, 59		3, 12	9
22		18, 54		12, 21		2, 78	8
23		18, 41		12, 4		2, 44	7
24		18, 27		11, 76		2, 9	6
25		18, 13		11, 47		1, 74	5
26		17, 98		11, 18		1, 40	4
27		17, 82		10, 89		1, 5	3
28		17, 66		10, 60		0, 70	2
29		17, 49		10, 30		0, 35	1
30		17, 32		10, 0		0, 0	0
	−	+	−	+	−	+	
	XI.	V.	X.	IV.	IX.	III.	

To find the aberration in $\left\{\begin{matrix}\text{longitude}\\\text{latitude}\end{matrix}\right\}$, multiply the quantities taken from this Table by $\left\{\begin{matrix}\text{secant}\\\text{sine}\end{matrix}\right\}$ of the ſtar's latitude.

Ex. Let the longitude of the ſun be $7^s.\ 5^\circ.\ 18'$, the longitude of a ſtar $5^s.\ 18^\circ.\ 14'$, and it's latitude $31^\circ.\ 10'$.

$7^s.\ 5^\circ.\ 18'$		
$5.\ 18.\ 14$		
$1.\ 17.\ 4$	$-13'',62$	
$31.\ 10$ ſec.	$1,169$	
Aberration in *Longitude*	$-15,92$	Product.
$1^s.\ 17^\circ.\ 4' - 3^s = 10^s.\ 17^\circ.\ 4'$	$-14'',65$	
$31.\ 10$ ſine	$0,5175$	
Aberration in *Latitude*	$-7,58$	Product.

514. When the earth is at *A*, the ſtar is in conjunction, and it's apparent place at *a*; therefore the angle *AxE* deſcribed by the earth from conjunction, or the angle *sSa*, ſhows the elongation of the ſtar from the ſun.

To find the aberration in *Right Aſcenſion* and *Declination*, we ſhall, in part, follow the method given by M. CAGNOLI in his Trigonometry, as being the moſt convenient for practice, and from which M. de LAMBRE has computed a ſet of Tables, by which the aberration may, at any time, be very readily found.

FIG. 112. 515. To find the aberration in *Declination*. Let *abcd* be the ellipſe of aberration, and *P* the pole of the ecliptic, as deſcribed in the laſt figure; on the major axis *ac* deſcribe the circle *apcq*, which we will now ſuppoſe to lie in the plane of the ellipſe, and then every point of this circle will be projected into the ſame point of the ellipſe as before; let *R* be the pole of the equator, and perpendicular to *RS* draw the diameter *MN* of the ellipſe; alſo draw *BMC*, *ϒNW* perpendicular to *ac*, and *ϒB* will be the correſponding diameter of the circle; draw *FS* perpendicular to *Bϒ*, *FQD* perpendicular to *ac*, and *QH* perpendicular to *MS*; from any point *X* let fall *XsE* perpendicular to *ac*; draw *Xz*, *st*, perpendicular to *Bϒ* and *MN* reſpectively, and *sv* an ordinate to the diameter *MN*.

516. As

516. As the point F of the circle lies at the diſtance of 90° from the diameter $B\Upsilon$, the diameter FSf will be projected into a diameter $\mathfrak{Q}Sr$ which will be conjugate to MSN, and therefore a tangent at $\mathfrak{Q}$ is parallel to MN; hence $\mathfrak{Q}H$ is the greateſt perpendicular on MN, and conſequently it is the greateſt aberration in declination; for as MN is the projection of $B\Upsilon$, which is perpendicular to the circle of declination RS, there can be no aberration in MN; alſo st is the aberration in declination at any point s. Now when the apparent place of the ſtar is at a, the ſtar is (513) then in conjunction; and as the motion of the ſun is (510) equal to the motion of the ſtar in the circle $apcq$, whilſt the ſtar moves from s to $\mathfrak{Q}$ in the ellipſe, it's motion in the circle would be XF, which therefore repreſents the ſun's motion in the ſame time, or the motion from the time when the ſtar is at s to the time when the aberration in declination is the greateſt. Alſo (514) the arc Fa ſhows the elongation of the ſtar from the ſun when the ſtar appears at $\mathfrak{Q}$, and Xa the elongation when at s.

517. When the ſtar is at s, st is the aberration in declination; and as the poſition of st to sv is conſtant, st varies as sv; but sv is the projection of Xz, and therefore in a given ratio to it; hence st varies as Xz the ſine of $X\Upsilon$, or coſine of XF; that is, the aberration in declination at any time is as the coſine of the ſun's diſtance from the point where it was when the aberration in declination was the greateſt.

To find $\mathfrak{Q}H$, we have, by the property of the ellipſe, $\mathfrak{Q}H \times SM = Sd \times Sc$; hence $\frac{\mathfrak{Q}H}{Sc} = \frac{Sd}{SM} = \frac{Sd \times BS}{SM \times BS} =$ (becauſe $\frac{CM}{CB} = \frac{Sd}{Sq} = \frac{Sd}{SB}$) $\frac{CM \times BS}{SM \times CB}$ $= \frac{CM}{SM}$ divided by $\frac{CB}{BS} = \frac{\text{ſin. } MSa}{\text{ſin. } BSa}$; conſequently $\mathfrak{Q}H : cS ::$ ſin. $MSa :$ ſin. $BSa ::$ ſin. $PSR :$ cos. FSa, hence $\mathfrak{Q}H = \frac{20'' \times \text{ſin. } PSR}{\text{cos. } FSa}$, the greateſt aberration in declination.

518. Let P be the pole of the equator $\mathfrak{Q}EW$, O the pole of the ecliptic CEV, S the place of the ſtar, $PSAM$ a circle of declination, OSL a circle of latitude; then L has the ſame longitude as the ſtar, and therefore (513) it marks the place of the ſun when the aberration in latitude is nothing. Draw the circle STR perpendicular to PSA, and $\mathcal{T}$ will be the place of the ſun when the aberration in declination is the greateſt; for by *Conics*, $WN : W\Upsilon$ FIG. 112. $:: Sb : Sp ::$ (511) ſin. ſtar's lat. : rad. alſo $WN : W\Upsilon ::$ tan. NSW, or PSR, : tan. ΥSW, or cot. FSa; hence ſin. ſtar's lat. : rad. :: tan. PSR : cot. FSa. But ſin. SL (the ſtar's lat.) : rad. :: cot. TSL, or tan. LSM, : cot. $\mathcal{T}L$; hence FIG. 113. the three firſt terms in theſe two laſt proportions being reſpectively equal, the

arc aF (Fig. 112) $= \mathcal{T}L$ (Fig. 113); and as aF repreſents the motion of the ſun from the time when the aberration in declination is the greateſt to conjunction, and L repreſents the place of the ſun at conjunction, $\mathcal{T}$ muſt be the place at the greateſt aberration. Hence, by the laſt Article, the greateſt aberration in declination $= \frac{20'' \times \text{ſin. } MSL}{\text{cos. } L\mathcal{T}}$. But in the triangle $S\mathcal{T}L$, cos. $\mathcal{T}SL$, or ſin. MSL, $=$ ſin. $L\mathcal{T}S \times$ cos. $L\mathcal{T}$; hence the greateſt aberration in declination becomes $20'' \times$ ſin. $L\mathcal{T}S$. Alſo in the triangle $E\mathcal{T}R$, ſin. $E\mathcal{T}R$, or ſin. $L\mathcal{T}S$, $= \frac{\text{ſin. } ER \times \text{ſin. } ER\mathcal{T}}{\text{ſin. } E\mathcal{T}} =$ (becauſe the meaſure of $ER\mathcal{T}$ is AS, and AE is the complement of ER) $\frac{\text{cos. } AE \times \text{ſin. } SA}{\text{ſin. } E\mathcal{T}}$. Hence we get the greateſt aberration in declination $= 20'' \times$ cos. right aſcenſion $\times$ ſin. dec. divided by the ſin. of the ſun's longitude at the time when the aberration is greateſt ſubtractive. Therefore, as the aberration at any other time is (517) as the coſine of the ſun's diſtance from that place where it was when the aberration was the greateſt, if L be the ſun's longitude at the time of the greateſt aberration in declination ſubtractive, S it's longitude at any other time, A the ſun's right aſcenſion, D it's declination, O the obliquity of the ecliptic, the aberration at that time $= \frac{-20'' \times \text{cos. } A \times \text{ſin. } D \times \text{cos. } \overline{L-S}}{\text{ſin. } L} =$

$\frac{20'' \times \text{ccs. } A \times \text{ſin. } D \times \overline{\text{cos. } L \times \text{cos. } S + \text{ſin. } L \times \text{ſin. } S}}{\text{ſin. } L} =$ (becauſe $\frac{\text{cos. } L}{\text{ſin. } L}$ $=$ cot. L) $- 20'' \times$ cos. $A \times$ ſin. $D \times$ cot. $L \times$ cos. $S - 20'' \times$ cos. $A \times$ ſin. $D \times$ ſin. S. But (MAUDUIT's Trigonometry, or CRAKELT's Tranſlation) cot. $E\mathcal{T}$, or cot. $L = \frac{\text{cot. } ER\mathcal{T} \times \text{ſin. } E}{\text{ſin. } ER} + \text{cos. } E \times \text{cot. } ER =$ (becauſe cot. $ER = -\text{tan. } A$) $\frac{\text{cot. } D \times \text{ſin. } O}{\text{cos. } A} - \text{cos. } O \times \text{tan. } A$; hence the aberration in declination becomes $- 20'' \times$ ſin. $D \times$ cos. $S \times$ cot. $D \times$ ſin. $O + 20'' \times$ cos. $A \times$ ſin. $D \times$ cos. $S \times$ cos. $O \times$ tan. $A - 20'' \times$ cos. $A \times$ ſin. $D \times$ ſin. $S =$ (becauſe ſin. $D \times$ cot. $D =$ cos. D, and cos. $A \times$ tan. $A =$ ſin. A) $- 20'' \times$ ſin. $O \times$ cos. $D \times$ cos. $S + 20'' \times$ cos. $O \times$ ſin. $A \times$ ſin $D \times$ cos. $S - 20'' \times$ cos. $A \times$ ſin. $D \times$ ſin. $S = -20'' \times$ ſin. $O \times$ cos. $D \times$ cos $S - 20'' \times$ ſin. $D \times \overline{\text{cos. } A \times \text{ſin. } S - \text{cos. } O \times \text{ſin. } A \times \text{cos. } S}$. For ſouth declination we muſt change the ſigns. But by Trigonometry, cos. $D \times$ cos. $S = \frac{1}{2}$ cos. $\overline{S+D} + \frac{1}{2}$ cos. $\overline{S-D}$, and cos. $A \times$ ſin. $S = \frac{1}{2}$ ſin. $\overline{A+S} - \frac{1}{2}$ ſin. $\overline{A-S}$, alſo ſin. $A \times$ cos. $S = \frac{1}{2}$ ſin. $\overline{A+S} + \frac{1}{2}$ ſin. $\overline{A-S}$; hence the aberration in *Declination* $=$

$+ 10''$

$$\left.\begin{array}{l} +\ 10'' \times \overline{1+\cos.\ O} \times \text{fin.}\ \overline{A-S} \times \text{fin.}\ D \\ -\ 10'' \times \overline{1-\cos.\ O} \times \text{fin.}\ \overline{A+S} \times \text{fin.}\ D \\ -\ 10'' \times \text{fin.}\ O \times \cos.\ \overline{S-D} \\ -\ 10'' \times \text{fin.}\ O \times \cos.\ \overline{S+D}. \end{array}\right\} = \left\{\begin{array}{l} +19'',17 \times \text{fin.}\ \overline{A-S} \times \text{fin.}\ D \\ -\ 0,83 \times \text{fin.}\ \overline{A+S} \times \text{fin.}\ D \\ -\ 3,98 \times \cos.\ \overline{S-D} \\ -\ 3,98 \times \cos.\ \overline{S+D}. \end{array}\right.$$

The two laſt terms muſt have their ſigns changed, when the declination is ſouth.

519. To find the ſun's place when the aberration is the greateſt, we have in the triangle $LS\curlyvee$, fin. SL : rad. :: cot. $\curlyvee SL$: cot. $\curlyvee L$; therefore knowing the longitude of the ſtar, or of the point L, the longitude of $\curlyvee$ the place of the ſun is known. Hence we find the ſun's longitude when the aberration is greateſt ſubtractive.

520. To find the aberration in *Right Aſcenſion*. Let S be the true place of the ſtar, $abcd$ the ellipſe of aberration, $apcq$ the circumſcribing circle, P the pole of the ecliptic and R that of the equator, and let MSN be a conjugate diameter to ASB; draw FNC, DAV perpendicular to ca, join VS, draw CSK which muſt be perpendicular to VS, and draw MG perpendicular to AB; alſo from any point $\mathfrak{Q}$ draw $\mathfrak{Q}sW$ perpendicular to ca, and $\mathfrak{Q}\curlyvee$, sv perpendicular to SV, SA reſpectively, and sr an ordinate to the diameter AB. Now it is manifeſt that A is the apparent place of the ſtar when the aberration in right aſcenſion is nothing, and M when it is greateſt, becauſe a tangent at M is parallel to AB. By the property of the ellipſe, $MG \times AS = dS \times cS$, therefore $AS : cS$ or $SV :: dS : MG$, hence $\frac{AD}{SV} : \frac{AD}{AS} :: dS : MG$; but $AD : dS :: VD : Sq$, therefore $\frac{VD}{SV} : \frac{AD}{AS} :: Sq : MG$, that is, the ſine of Va : the ſine of ASa, or cos. PSR, :: $20'' : MG = \frac{20'' \times \cos.\ PSR}{\text{fin.}\ Va}$, the greateſt aberration in right aſcenſion. If the ſtar be at any other point s, then sv is the aberration in right aſcenſion; but sv is in a given ratio to sr, and sr is in a given ratio to $\mathfrak{Q}\curlyvee$, becauſe $\mathfrak{Q}\curlyvee$ is projected into sr; hence sv varies as $\mathfrak{Q}\curlyvee$ the ſine of $\mathfrak{Q}V$, or coſine of $K\mathfrak{Q}$ the diſtance of the ſun from that point where it was when the aberration was greateſt. Now tan. ASD, or cot. PSR : tan. VSa :: ($AD : VD$ ::) ſin. ſtar's lat. : rad. (511); but tan. ML : tang. MSL :: ſin. ſtar's lat. : rad. hence, as $PSR = MSL$, the tang. $VSa \times$ tan. LM is conſtant, therefore LM is the complement of VSa; hence $LM = Ka$ th elongation of the ſun from the ſtar when the aberration is greateſt; therefore M is the place of the ſun at that time; the longitude of which put $= L$ at the time when the aberration is greateſt ſubtractive. Hence the greateſt aberration

FIG. 114.

FIG. 113, 114.

ration in right aſcenſion $= \frac{20'' \times \text{cos. } MSL}{\text{cos. } LM}$. This is the aberration at the ſtar, and therefore reduced to the equator it (13) becomes $\frac{20'' \times \text{cos. } MSL}{\text{cos. } ML \times \text{cos. } SA}$.

FIG. 113. But $\frac{\text{cos. } MSL}{\text{cos. } ML} = \text{ſin. } M = \frac{\text{ſin. } AE}{\text{ſin. } ME} = \frac{\text{ſin. } A}{\text{ſin. } L}$; therefore the greateſt aberration ſubtractive becomes $\frac{-20'' \times \text{ſin. } A}{\text{cos. } D \times \text{ſin. } L}$; hence the aberration in right aſcenſion at any other time $= \frac{-20'' \times \text{ſin. } A \times \text{cos. } \overline{L-S}}{\text{cos. } D \times \text{ſin. } L} = \frac{-20'' \times \text{ſin. } A}{\text{cos. } D \times \text{ſin. } L} \times \overline{\text{cos. } L \times \text{cos. } S + \text{ſin. } L \times \text{ſin. } S} =$

$\frac{-20'' \times \text{ſin. } A \times \text{cot. } L \times \text{cos. } S - 20'' \times \text{ſin. } A \times \text{ſin. } S}{\text{cos. } D} =$ (becauſe cot. $L =$ cos. $O \times$ cot. A) $\frac{-20'' \times \text{ſin. } A \times \text{cos. } S \times \text{cos. } O \times \text{cot. } A - 20'' \times \text{ſin. } A \times \text{ſin } S}{\text{cos. } D}$

$= \frac{-20'' \times \text{cos. } O \times \text{cos. } S \times \text{cos. } A - 20'' \times \text{ſin. } A \times \text{ſin. } S}{\text{cos. } D}$. Now if we augment A by 90°, or 3 ſigns, the numerator of this fraction becomes the ſame as the coefficient of ſin. D in the aberration of declination, becauſe the ſin. $A = \text{cos. } \overline{A+3s}$, and cos. $A = \text{ſin. } \overline{A+3s}$. But to reduce this further, we have cos. $A \times$ cos. $S = \frac{1}{2}$ cos. $\overline{A+B} + \frac{1}{2}$ cos. $\overline{A-B}$, and ſin. $A \times$ ſin. $S = \frac{1}{2}$ cos. $\overline{A-B} - \frac{1}{2}$ cos. $\overline{A+B}$; hence the aberration in *Right Aſcenſion* $= -\frac{10'' \times \overline{1 + \text{cos. } O} \times \text{cos. } \overline{A-S} - 10'' \times \overline{1 - \text{cos. } O} \times \text{cos. } \overline{A+S}}{\text{cos. } D} =$

$-\overline{19'',17 \times \text{cos. } A-S - 0'',83 \times \text{cos. } A+S} \times \text{ſec. } D.$

521. As M (520) is the place of the ſun when the aberration in right aſcenſion is the greateſt, we have cos. AEM : tan. AE the ſtar's right aſcenſion :: rad. : tan. EM the ſun's longitude. Hence we can find the ſun's longitude when the aberration is greateſt ſubtractive.

522. From theſe expreſſions for the aberration in right aſcenſion and declination, M. de LAMBRE has computed the following TABLES, by which the aberration of a ſtar at any time may be very readily found.

GENERAL

GENERAL TABLES FOR THE ABERRATION OF THE FIXED STARS.

Table I. Arg. $A-S$.					Table II. Arg $A+S$.					Tab. III. Arg. $S+D$ & $S-D$					
S	O. VI.	I. VII.	II. VIII	S	S	O. VI.	I. VII.	II. VIII	S	S	O. VI.	I VII.	II. VIII	S	
D	− +	− +	− +	D	D	+ −	+ −	+ −	D	D	− +	− +	− +	D	
0	19″,17	16″,60	9″,59	30	0	0″,83	0″,72	0″,41	30	0	3″,98	3″,45	1″,99	30	
1	19,17	16,43	9,30	29	1	0,83	0,71	0,40	29	1	3,98	3,42	1,93	29	
2	19,16	16,26	9,00	28	2	0,82	0,70	0,39	28	2	3,98	3,38	1,87	28	
3	19,15	16,08	8,70	27	3	0,82	0,69	0,38	27	3	3,98	3,34	1,81	27	
4	19,13	15,89	8,40	26	4	0,82	0,68	0,37	26	4	3,97	3,30	1,75	26	
5	19,10	15,71	8,10	25	5	0,82	0,67	0,35	25	5	3,97	3,26	1,68	25	
6	19,07	15,51	7,80	24	6	0,82	0,67	0,33	24	6	3,96	3,22	1,62	24	
7	19,03	15,31	7,49	23	7	0,82	0,66	0,32	23	7	3,95	3,18	1,56	23	
8	18,99	15,11	7,19	22	8	0,82	0,65	0,30	22	8	3,94	3,14	1,49	22	
9	18,94	14,90	6,87	21	9	0,82	0,64	0,29	21	9	3,93	3,10	1,43	21	
10	18,88	14,69	6,56	20	10	0,82	0,63	0,28	20	10	3,92	3,05	1,36	20	
11	18,82	14,47	6,24	19	11	0,82	0,62	0,27	19	11	3,91	3,01	1,30	19	
12	18,75	14,25	5,93	18	12	0,82	0,61	0,25	18	12	3,90	2,97	1,23	18	
13	18,68	14,02	5,61	17	13	0,81	0,61	0,24	17	13	3,89	2,92	1,17	17	
14	18,60	13,79	5,28	16	14	0,81	0,60	0,23	16	14	3,87	2,87	1,10	16	
15	18,52	13,56	4,96	15	15	0,80	0,58	0,22	15	15	3,85	2,82	1,03	15	
16	18,43	13,32	4,64	14	16	0,80	0,57	0,20	14	16	3,83	2,77	0,97	14	
17	18,33	13,08	4,31	13	17	0,80	0,56	0,19	13	17	3,81	2,72	0,90	13	
18	18,23	12,83	3,99	12	18	0,79	0,55	0,17	12	18	3,79	2,67	0,83	12	
19	18,13	12,58	3,66	11	19	0,78	0,54	0,15	11	19	3,77	2,62	0,76	11	
20	18,02	12,32	3,33	10	20	0,78	0,53	0,14	10	20	3,74	2,56	0,69	10	
21	17,90	12,07	3,00	9	21	0,77	0,52	0,12	9	21	3,72	2,51	0,63	9	
22	17,78	11,80	2,67	8	22	0,76	0,51	0,11	8	22	3,70	2,46	0,56	8	
23	17,65	11,54	2,34	7	23	0,76	0,50	0,10	7	23	3,67	2,40	0,49	7	
24	17,52	11,27	2,00	6	24	0,75	0.49	0,09	6	24	3,64	2,34	0,42	6	
25	17,38	11,00	1,67	5	25	0,75	0,47	0,07	5	25	3,61	2,28	0,35	5	
26	17,23	10,72	1,34	4	26	0,75	0,46	0,06	4	26	3,58	2,23	0,28	4	
27	17,08	10,44	1,00	3	27	0,74	0,45	0,05	3	27	3,55	2,17	0,21	3	
28	16,93	10,16	0,67	2	28	0,73	0,44	0,03	2	28	3,52	2,11	0,14	2	
29	16,77	9,87	0,33	1	29	0,72	0,43	0,02	1	29	3,49	2,05	0,07	1	
30	16,60	9,59	0,00	0	30	0,72	0,41	0,00	0	30	3,45	1,99	0,00	0	
D	− +	− +	− +	D	D	+ −	+ −	+ −	D	D	− +	− +	− +	D	
S	XI. V.	X. IV.	IX. III.	S	S	XI. V.	X. IV.	IX. III.	S	S	XI. V.	X. IV.	IX. III.	S	

USE OF THE TABLES.

$A =$ the right aſcenſion
$D =$ the declination } of the ſtar
$S =$ the longitude of the ſun.

Enter Table I. with the argument $A - S$, and Table II. with $A + S$, and the ſum of the two correſponding numbers multiplied by the ſecant of D will be the aberration in *Right Aſcenſion*.

Enter Table I. with the argument $A - S + 3$ ſigns, and Table II. with $A + S + 3$ ſigns, and the ſum of the two correſponding numbers multiplied by the ſine of D will be the firſt part of the aberration in declination.

Enter Table III. with the arguments $S + D$ and $S - D$, and you will have two other parts of the aberration in declination; and the ſum of theſe three parts will give the whole aberration in *Declination*.

If the declination of the ſtar be ſouth, add ſix ſigns to $S + D$ and $S - D$.

Ex. To find the aberration of α *Aquilæ*, on May 10, 1795, at 12 o'clock in the evening.

$A = 9^s\ 25^\circ.\ 12'$
$S = 1.\ 20.\ 12$

$A - S = 8.\ 5.\ 0$ Table I. . . . . $+ 8'',1$
$A + S = 11.\ 15.\ 24$ Table II. . . . . $+ 0,8$

$+ 8,9$
$D = 8^\circ.\ 20'$ ſecant . . . . . $1,011$

Aberration in *Right Aſcenſion* . . . . . $+ 8,998$ Product.

$A-S+3$ signs . .	11°. 5°. 0′ Table I. . . .	− 17″,38
$A+S+3$ signs . .	2. 15. 24 Table II. . .	+ 0, 21
		− 17, 17
$D=0^s$. 8°. 20′ fine		0, 145
$S=1$. 20. 12		− 2, 49 Product.
$S+D=1^s$. 28°. 32′ Table III.		− 2, 08
$S-D=1$. 11. 52 Table III.		− 7, 97
Aberration in *Declination*		− 7, 54

If the ſtar's declination had been *ſouth*, then

$S+D+6$ ſigns $=7^s$. 28°. 32′ Table III. .		+ 2″,08
$S-D+6$ ſigns $=7$. 11. 52 Table III. .		+ 2, 97
Firſt Part		− 2, 49
Aberration in *Declination*		+ 2, 56

The aberration of a ſtar applied to it's apparent place gives the true place.

523. The following Rules for finding the aberration of a ſtar are given by Dr. MASKELYNE.

For the Aberration in Longitude.

Coſin. lat. : rad. :: 20″ : *M*, or maximum.

Aberration is 0 tending to exceſs when the ſun's longitude is 3ˢ greater than that of the ſtar; or the argument of aberration is = the ſun's long. − the ſtar's long. − 3ˢ.

For the Aberration in Latitude.

Rad. : ſin. lat. :: 20″ : *M*, or maximum.

Aberration is 0 tending to exceſs when the ſun's longitude is oppoſite that of the ſtar; or the argument is always the ſun's long. − the ſtar's long. ± 6ˢ.

For the Aberration in Right Aſcenſion.

Sin. lat. : rad. :: cot. P, or $\angle$ poſition, : tan. Z.
Cos. declin. $\times$ ſin. Z : cos. $P \times$ rad. :: 20″ : M.

If the ſtar be in 1st or laſt quad. of long. with N. lat. } Z added to the ſtar's
——— or in 2d or 3d quad. of long. with S. lat. } long. gives ⊙.

If the ſtar be in 1st or laſt quad. of long. with S. lat. } Z taken from the
——— or in 2d or 3d quad. of long. with N. lat. } oppoſite to the ſtar's long. gives ⊙.

N. B. ⊙ is the ſun's longitude when the aberration is 0 tending to exceſs.

For the Aberration in North polar diſtance.

Sin. ſtar's lat. : rad. :: tan. P : tan. Z .
Sin. Z : ſin. P :: 20″ : M.

If the ſtar be in 1st or laſt quad. of long. with N. lat.
——— 2d or 3d quad. of long. with N. lat.
——— 2d or 3d quad. of long. with S. lat.
——— 1st or laſt quad. of long. with S. lat.
} ⊙ = { ſtar's long. $- Z$. / ſtar's long. $+ Z$. / op. ſtar's long. $- Z$. / op. ſtar's long. $+ Z$.

N.B. If $\angle$ of poſition, or P, be obtuſe, for Z uſe it's ſupplement to 180°.

⊙ thus found ſubtracted from the ſun's longitude at any time gives the argument of aberration, whoſe ſine multiplied into the maximum gives the aberration itſelf; and if the argument be under ſix ſigns the aberration is poſitive, if above ſix ſigns it is negative.

524. Dr. BRADLEY has ſhown the agreement of his theory with obſervation, which we ſhall here put down for the ſatisfaction of the reader.

γ Draconis. 1727	D.	Aber. in dec. by Obſerv.	Aber. in dec. by Theory
Oct.	20	4″,5	4″,5
Nov.	17	11,5	12
Dec.	6	17,5	18,5
——	28	25	26
1728			
Jan.	24	34	34
Feb.	10	38	37
March	7	39	39

γ Draconis. 1728	D.	Aber. in dec. by Obſerv.	Aber. in dec. by Theory
March	24	37″	38″
April	6	36	36,5
May	6	28,5	29,5
June	5	18,5	20
——	15	17,5	17
July	3	11,5	11,5
Auguſt	2	4	4
Sep.	6	0	0

η Urſæ maj. 1727	D.	Aber. in dec. by Obſerv.	Aber. in dec. by Theory
Sep.	14	29″,5	28″,5
——	24	24,5	25,5
Oct.	16	19,5	19,5
Nov.	14	11,5	10,5
Dec.	14	4	3
1728			
Feb.	17	2	3
March	21	11,5	10,5

η Urſæ maj. 1728	D.	Aber. in dec. by Obſerv.	Aber. in dec. by Theory
April	16	18″,5	18″,
May	5	24,5	23,5
June	5	32	31,5
——	25	35	34,5
July	17	36	36
Auguſt	2	35	35,5
Sep.	20	26,5	26,5

Dr. BRADLEY further obſerves, that in above 70 obſervations made in a year on *γ Draconis*, there was but one (and that is noted very dubious on account of clouds) which differed more than 2″ from the theory, and that did not differ 3″. And in about 50 obſervations made in a year on *η Urſæ majoris*, he did not find a difference of 2″, except in one marked doubtful on account of the undulation of the air, &c. and that did not differ 3″. This agreement between the theory and obſervation leaves no room to doubt but that the cauſe is rightly aſſigned. And if this be the caſe, the annual parallax of the fixed ſtars muſt be extremely ſmall. " I believe, ſays the DR. that I may venture to ſay that in either of the above-mentioned ſtars, it does not amount to 2″. I believe if it were 1″ I ſhould have perceived it in the great number of obſervations that I made, eſpecially on γ Draconis; which agreeing with the theory (without allowing any thing for parallax) nearly as well in conjunction with, as in oppoſition to this ſtar, it ſeems very probable that the parallax is

not ſo great as one ſingle ſecond; and conſequently that it is above 400000 times further from us than the ſun." The obſervations of Mr FLAMSTEAD of the different diſtances of the pole ſtar from the pole at different times of the year, and which was looked upon as a proof of it's annual parallax, was undoubtedly owing to this cauſe. For he concluded that the ſtar was 35″, 40″ or 45″ nearer the pole in December than in May or July; and according to this hypotheſis, it ought to appear 40″ nearer in December than in June. This agreement is greater than could have been expected from obſervations made with his inſtrument.

525. Hence Dr. BRADLEY deduced the following concluſions. 1. That the light of all the fixed ſtars arrives at the earth with equal velocities; for the major axis of the ellipſe is the ſame in all the ſtars, that is 40″ according to his laſt determination. 2. That unleſs their diſtances from us are all equal, which is very improbable, their lights are propagated *uniformly* to all diſtances from them. 3. That light moves from the ſun to the earth in 8′. 7″,5, and it's velocity is to the velocity of the earth in it's orbit as 10314 : 1. 4. That the time thus determined can ſcarce err from the truth by above 5″ or 10″ at moſt, which is ſuch a degree of exactneſs as can never be expected from the eclipſes of *Jupiter's* ſatellites. 5. That as this velocity of ſtar light comes out about a mean of the ſeveral velocities found from the eclipſes of Jupiter's ſatellites, we may reaſonably conclude that the velocities of theſe reflected lights are equal to the velocity of direct light. 6. And as it is highly probable that the velocity of the ſun's emitted light is equal to that of ſtar light, it follows that it's velocity is not altered by reflection into the ſame medium.

On the Aberration of Light in the Planets.

FIG. 115.

526. Let S be the ſun, T the earth, P the correſponding place of the planet, and let us ſuppoſe Tt to be the direction in which the earth is moving, parallel to which draw $P\mathcal{Q}$, and whilſt light moves from P to T let $P\mathcal{Q}$ be equal to the ſpace through which the earth has moved, and (504) $\mathcal{Q}$ is the apparent place of the planet. Now let Pp be the motion of the planet in the ſame time, then $\mathcal{Q}$ being the apparent and p the correſponding true place, the angle $\mathcal{Q}Tp$ is the aberration ariſing from the progreſſive motion of light and the motion of the planet. As $P\mathcal{Q}$, Pp repreſent the motions of the earth and planet, $\mathcal{Q}p$ repreſents their relative motion; hence the motion of the planet about the earth in the time in which light comes from the planet to the earth is the aberration. Let $ST = 1$, $PT = d$, $m =$ the angle deſcribed by the planet about the earth, or it's geocentric motion, either in latitude, longitude, right aſcenſion, or declination,

nation, in 24 hours; then $1 : d :: 8'.\ 7'',5 : 8'.\ 7'',5\ d$ the time light is moving from P to $\mathcal{T}$; consequently $24h. : 8'.\ 7'',5\ d :: m$: the aberration $= \frac{8'.\ 7'',5\ dm}{24h.} = 0{,}00564\ dm.$ Thus we find the aberration of a planet either in latitude, longitude, right ascension or declination. The geocentric motion may be taken from the *Nautical Almanac*, and the distance need not be calculated to any very great degree of accuracy. We may also further observe, that when $m = 0$, or the planet is stationary, the aberration becomes equal to nothing.

Ex. 1. On May 1, 1791, at noon, what is the aberration in longitude of *Mars*?

Here $SP = 1{,}5237$ the mean distance, the longitude of the sun is $1^s.\ 11°$, and the geocentric longitude of Mars is $0^s.\ 29°.\ 19'$; hence the angle $PTS = 11°.\ 41'$, and consequently $PT = 2{,}489 = d$; also $m = 44'.\ 50'' = 2690''$ from the *Nautical Almanac*; hence $0{,}00564\ dm = 37''\frac{2}{3}$ the aberration in longitude.

Ex. 2. For the *Moon*, $d = 0{,}00253$ the mean distance, $m = 13°.\ 10'.\ 35'' = 47435''$ the mean diurnal motion; hence $0{,}00564\ dm = 0'',67$ the aberration, which is so small that it may be neglected.

527. Dr. MASKELYNE observes, that since a planet, as affected by aberration, appears in the place where it should have appeared at the instant of the emission of it's light, exclusive of this cause of error, it follows that the most simple as well as the most elegant method of computing the apparent geocentric place of a planet, is to compute it's geocentric place by the common rules for that instant which precedes the given time by the interval of time taken up by light to move from the planet to the earth. For this purpose the distance of the planet need not be computed very accurately, and then the time may be found by Table XXIV. at the end of this Volume. The sun's longitude must be computed with the epoch of it's mean longitude advanced by 20″, because it always appears so much too backward in the ecliptic by aberration, and the Tables have been constructed without making any correction on this account, and consequently they show the epoch of the mean longitude 20″ too little.

528. If we suppose the planet and earth to describe circles which lie in the same plane, which will make no sensible difference, then if in $\mathcal{T}t$ produced we take $E\mathcal{T} = P\mathcal{Q}$, and draw Pa parallel to $p\mathcal{T}$, the angle $EPa = p\mathcal{T}\mathcal{Q}$ the aberration; draw also $\mathcal{T}ur$ parallel to Pp. By Art. 506. the angle $\mathcal{T}PE$

$=$ fin. $E \times 20''$; alſo fin. E : fin. ErT :: Tr : TE :: Tu : Ta * $= \frac{\text{fin. } ErT}{\text{fin. } E} \times Tu$; hence $Ea = TE \mp \frac{\text{fin. } ErT}{\text{fin. } E} \times Tu$, conſequently TE : $Ea = TE \mp \frac{\text{fin. } ErT}{\text{fin. } E} \times Tu$ :: angle TPE : angle EPa :: fin. $E \times 20''$: angle $EPa =$ fin. $E \times 20'' \mp \frac{\text{fin. } ErT \times Tu}{TE} \times 20''$; but TE : Tu :: velocity of the earth : velocity of the planet :: $\sqrt{SP}$: $\sqrt{1}$,† alſo fin. E, or PTt, $=$ cos. STP, and fin. ErT, or EPp $=$ cos. SPE, or SPT; hence by ſubſtitution, the angle $EPa =$ cos. $STP \times 20'' \mp \frac{\text{cos. } SPT}{\sqrt{SP}} \times 20''$ the aberration in longitude. The firſt term cos. $STP \times 20''$, or fin. $E \times 20''$, is (506) the aberration for a fixed ſtar; hence the other term, or the aberration which ariſes from the motion of the body, varies as the coſine of the angle at the body between the ſun and earth directly, and as the ſquare root of the diſtance of the body from the ſun inverſely. The firſt part is common to all the planets, the elongation being given. If we take the ſum of the aberrations when the planet is in conjunction and oppoſition, the laſt part will be deſtroyed by the oppoſition of it's ſigns, and as the cos. STP in each caſe $= 1$, the ſum of the two aberrations is always $= 40''$.

529. When p coincides with Q, or when a line joining the earth and planet continues parallel to itſelf, there is no aberration; this therefore happens when the planet is ſtationary. In this caſe (putting $a = SP$), cos. $STP \times 20'' - \frac{\text{cos. } SPT}{\sqrt{a}} \times 20'' = 0$, or $a \times$ cos. $STP^2 =$ cos. SPT^2, or $a \times 1 - \text{fin. } STP^2 = 1 - \text{fin. } SPT^2$; but a : 1 :: fin. STP : fin. $SPT = \frac{\text{fin. } STP}{a}$, which ſubſtituted for fin. SPT in the laſt equation, by reduction gives fin. $STP = \sqrt{\frac{a^3 - a^2}{a^3 - 1}}$, the ſame as in Art. 313.

* This is not accurately true, becauſe ua is not ſtrictly parallel to rE, but ſufficiently ſo for all practical purpoſes, as the angle EPT is very ſmall.

† This will appear when we treat on the Phyſical Principles of Aſtronomy.

530. M. de la

530. M. de la LANDE has calculated the following TABLE, ſhowing what we muſt apply to the true place of a planet in longitude to find the apparent place, in which the quantities are to be applied according to their ſigns. All the orbits are ſuppoſed to be circular, except that of *Mercury*. When the aberration is *negative*, the planet's motion is *direct*; when *poſitive*, it is *retrograde*.

Elong. from Sun	*Mars*	*Jupiter*	*Saturn*	*Georgian*	Elong.	*Venus*
O. conjun. 0°	−36″	−29″	−27″	−25″	Conj. ſup.	−43″,5
15	−35	−28	−26	−24	15°	−41
I. XI. 0	−32	−26	−24	−22	30	−34
15	−28	−23	−21	−19	45	−19
II. X. 0	−23	−19	−16	−15	Gr. elong.	−14
15	−18	−14	−12	−10	45°	− 9
III. IX. 0	−12	− 9	− 6	− 5	30	0
15	− 7	− 4	1	0	15	+ 3
IV. VIII. 0	− 3	+ 1	+ 4	+ 5	Conj. inf.	+ 3,5
15	0	+ 5	+ 8	+ 9		
V. VII. 0	+ 2	+ 9	+11	+13		
15	+ 3	+10	+12	+15		
VI. oppoſ. 0	+ 4	+11	+13	+15		

	MERCURY.		
	Aphelion	Mean Diſtance	Perihelion
Conjunc. ſuper.	−46″	−51″,5	−59″,5
5°	−45	−51	−58
10	−44	−48	−52
15	−41	−43	−41
20	−36	−33	
25	−29		
Greateſt Elong.	−18	−18	−19
25	− 7		
20	− 1	− 4	
15	+ 2	+ 4	+ 2
10	+ 4	+ 8	+13
5	+ 6	+11	+18
Conjunc. infer.	+ 6	+11,5	+19,5

531. M. de la

531. M. de la LANDE obſerves, that in the paſſage of *Mercury* over the ſun in 1782, the aberration retarded the phaſes by computation 6′. 34″, as will appear by augmenting it's longitude by 18″,8, the aberration at that time, and diminiſhing that of the ſun 20″, which is always it's aberration. Compute the phaſes by ſuppoſing each body to be at it's true place, and at it's apparent place at the ſame time, and the difference ſhows how much the aberration affects the time. Moreover, when we calculate the true geocentric place of a planet, we muſt add 20″ to the place of the ſun in the Tables of it's motion, the place of the ſun being put down as affected by aberration.

532. By Article 526, the aberration = 0,00564 *dm*, if the earth's diſtance from the ſun be unity; if therefore that diſtance be repreſented by 10, the aberration = 0,000564 *dm*, from which the following Table was conſtructed, to be entered with the diſtance of the planet from the earth, and the angle deſcribed by the planet about the earth in 24 hours, in latitude, longitude, right aſcenſion or declination.

If the diſtance of the body from the earth be greater than 10, as 37 for inſtance, find the value for 3 and then multiply it by 10, and to it add the value for 7.

A TABLE

A TABLE

To find the Aberration of a Planet *or* Comet, *in Latitude, Longitude, Right Aſcenſion or Declination.*

Diurnal Motion	Diſtance from the Earth; that of the Sun being 10.								
	2	3	4	5	6	7	8	9	10
D. M.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.
0. 8	0,5	0,8	1,1	1,4	1,6	1,9	2,2	2,4	2,71
0. 16	1,1	1,6	2,2	2,7	3,3	3,8	4,3	4,9	5,41
0. 24	1,6	2,4	3,3	4,1	4,9	5,7	6,5	7,3	8,12
0. 32	2,2	3,3	4,3	5,4	6,5	7,6	8,7	9,7	10,83
0. 40	2,7	4,1	5,4	6,8	8,1	9,5	10,8	12,2	13,53
0. 48	3,3	4,9	6,5	8,1	9,8	11,4	13,0	14,6	16,24
0. 56	3,8	5,7	7,6	9,5	11,4	13,0	15,2	17,1	18,95
1. 4	4,3	6,5	8,7	10,8	13,0	15,2	17,3	19,5	21,66
1. 12	4,9	7,3	9,8	12,2	14,6	17,1	19,5	21,9	24,36
1. 20	5,4	8,1	10,8	13,5	16,2	19,0	21,7	24,4	27,07
1. 28	6,0	8,9	11,9	14,9	17,9	20,8	23,8	26,8	29,78
1. 36	6,5	9,8	13,0	16,2	19,5	22,7	26,0	29,2	32,48
1. 44	7,0	10,6	14,1	17,6	21,1	24,6	28,2	31,7	35,19
1. 52	7,6	11,4	15,2	19,0	22,7	26,5	30,3	34,1	37,90
2. 0	8,1	12,2	16,2	20,3	24,4	28,4	32,5	36,6	40,61
2. 8	8,7	13,0	17,3	21,7	26,0	30,3	34,7	39,0	43,31
2. 16	9,2	13,8	18,4	23,0	27,6	32,2	36,8	41,4	46,02
2. 24	9,8	14,6	19,5	24,4	29,2	34,1	39,0	43,9	48,73
2. 32	10,3	15,4	20,6	25,7	30,9	36,0	41,2	46,3	51,43
2. 40	10,8	16,3	21,7	27,1	32,5	37,9	43,3	48,7	54,14
2. 48	11,4	17,1	22,8	28,4	34,1	39,8	45,5	51,2	56,85
2. 56	11,9	17,9	23,8	29,8	35,7	41,7	47,6	53,6	59,55
3. 0	12,2	18,3	24,4	30,5	36,5	42,6	48,7	54,8	60,91

Ex. Suppoſe the diſtance of a comet from the earth to be 43, and it's apparent motion in 24 hours to be 2°. 15′ in longitude; to find the aberration in longitude.

Enter with the diſtance 10 and daily motion 2°. 15′, and we get 45″,68, which multiplied by 4 gives 182″,7; and by entering with the diſtance 3 we get 13″,7; hence the aberration is 196″,4.

To reduce the place of the body computed from the Tables to the apparent place, *add* the aberration, if the latitude, longitude, right aſcenſion or declination of the body *decreaſe*, but *ſubtract*, if it *increaſe*; and the contrary, to reduce the apparent to the computed place

CHAP.

CHAP. XXIII.

ON THE PROJECTION FOR THE CONSTRUCTION OF SOLAR ECLIPSES.

Art. 533. AS the ecliptic is inclined to the equator and cuts it in two oppoſite points, the ſun keeps continually approaching to one pole and receding from the other by turns, and therefore to a ſpectator at the ſun, the poles muſt appear and diſappear by turns. When the ſun is on the north ſide of the equator, the north pole muſt appear; and when on the ſouth ſide, the ſouth pole. When the ſun is in the equator, the plane of illumination is perpendicular to the equator, and conſequently the poles will lie in the circumference of the circle of illumination; when the ſun comes to the tropic, the pole will appear in the middle of it's path over the circle of illumination; and when the ſun comes to the next equinox, the pole will appear on the other ſide of the circle of illumination. When the ſun gets on the other ſide of the equator, this pole will diſappear, and the other will appear in like manner. Hence to a ſpectator at the ſun, the apparent motion of the pole P is the ſame as if the axis Pp of the earth had an annual conical motion $Pr\mathfrak{Q}s$, $pnqm$ about an axis GOF perpendicular to the ecliptic EOC, the angle POG being equal to the greateſt declination of the ſun. As theſe circles $Pr\mathfrak{Q}s$, $pnqm$ are parallel to the ecliptic, their planes will paſs through the ſun, and therefore to a ſpectator at the ſun the apparent motion of the poles will be in the ſtraight lines $P\mathfrak{Q}$, pq; and as P moves as faſt in the circle $Pr\mathfrak{Q}s$ as the ſun does in the ecliptic, if P be the place of the pole at the equinox, and we take the arc Pv equal to the ſun's diſtance from that equinox, and draw vo perpendicular to $P\mathfrak{Q}$, o will be the apparent place of the pole at that time. It is manifeſt that Pv may be ſet off upon any circle deſcribed on $P\mathfrak{Q}$. Hence alſo the angle which the axis vOw makes with the plane of illumination muſt be equal to the declination of the ſun. As this apparent motion of the pole over the enlightened diſc of the earth is cauſed by the motion of the earth in it's orbit, the motion of the pole over the diſc will be in a direction contrary to the diurnal motion of the diſc; if therefore P be the poſition of the pole at the vernal equinox, and $Pr\mathfrak{Q}$ be it's motion over the diſc of the earth to the next equinox, the diurnal motion of the diſc will be made in the contrary direction. FIG. 116.

534. When

534. When the ſun, and conſequently the ſpectator who is ſuppoſed to be at the ſun, is in the equator, the ſpectator being in the plane of the equator, and, as to ſenſe, in the plane of all the circles parallel to it, they will all appear to be projected upon the circle of illumination into right lines parallel to each other. But when the ſun, and conſequently the ſpectator, is out of the equator, the equator, and all the circles parallel to it, being ſeen obliquely, will appear to be projected into ellipſes upon the plane of illumination, as the eye may be conſidered at an infinite diſtance; and as the eye has the ſame relative ſituation to all theſe circles, the ellipſes muſt be all ſimilar. When the ſun is on the north ſide of the equator, that part of the ellipſe which is the projection of that part of the circle which lies between the north pole and equator on the enlightened hemiſphere will be concave to the pole; but when the ſun is on the other ſide of the equator, that part will be convex. That is,

FIG. 117. let P be the north pole on the enlightened hemiſphere, the ſun being on the north ſide of the equator, and $vxyz$, $ambn$, the ellipſes into which the equator and any parallel to it are projected; then amb is that part of the ellipſe which the place on this parallel deſcribes in the *day*, and the other part bna is that which is deſcribed in the *night*; and the place is at m at 12 at noon, and at n at 12 at midnight. In this caſe, the other pole p muſt be conſidered as being on the other, or dark ſide of the earth. But if P be ſuppoſed on the dark ſide, and conſequently p on the light ſide, or if the ſun be on the ſouth ſide of the equator, n will be 12 at noon, and m will be 12 at midnight.

FIG. 118. For if Pp be the axis, LN the plane upon which the circle ab is to be projected, E the ſun on that ſide next to the north pole; then drawing Eam, Enb, the point a anſwering to noon, the ſun being on the meridian, is projected at m, and the point b, anſwering to midnight, is projected at n; but when the ſun is on the other ſide of ab, as at e, a is projected to n' and b to m', therefore n' repreſents noon and m' midnight. On account of the great diſtance of the ſun compared with the radius of the earth, the lines Ea, Eb, and ea, eb may be conſidered as parallel, and therefore the circle ab is orthographically projected upon the plane LN into an ellipſe, whoſe minor axis is mn, or $m'n'$.

535. The next thing to be done is to determine the magnitude of the ellipſe into which the circle ab is projected, and it's poſition upon the plane of illumination. Let Pp repreſent the axis of the earth, $asbt$ a circle of latitude to any place, $LPNp$ the meridian paſſing through the ſun, and LON the plane upon which the projection is made; then (533) the angle LOP is equal to the ſun's declination; draw am, bn, vr perpendicular to LO, and (534) mn is the minor axis of the ellipſe; let vs be that radius of the circle ab which is parallel to the plane of projection, and it will be projected into a line equal to

FIG. 119.

itſelf,

itſelf, and conſequently it will be the major axis; hence $2vs$, or $2va$, or 2 cos. lat. is the major axis of the ellipſe; but mn (the projection of ab upon LN) : ab :: ſin. mab, or POL the dec. : radius; that is, the axis major : axis minor :: rad. : ſin. declination. And to find the diſtance Or from the center of projection to the center of the ellipſe, we have, rad. $=1$: cos. vOr the dec. :: $vO : Or = vO \times$ cos. dec. $=$ ſin. lat. $\times$ cos. dec. But (541) the radius of the projection is the horizontal parallax of the moon diminiſhed by the horizontal parallax of the ſun; the radius therefore thus expreſſed being multiplied by the quantities whoſe values are expreſſed when radius is ſuppoſed to be unity, give the value in terms of that radius; hence if hor. par. $\mathbb{C}$ — hor. par. $\odot = h$, then $h \times$ cos. lat. $=$ the ſemi-axis major of the ellipſe; $h \times$ cos. lat. $\times$ ſin dec. $=$ the ſemi-axis minor; and $Or = h \times$ ſin. lat. $\times$ cos. dec. Thus we have gotten the dimenſions and poſition of the ellipſe in terms of the radius of projection. Hence we have the following conſtruction for the apparent ellipſe deſcribed by any place on the earth's ſurface to a ſpectator at the ſun.

536. Let $GCFE$ be that half of the earth which is illuminated, EC the plane of the ecliptic, GOF perpendicular to it; take $G\mathbb{Q} = GV$ equal to the ſun's greateſt declination, join $\mathbb{Q}V$, and on it deſcribe the ſemicircle $VK\mathbb{Q}$, and take Vh equal to the ſun's diſtance from the vernal equinox correſponding to the pole at V, and draw hP perpendicular to $V\mathbb{Q}$, and P (533) is the place of the pole, which we will ſuppoſe to be on the enlightened diſc of the earth. Put $c =$ cos. Vh, $n =$ ſin. VG, $m =$ it's coſine, to radius unity; then. FIG. 120.

$$\begin{array}{l} Pc : cV :: c : 1 \\ cV : OG :: n : 1 \\ \hline \therefore Pc : OG\,(=h) :: c \times n : 1 \end{array}$$

; hence $Pc = h \times c \times n$.

Alſo $h \times m = Oc$; hence $h \times m : h \times c \times n ::$ rad. : tan. $POc = \frac{c \times n}{m} = c \times$ tan. 23°. 28′ $=$ 0,4341208 $\times c$. Draw POp, and upon OP take $Or = h \times$ ſin. lat. $\times$ cos. dec.; draw bra perpendicular to OP, and take $ra = rb = h \times$ cos. lat. and $rm = rn = h \times$ cos. lat. $\times$ ſin. dec. and deſcribe an ellipſe $ambn$, and (535) it will repreſent the apparent diurnal path of the place to a ſpectator at the ſun, for the given declination of the ſun. If x and z be the points where the ellipſe touch the circle $GCFE$, the part $xambz$ will (534) be on the illuminated part of the earth, and therefore viſible to a ſpectator at the ſun, and the part znx on the dark part, P being the north pole, and the ſun's declination north; but if the declination be ſouth, xnz will be the part on the illuminated ſide of the earth, and $zbmax$ on the dark part. Let the declination be north, and a the weſt ſide of the diſc; then to find where the given place

on

on the earth's ſurface, is at any time, we may obſerve that the place deſcribing the circle which is projected into the ellipſe *ambn* moves uniformly in that circle, from the uniform motion of the earth about it's axis; let therefore *ayb* be a circle; then if every ordinate be diminiſhed in the ratio of *yr* : *mr*, the circle will be projected into the ellipſe *amb*; this ſemicircle may therefore repreſent the half of the diurnal motion of the given place, ſo far as it is neceſſary to obtain the correſponding poſitions of the place in the ellipſe. For divide the ſemicircle *ayb* into 12 equal parts from *a*, at 7, 8, 9, 10, 11, 12, 1, 2, 3, 4, 5, 6, repreſenting the poſitions of the given place from *a* at ſix o'clock in the morning to *b* at ſix in the evening, and theſe figures will repreſent the poſitions of the given place at the reſpective hours denoted by the figures; and if the dotted lines be drawn perpendicular to *ab*, the correſponding points denoted by the ſame figures will repreſent the poſitions of the place in the ellipſe. This ellipſe may be very accurately deſcribed by diminiſhing each ordinate of the circle perpendicular to *ab* in the ratio of *yr* to *mr*, by taking a proper number of ordinates and then deſcribing a curve through all the points; and if theſe lines be continued to the other half of the ellipſe, the hours, as there marked, will correſpond to the poſitions of the given place. If each diviſion of the ſemicircle be divided into 10 equal parts, and ordinates be drawn to *ab*, the ellipſe will be divided into every ſix minutes; and if the ſcale be large enough, and theſe diviſions on the ellipſe be ſubdivided into ſix equal parts, the ellipſe will be divided into minutes, for there will be no occaſion to uſe the circle for this laſt ſubdiviſion. Thus we can always find the apparent poſition of any place on the earth's ſurface to a ſpectator at the ſun.

537. Draw 11*w* and 11*d* perpendicular to *ry*; then 11*w* = 11*d* is the ſine of 15° to the radius *ra*; and by the principles of projection, *yr* : *mr* :: *yd* : *mw*, therefore as *yd* is the verſed ſine of 15° to the radius *yr*, *mw* muſt be the verſed ſine of 15° to the radius *mr*; hence if we take the ſine and verſed ſine of 15° to radius unity, and multiply them into *ra* and *rm* reſpectively, they will give the values of 11*w* and *mw*; and if *rm* be multiplied into the coſine of 15°, it gives *rw*. The ſame for any other angle.

538. By Art. 535. $Or = h \times$ ſin. lat. $\times$ cos. dec. and $ra = h \times$ cos. lat. hence $Or : ra ::$ ſin. lat. $\times$ cos. dec. : cos. lat. conſequently $Or = ra \times \frac{\text{ſin. lat.} \times \text{cos. dec.}}{\text{cos. lat.}} = ra \times$ tan. lat. $\times$ cos. dec. therefore if *ra* and cos. dec. be conſtant, *Or* varies as tan. lat. Alſo (535) the radius of projection muſt vary inverſely as the coſine of the latitude.

539. Having determined the ſituation of the ellipſe for any one latitude in reſpect to the center of projection, as the ellipſes for all latitudes are ſimilar if the declination be given, we may make uſe of the ſame ellipſe for all latitudes,

tudes, only by altering Or in a proper ratio; for (538) if ra and the declination remain conſtant, rO varies as the tangent of latitude. Hence take $rO : rO'$ as the tangent of the latitude for which the projection was made : tangent of any other latitude, and O' will be the center of projection, whoſe radius is (538) alſo known ; and $ambn$ is the ellipſe for that latitude.

540. Let e be any poſition of the given place, and join eO; then the angle under which EO appears at the ſun is the ſun's horizontal parallax; alſo the angle under which eO appears at the ſun muſt be the parallax in altitude at the point e, for the ſun being vertical to O, the arc correſponding to eO is the zenith diſtance of the ſun at the given place, and eO is the ſine of that arc from the nature of the projection; but (154) the hor. parallax : parallax at any altitude :: rad. : ſine of the zenith diſtance :: $OE : Oe$; hence if OE repreſent the horizontal parallax, Oe will repreſent the parallax in altitude at e. Alſo as e repreſents the zenith of the given place, eO repreſents the vertical circle paſſing through the ſun. The uſe of this projection is to conſtruct the phaſes and times of a ſolar eclipſe, as we ſhall now proceed to explain.

541. Let S be the center of the ſun, xw the enlightened hemiſphere of the earth, which we muſt conceive to be perpendicular to SC; draw SD, SV tangents to two oppoſite points of the earth, and let $ambn$ be the apparent ellipſe (536) deſcribed by any point m on the earth's ſurface; let OC be the diſtance of the moon from the earth, and vd, $a'm'b'n'$ be the projection of VD, $ambn$ upon a plane at the moon perpendicular to SO, to an eye at S, and $a'm'b'n'$ will be the apparent motion of the center of the ſun at S to the ſpectator deſcribing $ambn$. The curve $a'm'b'n'$ may be conſidered as an ellipſe; for the angle DSC being only $8\frac{3}{4}''$, DS, CS may be reckoned as parallel, and therefore the projection of DV upon a plane parallel to it may be conſidered as an orthographic projection, and conſequently the two figures may in all reſpects be conſidered as ſimilar. Let LM be the orbit of the moon; then if we know at any time the point of the ellipſe $ambn$ where the ſpectator is, we know the correſponding point where the center of the ſun is in the ellipſe $a'm'b'n'$; if therefore we determine at the ſame time the point where the moon is in it's orbit LM, we ſhall know the apparent ſituation of the moon in reſpect to the ſun. Hence if we find two points, one in the ellipſe $a'm'b'n'$ where the center of the ſun is, and another in LM where the center of the moon is at the ſame time, and about theſe centers, with radii equal to the apparent ſemidiameters of the ſun and moon, we deſcribe two circles, they will repreſent the apparent ſituations of the two diſcs. If that of the moon fall upon the ſun, it ſhows how much the ſun is eclipſed at that inſtant. Now the angle $OVv = COV - OSV$, that is, the radius of projection is equal to the difference of the horizontal parallaxes of the moon and ſun. The projection Oe' of Ce is

FIG. 121.

is the parallax in altitude of the moon from the ſun, ſuppoſing the moon to be at the ſame altitude as the ſun; for the radius Ov repreſents the difference of the horizontal parallaxes of the ſun and moon, or the horizontal parallax of the moon from the ſun; and as the parallax of each varies (154) as the ſine of the apparent zenith diſtance, the difference of the parallaxes muſt vary as the ſine of their common apparent zenith diſtance; hence $Ov : Oe' ::$ difference of the horizontal parallaxes : difference of the parallaxes at their common apparent altitude; therefore if Ov repreſent the third term, Oe' will repreſent the fourth. In an eclipſe of the ſun therefore this will be nearly true, but not accurately ſo, except when the ſun and moon are atthe ſame altitude. The place of the pole of the earth is here ſuppoſed to be fixed during the time of the eclipſe, and conſequently the earth is ſuppoſed to be immoveable for that time; the ſun's declination is alſo ſuppoſed to be conſtant for the ſame time; but as theſe circumſtances do not take place, the projected path of the ſpectator will not be accurately an ellipſe. M. de la CAILLE obſerves, that in this projection, all the errors ariſing from the finite diſtances of the ſun and moon are ſuppoſed to be compenſated, by making the ſemidiameter of the projection equal to the difference of their horizontal parallaxes; whereas only a part of the lines ſhould be diminiſhed in that ratio. The ſun alſo not being at an infinite diſtance, the projection will not be an accurate ellipſe. The ſpheroidical figure of the earth is alſo here not conſidered. All theſe circumſtances tend to render the method of determining the phaſes of an eclipſe by this conſtruction ſubject to a certain degree of inaccuracy; but if the conſtruction be made upon a large ſcale, it will be ſufficiently accurate, when we only want to predict an eclipſe. If S be a fixed ſtar, the ſame conſtruction will give the time of it's occultation by the moon. In this caſe, as the fixed ſtar has no parallax, the radius of projection is equal to the horizontal parallax of the moon. This projection was firſt given by Mr. FLAMSTEAD.

542. If we make this projection upon a plane at the orbit of *Venus* or *Mercury*, the radius of projection muſt be taken equal to the difference of the horizontal parallaxes of Venus or Mercury and the ſun; and by proceeding as for the moon, we may determine the times of the phaſes of the tranſits of Venus and Mercury over the ſun's diſc; but this is beſt done by calculation, as will be afterwards explained.

CHAP.

CHAP. XXIV.

ON ECLIPSES OF THE SUN AND MOON, AND OCCULTATIONS OF FIXED STARS BY THE MOON.

Art. 543. AN eclipſe of the *Moon* is cauſed by it's entering into the earth's ſhadow, and conſequently it muſt happen when the moon is in oppoſition to the ſun, or at the full moon. An eclipſe of the *Sun* is cauſed by the interpoſition of the moon between the earth and ſun, and therefore it muſt happen when the moon is in conjunction with the ſun, or at the new moon. If the plane of the moon's orbit coincided with the plane of the ecliptic, there would be an eclipſe at every oppoſition and conjunction; but the plane of the moon's orbit being inclined to the ecliptic, there can be no eclipſe at oppoſition or conjunction, unleſs at that time the moon be at, or near to the node. For let *Mamb* be the orbit of the moon, *Mcmd* the plane of the earth's orbit, or that plane in which the ſun *S*, *S'* appears as ſeen from the earth, and let theſe two planes be inclined to each other, ſo that we may conceive the part *Mam* to lie above *Mcm*, and the part *mbM* below *mdM*; and *M*, *m* are the nodes. Now if when the moon is at *M* the ſun be in conjunction at *S*, the three bodies are then in the ſame plane, and therefore the moon muſt interpoſe between the earth and ſun, and cauſe an eclipſe of the ſun. But if the moon be at *M'* when the ſun comes into conjunction at *S'*, *M'* is now elevated above the line joining *E* and *S'*, and the further *M'* is from *M*, the more elevated will *M'* appear above *S'*, ſo that *M'* may be ſo far from *M*, that the moon may not at all interpoſe between *E* and *S'*, in which caſe there will be no eclipſe of the ſun. Whether therefore there will, or will not be an eclipſe of the ſun at the conjunction, depends upon the diſtance of the moon from the node at that time. If the moon be at *m* at the time of oppoſition, then the three bodies being in the ſame plane, the ſhadow *EV* of the earth muſt fall upon the moon, and the moon muſt ſuffer an eclipſe. But if the moon be at *m'* at the time of oppoſition, *m'* may be ſo far below the ſhadow *Ev* of the earth, that the moon may not paſs through it, in which caſe there will be no eclipſe. Whether therefore there will be a lunar eclipſe at the time of oppoſition, depends upon the diſtance of the moon from the node at that time. If the two planes coincided, there would evidently be a central interpoſition every conjunc-

FIG. 122.

tion and oppoſition, and conſequently a total eclipſe. METON, who lived about 430 years before CHRIST, obſerved, that after 19 years the new and full moons returned again on the ſame day of the month. The ancient Aſtronomers alſo obſerved, that at the end of 18 years 10 days, a period of 223 lunations, there was a return of the ſame eclipſes; and hence they were enabled to foretel when they would happen. This is mentioned by PLINY the Naturaliſt, Lib. II. Ch. 13. and by PTOLEMY, Lib. IV. Ch. 2. This reſtitution of eclipſes depends upon the return of the following elements to the ſame ſtate.—1. The ſun's place. 2. The moon's place. 3. The place of the moon's apogee. 4. The place of the aſcending node of the moon. The exact reſtitution of theſe can never take place; but it ſo nearly happens in the above time, as to produce eclipſes remarkably correſponding. In this manner Dr. HALLEY predicted and publiſhed a return of eclipſes from 1700 to 1718, many of them corrected from obſervations; together with the following elements.—1. The apparent time of the middle. 2. The ſun's anomaly. 3. The annual argument. 4. The moon's latitude He ſays, that in this period of 223 lunations there are 18 years 10 or 11 days (according as there are five or four leap-years) $7h.\ 43'\frac{1}{4}$; that if we add this time to the middle of any eclipſe obſerved, we ſhall have the return of a correſponding one, certainly within $1h.\ 30'$; and that by the help of a few equations, we may find the like ſeries of eclipſes for ſeveral periods.

To Calculate an Eclipſe of the Moon.

544. The firſt thing to be done, is to find the time of the *mean* oppoſition. To get which, from the Tables of Epacts*, amongſt the Tables of the moon's motion, take out the epact for the year and month, and ſubtract the ſum from $29d.\ 12h.\ 44'.\ 3''$ one ſynodic revolution of the moon, or two if neceſſary, ſo that the remainder may be leſs than a revolution, and that remainder gives the time of the mean conjunction. If to this we add $14d.\ 18h.\ 22'.\ 1'',4$ half a revolution, it gives the time of the next mean oppoſition; or if we ſubtract, it gives the time of the preceding mean oppoſition. If it be leap-year, in January and February ſubtract a day from the ſum of the epacts, before you make the ſubtraction. When the day of the mean conjunction is 0, it denotes the laſt day of the preceding month.

Ex.

* The epact for any year is the age of the moon at the beginning of the year from the laſt *mean* conjunction, that is, from the time when the mean longitudes of the ſun and moon were laſt equal. The epact for any month is the age which the moon would have had at the beginning of the month, if it's age had been nothing at the beginning of the year.

Ex. To find the time of the *mean* new and full moons in February, 1795.

Epact 1795 . . .	9[d].	11[h].	6′.	17″
February	1.	11.	15.	57
	10.	22.	22.	14
	29.	12	44.	3
Mean new moon . .	18.	14.	21.	49
	14.	18.	22.	1,4
Mean full moon . .	3.	19.	59.	47,6

545. To determine whether an eclipſe may happen at oppoſition, find the *mean* longitude of the earth at the time of *mean* oppoſition, and alſo the longitude of the moon's node; then, according to M. CASSINI, if the difference between the *mean* longitudes of the earth and the moon's node be leſs than 7°. 30′, there *muſt* be an eclipſe; if it be greater than 14°. 30′, there *cannot* be an eclipſe; but between 7°. 30′ and 14°. 30′ there *may*, or may *not* be an eclipſe. M. de LAMBRE makes theſe limits 7°. 47′ and 13°. 21′.

Ex. To find whether there will be an eclipſe at the full moon on February 3, 1795.

Sun's mean long. at 3[d]. 19[h]. 59′. 47″,6 (543)	10[s].	13°.	27′.	20″,8
Mean long. of the earth	4.	13.	27.	20, 8
Long. of the moon's node	4.	8.	1.	48, 5
Difference	0.	5.	25.	32, 3

Hence there muſt be an eclipſe.

Examine thus all the new and full moons for a month before and a month after the time at which the ſun comes to the place of the nodes of the lunar orbit, and you will be ſure not to miſs any eclipſes. Or having the eclipſes for the laſt 18 years, if you add to the times of the middle of theſe eclipſes, 18*y*. 10*d*. or 11*d*. 7*h*. 43′$\frac{3}{4}$, it will give the times when you may expect the eclipſes will return.

546. To the time of *mean* oppoſition, compute * the true longitudes of the ſun and moon, and the moon's true latitude; and find, from the Tables of their motions, the horary motions of the ſun and moon in longitude, and the difference *(d)* of their horary motions is the relative horary motion of the moon in reſpect to the ſun, or the motion with which the moon approaches to, or recedes from the ſun; find alſo the moon's horary motion in latitude; and ſuppoſe at the time *(t)* of *mean* oppoſition, the moon is at the diſtance *(m)* from oppoſition; then, $d : m ::$ 1 hour : the time *(w)* between t and the oppoſition, which added to, or ſubtracted from the time t, according as the moon is not yet got into oppoſition, or is beyond it, gives the time of the ecliptic oppoſition.

547. To find the place of the moon in oppoſition, let n be the moon's horary motion in longitude; then, 1 hour : $w :: n$: the increaſe of the moon's longitude in the time w, which applied to the moon's longitude at the time of the mean oppoſition, gives the true longitude of the moon at the time of the ecliptic oppoſition. The oppoſite to that muſt be the true longitude of the ſun. Find alſo the moon's true latitude at the time of oppoſition, by ſaying, 1 hour : $w ::$ the horary motion in latitude : the motion in latitude in the time w, which applied to the moon's latitude at the time of the mean oppoſition, gives the true latitude at the time of the true oppoſition †. In like manner you may compute the true time of the ecliptic conjunction, and the places of the ſun and moon for that time, when you calculate a ſolar eclipſe.

548. With the ſun's horary motion in longitude, and the moon's in longitude and latitude, find the inclination of the *relative* orbit, and the horary motion upon it. To do this, let LM be the horary motion of the moon in longitude, SM that of the ſun; draw Ma perpendicular to LM and equal to the moon's horary motion in latitude; take $Sb = Ma$ and parallel to it, and join La, Lb; then La is the moon's *true* orbit, and Lb it's *relative* orbit in reſpect to the ſun. Hence LS (the difference of the horary motions in longitude) : Sb (the moon's horary motion in latitude) :: radius : tan. bLS the inclination of the relative orbit; and cos. bLS : radius :: LS : Lb the horary motion in the relative orbit. By Logarithms the calculations are thus.

FIG. 123.

Log.

* The method of doing this will be explained in the Introduction to the Tables, in the ſecond Volume.

† For greater certainty you may compute again from the Tables the places of the ſun and moon, and if they be not exactly in oppoſition, which probably may not be the caſe, as the moon's longitude does not increaſe uniformly, repeat the operation. This accuracy however in eclipſes is generally unneceſſary; for the beſt lunar Tables cannot be depended upon to give the moon's longitude nearer than 30″; therefore the probable error from the Tables is vaſtly greater than that which ariſes from the motion in longitude not being uniform. Unleſs therefore very great accuracy be required, this operation is unneceſſary.

Log. $Sb + 10, -$ log. $LS =$ log. tan. bLS.
Log. $LS + 10, -$ log. cos. $bLS =$ log. Lb.

M. de la LANDE obſerves, that if we add 8″ to the difference of the horary motions in longitude it will give the horary motion in the *relative* orbit; for in a right angled triangle, of which the baſe is the difference of the horary motions in longitude, which is about half a degree, and the angle at the baſe about $5\frac{2}{3}°$, the difference between the baſe and hypothenuſe will always be about 8″.

549. At the time of oppoſition, find, from the Tables, the moon's horizontal parallax, it's ſemidiameter, and the ſemidiameter of the ſun, the horizontal parallax of which we may here take = 9″.

550. To find the ſemidiameter of the earth's ſhadow at the moon, ſeen from the earth. Let AB be the diameter of the ſun, TR the diameter of the earth, O and C their centers; draw AT, BR to meet at I, and join OCI; let FGH be the diameter of the earth's ſhadow at the diſtance of the moon, and join OT, CF. Now the angle $FCG = CFA - CIA$, but $CIA = OTA - TOC$, therefore $FCG = CFA - OTA + TOC$, that is, *the angle under which the ſemidiameter of the earth's ſhadow appears at the moon, is equal to the ſum of the horizontal parallaxes of the ſun and moon diminiſhed by the apparent ſemidiameter of the ſun.* In eclipſes of the moon, the ſhadow is found to be a little greater than this Rule gives it, owing to the atmoſphere of the earth. This augmentation of the ſemidiameter is, according to M. CASSINI, 20″; according to M. MONNIER, 30″; and according to M. de la HIRE, 60″. MAYER thinks the correction is about $\frac{1}{60}$ of the ſemidiameter of the ſhadow, or that you may add as many ſeconds as the ſemidiameter contains minutes. Some Computers always add 50″.; but this muſt be ſubject to ſome uncertainty. FIG. 124.

551. As the angle CIT ($= OTA - TOC$) is known, we have ſin. TIC : cos. TIC :: TC : CI the length of the earth's ſhadow. If we take the angle ATO = 16′. 3″ the mean ſemidiameter of the ſun, TOC = 9″ the horizontal parallax of the ſun, we have CIT = 15′. 54″; hence ſin. 15′. 54″ : cos. 15′. 54″, or 1 : 212,6, :: TC : CI = 216,2 TC.

552. Let PQ repreſent the ſection of the earth's ſhadow at the moon, CN the ecliptic, NL the moon's orbit; draw Cn perpendicular to CN, and Cm perpendicular to NL, and let the moon at m juſt touch the earth's ſhadow at r externally, ſo that Cm may be the ſum of the radii of the moon and earth's ſhadow; then to determine when this happens, we may take the angle at N = 5°. 17′, which is very nearly it's value in all eclipſes, the inclination of the lunar orbit being at that time always greateſt, as will afterwards be ſhown; hence, ſine 5°. 17′ FIG. 125.

5°. 17′ : rad. :: ſin. Cm : ſin. CN; now the greateſt value of Cm is about 1°. 3′. 30″; hence the correſponding value of $CN = 11°. 34'$; when therefore CN is greater than that quantity, there can be no eclipſe. According to M. CASSINI, if the latitude Cn of the moon at the time of the ecliptic conjunction exceed the ſum of the ſemidiameters of the earth's ſhadow and moon by 18″, there will be no eclipſe; but if it do not exceed that ſum by 16″, there will be an eclipſe. If $Cm = Cr - rm$, or the limbs touch internally, the eclipſe will be juſt total; hence if the diſtance of the moon's node from the place of the earth be leſs than the computed value of CN in this caſe, there muſt be a total eclipſe of ſome duration. If therefore it was before doubtful, and it now appears that there will be an eclipſe, proceed as follows to compute it.

FIG. 126, 127. 553. Let APB be that half of the earth's ſhadow where the moon paſſes through, NL the relative orbit of the moon, one figure repreſenting a partial eclipſe, and the other a total one; draw Cmr perpendicular to NL, and let z be the center of the moon at the beginning of the eclipſe, m at the middle, x at the end, v at the beginning of total darkneſs, w at the end; alſo let AB be the ecliptic, and Cn perpendicular to it. Now in the right angled triangle Cnm, we know Cn the latitude of the moon at the time of the ecliptic conjunction, and (548) the angle Cnm* the complement of the angle which the relative orbit of the moon makes with the ecliptic; hence, radius : cos. Cnm :: Cn : nm, which we call the *Reduction*; and radius : ſine Cnm :: Cn : Cm. By Logarithms the calculations are thus.

$$\text{Log. cos. } Cnm + \text{log. } Cn - 10, = \text{log. } nm.$$
$$\text{Log. ſin. } Cnm + \text{log. } Cn - 10, = \text{log. } Cm.$$

The horary motion *(h)* of the moon upon it's relative orbit being known, we know the time of deſcribing mn, by ſaying, h : mn :: 1 *hour* : *the time of deſcribing* mn; the computation of this is moſt readily performed by logiſtic† logarithms. Hence, knowing the time of the ecliptic conjunction at n, we know the time of the middle of the eclipſe at m. Next, in the right angled triangle Cmz, we know Cm, and Cz the ſum of the ſemidiameters of the earth's ſhadow and the moon, to find mz, which is done thus by logarithms; as $mz = \sqrt{Cz^2 - Cm^2} = \sqrt{Cz + Cm \times Cz - Cm}$, the *log. of* $mz = \frac{1}{2} \times$ *log.* $Cz + Cm$ + *log.* $Cz - Cm$. Hence the horary motion of

* If the moon at n have north or ſouth latitude increaſing, the angle Cnm is to be ſet off to the right; otherwiſe to the left of Cn.

† For the nature and uſe of theſe Logarithms, ſee the Table at the end of this Volume.

of the moon being known, we know the time of deſcribing zm, which *ſubtracted* from the time at m gives the time of the beginning, and *added*, gives the time of the end. In the ſame manner, in the right angled triangle Cmv, we know Cm, and Cv the difference of the ſemidiameters of the earth's ſhadow and moon; hence by Logarithms, the *log. of* $mv = \frac{1}{2} \times \overline{log.\ Cv + Cm} + log.\ \overline{Cv - Cm}$; from whence, as before, we know the time of deſcribing mv, which *ſubtracted* from the time at m gives the time of the beginning of total darkneſs, and *added*, gives the time of the end. The magnitude of the eclipſe at the middle is repreſented by tr, which is the greateſt diſtance of the moon within the earth's ſhadow, and this is meaſured in terms of the diameter of the moon, conceived to be divided into 12 equal parts, called *Digits*, or *Parts deficient*; to find which, we know Cm, the difference between which and Cr gives mr, which added to mt, or if m fall *out* of the ſhadow take the difference between mr and mt, and we get tr; hence to find the number of digits eclipſed, ſay, $mt : tr ::$ 6 *digits*, *or* 360′, (it being uſual to divide a digit into 60 equal parts, and call them minutes,) : *the digits eclipſed.* If the latitude of the moon be north, we uſe the *upper* ſemicircle; if ſouth, we take the *lower*.

554. If the earth had no atmoſphere, when the moon was totally eclipſed it would be inviſible; but we have ſhown (201) that by the refraction of the atmoſphere, ſome rays will be brought to fall on the moon's ſurface, upon which account the moon will be viſible at that time, and appear of a duſky red colour. M. MARALDI (*Mem. de l'Acad.* 1723) has obſerved, that, in general, the moon's umbra at a certain diſtance is divided by a kind of penumbra, from the refraction of the earth's atmoſphere. This will account for the circumſtance of the moon being more viſible in ſome total eclipſes than in others. It is ſaid that the moon, in the total eclipſes in 1601, 1620 and 1642, entirely diſappeared.

555. An eclipſe of the moon ariſing from it's real deprivation of light, it muſt appear to begin at the ſame inſtant of time to every place on that hemiſphere of the earth which is next the moon. Hence it affords a very ready method of finding the difference of longitudes of places upon the earth, as will be afterwards explained. The moon enters the penumbra of the earth before it comes to the umbra, and therefore it gradually loſes it's light; and the penumbra is ſo dark juſt at the umbra, that it is difficult to aſcertain the exact time when the moon's limb touches the umbra, or when the eclipſe begins. When the moon has entered into the umbra, the ſhadow upon it's diſc is tolerably well defined, and you may determine to a conſiderable degree of accuracy the time when any ſpot enters into the umbra. Hence the beginning and end of a lunar eclipſe are not ſo proper to determine the longitude from, as the times at which the umbra touches any of the ſpots.

EXAMPLE

EXAMPLE I.

A Computation of a Partial Eclipſe of the Moon, on February 3, 1795; *for the Meridian of the Royal Obſervatory at Greenwich.*

The time of the mean full moon is at 3*d*. 19*h*. 59′. 47″,6 by Art. 544.

By Art. 545. it appears that there will be an eclipſe.

By computation (546) the mean time of the ecliptic oppoſition is at 12*h*. 46′. 18″, from which ſubtract the equation of time 14′. 20″, and we have 12*h*. 31′. 58″ the apparent time at Greenwich.

To this time compute (547) the moon's place in the ecliptic, and it will be found 4ˢ. 15°. 15′. 57″, the oppoſite point to which is 10ˢ. 15°. 15′. 57″ the place of the ſun. Compute alſo the moon's latitude *Cn*, and it will be found 37′. 39″ N. aſcending.

By the Tables, the horary motion of the moon in latitude is 2′. 57″; the horary motion of the ſun is 2′. 32″, and of the moon 32′. 9″ in longitude; hence the horary motion of the moon from the ſun in longitude is 29′. 37″; conſequently (548) the horary motion of the moon from the ſun on the relative orbit is 29′. 45″; alſo the inclination of the relative orbit is 5°. 41′. 27″. The reduction *nm* (553) is 3′. 44″; reduce this into time by Logiſtic Logarithms, and the operation is thus;

3′. 44″	. . . . .	1,2061
29. 45	. . . . .	0,3047
7. 32	time of deſcribing *mn* .	0,9014

The neareſt approach *Cm* of the centers is 37′. 28″.

From 12*h*. 31′. 58″ ſubtract 7′. 32″ and it leaves 12*h*. 24′. 36″, the *Middle* of the eclipſe.

By the Tables, the horizontal parallax of the ſun is 0′. 9″, and of the moon (173) 56′. 30″; alſo the apparent ſemidiameter of the ſun is 16′. 16″, and of the moon 15′. 24″. Hence, hor. par. ⊙ + hor. par. ☾ − ſemidiam. ⊙ + 50″ = 41′. 13″, the ſemidiameter (550) of the earth's ſhadow increaſed by 50″ for refraction. Hence by Article 553,

Semid.

Semid. ☾ + ſemid. ☉'s ſhad. 56′. 37″ =	3397″
Neareſt app. of centers . . 37. 28 =	2248
Sum . . . . . =	5645 — log. 3,751664
Difference . . . . . =	1149 — log. 3,060320
	2)6,811984
Log. of 2546″,8 = 42′. 26″,8 mot. of half duration	3,405992

Reduce this into time by the logiſtic Logarithms.

29′. 45″	0,3047
42. 27	0,1503
1*h*. 25. 37 half duration . .	0,8456

Subtract this from and add it to 12*h*. 24′. 26″, and we get 10*h*. 58′. 49″ for the *Beginning*, and 13*h*. 50′. 3″ for the *End*.

From $Cr = 41'.\ 13''$ ſubtract $Cm = 37'.\ 28''$, and we get $mr = 3'.\ 45''$; hence (553) $mr + mt = rt = 19'.\ 9''$ the parts deficient; conſequently $15'.\ 24'' : 19'.\ 9''$ :: 6*d*. or 360′ : 7*d*. 27′. 36″ the digits eclipſed.

By logiſtic Logarithms the computation is thus;

19′. 9″ log. + 1, . . .	1,4960
15. 24	0,5906
7^{d}. 27. 36	0,9054

Hence the times of this eclipſe are February 3, 1795, the

Beginning at . .	10^{h}. 58′. 49″	apparent time at Greenwich.
Middle	12. 24. 26	
End	13. 50. 3	
Duration	2. 51. 12	

Digits eclipſed . . 7^{d}. 27. 36 on the moon's ſouth limb, as repreſented in Fig. 126. which was conſtructed for this eclipſe.

EXAMPLE II.

A Computation of a Total Eclipſe of the Moon, on December 3, 1797; *for the Meridian of the Royal Obſervatory at Greenwich.*

By Art. 545. it appears that there will be an eclipſe at this full moon.

By computation (546) the mean time of the ecliptic oppoſition is $3^d.\ 16^h.\ 16'.\ 46''$; to which add $9'.\ 18''$ the equation of time, and you get $3^d.\ 16^h.\ 26'.\ 4''$ for the apparent time.

To this time compute (547) the moon's place in the ecliptic, and it will be found to be $2^s.\ 12^\circ.\ 35'.\ 19''$; conſequently the ſun's place is $8^s.\ 12^\circ.\ 35'.\ 19''$. Compute alſo the moon's latitude Cn, and it will be found $4'.\ 55''$ S. decreaſing.

By the Tables, the horary motion of the moon in latitude is $3'.\ 15''$; the horary motion of the ſun is $2'.\ 32''$, and of the moon $35'.\ 14''$ in longitude; hence the horary motion of the moon from the ſun in longitude is $32'.\ 42''$; conſequently the horary motion of the moon from the ſun on the relative orbit (548) is $32'.\ 50''$; alſo the inclination of the relative orbit is $5^\circ.\ 40'.\ 34''$.

The reduction nm (553) is $0'.\ 29''$; reduce this into time by the logiſtic Logarithms, and the operation is thus.

$52'.\ 50''$	. . . . .	0,2618
$0.\ 29$	. . . . .	2,0939
$0.\ 53$	time of deſcribing mn . .	1,8321

The neareſt approach Cm of the centers is $4'.\ 54''$.

To $16^h.\ 26'.\ 4''$ add $53''$ and it gives $16^h.\ 26'.\ 57''$ for the *Middle* of the eclipſe.

By the Tables, the horizontal parallax of the ſun is $0'.\ 9''$, and of the moon (173) $59'.\ 9''$; alſo the apparent ſemidiameter of the ſun is $16'.\ 17''$, and of the moon $16'.\ 6''$. Hence hor. par. $\odot$ + hor. par. $\leftmoon$ − ſemid. $\odot$ + $50'' = 43'.\ 51''$, the ſemidiameter (550) of the earth's ſhadow increaſed by $50''$ for refraction. And as Cr $(= 43'.\ 51'')$ is greater than $Cm + ms$ $(= 21')$, the eclipſe muſt be total.

Hence by Article 553,

Semid.

Semid. ☾ + ſemid. ☉'s ſhad. 59′. 57″ = 3597″
Neareſt app. of the centers . . 4. 54 = 294

Sum . . . . . 3891 + log. 3,5900612
Difference . . . . . 3303 − log. 3,5189086

2)7,1089698

Log. of 3585″ = 59′. 45″ mot. of half duration . 3,5544849

Reduce this into time by the logiſtic Logarithms; but becauſe the fourth term, in this caſe, would come out a greater quantity than that to which the Table extends, we will take the half of 59′. 45″, and then double the concluſion;

32′. 50″ 0,2618
29. 52,5 0,30285

54. 35,5 0,04105

Hence 1*h*. 49′. 11″ is the half duration; which ſubtracted from and added to 16*h*. 26′. 57″, gives 14*h*. 37′. 46″ for the *Beginning*, and 18*h*. 16′. 8″ for the *End*.

By the ſame Article, we find the time of half the duration of total darkneſs thus:

Semid. ☉'s ſhad. − ſemid. ☾ 27′. 45″ = 1665″
Neareſt app. of the centers 4. 54 = 294

Sum . . . . . 1959 − log. 3,2920344
Difference . . . . . 1371 − log. 3,1370375

2)6,4290719

Log. of 1639″ = 27′. 19″ mot. of ½ dur. of tot. dark. 3,2145359

Reduce this into time by the logiſtic Logarithms.

52′. 50″	0,2618
27. 19	0,3417
49. 55 half duration of total darkneſs	0,0799

Subtract this from and add it to 16*h*. 26′. 57″, and it gives 15*h*. 37′. 2″ for the beginning of total darkneſs, and 17*h*. 16′. 52″ for the end.

From $Cr = 43'.\ 51''$ ſubtract $Cm = 4'.\ 54''$, and we get $mr = 38'.\ 57''$, to which add $tm = 16'.\ 6''$, and we get $tr = 55'.\ 3''$ the parts deficient; hence $16'.\ 6'' : 55'.\ 3'' :: 6^d$, or 360′, $: 20^d.\ 31'$ the digits eclipſed. The operation by logiſtic Logarithms is thus.

55′. 3″ log. + 1,	1,0374
16. 6	0,5713
$20^d.$ 31. 0	0,4661

Hence the times of this eclipſe are December 3, 1797, the

Beginning at	14*h*. 37′. 46″	apparent time at Greenwich.
Total darkneſs begins	15. 37. 2	
Middle	16. 26. 57	
Total darkneſs ends	17. 16. 52	
End of the eclipſe	18. 16. 8	
Duration of total darkneſs	1. 39. 50	
Duration of the whole eclipſe	3. 38. 22	
Digits eclipſed	$20^d.$ 31. 0	

If the time correſponding to the difference between the meridian of Greenwich and that of any other place, be applied to the times here found, it will give the times at that place.

556. Inſtead of *computing* the firſt eclipſe, it may be *conſtructed* thus. Having a ſcale of minutes and ſeconds, with the center C and radius $CB = 41'.\ 13''$, the ſemidiameter of the ſhadow, deſcribe a circle; draw Cn perpendicular to ABC and equal to 37′. 19″ the moon's latitude at the ecliptic conjunction; make the angle $CnN = 84°.\ 18'.\ 33''$ the complement of the angle which the relative orbit makes with the ecliptic, and produce Nn to L; with a radius $= 56'.\ 37''$, the ſum of the ſemidiameters of the earth's ſhadow and moon, ſet off Cz, Cx; let

FIG. 126.

fall

fall the perpendicular *Cm* upon *NL*; and with the centers *z*, *m*, *x*, and radius = 15′. 24″, the ſemidiameter of the moon, deſcribe the circles repreſenting the moon. To find the beginning, middle and end, mark the point *n* 12*h*. 32′ the time of the ecliptic conjunction; and with a radius equal to the relative horary motion of the moon upon *NL*, ſet off that extent from *n* both ways, and divide each interval into as many equal parts as you conveniently can, and continue theſe diviſions to *z* and *x*, and the times correſponding to the points *z*, *m*, *x* ſhow the beginning, middle and end of the eclipſe. And if *tr* be meaſured upon the ſcale, it will ſhow the digits eclipſed. This method will give the time ſufficiently near, when you only want to predict the eclipſe, as you may depend upon the time to a minute, if the radius *CB* be ſix or ſeven inches. You may proceed in the ſame manner if the eclipſe be total.

On an Eclipſe of the Sun.

557. An eclipſe of the ſun is cauſed by the interpoſition of the moon between the ſun and ſpectator, or by the ſhadow of the moon falling on the earth at the place of the obſerver. The different kinds of eclipſes will be beſt explained by a Figure. Let *S* be the ſun, *M* the moon, *AB* or *A′B′* the ſurface of the earth; draw tangents *pxvs*, *qzvr* from the ſun to the *ſame* ſide of the moon, and *xvz* will be the moon's *umbra*, in which no part of the ſun can be ſeen; if tangents *ptbd*, *qwac* be drawn from the ſun to the *oppoſite* ſides of the moon, the ſpace comprehended between the *umbra* and *wac*, *tbd*, is called the *penumbra*, in which part of the ſun only is ſeen. Now it is manifeſt, that if *AB* be the ſurface of the earth, the ſpace *mn* where the umbra falls will ſuffer a *total* eclipſe; the part *am*, *bn* between the boundaries of the umbra and penumbra will ſuffer a *partial* eclipſe; but to all the other parts of the earth there will be no eclipſe. Now let *A′B′* be the ſurface of the earth, the earth being, at different times, at different diſtances from the moon; then the ſpace within *rs* will ſuffer an *annular* eclipſe; for if tangents be drawn from any point *o* within *rs* to the moon, they muſt evidently fall within the ſun, therefore the ſun would appear all round about the moon in the form of a ring; the parts *cr*, *sd*, will ſuffer a *partial* eclipſe; and the other parts of the earth will ſuffer no eclipſe. In this caſe there can be no total eclipſe any where, as the moon's umbra does not reach the earth. According to M. du SEJOUR, an eclipſe can never be annular longer than 12′. 24″, nor total longer than 7′. 58″. FIG. 128.

558. The umbra *xvz* is a cone, and the penumbra *wcdt* the fruſtrum of a cone whoſe vertex is *V*. Hence if theſe be both cut through their common

axis

axis perpendicular to it, the ſection of each will be a circle having a common center in the line joining the centers of the ſun and moon, and the penumbra includes the umbra.

559. The moon's mean motion about the center of the earth is at the rate of about 33′ in an hour; but 33′ of the moon's orbit is about 2280 miles, which therefore we may conſider as the velocity with which the moon's ſhadow paſſes over the earth; but this is the velocity upon the ſurface of the earth where the ſhadow falls perpendicularly upon it, it being the velocity perpendicular to Mv; in every other place the velocity over the ſurface will be increaſed in the proportion of the ſine of the angle which Mv makes with the ſurface in the direction of it's motion, to radius. But the earth having a rotation about it's axis, the relative velocity of the moon's ſhadow over any given point of the ſurface will be different from this; if the point be moving in the direction of the ſhadow, the velocity of the ſhadow in reſpect to that point will be diminiſhed, and conſequently the time the ſhadow is paſſing over it will be increaſed; but if the point be moving in a direction contrary to that of the ſhadow, as is the caſe when the ſhadow falls on the other ſide of the pole, the time will be diminiſhed. The length of a ſolar eclipſe is therefore affected by the earth's rotation about it's axis.

FIG. 129. 560. Let E be the center of the earth $\mathcal{T}V$, S the ſun, ES the ecliptic, to which draw $E\mathcal{T}$ perpendicular; join $\mathcal{T}S$, and draw $\mathcal{T}w$ parallel to ES; let Lm be the diſtance of the center m of the moon xy in conjunction from the ecliptic at the time it touches the circle vz repreſenting the apparent magnitude of the ſun, whoſe center s is in $\mathcal{T}S$; and let Lm interſect $\mathcal{T}w$ in t. Then $Lm = Lt - ts + sm$; or if we take the angles under which theſe lines appear as ſeen from the earth, Lt is meaſured by the angle $L\mathcal{T}t$, or $\mathcal{T}LE$ the hor. par. of the moon, ts is meaſured by $t\mathcal{T}s$, or $\mathcal{T}SE$ the hor. par. of the ſun, and sm is the ſum of the apparent ſemidiameters of the ſun and moon; hence the angular diſtance $Lm =$ hor. par. ☾ − hor. par. ☉ + ſemid. ☉ + ſemid. ☾. According to M. CASSINI, if the latitude of the moon at the time of the ecliptic conjunction exceed this quantity by 28″, there can be no eclipſe; but if it do not exceed it by more than 26″ there muſt be an eclipſe. Or the ecliptic limits may be found thus. Find (545) the time of the mean conjunction, and at that time find the ſun's mean longitude, and alſo the longitude of the moon's node; and if the difference of theſe be leſs than 21°, but greater than 15° there *may* be an eclipſe; but if the difference be leſs than 15°, there *muſt* be an eclipſe, according to M. CASSINI. The ecliptic limits may alſo be found in this manner. Let NE be the ecliptic, NM the moon's orbit, Lr the radius of the earth, and mr the radius of the moon's penumbra juſt paſſing by the earth and touching it, and draw Lv perpendicular to LN; then Lm

FIG. 130.

when

when greateſt is 1°. 34′. 27″, and taking the angle $N = 5°. 17′$, we have, ſin. 5°. 17′ : rad. :: ſin. 1°. 34′. 27″ : ſin. $NL = 17°. 21′. 27″$, and as the value of mv is only 9′, we may take this value of LN to be the earth's diſtance from the node at the time of the ecliptic conjunction; if therefore that diſtance be leſs than 17°. 21′. 27″ at the time of the ecliptic conjunction, there may be an eclipſe.

561. An eclipſe of the ſun, or rather of the earth, without reſpect to any particular place, may be calculated exactly in the ſame manner as an eclipſe of the moon, that is, the times when the moon's umbra or penumbra firſt touches and leaves the earth; but to find the times of the beginning, middle and end at any particular place, the apparent place of the moon, as ſeen from thence, muſt be determined, and conſequently it's parallax in latitude and longitude muſt be computed, which renders the calculation of a ſolar eclipſe extremely long and tedious. We ſhall endeavour to render the whole operation as clear as poſſible, by precept and example.

To Calculate an Eclipſe of the Sun *for any particular Place.*

562. Having determined (560) that there will be an eclipſe ſomewhere upon the earth, compute, by the Aſtronomical Tables, the true longitudes of the ſun and moon, and the moon's true latitude, at the time of mean conjunction (544); find alſo the horary motions of the ſun and moon in longitude, and the moon's horary motion in latitude; and compute the time of the ecliptic conjunction of the ſun and moon, in the ſame manner (546) as the time of the ecliptic oppoſition was computed. At the time of the ecliptic conjunction, compute (547) the ſun's and moon's longitude, and the moon's latitude; find alſo the equatorial horizontal parallax of the moon from the Tables of the moon's motion, and reduce it (173) to the horizontal parallax for the given latitude, from which ſubtract the ſun's horizontal parallax, and you get the horizontal parallax of the moon from the ſun; reduce alſo (173) the apparent latitude of the place on the ſpheroid to the latitude on a ſphere.

563. To this reduced latitude of the place, and the correſponding horizontal parallax of the moon from the ſun, (which we here uſe inſtead of the horizontal parallax of the moon, as we want to find what effect the parallax has in altering their apparent relative ſituations,) at the time of the ecliptic conjunction, compute (164) the moon's parallax in latitude and longitude from the ſun; the parallax in latitude applied to the true latitude gives the apparent latitude *(L)* of the moon from the ſun; and the parallax in longitude ſhows the apparent difference *(D)* of the longitudes of the ſun and moon.

564. Let

FIG. 131.

564. Let S be the ſun, EC the ecliptic; take $SM = D$, draw MN perpendicular to MS, and take it $= L$, then N is the apparent place of the moon, and $SM = \sqrt{D^2 + L^2}$ is the apparent diſtance of the moon from the ſun.

565. If the moon be to the *eaſt* of the nonageſimal degree, the parallax *increaſes* the longitude; if to the *weſt*, it *diminiſhes* it; hence if the *true* longitudes of the ſun and moon be equal, in the former caſe the apparent place will lie from S towards E, and in the latter towards C. To ſome time, as an hour, *after* the true conjunction if the apparent place be towards C, or if the moon be to the *weſt* of the nonageſimal degree; or *before* the true conjunction if the apparent place be towards E, or if the moon be to the *eaſt* of the nonageſimal degree, find the ſun's and moon's true longitude, and the moon's true latitude, from their horary motions; and to the ſame time compute the moon's parallax in latitude and longitude from the ſun; apply the parallax in latitude to the true latitude, and it gives the apparent latitude (l) of the moon from the ſun; take the difference of the ſun's and moon's true longitude, and apply the parallax in longitude, and it gives the apparent diſtance (d) of the moon from the ſun in longitude. From S ſet off $SP = d$, and to EC erect the perpendicular $P\mathfrak{Q}$ equal to l, and $\mathfrak{Q}$ is the apparent place of the moon at one hour from the true conjunction; and $S\mathfrak{Q} = \sqrt{d^2 + l^2}$ is the apparent diſtance of the moon from the ſun; draw the ſtraight line $N\mathfrak{Q}$, and it will repreſent the relative apparent path of the moon, conſidered as a ſtraight line, in general it being very nearly ſo; it's value alſo repreſents the relative horary motion of the moon in the apparent orbit, the relative horary motion in longitude being MP.

566. The difference between the moon's apparent diſtance in longitude from the ſun at the time of the true ecliptic conjunction, and at the interval of an hour, gives the apparent horary motion (r) in longitude of the moon from the ſun; the difference (D) between the true longitude at the ecliptic conjunction, and the moon's apparent longitude is the apparent diſtance of the moon from the ſun in longitude at the true time of the ecliptic conjunction; hence, $r : D ::$ 1 hour : the time from the true to the apparent conjunction, conſequently we know the time of the apparent conjunction. To find whether this time is accurate, we may compute (from the horary motions of the ſun and moon) their true longitudes, and the moon's parallax in longitude from the ſun, and apply it to the true longitude and it gives the apparent longitude, and if this be the ſame as the ſun's longitude, the time of the apparent conjunction is truly found; if they be not the ſame, find from thence the true time, as before. To the true time of the apparent conjunction, find the moon's true latitude from it's horary motion, and compute the parallax in latitude, and you get the apparent latitude at the time of the apparent conjunction. Draw SA perpendicular to CE and equal to this apparent latitude; then the point A will

pro-

probably not fall in NQ; firſt let it fall in QN, to which draw SB perpendicular, and NR parallel to PM. Then knowing NR ($=PM$), and QR ($=QP \sim MN$), we have

$$NR : RQ :: \text{rad.} : \text{tan. } QNR, \text{ or } ASB$$
$$\text{Sin. } QNR : \text{rad.} :: QR : QN$$

The time of deſcribing NQ in the apparent orbit being equal to the time from M to P in longitude, NQ is the horary motion in the apparent orbit.

$$\text{Rad.} : \text{ſin. } ASB :: AS : AB.$$
$$\text{Rad.} : \text{cos. } ASB :: AS : SB.$$

567. At the apparent conjunction the moon appears at A, which time (566) is known; when the moon appears at B, it is at it's neareſt diſtance from the ſun, and conſequently the time is that of the greateſt obſcuration, (uſually called the time of the middle,) provided there is an eclipſe, which will always be the caſe when SB is leſs than the *ſum* of the apparent ſemidiameters of the ſun and moon. If therefore it appears that there will be an eclipſe, we proceed thus to find it's quantity, and the beginning and end. As we may conſider the motion to be uniform, $QN : AB$ :: the time of deſcribing NQ : the time of deſcribing AB, which added to or ſubtracted from the time at A, (according as the apparent latitude is decreaſing or increaſing,) gives the time of the greateſt obſcuration. Or inſtead of taking QN, and the time of deſcribing it, we may take An (569) and the correſponding time, which will be more accurate.

568. From the ſum of the apparent ſemidiameters of the ſun and moon ſubtract BS, and the remainder ſhows how much of the ſun is covered by the moon, or the parts deficient; hence, ſemid. $\odot$: parts deficient :: 6 digits : the digits eclipſed. If SB be leſs than the *difference* of the ſemidiameters of the ſun and moon, and the moon's ſemidiameter be the *greater*, the eclipſe will be *total*; but if it be the *leſs*, the eclipſe will be *annular*, the ſun appearing all round the moon; if B and S coincide, the eclipſe will be *central*.

569. Let A fall out of QN; and to increaſe the accuracy, near to the apparent conjunction, that is within 10 or 15 minutes, calculate the apparent longitude mS of the moon from the ſun, and the apparent latitude mn; draw nr parallel to Sm; and in the triangle Anr, find the angle Anr which is equal to ASB, and compute SB, AB as before. But except in caſes where very great accuracy is required, this is unneceſſary. If NQ were a perfect ſtraight line, the firſt operation would give the correct values of AB, BS. KEPLER, in an eclipſe in 1598, found a curvature of more than 3′ in three hours, be-

 cauſe

cauſe the moon was very near the nonageſimal. In the eclipſe in 1764, M. de la LANDE found a curvature of 26″, but he does not ſay in what time. It is owing to this circumſtance, that is, the curvature of $N\mathfrak{Q}$, that it is neceſſary to find another point near to A, in order to determine accurately the values of AB, SB. Having determined the value of SB, and the time of the greateſt obſcuration, we thus find the beginning and end.

570. Produce, if neceſſary, $\mathfrak{Q}N$, and take SV, SW equal to the ſum of the apparent ſemidiameters of the ſun and moon, at the beginning and end reſpectively; then $BV = \sqrt{SV^2 - SB^2}$ (B being now ſuppoſed in $\mathfrak{Q}N$), and $BW = \sqrt{SW^2 - SB^2}$; then to find the times of deſcribing theſe ſpaces, ſay, as the hourly motion of the moon in the apparent orbit, or $N\mathfrak{Q}$, : BV :: 1 hour : the time of deſcribing VB; and $N\mathfrak{Q}$: BW :: 1 hour : the time of deſcribing BW, which times reſpectively ſubtracted from and added to the time of the greateſt obſcuration, give nearly the times of the beginning and end. But if accuracy be required, this method will not do; for it ſuppoſes VW to be a ſtraight line, which ſuppoſition will cauſe errors, too conſiderable in general to be neglected, and will never do where great accuracy is required. It may however always ſerve as a rule to aſſume the time of the beginning and end. Hence it follows, that the time of the greateſt obſcuration at B, is not neceſſarily equidiſtant from the beginning and end.

571. If the eclipſe be total, take Sv, Sw equal to the difference of the ſemidiameters of the ſun and moon, and then $Bv = Bw = \sqrt{Sv^2 - SB^2}$, from whence we may find the times of deſcribing Bv, Bw, as before, which we may conſider as equal, and which applied to the time of the greateſt obſcuration at B, give the time of the beginning and end of the total darkneſs.

572. To find more accurately the time of the beginning and end of the eclipſe, we muſt proceed thus. At the eſtimated time of the beginning, find, from the horary motions, and the computed parallaxes, the apparent latitude MN of the moon, and it's apparent longitude MS from the ſun, and we have $SN = \sqrt{SM^2 + MN^2}$, and if this be equal to the apparent ſemid. ☾ + ſemid. ☉ (which ſum call S) the eſtimated time is the time of the beginning; but if SN be not equal to S, aſſume (as the error directs) another time at a ſmall interval from it, *before*, if SN be *leſs* than S, but *after*, if it be *greater*; to that time compute again the moon's apparent latitude mn, and apparent longitude Sm from the ſun, and find $Sn = \sqrt{Sm^2 + mn^2}$; and if this be not equal to S, proceed thus; as the difference of Sn and SN : the difference of Sn and SL ($= S$) :: the above aſſumed interval of time, or time of the motion through Nn, : the time through nL, which added to or ſubtracted from the time at n, according as Sn is greater or leſs than SL, gives the time of the beginning.

FIG. 132.

The

The reaſon of this operation is, that as Nn, nL are very ſmall, they will (266) be very nearly proportional to the differences of SN, Sn, and Sn, SL. But as the variation of the apparent diſtance of the ſun from the moon is not exactly in proportion to the variation of the differences of the apparent longitudes and latitudes, in caſes where the utmoſt accuracy is required, the time of the beginning thus found (if it appear to be not correct) may be corrected, by aſſuming it for a third time, and proceeding as before. This correction however will never be neceſſary, except where extreme accuracy is required in order to deduce ſome conſequences from it. But the time thus found is to be conſidered as accurate, only ſo far as the Tables of the ſun and moon can be depended upon for their accuracy; and the beſt lunar Tables are ſubject to an error of 30″ in longitude, which, in this eclipſe, would make an error of about a minute an half in the time of the beginning and end. Hence accurate obſervations of an eclipſe compared with the computed time, furniſhes the means of correcting the lunar Tables, as will be afterwards explained. In the ſame manner the end of the eclipſe may be computed.

EXAMPLE.

To Compute the Times of the Solar *Eclipſe on April* 3, 1791; *for the Royal Obſervatory at Greenwich.*

The time of the mean conjunction (544) is April 3, 2*h*. 58′. 15″ mean time, at which time we find

Mean long. of the ſun	0ˢ. 11°. 51′. 16″
Long. of the moon's deſc. node .	0. 22. 14. 44
Mean long. of ☉ from ☾'s node .	0. 10. 23. 28

Hence (560) there muſt be an eclipſe ſomewhere upon the earth.

To the mean time of the new moon, compute the ſun's and moon's true longitudes, and they will be found to be 0ˢ. 13°. 47′. 43″, and 0ˢ. 14°. 49′. 24″; compute alſo the moon's true latitude, and it will be found to be 38′. 49″ N. deſcending. At the ſame time, the ſun's horary motion is found to be 2′. 28″, the moon's horary motion in longitude is 30′. 12″, and in latitude 2′. 46″ decreaſing; hence the moon's horary motion in longitude from the ſun is 27′. 44″.

By proceeding as directed in Article 547, we find the mean time of the ecliptic conjunction of the ſun and moon to be $3d.\ 0h.\ 44'.\ 48''$, from which ſubtract $3'.\ 18''$ the equation of time, and it gives the apparent time $3d.\ 0h.\ 41'.\ 30''$; at which time, the ſun's and moon's longitude in the ecliptic is $0^s.\ 13^\circ.\ 42'.\ 14''$; and the moon's true latitude is $44'.\ 59''$ N. deſcending. The horizontal parallax of the moon is $54'.\ 46''$, and of the ſun, $9''$; hence the horizontal parallax of the moon from the ſun is $54'.\ 37''$; therefore (173, 164) the moon's parallax in longitude from the ſun is $-20'.\ 56''$, and it's parallax in latitude from the ſun is $-33'.\ 44''$; hence $-20'.\ 56''$ is the apparent diſtance of the moon from the ſun in longitude; alſo the apparent latitude from the ſun is $11'.\ 15''$ north.

As the moon is to the weſt of the nonageſimal degree, aſſume 1 hour after, or $3d.\ 1h.\ 41'.\ 30''$, at which time (from the horary motions of the ſun and moon) the ſun's true longitude is found to be $0^s.\ 13^\circ.\ 44'.\ 42''$, the moon's true longitude on the ecliptic $0^s.\ 14^\circ.\ 12'.\ 26''$, and true latitude $42'.\ 13''$ north deſcending. The moon's parallax in latitude is $-30'.\ 41''$; hence the moon's apparent latitude is $11'.\ 32''$; alſo it's parallax in longitude from the ſun is $-28'.\ 50''$; but the moon's true longitude exceeds the ſun's by $0^\circ.\ 27'.\ 44''$; therefore the apparent diſtance of the moon from the ſun in longitude is $-1'.\ 6''$ Hence

Moon's apparent diſt. in long. at $0^h.\ 41'.\ 30'' = -20'.\ 56''$
$1.\ 41.\ 30 = -\ 1.\ 6$

Apparent hor. mot. ☾ from ☉ in long. . . . $19.\ 50 = MP.$

FIG. 131.

Hence $19'.\ 50'' : 20'.\ 56'' :: 1$ hour : $1h.\ 3'.\ 20''$, which added to the time of the true conjunction $0h.\ 41'.\ 30''$, gives $1h.\ 44'.\ 50''$, the time of the apparent conjunction. Alſo the apparent horary motion in latitude is $17'' = R\mathcal{Q}$; hence $\mathcal{Q}N$ is very nearly equal to MP.

At this time (from the horary motions) the ſun's true longitude is found to be $0^s.\ 13^\circ.\ 44'.\ 50''$, the moon's $0^s.\ 14^\circ.\ 14'.\ 7''$, and the moon's true latitude $42'.\ 4''$; hence the moon's true longitude is greater than the ſun's by $29'.\ 17''$. The moon's parallax in latitude from the ſun is $-30'.\ 32''$, and in longitude $-29'.\ 15''$; hence the moon's apparent latitude is $11'.\ 32''$ north; alſo the apparent longitude from the ſun is $29'.\ 17'' - 29'.\ 15'' = 2''$, which is what the moon's apparent longitude exceeds the ſun's true longitude.

This difference ſhows the apparent conjunction, found above, to be very nearly true; and to get it accurate, ſay, $19'.\ 50'' : 2'' ::$ 1 hour : $6''$, which (as

(as the moon's apparent longitude is the greater) ſubtracted from $1^h.\ 44'.\ 50''$ gives $1^h.\ 44'.\ 44''$, the true time of the apparent conjunction, at which time the moon's apparent longitude is $0^s.\ 13^\circ.\ 44'\ 50''$, the ſame as the ſun's true longitude, that not having ſenſibly varied in $6''$ of time. The apparent latitude is $11'.\ 32'',25$. Now at $1^h.\ 41'.\ 30''$ the moon's apparent diſtance in longitude from the ſun has been ſhown to be $1'.\ 6''$; and at $1^h.\ 44'.\ 44''$ the longitude of the ſun, and the moon's apparent longitude are equal; therefore in $3'.\ 14''$ the apparent motion of the moon from the ſun was $1'.\ 6'' = 66''$; let this $= Sm$, or nr; alſo at $1^h.\ 41'.\ 30''$, the apparent latitude $mn = 11'.\ 32''$, and at $1^h.\ 44'.\ 44''$ it was $11'.\ 32'',25 = SA$; therefore $Ar = 0'',25$. Hence

$$66'' : 0'',25 :: \text{rad.} : \text{tan. } Anr, \text{ or } SAB = 13'.\ 1''.$$

As the angle Anr is ſo very ſmall, we may take $An = rn = 66''$ without any ſenſible error; and for the ſame reaſon SB may be taken $= SA = 11'.\ 32''$.

$$\text{Rad.} : \text{ſin. } 13'.\ 1'' :: 11'.\ 32'' : AB = 2'',6.$$

Hence $An = 66'' : AB = 2'',6 :: 3'.\ 14'' : 8''$ the time through BA, which taken from $1^h.\ 44'.\ 44''$ gives $1^h.\ 44'.\ 36''$ the time of the greateſt obſcuration at B.

The moon's horizontal ſemidiameter is $14'.\ 56''$, and it's altitude at the time of the greateſt obſcuration (determined by a globe, which is ſufficiently near for this purpoſe) is about 38°; hence the augmentation of the diameter is $9''$, conſequently the apparent ſemidiameter of the moon is $15'.\ 5''$, which added to $15'.\ 59''$ the ſun's ſemidiameter, gives $31'.\ 4''$, from which ſubtract $SB = 11'.\ 32''$, and the remainder is $19'.\ 32''$ the parts deficient; hence $15'.\ 59'' : 19'.\ 32'' :: 6$ digits : $7^d.\ 19'.\ 57''$ the digits eclipſed at the time of the greateſt obſcuration.

To find the time of the beginning, we muſt firſt get the time (570) nearly. The value of $SB = 11'.\ 32'' = 692''$; and as the apparent ſemidiameter of the moon is now $15'.\ 6''$, we have $SV = 31'.\ 5'' = 1865''$; hence $BV = 1732''$. Now as MP is, in this caſe, nearly equal to QN, we may, for the purpoſe we here want it, aſſume the apparent horary motion of the moon from the ſun in the apparent orbit equal to that in longitude, which is $19'.\ 50'' = 1190''$; hence $1190'' : 1732'' ::$ 1 hour : $1^h.\ 27'.\ 20''$, which ſubtracted from $1^h.\ 44'.\ 36''$ (the time at B) gives $0^h.\ 17'.\ 16''$ the time of the beginning, nearly. Let us therefore aſſume the beginning at $0^h.\ 17'$, at which time we find (from the horary motions of the ſun and moon) the ſun's true longitude to be $0^s.\ 13^\circ.\ 41'.\ 15''$, and the moon's $0^s.\ 13^\circ.\ 29'.\ 55''$, whoſe difference is $11'.\ 20''$ their true diſtance in longitude; but the moon's parallax in longitude is

$-17'.45''$; hence their apparent diſtance in longitude is $29'.5'' = 1745''$. At the ſame time the moon's true latitude is $46'.7''$, and it's parallax in latitude $-35'.10''$; hence the apparent latitude of the moon from the ſun is $10'.57''$; therefore $\delta N = \sqrt{1745^2 + 657^2} = 1864'' = 31'.4''$, which being leſs than $31'.5'$ ſhows that the eclipſe is begun.

Let us next aſſume $0h.16'$; and by proceeding in the ſame manner, we find $Sn = 1883'' = 31'.23''$; therefore the eclipſe is not begun.

Hence $31'.23'' - 31'.4'' = 19'' : 31'.5'' - 31'.4'' = 1'' ::$ 1 minute : $3''$, which ſubtracted from $0h.17'$, gives $0h.16'.57''$ for the beginning of the eclipſe.

If to $1h.44'.36''$ we add $1h.27'.20''$, we have $3h.11'.56''$; we will therefore aſſume $3h.12'$ for the end; and by proceeding as before, we find the apparent diſtance of the moon from the ſun in longitude to be $30'.37''$, and the moon's apparent latitude $10'.48''$; hence the moon's apparent diſtance from the ſun is $\sqrt{1837^2 + 648^2} = 1948'' = 32'.28''$; but the ſum of the apparent ſemidiameters of the ſun and moon is now $31'.2''$; conſequently the eclipſe is ended.

Let us next aſſume the time $3h.6'$, and the apparent diſtance of the moon from the ſun in longitude is $28'.28''$, and in latitude $10'.55''$; hence the moon's apparent diſtance from the ſun is $\sqrt{1708^2 + 655^2} = 1829'' = 30'.29''$, therefore the eclipſe is not ended.

Hence $32'.28'' - 30'.29'' = 1'.59'' : 31'.2'' - 30'.29'' = 33'' :: 6' : 1'.39''$, which added to $3h.6'$, gives $3h.7'.39''$ for the end.

Hence at the Royal Obſervatory at Greenwich, the Tables give the times of the eclipſe on April 3, 1791,

Beginning . . .	$0^h.16'.57''$	} apparent time.
Greateſt obſcuration	1. 44. 36	
End	3. 7. 39	
Digits eclipſed .	$7^d.19.57$	

If it be required to compute the eclipſe for any other place, inſtead of the latitude of Greenwich uſe the latitude of the place; and reduce the apparent time at Greenwich to the apparent time at the place, according to the difference of the meridians.

573. To find what point of the ſun's limb will firſt be touched by the moon, let P be the pole of the ecliptic ES, Z the zenith, S, M, the centers of the ſun and moon when their limbs are in contact at a, and draw MD perpendicular to ES. By Art. 164, PZ is the altitude of the nonageſimal degree, and SPZ is the ſun's diſtance from that point, both which are found in the computation of the parallax; alſo MD is the apparent latitude of the moon; hence

FIG. 133.

Rad. : tan. PZ :: ſin. SPZ : tan. PSZ
Tan. SM : rad. :: tan. DS : cos. DSM

If L be the longitude of the nonageſimal degree, then $ZSD = 90° - PSZ$, when the ſun's longitude is between L and $L+180°$; otherwiſe $ZSD = 90° + PSZ$; and $ZSD \pm MSD$ (according as the moon's viſible latitude is ſouth or north) gives ZSM the diſtance of the point of the limb of the ſun firſt touched by the moon from the higheſt point of the ſun's diſc.

In this eclipſe, $PZ = 50°.\ 7'$, and $SPZ = 25°.\ 16'$; hence $PSZ = 27°.\ 3'$, which (in this caſe) added to 90° gives $117°.\ 3' = ZSD$; alſo $DSM = 20°.\ 42'$, which (as the moon's apparent latitude is north) ſubtracted from $117°.\ 3'$ gives $ZSM = 96°.\ 21'$, the moon's diſtance from the zenith of the ſun at the beginning of the eclipſe. In like manner, the diſtance at the middle and end of the eclipſe may be found, and thence the apparent path of the moon over the ſun's diſc in reſpect to the horizon may be deſcribed (578).

574. In the computation of this eclipſe, the moon's true latitude and longitude was at firſt computed from the Tables, and afterwards determined from the horary motions; but as the horary motions may be ſubject to a ſmall variation in the duration of an eclipſe, in caſes where the utmoſt accuracy is required, the true latitude and longitude ſhould be computed every time from the Tables; in ſuch caſes, the decimals of the ſeconds ſhould alſo be taken into conſideration, which in this Example were omitted. When we want only to predict an eclipſe, the method here practiſed will always be ſufficiently accurate, the error being always very ſmall, compared with the error to which the Tables are ſubject, which may, at it's maximum, be 30″ in longitude. We have followed the ſame method in computing the occultation of a fixed ſtar by the moon; that computation therefore may, if neceſſary, be rendered more correct, in the ſame manner.

To conſtruct a Solar Eclipſe, by the Principles of Projection delivered in the laſt Chapter.

575. According to this projection (541), the apparent ellipſe deſcribed by any point on the earth's ſurface, to any eye at the center of the ſun, is projected upon a plane at the moon perpendicular to a line joining the earth and ſun; and the point of the ellipſe of projection, correſponding to any point of the other ellipſe where the ſpectator is, is the point where the center of the ſun appears to the ſpectator. The center of projection is in the ecliptic. If the lunar orbit be properly laid down and divided, ſhowing where the center

ter of the moon is at any time, we ſhall then have the relative ſituation of the centers of the ſun and moon at any time ſeen from the given place of the ſpectator. From theſe principles of projection, we thus conſtruct the ſolar eclipſe which we have here calculated; aſſuming ſuch elements as are neceſſary, from that calculation.

FIG. 134. 576. Take (541) a radius OE equal to $54'.\ 37''$, the difference of the ſun's and moon's horizontal parallaxes, and divide it into minutes, and deſcribe the ſemicircle EGC repreſenting half the circle of projection, EOC repreſenting the ecliptic, to which draw OG perpendicular. Find (536) P the projected north pole; from the ſcale OE, take $Or = 54'.''37 \times$ ſin. lat. $\times$ cos. dec. and in a line perpendicular to Or ſet off both ways $r6 = h \times$ cos. lat. and $rm = rn = h \times$ cos. lat. $\times$ ſin. dec. and deſcribe the ellipſe $m6n6$, and divide it into hours, by Art. 536; and then ſubdivide thoſe hours which you will want to make uſe of, as far as you conveniently can for the ſize of the figure. From the ſcale take Ov equal $44'.\ 59''$, the moon's true latitude north deſcending at the time of the ecliptic conjunction, and draw LvM making an angle with Ov equal to $84°.\ 18'$, the complement of the angle which the relative orbit makes with the ecliptic, on the *left* ſide, if the latitude be north or ſouth *decreaſing*, and on the *right*, if *increaſing*; in this Example, it is on the left ſide; and LvM will repreſent the moon's relative orbit. Mark upon the moon's orbit at the point v, $41'.30''$, that being the time after 12 o'clock at which the true ecliptic conjunction happens; and with an extent $= 27'.\ 52''$, the horary motion of the moon from the ſun in it's relative orbit, ſet off the hours each way from v, and ſubdivide them into minutes, or as far as the ſize of the figure will permit. Now to find the time of the middle of the eclipſe, take the compaſs, and find, by trial, what two correſponding times, as at z and x, upon the ellipſe and moon's orbit are neareſt together, which will give the time of the *greateſt obſcuration*, becauſe the centers of the ſun and moon are then at the leaſt diſtance. To find the time of the beginning, take, with the compaſs, from the ſcale an extent equal to $31'.\ 5''$, the ſum of the ſemidiameters of the ſun and moon, and by trial find two correſponding times, as at s and t, at that diſtance, and it gives the time of the *beginning*; and if you find two correſponding times, as at y and w, at the diſtance $31'.\ 2''$, the ſum of the ſemidiameters at the end, it gives the time of the *end*; or you may omit the variation of the diameter of the moon in the interval. For the beginning muſt be when the centers of the ſun and moon arrive at the diſtance of the ſum of their ſemidiameters; and the end muſt be when they have receded till they have got to that diſtance. To find the digits eclipſed at the greateſt obſcuration, take eu from the ſcale, and ſay, $ze : eu ::$ 6 digits : the digits eclipſed. To find the digits eclipſed at any other time, take, with the compaſs, the interval

at

at that time on the ellipſe and on the moon's orbit, and apply it to the ſcale, and then ſay, ze : that diſtance :: 6 digits : the digits eclipſed.

577. If by taking the interval of two correſponding times, it appears that it is always greater than the ſum of the ſemidiameters of the ſun and moon, it ſhows that there will be no eclipſe at that place.

578. From this conſtruction, the poſition of the moon in reſpect to the zenith of the ſun's diſc may be found, and thence the apparent path of the moon over the ſun in reſpect to the horizon. For (540) a line drawn from O to any point of the ellipſe, where the ſpectator is, being vertical, from the principles of the projection, the angles Ots, Ozx, Owy, ſhow the angular diſtance about the center of the ſun from it's vertex to the center of the moon at the beginning, middle and end of the eclipſe. Hence let C be the center of the ſun $Zbda$, Z it's zenith; and make the angles ZCP, $ZC\mathfrak{Q}d$, $ZCR = Ots$, Ozx, Owy in Fig. 134. reſpectively; take CP, CR equal to the ſum of the ſemidiameters of the ſun and moon, $dCs =$ the digits eclipſed, and $S\mathfrak{Q} = aP$, and with the centers P, $\mathfrak{Q}$, R and radii $Pa = \mathfrak{Q}s = Rb$ equal to the ſemidiameter of the moon deſcribe three circles, and they will repreſent the ſituation of the moon at the beginning, middle and end of the eclipſe in reſpect to the vertex Z of the ſun, and conſequently in reſpect to the horizon; hence if we deſcribe a circle through P, $\mathfrak{Q}$, R, and with the ſame radius deſcribe rst parallel to it, it muſt very nearly repreſent the boundary of the eclipſe, or of the extreme part of the moon's limb as it paſſes over the ſun, in reſpect to the horizon. FIG. 135.

579. The eclipſe may alſo be thus calculated from the projection. Aſſume the time at t of beginning, as determined by the conſtruction; draw tc perpendicular to OP, and join Ot, Os. The time from t to m being given, convert it into degrees $a°$; then (537) ſin. $a° \times rb = tc$, and cos. $a° \times rm = rc$, but (535) Or is known, hence Oc is known; therefore in the right angled triangle Oct, we know Oc, ct, to find cOt, and Ot; but (536) POv is given, therefore $cOt \pm POv = tOv$ is known; alſo Ov and the angle Ovs are known by the conſtruction; and the time from s to v being given, and alſo the moon's relative horary motion in LM, we know vs; hence in the triangle Ovs, we know Ov, vs and the angle Ovs, to find Os, and the angle vOs; hence we find $tOs = tOv \pm vOs$; and laſtly, in the triangle tOs, we know tO, Os and the angle tOs, to find ts, and if this be equal to the ſum of the ſemidiameters of the ſun and moon, the aſſumed time is true; if it be not equal to the ſum, aſſume another time for the beginning, and find another value of ts, and proceed with theſe two as in Article 572. In like manner we may find the end But this method is not (541) ſubject to the ſame accuracy as the method of calculation which we have already given. FIG. 134.

580. Sir

580 Sir. I. NEWTON ſuppoſes that the aberration of rays in the focus of a teleſcope makes the image appear greater than it ought; and hence different teleſcopes will give different meaſures of the ſun's diameter, and conſequently make the eclipſe appear to begin at different times. That teleſcope which gives the diameter the leaſt, is the moſt perfect inſtrument. The excellent tranſit teleſcope at Greenwich makes the diameter of the ſun leſs by 6″ than that given by MAYER in his Tables, as Dr. MASKELYNE has found by his obſervations. The diameter of the ſun aſſumed in theſe calculations has therefore been taken 6″ leſs than that which MAYER determined. M. du SEJOUR ſuppoſes that the rays of light coming from the ſun are inflected as they paſs by the moon, which he attributes to the refraction which they ſuffer in paſſing through the moon's atmoſphere; on this account the apparent contact of the limbs will not take place ſo ſoon as it otherwiſe would; this would be the ſame as a diminution of the moon's diameter; which of theſe hypotheſes ought to be admitted M. du SEJOUR endeavoured to determine from the obſervations of Mr. SHORT, on the ſolar eclipſe, April 1, 1764, upon the diſtance of the horns of the moon, but he could deduce nothing ſatisfactory from thence. He ſuppoſed the inflection 3″,291, and the diameter of the moon to be diminiſhed by the ſame quantity, and calculated upon each ſuppoſition a great many diſtances of the horns, and compared them with the obſerved diſtances; but he could not decide between the two hypotheſes. An inflection of 1″,8, and a diminution of 1″,5 of the ſemidiameter, he found would ſatisfy ſome obſervations, and he ſeemed to think this concluſion moſt likely to be neareſt the truth; but he came at laſt to no determination upon the ſubject. All the requiſite obſervations ſeem not to be capable of being made to that degree of accuracy which is neceſſary to ſettle ſo nice a matter. M. du SEJOUR therefore propoſed the following method to determine whether the rays of light paſſing by the limb of the moon ſuffer any deviation. Take a teleſcope mounted upon a polar axis, with a wire micrometer annexed to it. When two ſtars come into the field of view together, and one of them is to be eclipſed by the moon, open the wires and bring one ſtar upon one of the wires and the other ſtar upon the other, and thus follow the ſtars until one of them be eclipſed, and at the inſtant before it diſappears, obſerve whether it's diſtance from the other ſtar is changed, that is, whether it be off the wire, the other ſtar remaining upon it's wire; if this be found to be the caſe, the rays muſt have ſuffered a deviation. *Traité Analytique*, pag. 420. I do not find that an obſervation of this kind has been ever made.

To trace out the Path on the Surface of the Earth, where the Eclipſe will be central, or for any number of digits.

581. Let *EACD* be the enlightened hemiſphere of the earth, *O* the center of the diſc, or that point to which the ſun is vertical; *EOC* the plane of the ecliptic, *OG* perpendicular to it; *P* the north pole; join *PO*, and let *LM* repreſent the path of the center of the penumbra, conceived to be upon the plane into which the diſc would be orthographically projected; then from the nature of that projection, the angles at *O* upon the ſurface will be equal to the angles into which they are projected. Now the place upon which the center of the moon is projected is manifeſtly that point on the earth where the eclipſe is central, becauſe the projection is made by lines drawn to the center of the ſun. Let *Z* be the projected center of the moon at any time, or the real center of the penumbra, and *PB* any given meridian. Now we know *Ov* the moon's latitude at the time of conjunction, and (576) the angle *OvZ*; and as the time, when the center of the penumbra is at *Z*, is given, the time through *Zv* is known, and the relative horary motion of the moon being known, *vZ* will be known; hence we can find *ZO*, and the angle *ZOv*; find (536) the angle *POv*, and we ſhall have the angle *POZ*. Now conſider *PO* and *ZO* as two circles upon the earth's ſurface, then the angle *POZ* between them is equal to the angle *POZ* of projection, and therefore known; alſo the arc *PO* is the complement of the ſun's declination; and to find the arc *ZO*, we muſt conſider *ZO* in the projection to be the ſine of the arc projected; hence the arc *ZO* is that whoſe ſine is to radius as *OZ* to *OA*, therefore we know the ſine of the arc *ZO*, and conſequently we get *ZO* itſelf; hence in the ſpherical triangle *OPZ*, we know *PO*, *OZ* and the angle *POZ*, to find *PZ* the complement of the latitude of the place where the eclipſe is central. Find alſo the angle *OPZ*; then the time at the meridian *PB* being known, the angle *OPB* (the ſun's diſtance from the meridian) is known; hence we know the angle *BPZ* the longitude of the point *Z* from the meridian *PB*; therefore the latitude and longitude of *Z* being known, the point *Z* is determined where the eclipſe was central at the given time. Make this calculation for every quarter or half hour, for all the time the penumbra is deſcribing *de*, and you will trace out upon the ſurface of the earth the path of the center of the penumbra, or that tract where the eclipſe is central. If we bring *Z* to *d*, we get the place where the ſun riſes centrally eclipſed; and if *Z* be brought to *e*, we ſhall find where the ſun ſets centrally eclipſed. If *Z* coincide with *r*, we get the place where the ſun is centrally eclipſed upon the meridian. Let *y* be the center of

FIG. 136

the penumbra when it firſt touches the earth, and x the center when it leaves the earth, and draw Ow perpendicular to LM. Then knowing Ov and the angle Ovw, we can find Ow and vOw; alſo $Oy =$ ſemid. $\ominus +$ ſemid. penumb. known; hence in the right angled triangle yOw, we get the angle yOw, and therefore we know yOv; and POv being already found, we know bOP; hence in the triangle bOP, we know bO $(= 90°)$, PO, and the angle POb; hence we find Pb the complement of the *latitude* of b; find alſo OPb, and we get bPB the *longitude* of b, from the given meridian PB; thus we get the place b where the eclipſe firſt begins at the ſun riſing. In like manner we get the place a where the eclipſe laſt ends at ſun ſetting.

Ex. In the ſolar eclipſe which we have here computed, let it be required to find that place upon the earth's ſurface where the ſun is centrally eclipſed at one o'clock, apparent time at Greenwich. In this caſe, $Ov = 44'.\ 59''$, and the angle $OvZ = 5°.\ 42'$, and as the time at v is $41'.\ 30''$, and the center of the penumbra is at Z at one o'clock, the time through $vZ = 19'.\ 30''$, which gives $vZ = 9'.\ 3''$; hence $ZO = 35'.\ 59''$, the angle $OZv = 81°.\ 22'$. and $ZOv = 13°.\ 56'$. Now the radius $OG = 54'.\ 56''$; hence the arc OZ upon the ſurface (correſponding to it's projection $= 35'.\ 59''$) $= 41°.\ 4'$; alſo $PO = 84°.\ 30'$, and (536) $POv = 23°.\ 22'$; hence $POZ = 9°.\ 26'$; conſequently $PZ = 45°.\ 43'$, the complement of which is $44°.\ 17'$ the latitude of the place; alſo $ZPO = 8°.\ 51'$; but at one o'clock apparent time at Greenwich, it's meridian BP makes an angle of $15°$ with PO, Greenwich being upon that meridian at 12 o'clock; hence $BPZ = 23°.\ 51'$ the longitude of the place, weſt from Greenwich. In like manner may any of the other phænomena be calculated.

FIG. 137. 582. Draw OW perpendicular to LM, and take $vc = ve$ equal to the ſum of the ſemidiameters of the ſun and moon, and draw dcf, xey parallel to LM; then df and xy will mark out the boundaries of the eclipſe, or the places where the limbs of the ſun and moon juſt appear in contact. So that if we take the moon at any place Z, and draw Zr perpendicular to df, and compute the latitude and longitude of the point r in the ſame manner as we did that of Z in the laſt Article, it will give the place where the limbs of the ſun and moon appear in contact outwardly. If we take Zr on the other ſide of LM, we ſhall, on that ſide, get the place where they appear in contact. If we do this for every quarter or half hour, we ſhall trace the path over the ſurface of the earth where the limbs of the ſun and moon appear in contact, or the boundaries of the eclipſe; thus we can lay down upon the earth's ſurface that tract over which the penumbra paſſes. If cv be divided into twelve equal parts, and ck be

be taken equal to three of them, for inſtance, and *tks* be drawn parallel to *LM*, and the place of *r'* be computed, it gives the place where the ſun will be three digits eclipſed; and in like manner as before may the tract on the earth's ſurface be marked out where the ſun will appear three digits eclipſed; in the ſame manner, we may trace out the path for any number of digits. If $va = vb$, and *ab* be the difference between the apparent diameters of the ſun and moon, and *waz*, *gbh* be drawn parallel to *LM*; then if the diameter of the moon be greater than that of the ſun, the ſpace between *wz* and *gh* is the limit for the total eclipſe; but if the diameter of the ſun be the greater, it will be the limit of the annular eclipſe. This method of delineating the lines of the phaſes upon the earth's ſurface ſuppoſes that the apparent neareſt diſtance of the centers of the ſun and moon, or of the correſponding horns upon the relative orbit of the moon and upon the ellipſe, is in a line perpendicular to the moon's relative orbit; but this is not accurately true, and therefore the delineation cannot be accurate. This line will have different poſitions at different places for the ſame phaſe. M. du SEJOUR propoſes therefore to take the mean angle. He found an error of 3°. 33′,5 in longitude, and of 35′. 49″ in time of the contact, for the latitude of 16°. 57′, by ſuppoſing it to happen upon a perpendicular to the relative orbit.

583. M. de la LANDE has given the following graphical method. Draw *LM* on a ſeparate piece of paper, and divide it, ſo that by moving it you may bring any hour to *v*. Then if, for inſtance, the orbit be moved to the right till the time of falling on *v* be one hour later than that for which the conſtruction was firſt made, it is in a proper poſition for a place 15° to the eaſt of that place. Let *pq* be any parallel of latitude, and divide it into hours, &c. then move the orbit *LM* until, with an extent of compaſs equal to the ſum of the ſemidiameters of the ſun and moon, you can make the points fall on the ſame hour both on *pq* and *LM*, and at the ſame time that it ſhall be the ſhorteſt diſtance between any two correſponding hours on *pq* and *LM*; then the difference of the hours ſhown at *v* in conſequence of the removing of the orbit ſhows the longitude of the place from that for which the projection was made, and the parallel *pq* ſhows the latitude; therefore the place on the earth's ſurface is determined where the limbs juſt come into contact, and alſo the time. This we may repeat for as many parallels of latitude as we pleaſe, and thus trace out the curve on the ſurface of the earth where the limbs juſt come into contact. In like manner, if we take an extent of compaſs $\frac{1}{12}$ of the ſun's diameter leſs, we may trace out the path where the ſun is one digit eclipſed; and ſo for any other phaſe.

584. But the general phænomena may be much more eaſily, and with ſufficient accuracy, determined by a common globe thus. Bring the ſun's place

S to

FIG. 138. S to the brazen meridian PV, ſet the hour index to 12, and bring the ſun to the zenith; and from that point on the globe, extend a thread SW perpendicular to the ecliptic EC. Upon a ſmall ſtraight rod LM let there be a moveable circle $adbm$, whoſe center r is in the middle of the rod, and let it's radius be to the radius of the globe as the ſemidiameter of the penumbra is to the horizontal parallax of the moon. Let there be two moveable upright pieces attached to the horizon of the globe, and capable of being fixed at any two points; at a diſtance Sv equal to the moon's latitude at the time of the ecliptic conjunction, let the rod LM cut the thread in a plane touching the globe at S, and making an angle MvS equal to the angle which the relative orbit makes with a perpendicular to the ecliptic, as before directed; and in this ſituation let the rod be fixed to the above mentioned pieces attached to the horizon; let alſo LM be divided (as in Art. 576.) into time, the time at the point v being that of the ecliptic conjunction; and let the horizon of the globe be fixed exactly parallel to the horizon; and upon a fine thread ab ſuſpend a ſmall conical weight, with an acute vertex; every thing being thus prepared, we proceed thus; firſt premiſing, that the circle $TEVC$ (coinciding with the horizon of the globe) contains all the places where the ſun is in the horizon, the weſtern hemiſphere containing thoſe where the ſun is riſing, and the eaſtern where the ſun is ſetting.

Carry the center of the penumbra to r where you find, by ſuſpending the line from it's periphery, that it's projection would juſt touch the earth at t; then turn the globe till the index ſhows the ſame hour as the point r, and the point t at that time is the place firſt touched by the penumbra, and the eclipſe there begins at ſun riſe. Carry the center of the penumbra to s, a quarter of an hour forwards for inſtance, and ſet the globe again to that time; and with the line find the points x, w, where the projection of the circumference of the penumbra would interſect the circle $TCVE$, and x, w, are two other points where the eclipſe begins at ſun riſe. By proceeding thus through the weſtern part, we may find as many points as we pleaſe where the eclipſe begins at ſun riſe, and by drawing a curve through them, we get the tract on the earth's ſurface where the eclipſe begins at the riſing of the ſun. But when x comes into the eaſtern part, we get the places where the eclipſe begins at ſun ſet; conſequently the curve paſſing through them is a continuation of the curve which paſſes through all the places where the eclipſe begins at ſun riſing, provided the whole of the penumbra does not fall upon the earth at the middle of the eclipſe; that is, if O be the middle of the general eclipſe, and OT be perpendicular to LM in the projection, when OT is leſs than the radius (R) of the penumbra. If $OT = R$, the curves touch, and if OT be greater than R, they are ſeparated; becauſe the penumbra then paſſes over a part of the earth

after

after it leaves the weſtern ſide before it comes to the eaſtern, and therefore in that interval the ſun is not eclipſed to any part of the circumference $TEVC$.

Draw bra perpendicular to LM, and ac parallel to it; and when w comes to c, the interſection manifeſtly begins to go back, and the part adb will then cut TCV, and therefore the interſections will ſhow the points in the horizon where the eclipſe ends at ſun ſet; for the velocity of the penumbra being greater than that of the earth about it's axis, the penumbra muſt leave the point which it then touches. And for this reaſon the eclipſe begins where bta cuts TCV. Hence as c is a point where the eclipſe ceaſes to begin at ſun riſe and begins to end at ſun ſet, the curve ſhowing the places when the eclipſe begins at ſun riſing is continued into the curve which ſhows the places where the eclipſe ends at ſun riſing; and as x and w meet again on the eaſtern ſide of the earth, the curves there meet again. When therefore OT is leſs than R, we have a curve in the form of Fig. 139. The velocity of the parts of the earth, from it's rotation, being greateſt towards the equator, the ovals will be more open towards A and B than towards C. FIG. 139.

If $OT = R$, the two ovals touch as in Fig 140; but that neareſt the pole is double. FIG. 140.

If OT be greater than R, the two ovals will be detached as in Fig. 141; but if the perpendicular Dz on LM (in the projection,) be leſs than R, the oval neareſt the pole will be double. Two curves AB, CD touching the ovals, and formed by the projection of a and b, will ſhow where the limbs of the ſun and moon were in contact. FIG. 141.

If LM fall beyond the earth, or if O lie on the other ſide of T, the curve will be like Fig. 139. until Dz be greater than R, in which caſe, the curve becomes a ſimple oval; and when $OT = R$, the oval vaniſhes. Theſe circumſtances, which were given by M. de LAMBRE, appear from this method of delineation. FIG. 138.

The projection of ba will give the middle of the eclipſe. The weſtern part of TCV which is cut by ba in the projection is the point where the eclipſe is in the middle at ſun riſe; and where the eaſtern part of the horizon is cut by it, it ſhows the point where the middle of the eclipſe is at ſun ſet. When a comes to c, the globe being adjuſted to that time, c is a point of the former kind, but here the eclipſe is only for a moment, or rather there is no eclipſe, the moon's limb only touching that of the ſun. As the umbra advances, ab will cut TCV at ſome other point, as w; to the time denoted by the center r, adjuſt the globe, and w is the place upon the earth where the middle of the eclipſe is at ſun riſing. And thus we may find any number of ſuch places, and draw a curve, as BDA, paſſing through all the places where the middle of the eclipſe is at the riſing of the ſun. As the line, in this caſe, muſt be ſuſpended from FIG. 139.

ba, it will be more convenient to have the penumbra divided into two parts through *ab*, ſo that they may be ſeparated.

FIG. 138. To trace out the center of the penumbra over the earth, bring, by the line, the center to coincide in the projection with *TCV* at *e*, and adjuſt the globe to that time, and *e* will be the place where the center of the umbra firſt touches the earth. Carry on the penumbra, a quarter of an hour for inſtance, and adjuſt the globe to the time, and project the center upon the earth, and it gives the point where the eclipſe is central at that time. Find thus as many points as you pleaſe, and draw a curve through them all, and you get the path of the center of the penumbra over the earth, ſhowing all thoſe places where the eclipſe was central.

If the penumbra be formed by 12 equidiſtant concentric wires, the phænomena of any one of the digits may be traced out in the ſame manner; that is, we can find, for inſtance, all the places where the ſun is three digits eclipſed at it's riſing and ſetting, and the tract where the ſun is three digits for the time of the eclipſe. The globe here uſed ſhould be one which has the hours marked on the equator.

The method of tracing out the different curves was, I believe, firſt given by M. de la CAILLE in his Aſtronomy. M. du SUJOUR has given an analytical method of laying down the curves in his *Traité Analytique*. But theſe are matters rather of curioſity, than of any real uſe in Aſtronomy. If we place the circle *ETCV* perpendicular to the horizon, and vertical to *S* a ſtrong lamp be fixed in the principal focus of a double convex lens, ſo that the rays may be thrown parallel upon the globe and perpendicular to *ETCV*, the ſhadow of the penumbra will give the points of projection required, inſtead of the plumb line. Thus we make a common globe anſwer the purpoſe of an Eclipſareon, invented by Mr. FERGUSON, and deſcribed in his Aſtronomy.

585. As there are not many perſons who have an opportunity of ſeeing a total eclipſe of the ſun, we ſhall here give the phænomena which attended that on April 22, 1715. Captain STANNYAN, at Bern in Switzerland, ſays, "the ſun was totally dark for four minutes and an half; that a fixed ſtar and planet appeared very bright; and that it's getting out of the eclipſe was preceded by a blood-red ſtreak of light, from it's left limb, which continued not longer than ſix or ſeven ſeconds of time; then part of the ſun's diſc appeared, all on a ſudden, as bright as *Venus* was ever ſeen in the night; nay brighter, and in that very inſtant gave a light and ſhadow to things, as ſtrong as moon light uſed to do." The inference drawn from theſe phænomena is, that the moon has an atmoſphere.

J. C. FACIS, at Geneva, ſays, "there was ſeen, during the whole time of the total immerſion, a whiteneſs, which ſeemed to break out from behind the moon,

moon, and to encompaſs it on all ſides equally; it's breadth was not the twelfth part of the moon's diameter. *Venus*, *Saturn*, and *Mercury* were ſeen by many; and if the ſky had been clear, many more ſtars might have been ſeen, and with them *Jupiter* and *Mars*. Some Gentlewomen in the country ſaw more than 16 ſtars; and many people on the mountains ſaw the ſky ſtarry, in ſome places where it was not overcaſt, as during the night at the time of the full moon. The duration of the total darkneſs was three minutes."

Dr. J. J. SCHEUCHZER, at Zurich, ſays, that "both planets and fixed ſtars were ſeen; the birds went to rooſt; the bats came out of their holes; and the fiſhes ſwam about; we experienced a manifeſt ſenſe of cold; and the dew fell upon the graſs. The total darkneſs laſted four minutes."

Dr. HALLEY,* who obſerved this eclipſe at London, has thus given the phænomena attending it. "It was *univerſally* obſerved, that when the laſt part of the ſun remained on it's eaſt ſide, it grew very faint, and was eaſily ſupportable to the naked eye, even through the teleſcope, for above a minute of time before the total darkneſs; whereas on the contrary, my eye could not endure the ſplendor of the emerging beams in the teleſcope from the firſt moment. To this perhaps two cauſes concurred; the one, that the pupil of the eye did neceſſarily dilate itſelf during the darkneſs, which before had been much contracted by looking on the ſun. The other, that the eaſtern parts of the moon, having been heated with a day near as long as thirty of ours, muſt of neceſſity have that part of it's atmoſphere replete with vapours, raiſed by the long continued action of the ſun; and by conſequence, it was more denſe near the moon's ſurface, and more capable of obſtructing the luſtre of the ſun's beams. Whereas at the ſame time the weſtern edge of the moon had ſuffered as long a night, during which time there might fall in dews, all the vapours that were raiſed in the preceding long day; and for that reaſon, that part of it's atmoſphere might be ſeen much more pure and tranſparent.

About two minutes before the total immerſion, the remaining part of the ſun was reduced to a very fine horn, whoſe extremities ſeemed to loſe their acuteneſs, and to become round like ſtars. And for the ſpace of about a quarter of a minute, a ſmall piece of the ſouthern horn of the eclipſe ſeemed to

* The DR. begins his account thus. Though it be certain from the principles of Aſtronomy, that there happens neceſſarily a central eclipſe of the ſun, in ſome part or other of the terraqueous globe, about twenty-eight times in each period of eighteen years; and that of theſe, no leſs than eight do paſs over the parallel of London, three of which eight are total with continuance: yet from the great variety of the elements, whereof the *calculus* of eclipſes conſiſts, it has ſo happened, that ſince March 20, 1140, I cannot find that there has been a total eclipſe of the ſun ſeen at London, though in the mean time the ſhade of the moon has often paſt over other parts of Great Britain.

to be cut off from the reſt by a good interval, and appeared like an oblong ſtar rounded at both ends: which appearance could proceed from no other cauſe, but the inequalities of the moon's ſurface, there being ſome elevated parts thereof near the moon's ſouthern pole, by which interpoſition, part of that exceedingly fine filament of light was intercepted.

A few ſeconds before the ſun was totally hid, there diſcovered itſelf round the moon a luminous ring, about a digit or perhaps a tenth part of the moon's diameter in breadth. It was of a pale whiteneſs, or rather pearl colour ſeeming to me a little tinged with the colours of the *iris*, and to be concentric with the moon; whence I concluded it was the moon's atmoſphere. But the great height of it, far exceeding that of our earth's atmoſphere; and the obſervations of ſome who found the breadth of the ring to increaſe on the weſt ſide of the moon, as the emerſion approached; together with the contrary ſentiments of thoſe, whoſe judgement I ſhall always revere, make me leſs confident, eſpecially in a matter whereto I gave not all the attention requiſite.

Whatever it was, this ring appeared much brighter and whiter near the body of the moon, than at a diſtance from it; and it's outward circumference, which was ill defined, ſeemed terminated only by the extreme rarity of the matter it was compoſed of; and in all reſpects reſembled the appearance of an enlightened atmoſphere viewed from far: but whether it belonged to the ſun or the moon, I ſhall not at preſent undertake to decide.

During the whole time of the total eclipſe, I kept my teleſcope conſtantly fixt on the moon, in order to obſerve, what might occur in this uncommon appearance, and I ſaw perpetual flaſhes or coruſcation of light, which ſeemed for a moment to dart out from behind the moon, now here, now there, on all ſides, but more eſpecially on the weſtern ſide, a little before the emerſion: and about two or three ſeconds before it, on the ſame weſtern ſide, where the ſun was juſt coming out, a long and very narrow ſtreak of a duſky, but ſtrong red light, ſeemed to colour the dark edge of the moon, though nothing like it had been ſeen immediately after the immerſion. But this inſtantly vaniſhed upon the firſt appearance of the ſun, as did alſo the aforeſaid luminous ring.

As to the degree of darkneſs, it was ſuch, that one might have expected to have ſeen many more ſtars than were ſeen in London; the planets, *Jupiter*, *Mercury* and *Venus* were all that were ſeen by the gentlemen of the Society from the top of their houſe, where they had a free horizon: and I do not hear that any one in town ſaw more than *Capella* and *Aldebaran* of the fixed ſtars. Nor was the light of the ring round the moon capable of effacing the luſtre of the ſtars, for it was vaſtly inferior to that of the full moon, and ſo weak, that I did not obſerve it caſt a ſhade. But the under parts of the hemiſphere, particularly in the ſouth-eaſt under the ſun, had a crepuſcular brightneſs; and all round

us, ſo much of the ſegment of our atmoſphere as was above the horizon, and was without the cone of the moon's ſhadow, was more or leſs enlightened by the ſun's beams; and it's reflection gave a diffuſed light, which made the air ſeem hazy, and hindered the appearance of the ſtars. And that this was the real cauſe thereof, is manifeſt by the darkneſs being more perfect in thoſe places near which the center of the ſhade paſt, where many more ſtars were ſeen, and in ſome, not leſs than twenty, though the light of the ring was to all alike.

I forbear to mention the chill and damp, with which the darkneſs of this eclipſe was attended, of which moſt ſpectators were ſenſible, and equally judges; or the concern that appeared in all ſorts of animals, birds, beaſts and fiſhes upon the extinction of the ſun, ſince ourſelves could not behold it without ſome ſenſe of horror."

586. At an eclipſe of the ſun, the diſtance between the centers of the ſun and moon may be found at any time with a micrometer thus. Let ACB be the ſun, S it's center, ADB the moon, M it's center; take the diſtance AB of the horns with the micrometer; then we know Ae half that diſtance, and knowing SA from the Tables, we have $Se = \sqrt{SA^2 - Ae^2}$; for the ſame reaſon knowing AM from the Tables, we have $Me = \sqrt{AM^2 - Ae^2}$; hence $SM = Se + eM$ is known. If we thus take SN, $S\mathfrak{Q}$ the diſtances of the ſun and moon at any times, and calculate the apparent motion $N\mathfrak{Q}$ of the moon in the interval, we may find the apparent time of conjunction. M. du SEJOUR found it neceſſary to ſubtract $3'',5$ from the ſemidiameter of the ſun as given in the Tables he uſed, in order to make his calculations agree with obſervations, independent of the diminution of the moon's ſemidiameter by inflection (580, 587). In our calculations, we have taken the ſemidiameter of the ſun $3''$ leſs than that given in MAYER's Tables, according to Dr. MASKELYNE's determination. The diſtance of the centers may alſo be found by meaſuring the breadth Cx, which taken from Cz leaves xz; hence $MS = Mx + Sz - 2xz$ is known. FIG. 142.

587. Admitting the inflection of the rays of the ſun at the moon, an eclipſe will begin later and end ſooner, and therefore the duration will be diminiſhed. For if S be the ſun, M the moon, T the ſpectator; draw Tma a tangent to the ſun, and let a ray abT be inflected at b; then the eclipſe does not begin till the moon's limb gets to b; whereas, without inflection, it would have begun at the line Sma; for the ſame reaſon it ends ſooner. The duration however of an annular eclipſe, and the breadth of the annulus is increaſed by this cauſe. This M. du SEJOUR has found to agree with obſervation. FIG. 131. FIG. 143.

588. Let $Nanb$ repreſent the orbit of the earth, $Ncnd$ the plane of the moon's orbit inclined to it; take $vN = Nw = xn = nz = 17°.\ 21'$, $pN = Nr = sn = nt = 11°.\ 34'$, then (552, 560) all the time the earth is moving from v to w, and from x to z, it is within the ſolar ecliptic limits; and whilſt it moves from p FIG. 144.

to *r*, and from *s* to *t*, it is within the lunar ecliptic limits. Now if a conjunction happen at or very near the node, there will be a great ſolar eclipſe; but at the preceding oppoſition, the earth was not got into the lunar ecliptic limits, and at the next oppoſition it will be got beyond it; hence at each node there may happen only one ſolar eclipſe, and therefore in a year there *may* happen only two ſolar eclipſes.

There muſt be one conjunction happen in the time in which the earth is paſſing through the ſolar ecliptic limits, and conſequently there *muſt* be one ſolar eclipſe happen at each node; hence there *muſt* be two ſolar eclipſes at leaſt in a year.

If an oppoſition happen juſt before the earth gets into the lunar ecliptic limit, the next oppoſition may not happen till the earth is got beyond the limit on the other ſide of the node; conſequently there may not be a lunar eclipſe at the node; hence there *may not* be an eclipſe of the moon in the courſe of a year. When therefore there are only two eclipſes in a year, they muſt be both of the ſun.

If there be an eclipſe of the moon as ſoon as the earth gets within the lunar ecliptic limit, it will be got out of the limit before the next oppoſition; conſequently there can be only one lunar eclipſe at the ſame node. But as the lunar nodes move backwards about 19° in a year, the earth may come within the lunar ecliptic limits, at the ſame node, a ſecond time in the courſe of a year, and therefore there *may* be three lunar eclipſes in a year; and there can be no more.

If an eclipſe of the moon happen at or very near to the node, a conjunction may happen before and after, whilſt the earth is within the ſolar ecliptic limits; hence there may, at each node, happen two eclipſes of the ſun and one of the moon; and in this caſe, the eclipſes of the ſun will be ſmall, and that of the moon large. Thus when the eclipſes happen at each node only once, there may be ſix eclipſes in a year, four of which muſt be of the ſun and two of the moon. But if, as in the laſt caſe, an eclipſe ſhould happen at the return of the earth within the lunar ecliptic limits at the ſame node a ſecond time within the year, there may be ſix eclipſes, three of the ſun and three of the moon.

There may be ſeven eclipſes in a year. For twelve lunations are performed in 354 days, or in eleven days leſs time than a common year. If therefore an eclipſe of the ſun ſhould happen before January 11, and there be at that, and at the next node, two ſolar and one lunar eclipſe at each node; then the twelfth lunation from the firſt eclipſe will give a new moon within the year, and (on account of the retrograde motion of the moon's nodes) the earth may be got within the ſolar ecliptic limits, and there may be another ſolar eclipſe.

Hence

Hence when there are ſeven eclipſes in a year, five will be of the ſun and two of the moon.

As there are but ſeldom ſeven eclipſes in a year, the mean number will be about four.

The nodes of the moon move backwards about 19° in a year, which arc the earth deſcribes in about 19 days, conſequently the middle of the ſeaſons of the eclipſes happens every year about 19 days ſooner than in the preceding year.

589. The ecliptic limits of the ſun (560) are greater than thoſe of the moon (552), and hence there will be more ſolar than lunar eclipſes, in about the ſame proportion as the limit is greater, that is as 3 : 2 nearly. But more lunar than ſolar eclipſes are ſeen at any given place, becauſe a lunar eclipſe is viſible to a whole hemiſphere at once; whereas a ſolar eclipſe is viſible only to a part, and therefore there is a greater probability of ſeeing a lunar than a ſolar eclipſe. Since the moon is as long above the horizon as below, every ſpectator may expect to ſee half the number of lunar eclipſes which happen.

To Compute the Time of an Occultation of a Fixed Star *by the* Moon.

590. Find, from the precepts and Tables (given at the end of this Volume), the apparent longitude and latitude of the ſtar at the time when an occultation is expected.*

591. Compute (as will be explained in the Introduction to the Tables in the ſecond Volume) the time of the *mean* conjunction, and at that time the moon's true latitude; then (according to M. CASSINI) if the difference of the latitudes of the moon and ſtar exceed 1°. 37′, there can be no occultation; but if the difference be leſs than 51′, there muſt be one, ſomewhere on the earth; hence between 1°. 37′ and 51′ it is doubtful.

592. Compute the time of the true conjunction from that of the mean, from the moon's true longitude at the mean conjunction, and it's horary motion. But if, from the equation of the moon's orbit, it appears that there is a conſiderable interval between the mean and true conjunction, it will be better to aſſume ſome time as near as you can conjecture to the true conjunction, at which time compute the moon's true longitude and it's horary motion, and then

* Here, as in ſolar eclipſes, no method, previous to the calculation of the apparent place of the moon at the ecliptic conjunction, can be given, to determine whether there will be an occultation at any particular place, on account of the moon's parallax; from the courſe of the moon, however, we may conjecture when it is probable ſuch a phænomenon may happen. According to M. CASSINI, thoſe ſtars whoſe north and ſouth latitudes do not exceed 6°. 36′, may ſuffer an occultation *ſomewhere* upon the earth; and if the latitude do not exceed 4°. 32′, they may be eclipſed on *any* part of the earth.

then by applying the horary motion you will get the time of the true conjunction to a great degree of accuracy; whereas, from the conſiderable variation of the horary motion in the courſe of a few hours, the time of the true conjunction found in this manner from the mean, when the interval is conſiderable, will be ſubject to a proportional degree of error. If the true longitude be computed from the Tables every time it is uſed, there will be no occaſion for this aſſumption of a nearer time; but by the aſſumption, we may either avoid a certain degree of error, or the trouble of computing the longitude again from the Tables. If at the time of the true conjunction, the moon's true latitude be computed, and the difference between it's latitude and that of the ſtar exceed 1°. 19′, there can (according to M. CASSINI) be no occultation; but if it be leſs than 1°. 7′ there muſt be one; hence between 1°. 19′ and 1°. 7′ it is doubtful. It being determined that there may be an occultation, we proceed thus to compute it.

593. Having found, at the time of the ecliptic conjunction, the moon's true longitude, it's horary motion in longitude, and it's true latitude, compute it's horary motion in latitude; alſo it's parallax in latitude and longitude, (as in ſolar eclipſes,) and it's ſemidiameter.

594. The parallax in longitude at the true time of the ecliptic conjunction ſhows the apparent diſtance of the moon from the ſtar in longitude; and the parallax in latitude applied to the true latitude gives the apparent latitude, the difference between which and the ſtar's latitude, gives their apparent difference of latitudes, and if this be leſs than the moon's ſemidiameter, there will probably be an occultation, in which caſe, we proceed thus to find the time of immerſion and emerſion.

595. To find nearly the time of the *apparent* conjunction, ſay, as the moon's horary motion : the parallax in longitude :: 1 hour : the time from the true to the apparent conjunction nearly,* which added to or ſubtracted from the time of the true conjunction, according as the moon is to the weſt or eaſt of the nonageſimal degree, gives the time nearly of the apparent conjunction. If at the time of the true conjunction, the difference of the apparent latitudes of the moon and ſtar ſhould be very nearly equal to the ſemidiameter of the moon, it will be neceſſary to compute the difference at the apparent conjunction, in order to be ſure whether, or not, there will be an occultation.

596. It being found that there will be an occultation, we muſt aſcertain, as nearly as poſſible, the beginning and end; for this purpoſe, the Table at the

* This is not accurately the time, becauſe the parallax in longitude has varied a little, and therefore it muſt have made a ſmall variation of the horary motion in longitude; but great accuracy here is not neceſſary, as we only want the apparent time of conjunction in order to aſſume nearly the time of immerſion and emerſion.

the end of this ſubject is very uſeful; it was conſtructed and computed by the Rev. MALACHY HITCHINS, a gentleman well converſant in the theory and practice of Aſtronomy, who had the goodneſs to communicate it to me, with permiſſion to publiſh it; it's conſtruction and uſe we ſhall here explain.

597. Let C be the center of the moon, LM that diameter which is parallel to the ecliptic EP, to which draw dCF perpendicular; let vw repreſent the path of the ſtar behind the moon,* z the place at the apparent ecliptic conjunction, i at the immerſion, and e at the emerſion; aſſume a point r a little before the immerſion, and draw rx parallel to LM, and ram, xtn, isp perpendicular to EP, or ſecondaries to the ecliptic, and join Ci, Cr, and Cx; draw alſo zo parallel to LM, and join Co. Now the conſtruction of the Table is to repreſent the value of ct correſponding to any ſemidiameter Cx of the moon, and to any difference xt of the apparent latitudes of the moon and ſtar, by entering, with the former, the head of the Table, and with the latter, it's ſide. FIG. 145.

598. To find nearly the time of immerſion and emerſion, with the moon's ſemidiameter, and the difference Cz of the apparent latitudes at the apparent conjunction, enter the Table, and it gives zo; and as vz makes but a ſmall angle with zo, zo is generally nearly equal to zi, and alſo to ze; take therefore the horary motion of the moon, and find the time of deſcribing zo, and ſubtract it from and add it to the time at z, and it generally gives the time of the beginning and end ſufficiently near for the purpoſe here wanted.†

599. By applying this Rule, let us ſuppoſe that it gives the beginning at r inſtead of i. The true longitude and latitude of the moon at the time of the true conjunction being known, and their horary motions, find it's true latitude and longitude at the aſſumed time of beginning, and to that time compute the parallaxes‖ (173) in latitude and longitude, and apply them to the true latitude and longitude and we get the apparent latitude and longitude, the differences between which and the apparent latitude and longitude of the ſtar, give the apparent diſtance of the moon from the ſtar in latitude and longitude, or they give ar and dm; alſo cos. Cd app. lat. $\mathfrak{C} \times dm = Ca$; hence $Cr = \sqrt{Ca^2 + ar^2}$ is known, and if this be *equal* to the moon's ſemidiameter, the aſſumed is the true

* We here repreſent the moon as being fixed, and the ſtar in motion, making the ſame angle with the ecliptic as the apparent path of the moon does, which makes no alteration in the relative ſituation of the two bodies, and the repreſentation is more ſimple.

† More accurately, zo ſhould be reduced to the ecliptic, by (108) multiplying it by the ſecant of the apparent latitude dz, before the time of deſcribing it is found, but this is unneceſſary. Alſo zi and ze are not accurately equal, and the nearer z is to F the greater will be the error. But accuracy here is not neceſſary; and, in general, the rule will be ſufficiently accurate, and always be a very uſeful guide for eſtimating the time of beginning and end.

‖ The horizontal parallax of the moon is not here diminiſhed (as it was in ſolar eclipſes by the ſun's parallax), the ſtar not having any parallax.

true time of beginning; if *greater*, the occultation has not taken place; if *leſs*, it has.

600. With the difference ra ($=tx$) of apparent latitudes, and the moon's ſemidiameter Cx, take out Ct from the Tables, and we get $ta = Ca \sim Ct$; hence ſec. app. lat. $dC \times ta = mn$; and from the horary motion find the time of deſcribing mn, which added to or ſubtracted from the above aſſumed time of beginning, will give very nearly the true time of beginning.

601. To this time find, as before, the apparent difference (d) of latitudes, and apparent difference (D) of longitudes at the moon, whence we get $\sqrt{D^2 + d^2} = m$, the apparent diſtance of the ſtar from the moon's center; if this diſtance be *equal* to the moon's ſemidiameter, this ſecond aſſumed time is the time of the immerſion; if *greater*, the occultation has not taken place; if *leſs*, the immerſion is paſt. If therefore this ſecond aſſumed time be not true, we proceed as in Art. 572, and ſay, $Cr \sim m : Cx \sim m ::$ the interval of the aſſumed times : the interval between the ſecond aſſumed time and the time of the immerſion; this interval therefore applied to the ſecond aſſumed time, gives the time of the *Immerſion*.

The time of the *Emerſion* is found exactly in the ſame manner.

EXAMPLE.

To find the Time of the Occultation of Aldebaran *by the Moon, on January* 2, 1795, *at Greenwich.*

The apparent longitude of *Aldebaran* on January 2, 1795 is found to be $2^s.\ 6^\circ.\ 55'.\ 35''$, and it's latitude $5^\circ.\ 28'.\ 50''$ ſouth.

The time of the *mean* conjunction is at $9^h.\ 9'.\ 53''$, at which time the difference of the moon's and ſtar's latitudes is $1^\circ.\ 1'$, conſequently (591) there may be an occultation. But from the equation of the moon's orbit, the difference of the times of the mean and true conjunction will probably happen five hours ſooner; let us therefore aſſume $4^h.$ at which time the moon's true longitude is found to be $2^s.\ 6^\circ.\ 47'.\ 45''$, and it's horary motion $35'.\ 39''$; hence the time of the true ecliptic conjunction of the moon and Aldebaran is found to be $2^d.\ 4^h.\ 13'.\ 21''$ mean time, from which ſubtract $4'.\ 41''$ the equation of time, and we get the true ecliptic conjunction at $4^h.\ 8'.\ 40''$ apparent time, at which time the moon's true longitude is $2^s.\ 6^\circ.\ 55'.\ 35''$.

At

At the time of the true conjunction, the moon's true latitude is found by calculation to be 4°. 36′ ſouth, and it's horary motion in latitude is 1′. 35″ decreaſing; alſo the moon's horizontal parallax at Greenwich is found to be 59′. 16″, and hence (173) the parallax in longitude is 29′. 17″ *additive* to the true to get the apparent longitude, the ſtar being to the eaſt of the nonageſimal degree; alſo the parallax in latitude is 48′. 4″, increaſing the true latitude. The horizontal ſemidiameter of the moon is 16′. 11″, which increaſed by 5″, the augmentation of the ſemidiameter on account of the moon's altitude, gives 16′. 16″ for the apparent ſemidiameter.

The parallax 29′. 17″ in longitude (at the time of the true conjunction) ſhows (594) the moon's apparent diſtance from the ſtar in longitude; and the parallax 48′. 4″ in latitude applied to the true latitude 4°. 36′ gives 5°. 24′. 4″ for the moon's apparent latitude, which differs from 5°. 28′. 50″, the ſtar's latitude, by 4′. 46″, which being leſs than 16′. 16″ the moon's apparent ſemidiameter, there will be an occultation.

To find (595) nearly the time of the apparent conjunction, ſay, 35′. 19″ : 29′. 17″ :: 1 hour : 49′. 18″ the time nearly between the true and the apparent conjunction; and as the moon is to the *eaſt* of the nonageſimal degree, this ſubtracted from the time 4*h*. 8′. 40″ of the true conjunction leaves 3*h*. 19′. 22″ for the time of the apparent conjunction nearly.

With the moon's ſemidiameter 16′. 16″, and the difference 4′. 46″ of the ſtar's and moon's apparent latitudes, enter the Table, and it gives $zo = 15'.32''$; hence 35′. 39″ : 15′. 32″ :: 1 hour : 26′. 9″, which ſubtracted from and added to 3*h*. 19′. 22″, give 2*h*. 53′. 13″ for the beginning, and 3*h*. 45′. 31″ for the end, nearly.

The true longitude of the moon at 4*h*. 8′. 40″ being 2ˢ. 6°. 55′. 35″, and the horary motion 35′. 39″, the true longitude at 2*h*. 53′. 13″ is 2ˢ. 6°. 10′. 44″, and the parallax in longitude is 28′. 43″; hence the apparent longitude at that time is 2ˢ. 6°. 39′. 27″, which ſubtracted from 2ˢ. 6°. 55′. 35″ the ſtar's longitude, gives 16′. 8″ = *dm* for the apparent difference of longitudes of the moon and ſtar, which multiplied by 0,9955 (the coſine of the moon's apparent latitude) gives 16′. 4″ = 964″ = *Ca*. Alſo the moon's true latitude at 4*h*. 8′. 40″ is 4°. 36′, and the horary motion in latitude being 1′. 35″ decreaſing, the true latitude at 2*h*. 53′. 13″ is 4°. 37′. 55″, and the parallax in latitude being 51′. 27″, the apparent latitude is 5°. 29′. 22″, which differs from 5°. 28′. 50″, the ſtar's latitude, by 32″ = *ra*; hence $\sqrt{964^2 + 32^2} = 965'' = 16'.5''$ = *cr*, which is leſs than 16′. 16″ the moon's apparent ſemidiameter by 11″; hence the occultation at this time muſt have taken place.

With 32″ the difference of the apparent latitudes, and 16′. 16″ the moon's ſemidiameter, enter the Table, and we get 16′. 15″, the difference between

 which

which and Ca ($= 16'.\ 4''$) is $ta = 11''$,* the time of deſcribing which is $22''$, which ſubtracted from $2h.\ 53'.\ 13''$ gives $2h.\ 52'.\ 51''$ for the next aſſumed time of immerſion.

At this time we find (exactly as we found the ſame for the firſt aſſumed time) the difference Ca of the moon's apparent longitude and that of the ſtar, at the moon, to be $16'.\ 18'' = 978''$; and the difference ra of the apparent latitudes to be $33''$; hence $\sqrt{978^2 + 33^2} = 979'' = 16'.\ 19'' = Cr$, which is greater than $16'.\ 16''$ by $3''$, conſequently the occultation has not taken place.

Hence (601) $11'' + 3'' = 14'' : 3'' :: 22'' : 5''$, which added to $2h.\ 52'.\ 51''$ gives $2h.\ 52'.\ 56''$ for the time of *Immerſion*.

At $3h.\ 45'.\ 31''$, the aſſumed time of the emerſion, compute as before the apparent longitude and latitude of the moon, and we find $Ca = 15'.\ 44'' = 944''$ (a now lying on the other ſide of C), and $ar = 2'.\ 59'' = 179''$; hence $\sqrt{944^2 + 179^2} = 961'' = 16'.\ 1''$, which is leſs than $16'.\ 16''$ by $15''$, conſequently the emerſion is not yet arrived.

With $179''$ the apparent difference of latitudes, and $16'.\ 16''$ the moon's ſemidiameter, enter the Table, and we get $16'$, the difference between which and $Ca = 15'.\ 44''$ is $16''$, the time of deſcribing which is $32''$, which added to $3h.\ 45'.\ 31''$ gives $3h.\ 46'.\ 3''$ for the next aſſumed time of emerſion.

At this time, compute, as before, the apparent longitude and latitude of the moon, and we find $Ca = 16'.\ 3'' = 963''$, and $ar = 2'.\ 59'' = 179''$; hence $\sqrt{963^2 + 179^2} = 977'' = 16'.\ 17''$, which is greater than $16'.\ 16''$ by $1''$, conſequently the emerſion has taken place.

Hence (601) $15'' + 1'' = 16'' : 1'' :: 32'' : 2''$, which ſubtracted from $3h.\ 46'.\ 3''$ leaves $3h.\ 46'.\ 1''$ for the time of *Emerſion*.

Hence the apparent times at Greenwich, are,

Immerſion	$2^h.$	$52'.$	$56''$
Emerſion	$3.$	$46.$	1
Duration	$0.$	53	5

The

* This quantity is ſo ſmall, that if we were to reduce it to the ecliptic before we found the time of deſcribing it, it would make no ſenſible difference; the reduction is therefore here unneceſſary, as it is alſo in finding nearly the time of the emerſion.

The times thus calculated muſt be ſubject to the error of the Tables, as in ſolar eclipſes, but in ſomewhat a leſs degree, as the horary motion of the moon in reſpect to the ſtar is greater than that in reſpect to the ſun. Hence the computed times compared with the times by obſervation, afford the means of correcting the Tables.

The immerſion at *i* is about 30″ north of the moon's center, and the emerſion at *e*, is about 3′ north.

To determine, by Conſtruction, *the Time of an Occultation of a Fixed Star by the Moon.*

602. The moon's latitude and longitude being computed for the true time of conjunction, and the horary motion of the moon in latitude and longitude, find the latitude and longitude for one hour before or after, according as the occultation happens before or after. Take alſo the ſtar's latitude and declination, and find (105) the time of paſſing the meridian; find alſo the moon's ſemidiameter and horizontal parallax. Now it is manifeſt, that the ſtar may be uſed as the ſun, only inſtead of 12 upon the ellipſe we muſt put the hour of the ſtar's paſſage over the meridian; and as the ſtar has no paralax, the radius of projection will be equal to the moon's horizontal parallax. Hence with that radius deſcribe the ſemicircle *EGC*, erect *GO* perpendicular to *EC*, find (536) the poſition of the pole *P*, and deſcribe the ellipſe for the latitude of the place, and declination of the ſtar; and to find the moon's orbit, take *Ov* equal to the difference of the moon and ſtar's latitude at the time of the true conjunction, and take alſo *Ox* equal to the moon's horary motion in longitude, and draw *xy* perpendicular to *EC* and equal to the difference of the moon's and ſtar's latitude at one hour from conjunction, and the ſtraight line *MyvL* will repreſent the moon's orbit. Then with an extent of compaſs equal to the moon's ſemidiameter, find two points *c* and *d* marked the ſame inſtant, and it gives the time of immerſion; find alſo two other points *s* and *t* denoting the ſame point of time, and you have the time of emerſion; alſo if the neareſt diſtance *wc* of the correſponding points of time be taken and meaſured upon the ſcale, it will give the neareſt diſtance of the ſtar to the moon's center in the time of occultation. FIG. 146

Ex. To conſtruct the occultation which we before computed. The time of conjunction was at 4*h*. 8′. 40″ in the morning, in longitude 2ˢ. 6°. 55′. 35″; the moon's latitude was 4°. 36′ S. it's horary motion in longitude 35′. 39″, and in latitude 1′. 35″ decreaſing; it's ſemidiameter was 16′. 16″, and hori-

 zontal

zontal parallax 59′. 16″; alſo *Aldebaran's* latitude was 5°. 28′. 50″ ſouth, it's declination 16°. 5′. 6″, and it paſſed the meridian at 9*h*. 29′. 24″ in the afternoon. Hence take $OC = 59°. 16'$, find (536) the pole P at the given time, and deſcribe the ellipſe for the latitude of Greenwich and the ſtar's declination, and at the point n mark 9*h*. 29′ (neglecting the 24″), being the time when the ſtar comes to the meridian, and from it divide the ellipſe into hours, and ſubdivide them as far as you can conveniently. Take $Ov = 52'. 50''$ the difference of the ſtar's and moon's latitude at the time of conjunction, and $Ox = 35'. 39''$ the moon's horary motion in longitude, and erect the perpendicular $xy = 51'. 15''$ (the moon's latitude decreaſing 1′. 35″ in an hour), draw the right line $LvyM$, and it will repreſent the moon's orbit, and vy it's horary motion in it; at the point v mark 4*h*. 9′, and vy repreſenting one hour, divide LM into hours from it, and ſubdivide it as far as the ſcale will permit; and taking an extent of compaſs equal to 16′. 16″ the moon's ſemidiameter, it will give the points c, d correſponding to the time of *Immerſion*, and the points s, t correſponding to the time of *Emerſion*; and the correſponding neareſt diſtance is found to be nearly 2′ north of the moon's center.

603. When an occultation of a ſtar by the moon takes place, for three or four ſeconds of time before the ſtar diſappears, it ſometimes appears to be projected upon the diſc of the moon; M. du SEJOUR explains the phænomenon thus. Let S be the ſtar, bmn the moon, abc the paſſage of a ray of light through the moon's atmoſphere, and juſt paſſing by the limb of the moon at b; let cE be the direction of the ray after it emerges from the atmoſphere, and produce Ec to s. Then to an eye at E the ſtar would appear at s; but at the ſame time a ray of light from the moon's limb at b would be refracted through bc and then move to E and appear alſo at s; thus when the ray of light which comes from the ſtar becomes a tangent to the moon, the ſtar at that time *appears* alſo to be in contact with the moon. The refraction of the atmoſphere alone therefore is not ſufficient to account for this phænomenon, as ſome Aſtronomers have ſuppoſed. But if the light from the ſtar ſuffer a different degree of refraction from the ſolar light refracted from d, for inſtance, if the ſtar be higher than the center of the moon, and the refraction of the light from the moon be greater than the refraction of the light from the ſtar, the point b being elevated by refraction more than the ſtar, the ſtar will appear upon the moon's diſc before the occultation takes place. Or the ſame would happen if the ſtar were lower than the moon's center, and the refraction of the light from the ſtar the greater. From the different colours of the light from different ſtars, he thinks we may admit different degrees of refraction of their light. The irradiation of the light of the ſtars, by which ſome have

FIG. 147.

conjectured they might appear to encroach a little upon the moon's limb before they diſappear, would, he obſerves, affect all ſtars and at all altitudes; whereas this circumſtance does not always take place. He ſtates however this objection to his hypotheſis, that the aberration of light from all the ſtars appears, from obſervation, to be the ſame, and therefore the velocities of their light muſt be all equal; conſequently the light from all the ſtars ſuffers the ſame refraction, admitting that the refraction depends altogether on the velocity of the light. But there have been only a few ſtars whoſe aberrations have been determined by obſervation, and we are not aſſured but, in ſome ſtars, a very ſmall difference in their aberrations might be found; future Aſtronomers may ſettle this.

604. Lunar eclipſes are uſeful for finding the longitudes of places; ſolar eclipſes, and occultations are uſeful for the ſame purpoſe, and alſo for correcting the errors on the lunar Tables. All theſe things will be explained when we treat on the methods of finding the longitude.

A TABLE

A TABLE

Showing the visible Difference of Longitudes between the Moon and Star, at the Star, at the instant of the Star's Immersion or Emersion in Occultations.

Vis. diff. of lat. of ☾ and ✶	THE SEMIDIAMETERS OF THE MOON.													
M. S.	M. S.	M. S.	M. S.	M. S.	M. S.	M. S.	M. S.	M. S.	M. S.	M. S.	M. S.	M. S.	M. S.	M. S.
0. 0	14. 40	14. 50	15. 0	15. 10	15. 20	15. 30	15. 40	15. 50	16. 0	16. 10	16. 20	16. 30	16. 40
1. 0	14. 38	14. 48	14. 58	15. 8	15. 18	15. 28	15. 38	15. 48	15. 58	16. 8	16. 18	16. 28	16. 38
2. 0	14. 32	14. 42	14. 52	15. 2	15. 12	15. 22	15. 32	15. 42	15. 52	16. 3	16. 13	16 23	16. 33
3. 0	14. 22	14. 32	14. 42	14. 52	15. 2	15. 12	15. 22	15. 32	15. 43	15. 54	16. 4	16. 14	16. 24
4. 0	14. 7	14. 17	14. 27	14. 38	14. 48	14. 58	15. 9	15. 19	15. 29	15. 40	15. 50	16. 1	16. 11
4. 30	13. 58	14. 8	14. 18	14. 29	14. 39	14. 50	15. 0	15. 10	15. 21	15. 31	15. 42	15. 52	16. 3
5. 0	13. 47	13. 58	14. 9	14. 19	14. 30	14. 40	14. 51	15. 1	15. 12	15. 22	15. 33	15. 43	15. 54
5. 20	13. 40	13. 51	14. 2	14. 12	14. 23	14. 34	14. 44	14. 55	15. 5	15. 16	15. 26	15. 37	15. 47
5. 40	13. 32	13. 43	13. 54	14. 4	14. 15	14. 26	14. 36	14. 47	14. 58	15. 9	15. 19	15. 30	15. 40
6. 0	13. 23	13. 34	13. 45	13. 56	14. 7	14. 18	14. 29	14. 39	14. 50	15. 1	15. 12	15. 23	15. 33
6. 20	13. 14	13. 25	13. 36	13. 47	13. 58	14. 9	14. 20	14. 31	14. 42	14. 52	15. 3	15. 14	15. 25
6. 40	13. 4	13. 15	13. 26	13. 37	13. 48	14. 0	14. 11	14. 22	14. 33	14. 44	14. 55	15. 6	15. 17
7. 0	12. 54	13. 5	13. 16	13. 27	13. 38	13. 50	14. 1	14. 12	14. 23	14. 34	14. 45	14. 56	15. 7
7. 20	12. 42	12. 54	13. 5	13. 16	13. 27	13. 39	13. 50	14. 2	14. 13	14. 24	14. 35	14. 46	14. 57
7. 40	12. 30	12. 42	12. 54	13. 5	13. 16	13. 28	13. 39	13. 51	14. 2	14. 14	14. 25	14. 36	14. 47
8. 0	12. 18	12. 30	12. 42	12. 54	13. 5	13. 17	13. 28	13. 40	13. 51	14. 3	14. 14	14. 26	14. 37
8. 15	12. 8	12. 20	12. 32	12. 44	12. 55	13. 7	13. 19	13. 31	13. 42	13. 54	14. 5	14. 17	14. 29
8. 30	11. 57	12. 9	12. 21	12. 34	12. 45	12. 57	13. 9	13. 21	13. 33	13. 45	13. 56	14. 8	14. 20
8. 45	11. 46	11. 58	12. 11	12. 23	12. 35	12. 47	13. 0	13. 12	13. 24	13. 36	13. 47	13. 59	14. 11
9. 0	11. 35	11. 47	12. 0	12. 12	12. 25	12. 37	12. 49	13. 2	13. 14	13. 26	13. 38	13. 50	14. 2
9. 15	11. 23	11. 35	11. 48	12. 1	12. 14	12. 26	12. 39	12. 51	13. 3	13. 15	13. 27	13. 40	13. 52
9. 30	11. 10	11. 23	11. 36	11. 49	12. 2	12. 15	12. 28	12. 40	12. 52	13. 4	13. 16	13. 29	13. 41
9. 45	10. 57	11. 11	11. 24	11. 37	11. 50	12. 3	12. 16	12. 29	12. 41	12. 53	13. 5	13. 18	13. 31
10. 0	10. 44	10. 57	11. 11	11. 24	11. 38	11. 51	12. 4	12. 17	12. 29	12. 42	12. 54	13. 7	13 20
10. 15	10. 29	10. 43	10. 57	11. 11	11. 25	11. 38	11. 51	12. 4	12. 17	12. 30	12. 43	12. 56	13. 9
10. 30	10. 14	10. 29	10. 43	10. 57	11. 11	11. 24	11. 38	11. 51	12. 4	12. 18	12. 31	12. 44	12. 57
10. 45	9. 59	10. 13	10. 28	10. 42	10. 56	11. 10	11. 24	11. 37	11. 51	12. 5	12. 18	12. 31	12. 44
11. 0	9. 42	9. 57	10. 12	10. 27	10. 41	10. 55	11. 9	11. 23	11. 37	11. 51	12. 4	12. 18	12. 31

THE TABLE CONTINUED.

Vif. diff. of lat. of ☾ and ✳	*THE SEMIDIAMETERS OF THE MOON.*													
M. S.	M. S.	M. S.	M. S.	M. S.	M. S.	M. S.	M. S.	M. S.	M. S.	M. S.	M. S.	M. S.	M. S.	M. S.
0. 0	14. 40	14. 50	15. 0	15. 10	15. 20	15. 30	15. 40	15. 50	16. 0	16. 10	16. 20	16. 30	16. 40	
11. 15	9. 25	9. 40	9. 55	10. 11	10. 25	10. 40	10. 54	11. 8	11. 22	11. 37	11. 50	12. 4	12. 18	
11. 30	9. 6	9. 22	9. 38	9. 54	10. 9	10. 24	10. 39	10. 53	11. 7	11. 22	11. 36	11. 50	12. 4	
11. 45	8. 46	9. 3	9. 19	9. 36	9. 51	10. 7	10. 22	10. 37	10. 51	11. 6	11. 21	11. 35	11. 49	
12. 0	8. 26	8. 43	9. 0	9. 17	9. 33	9. 49	10. 5	10. 20	10. 35	10. 50	11. 5	11. 19	11. 33	
12. 15	8. 4	8. 22	8. 39	8. 57	9. 13	9. 30	9. 46	10. 2	10. 17	10. 33	10. 48	11. 3	11. 17	
12. 30	7. 40	7. 59	8. 17	8. 35	8. 53	9. 10	9. 27	9. 43	9. 59	10. 15	10. 31	10. 46	11. 1	
12. 45	7. 14	7. 35	7. 54	8. 13	8. 31	8. 49	9. 7	9. 23	9. 40	9. 57	10. 13	10. 29	10. 44	
13. 0	6. 47	7. 9	7. 29	7. 49	8. 8	8. 27	8. 45	9. 2	9. 20	9. 37	9. 54	10. 10	10. 26	
13. 15	6. 17	6. 40	7. 2	7. 23	7. 43	8. 3	8. 22	8. 40	8. 58	9. 16	9. 33	9. 50	10. 6	
13. 30	5. 44	6. 9	6. 32	6. 55	7. 16	7. 37	7. 57	8. 16	8. 35	8. 53	9. 11	9. 29	9. 46	
13. 45	5. 6	5. 34	6. 0	6. 24	6. 47	7. 9	7. 30	7. 51	8. 11	8. 30	8. 49	9. 7	9. 25	
14. 0	4. 22	4. 54	5. 23	5. 49	6. 15	6. 39	7. 3	7. 25	7. 45	8. 5	8. 25	8. 44	9. 3	
14. 10	3. 48	4. 24	4. 56	5. 25	5. 52	6. 17	6. 41	7. 4	7. 26	7. 47	8. 8	8. 28	8. 47	
14. 20	3. 7	3. 49	4. 25	4. 57	5. 27	5. 54	6. 19	6. 44	7. 7	7. 29	7. 50	8. 10	8. 30	
14. 30	2. 12	3. 8	3. 50	4. 27	4. 59	5. 29	5. 56	6. 22	6. 46	7. 9	7. 31	7. 52	8. 13	
14. 40	0. 0	2. 13	3. 9	3. 52	4. 28	5. 1	5. 30	5. 58	6. 24	6. 48	7. 11	7. 34	7. 55	
14. 50		0. 0	2. 14	3. 10	3. 53	4. 30	5. 2	5. 32	6. 0	6. 26	6. 50	7. 14	7. 36	
15. 0			0. 0	2. 15	3. 11	3. 54	4. 31	5. 4	5. 34	6. 2	6. 28	6. 52	7. 16	
15. 10				0. 0	2. 15	3. 12	3. 56	4. 33	5. 6	5. 36	6. 4	6. 30	6. 55	
15. 20					0. 0	2. 16	3. 13	3. 57	4. 34	5. 7	5. 38	6. 6	6. 32	
15. 30						0. 0	2. 17	3. 14	3. 58	4. 36	5. 9	5. 39	6. 8	
15. 40							0. 0	2. 17	3. 15	3. 59	4. 37	5. 11	5. 41	
15. 50								0. 0	2. 18	3. 16	4. 1	4. 39	5. 12	
16. 0									0. 0	2. 19	3. 17	4. 2	4. 40	
16. 10										0. 0	2. 20	3. 18	4. 3	
16. 20											0. 0	2. 20	3 19	
16. 30												0. 0	2. 21	
16. 40													0. 0	

605. Sometimes the planets are eclipſed by the moon, the calculations of which are made in the ſame manner as for the ſun, or a fixed ſtar; conſidering the relative horary motion of the moon in reſpect to the planet, in latitude and longitude, in order to get the relative orbit.

606. The planets ſometimes eclipſe the planets. *Mars* eclipſed *Jupiter*, January 9, 1501; *Venus* eclipſed *Mars*, October 3, 1590; *Mercury* was eclipſed by *Venus*, May 17, 1737.

607. The fixed ſtars are ſometimes eclipſed by the planets. GASSENDUS obſerved *Jupiter* eclipſe a fixed ſtar in the foot of *Gemini*, December 19, 1633. Mr. POUND obſerved *Jupiter* eclipſe *a* of *Gemini*, November 21, 1716, the middle of the eclipſe was at 19*h*. 55′. *Phil. Tranſ.* No. 350. In 1672, *Mars* eclipſed one of the ſtars in *Aquarius*. *Venus* eclipſed the *Lion's Heart*, in 1574, and 1598. The fixed ſtars are alſo obſerved to be ſometimes eclipſed by *Comets*; which are very uſeful obſervations, as they ſerve to aſcertain very accurately the place of the comet.

CHAP.

CHAP. XXV.

ON THE TRANSITS OF MERCURY AND VENUS OVER THE SUN'S DISC.

Art. 608. WHEN Dr. HALLEY was at St. Helena, whither he went for the purpoſe of making a catalogue of the ſtars in the ſouthern hemiſphere, he obſerved a tranſit of *Mercury* over the ſun's diſc; and, by means of a good teleſcope, it appeared to him that he could determine the time of the ingreſs and egreſs, without it's being ſubject to an error of 1″;* upon which he immediately concluded, that the ſun's parallax might be determined by ſuch obſervations, from the difference of the times of the tranſit over the ſun, at different places upon the earth's ſurface. But this difference is ſo ſmall in Mercury, that it would render the concluſion ſubject to a great degree of inaccuracy; in Venus however, whoſe parallax is nearly four times as great as that of the ſun, there will be a very conſiderable difference between the times of the tranſits ſeen from different parts of the earth, by which the accuracy of the concluſion will be proportionably increaſed. The DR. therefore propoſed to determine the ſun's parallax from the tranſit of Venus over the ſun's diſc, obſerved at different places on the earth; and as it was not probable that he himſelf ſhould live to obſerve the next tranſits, which happened in 1761 and 1769, he very earneſtly recommended the attention of them to the Aſtronomers who ſhould be alive at that time. Aſtronomers were therefore ſent from England and France to the moſt proper parts of the earth to obſerve both thoſe tranſits, from the reſult of which, the parallax has been determined to a very great degree of accuracy.

609. KEPLER was the firſt perſon who predicted the tranſits of Venus and Mercury over the ſun's diſc; he foretold the tranſit of Mercury in 1631, and the tranſits of Venus in 1631 and 1761. The firſt time *Venus* was ever ſeen upon

* Hence Dr. HALLEY concluded, that by a tranſit of *Venus*, the ſun's diſtance might be determined with certainty to the 500th part of the whole; but the obſervations upon the tranſits which happened in 1761 and 1769, ſhowed that the time of contact of the limbs of the Sun and Venus could not be determined to that degree of certainty.

upon the ſun was in the year 1639, on November 24, at Hoole near Liverpool, by our countryman Mr. HORROX, who was educated at Emanuel College in this Univerſity. He was employed in calculating an Ephemeris from the *Lanſberge* Tables, which gave, at the conjunction of Venus with the ſun on that day, it's apparent latitude leſs than the ſemidiameter of the ſun. But as theſe Tables had ſo often deceived him, he conſulted the Tables conſtructed by KEPLER, according to which, the conjunction would be at $8h.$ 1′ A. M. at Mancheſter, and the planet's latitude 14′. 10″ ſouth; but, from his own corrections, he expected it to happen at $3h.$ 57′ P. M. with 10′ ſouth latitude. He accordingly gave this information to his friend Mr. CRABTREE at Mancheſter, deſiring him to obſerve it; and he himſelf alſo prepared to make obſervations upon it, by tranſmitting the ſun's image through a teleſcope into a dark chamber. He deſcribed a circle of about ſix inches diameter, and divided the circumference into 360°, and the diameter into 120 equal parts, and cauſed the ſun's image to fill up the circle. He began to obſerve on the 23d, and repeated his obſervations on the 24th till one o'clock, when he was unfortunately called away by buſineſs; but returning at 15′ after three o'clock, he had the ſatisfaction of ſeeing Venus upon the ſun's diſc, juſt wholly entered on the left ſide, ſo that the limbs perfectly coincided. At 35′ after three, he found the diſtance of Venus from the ſun's center to be 13′. 30″; and at 45′ after three, he found it to be 13′; and the ſun ſetting at 50′ after 3 o'clock, put an end to his obſervations. From theſe obſervations, Mr. HORROX endeavoured to correct ſome of the elements of the orbit of Venus. He found Venus had entered upon the diſc at about 62°. 30′ from the vertex towards the right on the image, which, by the teleſcope, was inverted. He meaſured the diameter of Venus, and found it to be to that of the ſun, as 1,12 : 30, as near as he could meaſure. Mr. CRABTREE, on account of the clouds, got only one ſight of Venus, which was at $3h.$ 45′. Mr. HORROX* wrote a Treatiſe, entitled *Venus in Sole viſa*, but did not live to publiſh it; it was however afterwards publiſhed by HEVELIUS. GASSENDUS obſerved the tranſit of *Mercury* which happened on November 7, 1631, and this was the firſt which had ever been obſerved; he made his obſervations in the ſame manner that HORROX did after him. Since his time, ſeveral tranſits of Mercury have been obſerved, as they frequently happen; whereas only two tranſits of Venus have happened ſince the time of HORROX. If we know the time of the tranſit at one node, we can determine, in the following manner, when they will probably happen again at the ſame node.

610. The

* The difficulties which this very extraordinary perſon had to encounter with in his Aſtronomical purſuits, he himſelf has deſcribed, in a *Prolegomena* prefixed to his *Opera Poſthuma*, publiſhed by Dr. WALLIS.

610. The mean time from conjunction to conjunction of Venus or Mercury being known (324), and the time of one mean conjunction, we ſhall know the time of all the future mean conjunctions; obſerve therefore thoſe which happen near to the node, and compute the geocentric latitude of the planet at the time of conjunction, in which caſe, if it be leſs than the apparent ſemidiameter of the ſun, there will be a tranſit of the planet over the ſun's diſc; and we may determine the periods when ſuch conjunctions happen, in the following manner. Let $P =$ the periodic time of the earth, p that of Venus or Mercury. Now that a tranſit may happen again at the ſame node, the earth muſt perform a certain number of complete revolutions in the ſame time that the planet performs a certain number, for then they muſt come into conjunction again at the ſame point of the earth's orbit, or nearly in the ſame poſition in reſpect to the node. Let the earth perform x revolutions whilſt the planet performs y revolutions; then will $Px = py$, therefore $\frac{x}{y} = \frac{p}{P}$. Now $P = 365{,}256$, and for *Mercury*, $p = 87{,}968$; therefore $\frac{x}{y} = \frac{p}{P} = \frac{87{,}968}{365{,}256} =$ (by reſolving it into it's continual fractions) $\frac{1}{4}, \frac{6}{25}, \frac{7}{29}, \frac{13}{54}, \frac{33}{137}, \frac{46}{191}$, &c. That is, 1, 6, 7, 13, 33, 46, &c. revolutions of the earth are nearly equal to 4, 25, 29, 54, 137, 191, &c. revolutions of Mercury, approaching nearer to a ſtate of equality, the further you go. The firſt period, or that of one year, is not ſufficiently exact; the period of ſix years will ſometimes bring on a return of the tranſit at the *ſame* node; that of ſeven years more frequently; that of 13 years ſtill more frequently, and ſo on. Now there was a tranſit of Mercury at it's deſcending node, in May 1786; hence by continually adding 6, 7, 13, 33, 46, &c. to it, you get all the years when the tranſit may be expected to happen at that node. In 1789 there was a tranſit at the aſcending node, and therefore by adding the ſame numbers to that year you will get the years in which the tranſits may be expected to happen at that node. The next tranſits at the deſcending node will happen in 1799, 1832, 1845, 1878, 1891; and at the aſcending node, in 1802, 1815, 1822, 1835, 1848, 1861, 1868, 1881, 1894. For *Venus*, $p = 224{,}7$; hence $\frac{x}{y} = \frac{p}{P} = \frac{224{,}7}{365{,}256} = \frac{8}{13}, \frac{235}{382}, \frac{713}{1159}$, &c. Therefore the periods are 8, 235, 713, &c. years. The tranſits at the ſame node will therefore ſometimes return at 8 years, but oftener in 235, and ſtill oftener in 713, &c. Now in 1769 a tranſit happened at the deſcending node in June, and the next tranſits at the ſame node will be in 2004, 2012, 2247, 2255, 2490, 2498, 2733, 2741 and 2984. In 1639 a tranſit happened at the aſcending node in November, and the next tranſits at the ſame node

will be in 1874, 1882, 2117, 2125, 2360, 2368, 2603, 2611, 2846 and 2854. Theſe tranſits are found to happen, by continually adding the periods, and finding the years when they may be expected, and then computing, for each time, the ſhorteſt geocentric diſtance of Venus from the ſun's center at the time of conjunction, and if it be leſs than the ſemidiameter of the ſun, there will be a tranſit.

To compute the Time of the Tranſit of Venus *or* Mercury *over the Sun's diſc, and the Duration thereof, to a Spectator at the center of the Earth.*

FIG. 148. 611. Let A and P be two planets in conjunction, Pa, $A\mathfrak{Q}$, their cotemporary motions parallel to the ecliptic, and ab, $\mathfrak{Q}R$ perpendicular to it, then Pb, AR will be their real motions; take $Ar = Pa$, and $rs = ab$ and perpendicular to $A\mathfrak{Q}$; draw st parallel to $A\mathfrak{Q}$, and join sR, then sR will be the relative motion of A ſeen from P. For their relative motions in any directions muſt always be the ſum or difference of their real motions in thoſe directions, according as they move in different or in the ſame directions. In this figure, we have ſuppoſed them to move in the ſame direction, and Rt is the difference of their real motions in latitude, and ts in longitude; and by Trigonometry, $ts : Rt ::$ rad. : tan. Rst, the inclination of the relative orbit deſcribed by A; alſo, cos. Rst : rad. :: $st : sR$ the cotemporary motion in the relative orbit. If we apply this to the ſun and Venus, and $A\mathfrak{Q}$ repreſent the horary motion of Venus in longitude, Ar that of the ſun, and $\mathfrak{Q}R$ the horary motion of Venus in latitude; then the ſun having no motion in latitude, rR is the relative motion of Venus in reſpect to the ſun; hence $\mathfrak{Q}r : \mathfrak{Q}R ::$ rad. : tan. $Rr\mathfrak{Q}$, and cos. $Rr\mathfrak{Q}$: rad. :: $r\mathfrak{Q} : Rr$.

Ex. On July 3, 1769, at mean noon at Greenwich, the longitude of the earth was $8^s.\ 13^\circ.\ 3'.\ 14'',8$, and that of Venus $8^s.\ 12^\circ.\ 47'.\ 53'',7$, the difference of which is $15'.\ 21''$; the horary motion of the ſun (by the Tables) was $143'',46$; the diſtance of Venus from the ſun was 0,72626; hence (234) the horary motion of Venus in longitude was $237'',96$; therefore the difference $94'',5$ is the relative horary motion of Venus in reſpect to the earth in longitude; hence $94'',5 : 15'.\ 21'' ::$ 1 hour : $9^h.\ 44'.\ 45''$; therefore the conjunction was on June 3, at $9^h.\ 44'.\ 45''$ mean time, at which time the longitude of the earth was $8^s.\ 13^\circ.\ 27'.\ 16''$. The heliocentric latitude of Venus was alſo found (by the Tables) to be $6'.\ 27''$ north decreaſing, and it's horary motion in latitude $14'',06$; hence, 1 hour : $9^h.\ 44'.\ 45'' :: 14'',06 : 2'.\ 17''$, which ſubtracted from

from 6′. 27″ leaves 4′. 10″ the heliocentric latitude of Venus at the time of the ecliptic conjunction. The diſtance of the earth from the ſun was 1,01521; therefore the diſtance of Venus from the earth was 0,28895, the mean diſtance of the earth from the ſun being unity; hence 0,28895 : 0,72626 :: 4′. 10″ : 10′. 8″ the geocentric latitude of Venus at the ecliptic conjunction, which being leſs than the ſemidiameter of the ſun, there muſt be a tranſit of Venus over the ſun at the center of the earth, and conſequently ſomewhere upon the ſurface; we have therefore, in this caſe, no occaſion to compute the ſhorteſt diſtance of Venus from the center of the ſun, in order to determine whether there will be a tranſit. Alſo, 0,28895 : 0,72626 :: 94″,5 : 3′. 57″,52 the geocentric horary motion of Venus from the ſun in longitude; and 0,28895 : 0,72626 :: 14″,06 : 35″,42 the geocentric horary motion of Venus in latitude.

Now let the circle *DOK* repreſent the ſun, *C* it's center, *DCN* the ecliptic; and on *CO* (perpendicular to *DCN*) take *CV* = 10′. 8″, and *V* is the apparent place of Venus in conjunction; and let *SVN* repreſent the orbit of Venus as ſeen from the earth. Now the geocentric horary motion of Venus from the ſun in longitude is 3′. 57″,52, and the geocentric horary motion of Venus in latitude is 35″,42; hence (611), 3′. 57″,52 : 35″,42 :: rad. : tan. of 8°. 28′. 54″ the inclination of the relative orbit to the ecliptic; draw therefore *SVN* making the angle *CVN* = 81°. 31′. 6″, and *SVN* will be the apparent path of Venus ſeen from the earth. Draw *CM* perpendicular to *SE* and it will biſect it, therefore *M* is the middle of the tranſit. Now as *CV* = 10′. 28″, and the angle *VCM* = 8°. 28′. 54″, we have, rad. : ſin. 8°. 28′. 54″ :: 10′. 28″ : *VM* = 1′. 33″. FIG. 149.

The horary motion of Venus in it's relative orbit is found (611) by ſaying, coſine of inclination 8°. 28′. 54″ : rad. :: 3′. 57″,52 (the difference of the horary motions of Venus and the ſun ſeen from the earth in longitude) : 4′. 0″,15 the horary motion in it's relative orbit *SN*. Hence 4′. 0″,15 : 1′. 33″ :: 1 hour : 23′. 14″ the time of deſcribing *VN*, which added to 9*h*. 44′. 45″, gives 10*h*. 7′. 59″ for the middle of the tranſit.

In the triangle *VCM*, rad. : cos. *VCM* = 8°. 28′. 54″ :: *CV* = 10′. 28″ : *CM* = 10′. 21″ the neareſt diſtance of Venus from the ſun's center; hence in the triangle *SCM*, *CM* = 10′. 21″, and *SC* = 15′. 46″, therefore *SM* = 11′. 53″,6, to find the time of deſcribing which, ſay, 4′. 0″,15 : 1 hour :: *SM* = 11′. 53″,6 : 2*h*. 58′. 24″ the time of deſcribing *SM*, which ſubtracted from 10*h*. 7′. 59″, the time when Venus was at *M*, gives 7*h*. 9′. 35″ for the *beginning*, and added, gives 13*h*. 6′. 23″ for the *End*, mean time, according to the Tables. The effect of the menſtrual parallax has not been here conſidered, as it is in the following calculations, it being ſo extremely ſmall, compared with the errors to which the Tables are ſubject.

In

In the tranſit of *Mercury*, the variation of it's diſtance may be ſo great between the times of the ingreſs and egreſs, as ſenſibly to affect it's geocentric motion, and thereby render it neceſſary to be taken into computation. M. de l'Isle, in calculating the tranſit of Mercury, on November 7, 1756, found that the true middle of the paſſage was altered 11″,5 by this circumſtance, ſo that *DB* was performed in 23″ leſs time than *AD*.

A New Method of computing the Effect of Parallax, in accelerating or retarding the Time of the Beginning or End of a Tranſit of Venus, *or* Mercury *over the Sun's diſc. By* Nevil Maskelyne, *D.D. F.R.S. and* Aſtronomer Royal.

612. The ſcheme here given relates particularly to the tranſit of *Venus* over the ſun which happened in 1769. Let *C* repreſent the center of the ſun *LQ*, *P* the celeſtial north pole of the equator, *S* the ſouth pole, *PCS* a meridian paſſing through the ſun, *Z* the zenith of the place, *ADB* ☋ the relative path of Venus, ☋ being the relative place of the deſcending node, *A* the geocentric place of Venus at the ingreſs, *B* at the egreſs, and *D* at the neareſt approach to the ſun's center, as ſeen from the earth's center, and *o* the apparent place of Venus at the egreſs to an obſerver whoſe zenith is *Z*; draw *ouZ*, and *u* is the *true* place of Venus when the *apparent* place is at *o*, and *uo* is the parallax in altitude of Venus from the ſun; and the time of contact will be diminiſhed by the time which Venus takes to deſcribe *uB*; draw *n'o'honE* parallel to *AB*, meeting *ZB* produced in *E*, and *Bn*, *An'* tangents to the circle, and let *ChD* be perpendicular to *AB*. Now the trapezium *uoEB*, on account of the ſmallneſs of it's ſides, may be conſidered as rectilinear, and from the magnitude of *ZB* compared with *Bu*, *BE* may be conſidered as parallel to *uo*, and conſequently *uoEB* may be conſidered as a parallelogram, and therefore *Eo* may be taken equal to *Bu*. Now $Eo = En \overset{+}{\sim} no$, according as *E* falls without or within the circle *LQ* of the ſun's diſc; and by Trigonometry, *En* : *EB* :: ſin. *EBn* = coſ. *CBZ* : ſin. *BnE* = ſin. *BCD* = coſ. *CBD*, hence $En = \frac{EB \times \text{cos. } CBZ}{\text{cos. } CBD}$; and (by Euclid) $no = \frac{Bn^2}{no'} = \frac{Bn^2}{AB}$ very nearly; but *Bn* : *BE* :: ſin. *BEn* = ſin. *ZBD* : ſin. *BnE* = coſ. *CBD*, therefore $Bn^2 = \frac{BE^2 \times \text{sin. } ZBD^2}{\text{cos. } CBD^2}$; hence $no = \frac{BE^2 \times \text{sin. } ZBD^2}{AB \times \text{cos. } CBD^2}$. Put *h* = horizontal parallax of Venus from the ſun; and (154) *BE* = *h* × ſin. *Zo* = *h* × ſin. *ZB*; hence $uB = oE = En \overset{+}{\sim} no =$ *h* ×

Fig. 155.

$\frac{h \times \text{sin. } ZB \times \text{cos. } CBZ}{\text{cos. } CBD} \pm h^2 \times \text{sin. } ZB^2 \times \frac{\text{sin. } ZBD^2}{BA \times \text{cos. } CBD^2} = h \times \text{sin. } ZB \times$ cos. $CBZ \times$ sec. $CBD \pm \frac{h^2 \times \text{sin. } ZB^2 \times \text{sin. } ZBD^2 \times \text{sec. } CBD^2}{AB}$. Put $t'' =$ the time which Venus takes, by it's geocentric relative motion, to describe the space h; to find which, let m be the relative horary motion of Venus; then $m : h ::$ 1 hour $= 3600'' : t'' = \frac{h \times 3600''}{m}$. Hence to find the time of describing uB, we have, $h : h \times$ sin. $ZB \times$ cos. $CBZ \times$ sec. $CBD \pm \frac{h^2 \times \text{sin. } ZB^2 \times \text{sin. } ZBD^2 \times \text{sec. } CBD^2}{AB} :: t'' : t'' \times$ sin. $ZB \times$ cos. $CBZ \times$ sec. $CBD \pm \frac{t \times h \times \text{sin. } ZB^2 \times \text{sin. } ZBD^2 \times \text{sec. } CBD^2}{AB}$ the time of describing uB, or the effect of parallax in accelerating or retarding the time of contact; the upper sign is to be used when CBZ is acute, and the lower sign when it is obtuse. If CBZ be very nearly a right angle, but obtuse, it may happen that nE may be less than no, in which case, nE is to be taken from no, according to the rule. The principal part nE of the effect of parallax will increase or diminish the planet's distance from the sun's center, according as the angle ZBC is acute or obtuse; but the small part no of parallax will always increase the planet's distance from the center; take therefore the sum or difference of the effects, with the sign of the greater, as to increasing or decreasing the planet's distance from the center of the sun. Otherwise state the rule thus: Take the sum or difference of nE and no, according as ZBC is acute or obtuse, and the distance of the planet from the sun's center will always be increased in the first case and diminished in the second, except ZBC being obtuse, and near 90°, nE shall be less than no, and then the distance from the sun's center will be increased by the difference. If ZBC be acute, the part nE will retard the ingress and accelerate the egress; but if ZBC be obtuse, the part nE will accelerate the ingress and retard the egress. In like manner, the parallax affects the time of the planet's coming to any given distance from the sun's center before or after the middle of the transit. The second part of the correction will not exceed 9″ or 10″ of time in the transits of Venus in 1761 and 1769, where the nearest approach of Venus to the sun's center was about 10′. In the transits of *Mercury*, the first part alone will be sufficient, except the nearest distance be much greater.

CALCULATION. As $t \times$ sec. CBD is a constant quantity for the same transit, find it's logarithm, and it will be constant; and as $\frac{t \times h \times \text{sec. } CBD^2}{AB}$ is

alſo conſtant, find it's logarithm, and you get a ſecond conſtant logarithm. Then to find the firſt, or principal effect of parallax in time, to the *firſt* conſtant logarithm, add the log. ſine of the zenith diſtance ZB, and the log. coſine of CBZ, and the ſum is the logarithm of the firſt part of the effect of parallax; and to the *ſecond* conſtant logarithm, add twice the log. ſine of the zenith diſtance ZB, and twice the log. ſine of ZBD, and the ſum is the logarithm of the ſecond part of the effect of parallax.

613. From the Tables of the ſun's motion, the diſtance of the ſun from the earth at the time of the tranſit was 1,015214, the mean diſtance being unity; and the ſun's horary motion was 143″,457, uncorrected by the effect of the menſtrual parallax. From the Tables of the motion of Venus, the diſtance of Venus from the ſun was 0,7262648, it's mean diſtance being 0,72333, and it's mean horary motion was 240″,325; hence (233) it's true heliocentric horary motion was 238″,381.

FIG. 150. 614. To explain the effect of the menſtrual parallax, let S be the ſun, vw the earth's orbit, C the center of gravity of the earth E and moon M, MW the orbit of the moon, and Eu the orbit of the earth, which each deſcribes about their center of gravity C, whilſt that center deſcribes the orbit vw; join SC, SE, and on SE let fall the perpendicular Ca; then the angle ESC is the menſtrual parallax, which at it's maximum, or when CE is perpendicular to CS, is 7″,1, the ſun and moon being at their mean diſtances; let the former be repreſented by unity, and let $m =$ the mean diſtance of the moon. Then $CE : Ca :: 7″,1 : CSE = 7″,1 \times \frac{Ca}{CE} = 7″,1 \times$ ſin. elong. ☾ from ☉ when the ſun and moon are at their mean diſtances; but CSE muſt vary inverſely as the diſtance of the ſun; hence $CS : 1 :: 7″,1 \times$ ſin. elong. : $CSE = 7″,1 \times \frac{\text{ſin. elong.}}{CS}$, at any diſtance CS; alſo, by varying CE, the angle CSE muſt vary in proportion; but CE varies as CM; hence $m : CM :: 7″,1 \times \frac{\text{ſin. elong.}}{CS} : CSE = 7″,1 \times$ ſin. elong. $\times \frac{CM}{CS \times m} =$ (becauſe the hor. parallax of ☾ varies inverſely as it's diſtance) $7″,1 \times$ ſin. elong. $\times \frac{\text{mean hor. par. ☾}}{CS \times \text{hor. par. ☾}}$; hence the increment of this angle (the angle itſelf only being ſuppoſed variable) $= 7″,1 \times \dot{\text{elong.}} \times$ coſ. elong.* $\times \frac{\text{mean hor. par. ☾}}{CS \times \text{hor. par. ☾}}$; but in an hour, $\dot{\text{elong.}} =$ hor. mot. ☾ —

* Becauſe $\dot{\text{ſin.}} : \dot{\text{arc}} ::$ coſ. : rad., therefore $\dot{\text{ſin.}} = \dot{\text{arc}} \times$ coſ., the radius being unity.

☾ − hor. mot. ☉ = H; hence the horary motion of the menſtrual par. of ☉ in *Longitude* = 7″,1 × H × coſ. elong. × $\frac{\text{mean hor. par. ☾}}{CS \times \text{hor. par. ☾}}$.

615. To find the menſtrual horary motion in latitude, let vCw repreſent the ecliptic, E the earth, M the moon, C their center of gravity, and Ev perpendicular to Cv; then $CE : Ev :: 7'',1 : 7'',1 \times \frac{Ev}{CE} = 7'',1 \times$ ſin. ☾'s lat. which is the ſun's latitude at the mean diſtances of the ſun and moon; and if h = the moon's horary motion in latitude, by proceeding as before we get the horary motion of the menſtrual parallax in *latitude* = 7″,1 × coſ. lat. ☾ × h × $\frac{\text{mean hor. par. ☾}}{\text{☉'s diſt.} \times \text{hor. par. ☾}}$, and will be the ſame way as the ☾ moves in latitude. FIG. 151.

Hence the computation of theſe quantities, at the time of the tranſit in 1769, when the hor. par. ☾ was 61′. 28″; ☾'s hor. mot. 37′. 57″,8; ☉'s hor. mot. 2′. 23″,5; hence H = 35′. 34″,3; hor. mot. ☾ lat. 3′. 22″,5; ☉'s diſt. 1,015214; and the mean hor. par. ☾ 56′. 59″; alſo at the time of this tranſit, coſ. elong. = 1.

7″,1	log.	0,8513
56′. 59″	log. ſin.	8,2194
61. 28	co-ar. log. ſin.	1,7476
35. 34,3	log. ſin.	8,0148
1,015214	co-ar. log.	9.9935
0″,0671 hor. men. par. ☉ in *Long.*		8,8266

7″,1	log.	0,8513
56′. 59″	log. ſin.	8,2194
61. 28	co-ar. log. ſin.	1,7476
3. 22,5	log. ſin.	6,9920
1,015214	co-ar. log.	9,9935
0,0064 hor. men. par. ☉ in *Lat.*		8,8038

Hence the ſun's true horary motion in longitude is 143″,524.

616. Let C be the ſun, C ♋ P ♋ a ſemicircle in the plane of the orbit of *Venus*, ♋ ♎ ♋ a ſemicircle in the plane of the ecliptic; P the place of Venus, and FIG. 152.

and draw Pd perpendicular to $\odot C \varUpsilon$, and Pe perpendicular to the plane $\odot \mathfrak{Q} \varUpsilon$, and join ed; now

Rad. : cos. incl. ♀'s orb. :: Pd : de :: tan. $PC\varUpsilon$: tan. $eC\varUpsilon$ = tan. $PC\varUpsilon \times$ cos. incl.

Take the increments, and we get $\dot{eC\varUpsilon} = \dot{PC\varUpsilon} \times$ cos. incl. $\times$ sec. $\overline{PC\varUpsilon}^2 \times \overline{\text{cos. } eC\varUpsilon}^2$, becauſe the increment of the tangent of an angle = increm. arc $\times \overline{\text{sec.}}^2$, radius being unity. Alſo PC : Pd :: ſin. Pde = inclin. : ſin. PCe = helioc. lat. of ♀; but PC : Pd :: rad. : ſin. $P\varUpsilon$ or $PC\varUpsilon$, therefore rad. : ſin. $PC\varUpsilon$ :: ſin. inclin. : ſin. lat. PCe = ſin. incl. $\times$ ſin. $\dot{C}\varUpsilon$. Take the increments, and we get $\dot{PCe} = \dot{PC\varUpsilon} \times$ cos. $PC\varUpsilon \times$ ſec. $PCe \times$ ſin. 3°. 23′. 20″.

FIG. 153. 617. Let S be the ſun, $\mathcal{T}$, V, two cotemporary places of the earth and Venus; and after a ſmall ſpace of time let u and X be their cotemporary poſitions, the place u being affected by the motion of the earth about the center of gravity of the earth and moon during the intermediate time; then $uSX = \mathcal{T}SX - \mathcal{T}Su$ = hel. mot. ♀ in long. − hel. mot. ☉ in long. including the change of the menſtrual parallax; hence uSX (the app. hel. mot. ♀ from ☉ in long.) : SuX (the app. retrog. geoc. mot. of ♀ from ⊙ in long.) :: $Xu = \mathcal{T}V$: $SX = SV$, becauſe the angles being ſmall, they may be taken as their ſines.

FIG. 154. 618. Let alſo $\mathcal{T}e$ be the motion of the earth in a ſmall ſpace of time from $\mathcal{T}$ to e perpendicular to the plane of the ecliptic, and Vu the correſponding motion of Venus; where the motion $\mathcal{T}e$ is that about the center of gravity of the earth and moon in latitude. Then the heliocentric motion of Venus from the earth in latitude = $VSu + \mathcal{T}Se$ (the figure being adapted to the circumſtances of the tranſit in 1769).

FIG. 156. 619. Let VSd be the heliocentric motion of Venus from the earth in longitude (= uSX in Art. 617.), Vd being perpendicular to the SV; and let dSo be the heliocentric motion of Venus from the earth in latitude (= uSe in Art. 618), od being perpendicular to the ecliptic; then the hypothenuſe Vo will be the apparent heliocentric path of Venus relative to the earth, ſuppoſed to be at reſt; and oVd will be the angle which Venus's apparent heliocentric motion from the earth makes with the ecliptic, or which is the ſame, the angle which it's apparent geocentric motion from the ſun makes with the ecliptic. Now VSd (the hel. mot. of ♀ from ☉ in long.) : dSo (it's hel. mot. from ☉ in lat.) :: Vd : do :: rad. : tan. oVd; and Vd : Vo :: rad. : ſec. oVd.

FIG. 157. 620. Laſtly, let S, $\mathcal{T}$, V be the centers of the ſun, earth, and Venus, and $\mathcal{T}A$ = the ſemidiameter of the earth; then $VA = V\mathcal{T}$: SV :: $AS\mathcal{T}$ (= ⊙'s hor. par.) : $SAV = AV\mathcal{T} - AS\mathcal{T}$ = the diff. of the hor. par. of ⊙ and ♀.

Calculation from Article 616.

Hel. long. ♂ by obſ. at this tranſit . . .	8ˢ. 4°. 35′. 35″
Venus's hel. long. at mid. of tranſit . .	8. 13. 28. 13
Arg. of S. lat. of ♀, or diſt. of ♀ from ♂, it being ſo much ſhort of it as ſeen from ☉	0. 1. 7. 22

$PC♂ = 1°. 7'. 22''$	log. tan. 8,2922268
$Pde∠ ♂ = 3. 23. 20$	log. cof. 9,9992399
$eC♂ = 1. 7. 15$	log. tan. 8,2914667
Log. ratio cof. $eC♂$ to cof. $PC♂$	0,0000003
Log. ratio of the ſquare	0,0006006
Log. cof. 3°. 23′. 20″	9,9992399
Hel. hor. mot. ♀ in long. 238″,381 . .	log. 2,377272
Hel. hor. mot. ♀ on ecl. 237″,9644 . .	log. 2,376512
$P♂ = 1°. 7'. 22''$	log. ſin. 8,2921434
$∠ ♂ = 3. 23. 20$	log. ſin. 8,7716814
$Pe = 0. 3. 59$	log. ſin. 7,0638248

By this formula, $\overline{PCe} = \overline{PC♂} \times$ cof. $PC♂ \times$ ſec. $PCe \times$ ſin. 3°. 23′. 20″.

238″,381	log. 2,377272
3°. 23′. 20″	log. ſin. 8,7716814
1. 7. 22	log. cof. 9,9999166
	11,1488700
0. 3. 59	log. cof. 9,9999997
14″,0887 hel. mot. of ♀ in lat. . . .	log. 1,1488703

Hel. hor mot. ♀ red. to ecliptic	237″,9644
Hor. mot. ☉, including effect menſt. par. . .	143, 524
Hel. hor. mot ♀ from ☉ in long.	94, 4404

Calculation from Article 618, *and* 619.

94″,4404	log. 1,9751578
Hel. hor. mot. ♀ in lat. 14,0887	
——————☊—— 0,0064	
——————♀ á ☊—— 14,0951 . . .	log. 1,1490682
8°. 29′. 19″,17 ∠ rel. orb. makes with ecl. .	tan. 9,1739104
8°. 29′. 19″,17	co-ar. log. cof. 0,0047838
94″,4404	log. 1,9751578
95″,4864 app. hel. hor. mot. ♀ á ☊ . .	co-ar. log. cof. 1,9799416

Calculation from Article 617.

95″,4864	log. 1,9799416
$TV = 0{,}288949$	co-ar. log. 0,5391787
$SV = 0{,}726265$	log. 9,861095
240″,0023 app. geoc. mot. ♀ á ☉ . . .	log. 2,3802153

Calculation

Calculation from Article 620.

Assume the sun's mean horizontal parallax 8″,83, agreeable to what was determined from the observations of the transit in 1761; see the Precepts to MAYER's Tables, page 61 and 114.

8″,83	log.	0,945961
1,015214 ⊙'s dist. from ☉	log.	0,006515
8″,6985 hor. par. ⊙ on day of transit . .	log.	0,939446
$TV = 0{,}288949$	co-ar. log.	0,539179
$SV = 0{,}726265$	log.	9,861095
21″,864 ♀'s hor. par. á ⊙ during transit .	log.	1,339720

To find the apparent time taken by Venus to move over it's horizontal parallax from the sun.

As 240″,0023 ♀'s hor. mot. á ⊙ mean time in rel. orb. .	co-ar. log.	7,6197848
is to 3600″	log.	3,5563025
so is 1″	log.	0,0000000
to 14″,99986 time ♀ takes to move 1″ from ⊙ in rel. orb. . . .	log.	1,1760873
21″,864 hor. par. ♀ á ⊙	log.	1,339720
327″,95 mean time ♀ moves over it's hor. par. from ⊙ . . .	log.	2,515807

But 24 hours of apparent time = 24*h*. 0′. 10″ of mean time; hence to reduce the mean to apparent time,

$\frac{\text{App. time}}{\text{Mean time}} = \frac{24h.\ 0'.\ 0''}{24h.\ 0'.\ 10''} = \frac{8640}{8641}$ . . .	log.	0,000050
327″,95	log.	2,515807
327″,912 app. time ♀ moves over it's hor. par. á ⊙, which put = *t*. .	log.	2,515757

Time

	Time of obſ. of *firſt* int. cont. at *Wardhus*, red. to ☉'s center, by ſome former calculations . .	$9^h.\ 40'.\ 40''$ . . . D° *ſecond* int. cont.	$15^h.\ 23'.\ 4''$
	Diff. mer. E. of Greenwich	2. 4. 17	2. 4. 17
	App. times at Greenwich	7. 36. 23	13. 18. 47
	☉'s declinations . . .	22°. 25. 50 N.	22°. 27. 35 N.
FIG.	☉'s diſt. *PC* from N. pole	67. 34. 10	67. 32. 25
158.	☉'s diſt. *SC* from S. pole	112. 25. 50	112. 27. 35
	Angle betw. ecliptic and parallel to equator	7. 5. 20	6. 59. 27
	App. incl. ♀'s rel. orb. on ☉ to ecliptic *C* ☌ *B*	8. 29. 19	8. 29. 19
	Sum = ∠ bet. par. to equ. and ♀'s rel. orb. *PCD*	15. 34. 39	15. 28. 46

Otherwiſe,

	∠*LCP* of ecl. and mer. .	82°. 54'. 40''	83°. 0'. 33''
	It's ſupp. ☌ *CP* . . .	97. 5. 20	96. 59. 27
	☌ *CD* = comp. *C* ☌ *D*	81. 30. 41	81. 30. 41
	Diff. = *PCD* . . .	15. 34. 39	15. 28. 46

621. I ſhall take the difference of ſemidiameters of the ſun and Venus with M. de la LANDE = 15'. 15",1, which is what he found neceſſary to reconcile the total durations of the tranſits in 1761 and 1769 with the motion of the node of Venus's orbit in the interval, known nearly. By ſome calculations of this tranſit, I had found the chord deſcribed by Venus over the ſun between the two internal contacts, reduced to the center of the earth, to be = 1368",57. Hence the ſemi-chord is 684",285; with which, and the difference of ſemidiameters of the ſun and Venus 15'. 15",1 above-mentioned, I find the neareſt approach of Venus to the ſun's center, and the angle which Venus's path over the ſun ſeen from the center of the earth makes with the radius of the ſun's diſc at the two internal contacts, as follows.

915",1 +

915″,1 + 684″,285 = 1599,285 . . .	log. 3,2039531
915, 1 − 684, 285 = 230,815	log. 2,3632640
	2)5,5672171
607″,587 = 10′. 7″,587 = *CD*	2,7836085
915, 1	2,9614686
41°. 36′. 8″,5 = *CAD* = *CBD* . .	log. fin. 9,8221399

t = 327″,912	log. 2,51576
CAD = 41°. 36′. 8″	co-ar. cof. 0,12623
Firſt conſtant logarithm	2,64199

t = 327″,912	log. 2,51576
2 × co-ar. cof. *CAD*	0,25246
h = 21″,864	log. 1,33972
	4,10794
AB = 1368″,57	3,13627
Second conſtant logarithm	0,9717

	At 1ſt int. cont.	At 2d int. cont.
LCP = angle of ecl. and merid.	82°. 54′. 40″	83°. 0′. 33″
℘ *CP* it's ſupplement . . .	97. 5. 20	96. 59. 27
℘ *CD* = 90° − 8°. 29′. 19″ . .	− 81. 30. 41	− 81. 30. 41
PCD = ...	15. 34. 39	15. 28. 46
ACD = 90° − 41°. 36′. 8″ . .	48. 23. 52	*BCD* = 48. 23. 52
Diff. = *ACP* = ...	32. 49. 13	ſum *BCP* = 63. 52. 38

Now in the ſpherical triangle *ACP*, there is given *AC* = 15′. 15″, *PC* = 67°. 34′. 10″, and *ACP* = 32°. 49′. 13″; and in *BCP*, there is given *BC* = 15′. 15″, *PC* = 67°. 34′. 10″, and *BCP* = 63°. 52′. 38″. Let ℚ be the point where a perpendicular from *A* cuts *CP*; and *R* the point where a perpendicular from *B* cuts *CP*.

ACP =

$ACP = 32°.\ 49'.\ 13''$. cof. 9,92447 . . tan. 9,80953
$AC = 0.\ 15.\ 15$ . . tan. 7,64700

$CQ = 0.\ 12.\ 49$ . . tan. 7,57147 . . fin. 7,57150
$PC = 67.\ 34.\ 10$

$PQ = 67.\ 21.\ 21$ . . . . . co-ar. fin. 0,03484

$CPA = 0.\ 8.\ 57,4$ . . . . . tan. 7,41587

$BCP = 63°.\ 52'.\ 38''$. cof. 9,64374 . . tan. 10,30946
$BC = 0.\ 15.\ 15$. tan. 7,64700

$CR = 0.\ 6.\ 43$. tan. 7,29074 . . fin. 7,29074
$CP = 67.\ 34.\ 10$

$PR = 67.\ 27.\ 27$ . . . . . co-ar. fin. 0,03452

$BPC = 0.\ 14.\ 49,5$ . . . . . tan. 7,63472

Let fall the perpendicular PX from P, upon CA produced.

$ACP = 32°.\ 49'.\ 13''$. cof. 9,92447 . tan. 9,80953
$CP = 67.\ 34\ 10$. tan. 10,38427 . . . . . cof. 9,581567

$CX = 63.\ 50.\ 23$. tan. 10,30874 . fin. 9,95307 . . co-ar. cof. 0,355676

$CA = 0.\ 15.\ 15$

$AX = 63.\ 35.\ 8$ . . . co-ar. fin. 0,04788 . . . . . cof. 9,648224

$PAC = 147.\ 7.\ 21,6$ . . . . . tan. 9,81048 $PA = 67°.21'.21''$ cof. 9,585467

Let

Let fall the perpendicular PY from P, upon CB produced.

$BCP = 63°.52'.38''$. cof. 9,643745 . tan. 10,309460
$CP = 67.34.10$. tan. 10,384266 . . . . . cof. 9,581567
$CY = 46.50.47$. tan. 10,028011 . fin. 9,863039 . . co-ar. cof. 0,164971
$CB = 0.15.15$
$BY = 46.35.32$ . . . . co-ar. fin. 0,138775 . . . . . cof. 9,837074
$PBC = 116.1.42$ . . . . . tan. 10,311274 $PB = 67°.27'.28''$ cof. 9,583612

To compute the effect of parallax on the firſt internal contact at *Wardhus*.

$ZPC = 143°.32'.39''$
$APC = 0.8.57$
$ZPA = 143.23.42$ . . cof. 9,904588
Co-lat. $ZP = 19.37.25$ . . tan. 2,552118 . . . . cof. 9,974014
$PN = 15.58.21$ . . tan. 9,456706 . . co-ar. cof. 0,017099
$PA = 67.21.21$
$AN = 83.19.42$ . . . . . cof. 9,065130
$ZA = 83.27.51$ . . . . . cof. 9,056243
ZA . . . . . fin. 9,99717
Firſt conſtant logarithm 2,64199
From the next operation, $CAZ = 158°.45'.6''$ . . cof. 9,96942
Acceleration of contact, firſt part, $= 406'',05$ 2,60858

$ZPA =$

$ZPA = 143°.\ 23'.\ 42''$		tan. 9,870872
$PN = 15.\ 58.\ 21$		fin. 9,439611
$AN = 83.\ 19.\ 42$	co-ar.	fin. 0,002951
$ZAP = 11.\ 37.\ 44$		tan. 9,313434
$PAC = 147.\ 7\ 22$		

$ZAC = 158.\ 45.\ 6$; hence $ZAC - CAD = 117°.\ 8'.\ 58'' = ZAD$.

Second conſtant logarithm	0,972
2 × log. ſin. ZA	9,994
2 × log. ſin. ZAD	9,898
Retardation of contact, ſecond part, $= 7'',31$	0,864

Hence $406'',05 - 7'',31 = 398'',74 = 6'.\ 38'',74$ the whole effect of parallax in accelerating the ingreſs.

To compute the effect on the ſecond internal contact.

$ZPC = 128°.\ 8'.\ 51''$			
$BPC = 0.\ 14.\ 50$			
$ZPB = 127.\ 54.\ 1$	cof. 9,788373		
Co-lat. $PZ = 19.\ 37.\ 25$	tan. 9,552118		cof. 9,974014
$PN = 12.\ 21.\ 15$	tan. 9,340491	co-ar.	cof. 0,010174
$PB = 67.\ 27.\ 28$			
$BN = 79.\ 48.\ 43$			cof. 9,247678
$ZB = 80.\ 10.\ 47$			cof. 9,231866
ZB			ſin. 9,99359
Firſt conſtant logarithm			2,64199
From the next operation, $ZBC = 131°.\ 37'.\ 48''$			cof. 9,82238
Retardation of contact, firſt part, $= 287'',05$			2,45796

$ZPB =$

$ZPB = 127°.\ 54'.\ 1''$		tan.	10,108749
$PN = 12.\ 21.\ 15$		fin.	9,330320
$BN = 79.\ 48.\ 43$	co-ar.	fin.	0,006902
$PBZ = 15.\ 36.\ 6$		tan.	9,44597
$PBC = 116.\ 1.\ 42$			
$ZBC = 131.\ 37.\ 48$			

; hence $ZBC - DBC = 90°.\ 1'.\ 40'' = ZBD$.

Second conſtant logarithm	0,972
2 × log. ſin. ZB	9,978
2 × log. ſin. ZBD	0,000
Acceleration of contact, ſecond part, $= 8'',91$	0,950

Hence $287'',05 - 8'',91 = 278'',14 = 4'.\ 38'',14$ the whole effect of parallax, in retarding the egreſs. Hence the whole duration was lengthened $11'.\ 16'',88$ by parallax

To compute the effect of parallax on the firſt internal contact at *Otaheite*.

$ZSC = 33°.\ 57'.\ 0''$						
$CSA = 0.\ 8.\ 57$						
$ZSA = 34.\ 7.\ 57$		cof.	9,917895			
Co-lat. $ZS = 72.\ 30.\ 43$		tan.	10,501594		cof.	9,477855
$SN = 69.\ 9.\ 41$		tan.	10,419489	co-ar.	cof.	0,448871
$SA = 112.\ 38.\ 39$						
$NA = 43.\ 28.\ 58$					cof.	9,860686
$ZA = 52.\ 11.\ 54$					cof.	9,787412
ZA					fin.	9,89770
Firſt conſtant logarithm						2,64199
From the next operation, $ZAC = 9°.\ 45'.\ 27''$					cof.	9,99367
Retardation of contact, firſt part, $= 341'',48$						2,53336

$ZSA = 34°.\ 7'.\ 57''$		tan.	9,831152
$SN = 69.\ 9.\ 41$		fin.	9,970619
$NA = 43.\ 28.\ 58$	co-ar.	fin	0,162325
$SAZ = 42.\ 38.\ 4$		tan.	9,964096
$SAC = 32.\ 52.\ 37$			
$ZAC = 9.\ 45.\ 27$			

; hence $CAD - CAZ = 31°.\ 50'.\ 41'' = ZAD$.

Second conſtant logarithm	0,9717
2 × log. ſin. ZA	9,7954
2 × log. ſin. ZAD	9,4446
Retardation of contact, ſecond part, $= 1'',63$	0,2117

Hence the ingreſs is retarded $341'',48 + 1'',63 = 343'',11 = 5'.\ 43'',11$.

To compute the effect of parallax on the ſecond internal contact.

$ZSC =$	$48°.\ 32'.\ 0''$				
$CSB =$	$0.\ 14.\ 50$				
$ZSB =$	$48.\ 46.\ 50$	coſ.	9,818849		
Co-lat. $ZS =$	$72.\ 30.\ 43$	tan.	10,501594	coſ.	9,477855
$SN =$	$64.\ 26\ 44$	tan.	10,320443	co-ar. coſ.	0,365151
$SB =$	$112.\ 32.\ 32$				
$BN =$	$48.\ 5.\ 48$			coſ.	9,824696
$ZB =$	$62.\ 16.\ 21$			coſ.	9,667702
ZB				fin.	9,94703
Firſt conſtant logarithm					2,64199
From the next operation, $CBZ = 9°.\ 49'.\ 39''$				coſ.	9,99358
Acceleration of contact, firſt part, $= 382'',47$					2,58260

$ZSB =$

$ZSB = 48°.\ 46'.\ 50''$		tan: 10,057479
$SN = 64.\ 26.\ 44$		fin. 9,955291
$BN = 48.\ 5.\ 48$	co-ar.	fin. 0,128268
$SBZ = 54.\ 8.\ 39$		tan. 10,141038
$SBC = 63.\ 58.\ 18$		
$CBZ = 9.\ 49.\ 39$; hence $CBA + CBZ = 51°.\ 25'.\ 47'' = ZBD$.		

Second conſtant logarithm	0,9717
2 × log. ſin. ZB	9,8941
2 × log. ſin. ZBD	9,7862
Acceleration of contact, ſecond part, $= 4'',49$	0,6520

Hence the egreſs is accelerated $382'',47 + 4'',49 = 386'',96 = 6'.\ 26'',96$. Therefore the whole duration was diminiſhed $12'.\ 10'',07$.

The total duration at Whardus was lengthened by parallax $11'.\ 16'',88$, and diminiſhed at Otaheite by $12'.\ 10'',07$; hence the computed difference of the times is $23'.\ 26'',95$; but the obſerved difference was $23'.\ 10''$.

622. Hence the correct parallax may be accurately found as follows. Becauſe the obſerved difference of the total durations at Wardhus and Otaheite is $23'.\ 10''$, and the computed difference, from the aſſumed mean horizontal parallax of the ſun $8'',83$, is $23'.\ 26'',95$, the true parallax of the ſun is leſs than that aſſumed. Let the true parallax be to that aſſumed as $1 - e$ to 1, and (612) the firſt parts of the computed parallax will be leſſened in the ratio of $1 - e : 1$; and the ſecond parts, in the ratio of $\overline{1 - e}^2$ to 1, or of $1 - 2e$ to 1 nearly. All the firſt parts, viz. $406'',05$; $287'',05$; $341'',48$; $382'',47$, in all $= 1417'',05$, combine the ſame way to make the total duration longer at Wardhus than at Otaheite. As to the ſecond parts, the effects at Wardhus are $-7'',31$ and $-8'',91$, and at Otaheite are $+1'',63$ and $+4'',49$, in all $= -10'',10$. Therefore $1417'',05 \times \overline{1 - e} - 10'',10 \times \overline{1 - 2e} = 1390''$ the exceſs of the obſerved total duration at Wardhus above that at Otaheite; or $1417'',05 - 10'',10 - 1390'' = \overline{1417'',05 - 20'',20} \times e$; and $e = \frac{16'',95}{1396'',85} = 0,0121$. Hence the mean horizontal parallax of the ſun $= 8'',83 \times \overline{1 - 0,0121} = 8'',72316$; and t corrected $= 327'',912 \times \overline{1 - 0,0121} = 323'',944$, whoſe logarithm is 2,510649; and the log. of $327'',912$ − log. of $323'',944 = 2,515757 - 2,510469$. The

log. of the ratio of $1-e$ or 0,98794 to 1, or 323″,944 to 327″,912, is $\bar{1}$,99471; and the log. of the ratio of the ſquares of theſe quantities is $\bar{1}$,98894; or the correction of the *firſt* conſtant logarithm before uſed is $-$0,00529, and that of the *ſecond* conſtant logarithm before uſed is $-$0,0106.

I ſhall now calculate the effects of parallax over again at Wardhus and Otaheite, by means of theſe correcting logarithms.

Second internal contact at Wardhus.

	0,950		
	$-$ 0,011		
8″,690,939		2,45796	
		$-$ 0,00529	
283″,58 = 4′. 43″,58	2,45267		
$-$ 8, 69			
4. 34, 89			

First internal contact at Wardhus.

	0,864		
	$-$ 0,011		
7″,13	0,853	2,60898	
		$-$ 0,00529	
401″,13 = 6′. 41″,13	2,60329		
$-$ 7, 13			
6. 34, 00			

6′. 34″,00 firſt internal contact accelerated.
4. 34, 89 ſecond internal contact retarded.

11. 8, 89 total duration lengthened by parallax.
5^{h}. 53. 14 ——— obſerved.

5. 42. 5, 11 total duration reduced to the earth's center.

First internal contact at Otaheite.

	2,53336		0,2117
	− 0,00529		− 0,0106
337″,34 = 5′. 37″,34	2,52807	1″,59	0,2011
+ 1, 59			
5. 38, 93			

Second internal contact at Otaheite.

	2,58260		0,6520
	− 0,00529		− 0,0106
377″,84 = 6′. 17″,84	2,57731	4″,38	0,6414
+ 4, 38			
6. 22, 22			

5′. 38″,93 first internal contact retarded.
6. 22, 22 second internal contact accelerated.

12. 1, 15 total duration shortened by parallax.
5ʰ. 30. 4 ———— observed.

5. 42. 5, 15 total duration reduced to the earth's center.
5. 42. 5, 11 ———————————————— from obs. at Wardhus.

5. 42. 5, 13 mean = 20525″,13 log. 4,3122859

Venus's motion from the sun in 1″ of apparent time . . . . . $\bar{2}$,8239631

1368″,51 length of the path described by Venus on the Sun . . . 3,1362490
684″,26 = the semichord of Venus on the Sun.
915″,1 + 684″,26 = 1599,″36 . . . . . log. 3,2039462
915, 1 − 684, 26 = 230, 84 . . . . . log. 2,3633111

2)5,5672573

607″,61 = 10′. 7″,61 nearest approach of ♀ to ☉'s center 2,7836287

Firſt internal contact at Wardhus	9^h 34′. 10″,6
Effect of parallax	+ 6. 34, 0
Reduced to the earth's center	9. 40. 44, 6.

Second internal contact	15^h. 27′. 24″,6
Effect of parallax	− 4. 34, 89
Reduced to the earth's center	15. 22. 49, 71

Firſt internal contact at Otaheite . . .	21^h. 44′. 4″
Effect of parallax	− 5. 38, 93
Reduced to the earth's center	21. 38. 25, 07

Second internal contact	3^h. 14′. 8″
Effect of parallax	+ 6. 22, 22
Reduced to the earth's center	3. 20. 30, 22

623. If we would calculate the parallaxes for any other places, the conſtant logarithms to be uſed will be found, by ſubtracting 0,00529 from 264199 the firſt conſtant logarithm before uſed, and 0,1006 from 0,9717 the ſecond conſtant logarithm before uſed; which gives the firſt conſtant logarithm corrected 2,63670, and the ſecond conſtant logarithm corrected 0,9611.

624. The further the planet paſſes from the center of the ſun, the greater will be the angle *CBD*, and therefore (612) the greater will be the parallax, cæteris paribus. Hence the tranſit in 1769 is better to deduce the parallax from, than that in 1761; for by gaining a greater difference of times of the tranſit ſeen from different parts of the earth, any given error therein muſt leſs affect the concluſion.

625. Having explained the method of determining the ſun's parallax from the tranſit of Venus, we ſhall proceed to give the reſults deduced from the tranſits in 1761 and 1769. And firſt we ſhall give thoſe of Mr. J. SHORT, A. M. F. R. S. from a compariſon of the times of the ſame contact obſerved at the Cape of Good Hope, with thoſe in different parts of Europe.

626. The

626. The time of the internal contact at the Cape, in the tranſit of Venus in 1761, was 9*h*. 39′. 50″, and the difference of the longitudes of the Cape and Greenwich is 1*h*. 13′. 35″; hence the time at Greenwich of the contact at the Cape was 8*h*. 26′. 15″; but the obſerved time of the ſame contact at Greenwich was 8*h*. 19′, the difference of which, 7′. 15″, is the effect of parallax between the two places. Now if we ſuppoſe the ſun's horizontal parallax to be 8″,5, it appears, by computation, that the effect of the parallax at the Cape is 6′. 8″, by which time an obſerver at the Cape would ſee the contact *later* than at the center of the earth; and the effect of parallax at Greenwich is 1′. 12″, by which time an obſerver would ſee the contact *ſooner* than at the center of the earth; therefore the ſum, 7′. 20″, is the whole effect of parallax between the two places. Hence 7′. 20″ : 7′. 15″ :: 8″,5 : 8″,4 the ſun's horizontal parallax from theſe obſervations. Thus, knowing the difference of longitudes, by comparing the times at the Cape with the following places, Mr. SHORT deduced theſe horizontal parallaxes of the ſun. *Phil. Tranſ.* 1763.

Greenwich	8″,42
Leſkeard	8, 69
Rome	8, 61
Stockholm	8, 48
Abo	8, 58
Shirburn	8, 15
Paris	8, 42
Dronthein	8, 23
Hernoſand	8, 68
Torneo	8, 07
Savile Houſe	8, 57
Bologna	8, 41
Upſal	8, 50
Calmar	8, 86
Cajaneburg	8, 33

The mean of theſe reſults is 8″,47.

627. Mr. SHORT alſo determined the parallax from the whole time of the duration, in the following manner. He found the leaſt apparent diſtance of the center of Venus from the center of the ſun, to be 9′. 32″; from which, and the horary motion of Venus, he found the total time of duration at the center

of the earth to be 5^h. 58′. 1″. And from an assumed parallax of 8″,5, he computed the effect upon the observed time of the transit, and thence found the total duration at the center of the earth, which he compared with 5^h. 58″. 1, and thence deduced the parallax. *Phil. Trans.* 1762.

628. At Calcutta, the duration observed was 5^h. 50′. 36″. Now upon supposition that the horizontal parallax of the sun was 8″,5, the effect of the parallax was 7′. 30″ to *shorten* the duration; hence the duration at the center, from this assumed parallax, was 5^h. 58′. 6″. But the true time was found to be 5^h. 58′. 1″; this assumed parallax therefore gave the time too great by 5″. Now if we alter the parallax 1″, the time of duration will be altered 53″; hence 53″ : 5″ :: 1″ : 0″,094 the change of parallax corresponding to the difference 5″ of duration; this subtracted from 8″,5 gives 8″,4 for the parallax from this observation; and from a mean of sixteen observations of this kind, Mr. SHORT determined the parallax to be 8″,48. From the mean of all the observations computed by Mr. SHORT, he determined the parallax to be 8″,557.

629. Dr. HORNSBY, Savilian Professor of Astronomy in the University of Oxford, from the mean of a great number of computations of the same transit, found the parallax to be 9″,73. But from a mean of nine observations of the transit in 1769, he found the parallax to be 8″,65. Hence the mean of Mr. SHORT's and Dr. HORNSBY's conclusions give 8″,92 for the parallax. But if we take only those observations the most to be depended upon, from which Dr. HORNSBY computed in the first transit, the parallax at that time will be found to be only 8″,69; hence the mean result from the DOCTOR's two conclusions 8″,69, 8″,92, and of Mr. SHORT's 8″,557, is 8″,72 for the parallax at the times of the transits; and assuming 1,0152 for the distance of the sun from the earth at the time of the transit, we have 1 : 1,0152 :: 8,72 : 8,85 for the parallax at the mean distance. EULER made the mean parallax, 8″,68; M. PINGRE, 8″,8; M. LEXELL, 8″,63; M. du SEJOUR, 8″,81; M. de la LANDE, 8″,6. The mean of all these determinations is 8″,73; which agrees (page 413) very nearly with Dr. MASKELYNE's calculation from the observations at Wardhus and Otaheite. We may therefore suppose the mean horizontal parallax of the sun to be $8''\frac{3}{4}$, with a great probability of it's being extremely near to the truth. Hence the radius of the earth : the distance of the sun :: sin. $8''\frac{3}{4}$: rad. :: 1 : 23575.

630. The elements made use of by Mr. SHORT in his calculations were, the diameter of the sun = 31′. 31″, the diameter of Venus = 59″, the horary motion of Venus in it's path = 3′. 59″,8, the angle of the apparent orbit of Venus with the ecliptic = 8°. 30′. 10″, the nearest distance of the centers of Venus and the sun seen from the earth = 9′. 32″, and the difference of the horizontal parallaxes of Venus and the sun = 21″,35.

631. The

631. The effect of the parallax being determined, the tranſit affords a very ready method of finding the difference of the longitudes of two places where the ſame obſervations were made. For compute the effect of parallax in time, and reduce the obſervations at each place, to the time if ſeen from the center of the earth, and the difference of the times is the difference of the longitudes. For example, we have ſhown (page 416) that the times at Wardhus and Otaheite, at which the firſt internal contact would take place at the earth's center, are $9h.\ 40'.\ 44'',6$ and $21h.\ 38'.\ 25'',07$, the difference of which is $12h.\ 2'.\ 19'',53 = 180°.\ 34'.\ 53''$ the difference of the meridians. From the mean of 63 reſults from the tranſits of Mercury, Mr. SHORT found the difference of the meridians of Greenwich and Paris to be $9'.\ 15''$; and from the tranſit of Venus in 1761, to be $9'.\ 10''$, in time.

632. The tranſit of Venus affords a very accurate method of finding the place of the node. For by the obſervations made by Mr. RITTENHOUSE at Norriton in the United States of America, the leaſt diſtance CM was obſerved to be $10'.\ 10''$; hence, cof. $MCV = 8°.\ 28'.\ 54''$: rad. :: $CM = 10'.\ 10''$: $CV = 10'.\ 17''$ the geocentric latitude of Venus at the time of conjunction; and 0,72626 : 0,28895 :: $10'.\ 17''$: $4'.\ 5''$ the heliocentric latitude CV of Venus; hence, tan. $VNC = 3°.\ 23'.\ 35''$: rad. :: the heliocentric latitude $CV = 4'.\ 5''$: $CN = 1°.\ 8'.\ 52''$, which added to $2^s.\ 13°.\ 26'.\ 34''$ gives $2^s.\ 14°.\ 35'.\ 26''$ for the place of the aſcending node of the orbit of Venus. FIG. 149.

633. The time of the ecliptic conjunction may be thus found. Find, at any time (t), the difference (d) of longitudes of Venus and the ſun's center (Art. 636); find alſo the apparent geocentric horary motion (m) of Venus *from* the ſun in longitude, and then ſay, m : 1 hour :: d : the interval between the time t and the conjunction, which interval is to be added to or ſubtracted from t, according as the obſervation was made after or before the conjunction. In the tranſit in 1761, at $6h.\ 31'.\ 46''$ apparent time at Paris, M. de la LANDE found $d = 2'.\ 34'',4$, and $m = 3'.\ 57'',4$; hence $3'.\ 57'',4$: $2'.\ 34'',4$ :: 1 hour : $39'.\ 1''$, which ſubtracted from $6h.\ 31'.\ 46''$, becauſe at that time the conjunction was paſt, gives $5h.\ 52'.\ 45''$ for the time of conjunction from this obſervation. We may alſo thus find the latitude at conjunction. The horary motion of Venus in latitude was $35'',4$; hence $60'$: $39',1$ :: $35'',4$: $23''$ the motion in latitude in $39',1$, which ſubtracted from $10'.\ 1'',2$ the latitude obſerved at $6h.\ 31'.\ 46''$, gives $9'.\ 38'',2$ for the latitude at the time of conjunction.

On the Neceſſary Obſervations to be made in the Tranſit of Venus over the Sun's Diſc.

634. Previous to the time of the beginning of the tranſit, the obſerver ſhould have his teleſcope properly fixed, and prepared with black glaſſes to defend the eye, and ſhould know, from his computations, the point of the ſun's limb where Venus is expected to enter. Upon that part of the limb he ſhould keep his eye ſteadily fixed, and at the inſtant he ſuſpects the contact to take place, he muſt note the time, and proceed to obſerve, in order to be certain that he was not miſtaken. If he find that he was miſtaken, he muſt continue to wait for it, always noting the time when he ſuſpects it, in order that he may not miſs it when it really does happen. Venus having entered the ſun's diſc, wait for the internal contact, and note it's time. Do the ſame for the internal and external contact, when Venus paſſes off the diſc. In the tranſit in 1761, the Rev. Mr. HIRST, F. R. S. at Madraſs, obſerved a kind of penumbra, or duſky ſhade, which preceded the firſt external contact two or three ſeconds of time, and was ſo remarkable, that he was thereby aſſured that the contact was near, which happened accordingly. In the tranſit in 1769, Dr. MASKELYNE was very attentive to obſerve if this circumſtance took place, but he could perceive no ſuch effect. When Venus was a little more than half immerged into the ſun's diſc, he ſaw it's whole circumference completed, by means of a vivid, but narrow ill-defined border of light, which illuminated that part of it's circumference which was off the ſun; but this diſappeared about 2′ or 3′ before the internal contact. In the tranſit in 1769, Mr. HIRST had warning of the approach of Venus to the external contact, by the ſudden appearance of a violent coruſcation, ebullition, or agitation of the upper edge of the ſun, five or ſix minutes before the limb of Venus broke in upon the ſun. This he thinks might be owing to the atmoſphere of Venus. He did not, however, obſerve any kind of penumbra, as in the other tranſit. Some obſervers perceived, at the firſt external contact, a kind of watry pointed ſhadow, appearing to give a tremulous motion to that part of the ſun's limb. Moſt of the obſervers took notice of a tremulous motion of the ſun's limb, which rendered the true time of the contact uncertain to ſeveral ſeconds. Some Aſtronomers, at the laſt tranſit, obſerved a luminous creſcent at the times of the ingreſs and egreſs, which enlightened that part of Venus's circumference which was off the ſun, ſo that the whole circumference was viſible. At the internal contact, the limb of Venus ſeemed, to moſt of the obſervers, to be united to the ſun's

ſun's limb by a black protuberance or ligament, which was not broken by the thread of light, till ſome ſeconds after the regular circumference of Venus ſeemed to have coincided with the ſun's. Others obſerved that the thread of light between the limbs did not break inſtantaneouſly, the points of the threads darting into each other, and parting again, in a quivering manner, ſeveral times before they finally adhered. Perhaps the beſt way to get the time of the internal contact, is to judge by the eye, from that part of the circumference of Venus which is not diſturbed, when the regular circumference of Venus would juſt touch the ſun's limb. Hence it appears, that Dr. HALLEY was miſtaken, in ſuppoſing that the contacts could be obſerved to a ſecond of time; and accordingly the obſervations made by different obſervers, and reduced to the ſame meridian, differed more than was expected. The mean of all the obſervations however have counterbalanced this, and rendered the determination of the parallax of the ſun to be depended upon to very great accuracy.

635. After the firſt internal contact, the next obſervations are to determine the neareſt approach of their centers. This is beſt done with a micrometer fixed to the teleſcope, by meaſuring the horizontal diameter of the ſun, the diameter of Venus, and the neareſt diſtance of the exterior limb of Venus from the neareſt point of the ſun's limb; and this is done by bringing the limb of Venus up to the ſun's limb in different parts till you find that you have got the neareſt diſtance; or by Mr. DOLLOND's divided object glaſs micrometer, it is done by turning about the micrometer in it's own plane, and when, during this motion, Venus is carried parallel to a tangent to the neareſt point of the limb, or ſo as to continue to form a perfect internal contact, this is the poſition to meaſure the leaſt diſtance of their limbs; then ſubtract the ſemidiameter of Venus from the radius of the ſun, and you have the diſtance of their centers at that time. If the ſun be ſo near to the horizon, that it's vertical diameter is ſhortened by refraction, then, from the poſition of Venus, compute (207) how much that radius of the ſun, in which ſhe is, is ſhortened, and ſubtract the ſemidiameter of Venus from it. In thoſe countries where the middle can be obſerved, continue to obſerve the diſtance of Venus from the neareſt point of the ſun's limb till that diſtance increaſes no longer, and you then get the neareſt approach of their centers. If you cannot obſerve the middle, the leaſt diſtance may be thus found. Let vC, wC be two obſerved diſtances of the centers of Venus and the ſun; note the time when each obſervation was made, and you have the time through vw, and knowing the horary motion of Venus in it's apparent orbit, you will know vw. Hence you know Cv, Cw, vw, from which, compute the angle Cwv; therefore in the right angled triangle Chw, you know Cw, and the angle Cwh, to find Ch the leaſt diſtance required.

FIG. 159.

If ſeveral obſervations of this kind be made, the mean of the reſults will give the leaſt diſtance more accurately. If the teleſcope be mounted on a polar axis, it will be more convenient.

636. Secondly, the diſtance of Venus from the ſun's center may be found by a wire micrometer adapted to a teleſcope to meaſure the difference of right aſcenſions and declinations, or by Mr. DOLLOND's divided object glaſs micrometer; ſee my *Treatiſe on Practical Aſtronomy*, Chap. vi. Let P be the pole of the equator EQ, w the place of Venus; draw the great circles, PCF, PwD, and Cx parallel to DF. Then having determined the difference wx of declinations of Venus and the ſun's center, and the difference DF of their right aſcenſions, multiply DF by the coſine of FC the ſun's declination, and (13) you get Cx; knowing therefore wx, xC, in the right angled triangle wxC, you know wC. The diſtance of Venus from the ſun's center being twice taken, and the time between, you get the leaſt diſtance Ch as before. Having determined the difference of the right aſcenſions and declinations of the ſun and Venus at any time, you may find the difference of their longitudes by Art. 385.

637. The parallax of the ſun, from the tranſit of Venus, being determined from the difference of the times of the tranſits at two places, the concluſion will be moſt accurate when that difference is the greateſt poſſible. The places therefore to be choſen for the two obſervations ſhould be upon oppoſite meridians, and ſuch, that the middle of the tranſit may be when the ſun is upon the meridian; for under theſe circumſtances, the ingreſs at one place will be accelerated and the egreſs retarded, increaſing thereby the time of the tranſit, and the ingreſs at the other place will be retarded and the egreſs accelerated, by which the time of the tranſit will be diminiſhed; the difference therefore of the times of the tranſits at the two places will thus become the greateſt. As the tranſit muſt be obſerved under oppoſite meridians, it muſt happen in the day at one of the places, and at night at the other; the place therefore where it happens in the night muſt be ſo near to the north or ſouth pole, according as the declination of the ſun is north or ſouth, that the ingreſs may be obſerved before the ſun ſet, and the egreſs the next morning after it riſes. Hence the tranſits of Venus which happen in June are more convenient than thoſe which happen in December, becauſe there is a great choice of ſituations towards the north pole, which is not the caſe towards the ſouth. Dr. HALLEY made a miſtake, by ſetting off the axis of the planet's orbit on the *ſame* ſide of the ecliptic that the axis of the equator was ſituated, inſtead of the *contrary* ſide. By uſing therefore the difference of theſe two angles inſtead of their ſum, he made the difference of the times of the tranſit in 1761 ſeen at the Ganges and Port Nelſon (two places recommended by him for obſerving this tranſit) longer by 29″ than it ought, as computed by Dr. HORNSBY; ſee the *Phil. Tranſ.* 1763.

To

To determine at what Countries the Ingreſs and Egreſs are viſible.

638. Elevate the north or ſouth pole of the terreſtrial globe above the horizon equal to the ſun's declination at the time of the tranſit, according as the declination is north or ſouth. Bring Greenwich, for inſtance, to the meridian, and ſet the index to twelve. Now for the ingreſs, turn the globe and ſet it to the hour the ingreſs happens, and the globe will be in a proper poſition for that time, the ſun being vertical to that hemiſphere of the earth above the horizon of the globe. The beginning of the tranſit is therefore viſible to that hemiſphere. To thoſe places under the *weſtern* ſemicircle of the horizon, the ſun is then riſing, and therefore to ſuch places the tranſit begins at ſun riſe. To thoſe places under the *eaſtern* ſemicircle of the horizon, the ſun is then ſetting, and therefore the tranſit there begins at ſun ſet. To thoſe places lying under the meridian, the ingreſs begins at twelve o'clock.

Set the index, by turning the globe, to the time of the middle of the tranſit, and proceed as before. Then in the *weſtern* ſemicircle of the horizon you will ſee all thoſe places where the middle is at ſun riſe; and under the *eaſtern* ſemicircle of the horizon, thoſe where the middle is at ſun ſet. And the places under the meridian are thoſe where the tranſit is at the middle at twelve o'clock.

Turn the globe, and ſet the index to the time of the egreſs, and the egreſs will be viſible to all the countries above the horizon. The places lying under the meridian are thoſe where the tranſit ends at twelve o'clock. Under the *weſtern* ſemicircle of the horizon, lie the places where the end is at ſun riſe. And under the *eaſtern* ſemicircle of the horizon, lie the places where the end is at ſun ſet.

The times when any of theſe appearances happen at any other place, may be found, by taking the difference between the meridians, and converting it into time, and applying that difference to the time at Greenwich.

CHAP.

CHAP. XXVI.

ON THE NATURE AND MOTION OF COMETS.

Art. 639. COMETS are ſolid bodies, revolving in very excentric ellipſes about the ſun in one of the foci, and are therefore ſubject to the ſame laws as the planets, but differ in appearance from them; for as they approach the ſun, a tail of light, in ſome of them, begins to appear, which increaſes till the comet comes to it's perihelion, and then it decreaſes again, and vaniſhes; others have a light encompaſſing the nucleus, or body of the comet, without any tail. The moſt ancient philoſophers ſuppoſed comets to be like planets, performing their revolutions in ſtated times. ARISTOTLE, in his firſt book of *Meteors*, ſpeaking of comets, ſays, "But ſome of the Italians, called Pythagoreans, ſay, that a *Comet* is one of the *Planets*, but that they do not appear unleſs after a long time, and are ſeen but a ſmall time, which happens alſo to *Mercury*." SENECA alſo in *Nat. Queſt.* Lib. vii. ſays, "APOLLONIUS affirmed, that the *Comets* were, by the Chaldeans, reckoned among the *Planets*, and had their periods like them." SENECA himſelf alſo, having conſidered the phænomena of two remarkable comets, believed them to be ſtars of equal duration with the world, though he was ignorant of the laws that governed them; and foretold, that after ages would unfold all theſe myſteries. He recommended it to Aſtronomers to keep a catalogue of the comets, in order to be able to determine whether they returned at certain periods. Notwithſtanding this, moſt Aſtronomers from his time till TYCHO BRAHE, conſidered them only as meteors, exiſting in our atmoſphere. But that Aſtronomer, finding from his own obſervations on a comet, that it had no diurnal parallax, placed them above the moon. Afterwards KEPLER had an opportunity of obſerving two comets, one of which was very remarkable; and from his obſervations, which afforded ſufficient indications of an annual parallax, he concluded, "that comets moved freely through the planetary orbs, with a motion not much different from a rectilinear one; but of what kind he could not preciſely determine." HEVELIUS embraced the ſame hypotheſis of a rectilinear motion; but finding his calculations did not perfectly agree with his obſervations, he concluded, "that the path of a comet was bent in a curve line, concave towards the ſun." He ſuppoſed a comet to be generated in the atmoſphere of a planet, and to be diſcharged from it, partly by the rotation of the planet, and then to

revolve

revolve about the ſun in a parabola by the force of projection and it's tendency to the ſun, in the ſame manner as a projectile upon the earth's ſurface deſcribes a parabola. At length came the famous comet in 1680, which deſcending nearly in a right line towards the ſun, aroſe again from it in like manner, which proved it's motion in a curve about the ſun. G. S. Doërfell, Miniſter at Plaven in Upper Saxony, made obſervations upon this comet, and found that it's motion might be very well repreſented by a parabola, having the ſun in it's focus. He was ignorant however of all the laws by which the motion of a body in a parabola is regulated, and erred conſiderably in his parabola, making the perihelion diſtance about twelve times greater than it was. This was publiſhed five years before the *Principia*, in which work Sir I. Newton having proved that Kepler's law, by which the motions of the planets are regulated, was a neceſſary conſequence of his theory of gravity, it immediately followed, that comets were governed by the ſame law; and the obſervations upon them agreed ſo accurately with his theory, as to leave no doubt of it's truth. That the comets deſcribe ellipſes, and not parabolas or hyperbolas, Dr. Halley, (ſee his *Synopſis of the Aſtronomy of Comets*) advances the following reaſons.

"Hitherto I have conſidered the orbits of comets as exactly parabolic; upon which ſuppoſition it would follow, that comets, being impelled towards the ſun by a centripetal force, would deſcend as from ſpaces infinitely diſtant; and, by their ſo falling, acquire ſuch a velocity, as that they may again fly off into the remoteſt parts of the univerſe, moving upwards with a perpetual tendency, ſo as never to return again to the ſun. But ſince they appear frequently enough, and ſince none of them can be found to move with an hyperbolic motion, or a motion ſwifter than what a comet might acquire by it's gravity to the ſun, it is highly probable they rather move in very excentric elliptic orbits, and make their returns after long periods of time: for ſo their number will be determinate, and, perhaps, not ſo very great. Beſides, the ſpace between the ſun and the fixed ſtars is ſo immenſe, that there is room enough for a comet to revolve, though the period of it's revolution be vaſtly long. Now, the *latus rectum* of an ellipſis is to the *latus rectum* of a parabola, which has the ſame diſtance in it's perihelion, as the diſtance in the aphelion, in the ellipſis, is to the whole axis of the ellipſis. And the velocities are in a ſubduplicate ratio of the ſame: wherefore, in very excentric orbits, the ratio comes very near to a ratio of equality; and the very ſmall difference which happens, on account of the greater velocity in the parabola, is eaſily compenſated in determining the ſituation of the orbit. The principal uſe therefore of the Table of the elements of their motions, and that which indeed induced me to conſtruct it, is, that whenever a new comet ſhall appear, we may be able to

know, by comparing together the elements, whether it be any of thoſe which has appeared before, and conſequently to determine it's period, and the axis of it's orbit, and to foretel it's return. And, indeed, there are many things which make me believe, that the comet which APIAN obſerved in the year 1531, was the ſame with that which KEPLER and LONGOMONTANUS more accurately deſcribed in the year 1607; and which I myſelf have ſeen return, and obſerved in the year 1682. All the elements agree, and nothing ſeems to contradict this my opinion, beſides the inequality of the periodic revolutions: which inequality is not ſo great neither, as that it may not be owing to phyſical cauſes. For the motion of Saturn is ſo diſturbed by the reſt of the planets, eſpecially Jupiter, that the periodic time of that planet is uncertain for ſome whole days together. How much more therefore will a comet be ſubject to ſuch like errors, which riſes almoſt four times higher than Saturn, and whoſe velocity, though increaſed but a very little, would be ſufficient to change it's orbit, from an elliptical to a parabolical one. And I am the more confirmed in my opinion of it's being the ſame; for, in the year 1456, in the ſummer-time, a comet was ſeen paſſing retrograde between the earth and the ſun, much after the ſame manner; which, though nobody made obſervations upon it, yet, from it's period, and the manner of it's tranſit, I cannot think different from thoſe I have juſt now mentioned. And ſince looking over the hiſtories of comets, I find, at an equal interval of time, a comet to have been ſeen about Eaſter in the year 1305, which is an another double period of 151 years before the former. Hence, I think, I may venture to foretel that it will return again in the year 1758."

640. Dr. HALLEY computed the effect of *Jupiter* upon this comet in 1682, and found that it would increaſe it's periodic time above a year, in conſequence of which he predicted it's return at the end of the year 1758, or the beginning of 1759. He did not make his computations with the utmoſt accuracy, but, as he himſelf informs, *levi calamo*. M. CLAIRAUT computed the effects both of Saturn and Jupiter, and found that the former would retard it's return in the laſt period 100 days, and the latter 511 days; and he determined the time when the comet would come to it's perihelion to be on April 15, 1759, obſerving that he might err a month, from neglecting ſmall quantities in the computation. It paſſed the perihelion on March 13, within 33 days of the time computed. Now if we ſuppoſe the time ſtated by Dr. HALLEY to mean the time of it's paſſing the perihelion, then if we add to that 100 days, ariſing from the action of Saturn which he did not conſider, it will bring it very near to the time in which it did paſs the perihelion, and prove his computation of the effect of Jupiter to have been very accurate. If he mean the time when it would firſt appear, his prediction was very accurate, for it was firſt ſeen on

December

December 14, 1758, and his computation of the effects of Jupiter will then be more accurate than could have been expected, conſidering that he made his calculations only by an indirect method, and in a manner profeſſedly not very accurate. Dr. HALLEY therefore had the glory, firſt to foretel the return of a comet, and the event anſwered remarkably to his prediction. He further obſerved, that the action of Jupiter, in the deſcent of the comet towards it's perihelion in 1682, would tend to increaſe the inclination of it's orbit; and accordingly the inclination in 1682 was found to be 22′ greater than in 1607. A learned Profeſſor (Dr. LONG's Aſtronomy, p. 562) in Italy to an Engliſh gentleman writes thus. "Though M. de la LANDE, and ſome other French gentlemen, have taken occaſion to find fault with the inaccuracies of HALLEY's calculation, becauſe he himſelf had ſaid he only touched upon it ſlightly; nevertheleſs they can never rob him of the honour,—Firſt, of finding out that it was one and the ſame comet which appeared in 1682, 1607, 1531, 1456, and 1305.—Secondly, of having obſerved that the planet Jupiter would cauſe the inclination of the orbit of the comet to be greater, and the period longer. —Thirdly, of having foretold that the return thereof might be retarded till the end of 1758, or the beginning of 1759." From the obſervations of M. MESSIER upon a comet in 1770, M. EDRIC PROSPERIN, Member of the Royal Academies of Stockholm and Upſal, ſhowed, that a parabolic orbit would not anſwer to it's motions, and he recommended it to Aſtronomers to ſeek for the elliptic orbit. This laborious taſk M. LEXELL undertook, and has ſhown that an ellipſe, in which the periodic time is about five years and ſeven months, agrees very well with the obſervations. See the *Phil. Tranſ.* 1779. As the ellipſes which the comets deſcribe are all very excentric, Aſtronomers, for the eaſe of calculation, ſuppoſe them to move in parabolic orbits, for that part which lies within the reach of obſervation, by which they can very accurately find the place of the perihelion, it's diſtance from the ſun, the inclination of the plane of it's orbit to the ecliptic, and the place of the node. Previous therefore to the determination of the orbit of a comet from obſervation, we muſt premiſe ſuch particulars reſpecting the motion of a body in a parabola, as may be neceſſary for ſuch an inveſtigation. Several of the principles which we are here obliged to make uſe of, will be proved when we come to treat on the Phyſical Principles of Aſtronomy.

On the Motion of a Body in a Parabola.

FIG. 160. 641. Let APM be a parabola, S it's focus, A the vertex, P the place of the body, draw $P\mathcal{Q}$ perpendicular to AS, and PD perpendicular to the tangent PT, alſo SM perpendicular to AD. Now, by the property of the parabola, $\mathcal{Q}D$ is equal to half the latus rectum; hence if $AS=1$, then $\mathcal{Q}D=2$; alſo the angle $PSA=2\,PDA$; therefore if $\mathcal{Q}D$ be radius, $P\mathcal{Q}$ will be the tangent of PDA, or $\frac{1}{2}PSA$; hence to the radius AS, $P\mathcal{Q}$ will be twice the tangent of $\frac{1}{2}PSA$; therefore if $2t=P\mathcal{Q}$, t will be the tangent of (z) half the true anomaly PSA, to the radius $AS=1$. Alſo, by the property of the parabola, $A\mathcal{Q}\times 4\,AS=P\mathcal{Q}^2$, hence $A\mathcal{Q}=t^2$; alſo the area $A\mathcal{Q}P=\frac{4}{3}t^3$; and as $\mathcal{Q}S=1-t^2$, the area $\mathcal{Q}PS=t-t^3$; hence the area $ASP=\frac{1}{3}t^3+t$; alſo the area $ASM=\frac{4}{3}$. Now let a and b be the times in which the comet moves from A to M, and from A to P; then, as the areas deſcribed about S are proportional to the times, $a : b :: \frac{4}{3} : \frac{1}{3}t^3+t$, therefore $at^3+3at=4b$.

642. Hence if a, and the true anomaly be given, we have the time $b=\frac{1}{4}at^3+\frac{3}{4}at$. Alſo, becauſe $a : b :: \frac{4}{3} : \frac{1}{3}t^3+t$, therefore if the true anomaly, and conſequently t, be given in different parabolas, the times of deſcribing thoſe true anomalies from the perihelions will be in proportion to the times of deſcribing 90° from the perihelions.

643. If the times a and b be given, the true anomaly may be found from reſolving the cubic equation $t^3+3t=\frac{b}{\frac{1}{4}a}$, which may be done thus. In the

FIG. 161. right angled triangle CAB, let $AB=1$, $AC=\frac{b}{\frac{1}{2}a}$, and compute BC; then find two mean proportionals between $BC+AC$ and $BC-AC$, and their difference is the value of t.

644. Take the fluxion of $t^3+3t=\frac{4b}{a}$, and we have $\dot{t}=\frac{4}{3a}\times\frac{\dot{b}}{1+t^2}$; but $\dot{t}=\overline{1+t^2}\times\dot{z}$; hence we get $2\dot{z}=\frac{8}{3a}\times\frac{\dot{b}}{\overline{1+t^2}^2}=\frac{8}{3a}\times\overline{\text{col. } z}^4\times\dot{b}$ the variation of the true anomaly correſponding to any ſmall variation $\dot{b}$ of time expreſſed in decimals of a day, a being expreſſed in days.

FIG. 160. 645. Let SA be the mean diſtance of the earth from the ſun; then the area of the circle, deſcribed with that radius, will be 3,14159; alſo the area $AMS=\frac{1}{4}$. Now the velocity in the parabola : velocity in the circle :: $\sqrt{2}$: 1, and the areas deſcribed in the ſame time will be in the ſame ratio, becauſe

at

at A the motion in each orbit being perpendicular to SA, the areas deſcribed will be as the velocities, and it being ſo in one caſe, it muſt be always ſo, becauſe in each orbit reſpectively equal areas are deſcribed in equal times, as will be afterwards proved. But the times of deſcribing any two areas are as the areas directly, and the areas deſcribed in the ſame time inverſely; therefore $\frac{3,14159}{1}$: $\frac{4}{3\sqrt{2}}\left\{\frac{\sqrt{8}}{3}\right\}$ :: the time of the revolution in the circle $= 365d.\ 6h.\ 9'$: the time of deſcribing $AM = 109d.\ 14h.\ 46'.\ 20''$. Now as the time of deſcribing AM is in a given ratio to the time in the circle, which (as will be afterwards ſhown) varies as $AS^{\frac{3}{2}}$, therefore if $r =$ the perihelion diſtance in any other parabola, we have $1^{\frac{3}{2}} : r^{\frac{3}{2}} :: 109d.\ 14h.\ 46'.\ 20''$: the time of deſcribing 90° in that parabola from the perihelion. Hence, knowing the time correſponding to any true anomaly in that parabola whoſe perihelion diſtance $= 1$, we know the time correſponding to the ſame true anomaly in any other parabola, becauſe the (642) times of deſcribing 90° are as the times correſponding to the ſame true anomaly; therefore if n be the number of days correſponding to any given anomaly in that parabola whoſe perihelion diſtance is unity, then $nr^{\frac{3}{2}}$ will be the time t correſponding to the ſame anomaly in that whoſe perihelion diſtance is r; this may be readily found thus. Multiply the log. r by 3 and divide by 2, and to the quotient add the log. n, and the ſum will be the log. of the time required. Hence alſo $n = \frac{t}{r^{\frac{3}{2}}}$; therefore if from the log. t we ſubtract $\frac{3}{2}$ log. r, it gives the log. n of the number of days correſponding to the ſame anomaly in the parabola whoſe perihelion diſtance $= 1$; hence the anomaly will be found from the Table at the end of this Chapter, which exibits the times correſponding to the true anomaly for 200000 days from the perihelion, in that parabola whoſe perihelion diſtance is unity. This Table may be conſtructed by Art. 641. by taking $a = 109,\ 6154$, and aſſuming $b = 1,\ 2,\ 3,\ 4$, &c. and finding the correſponding values of t. Dr. HALLEY firſt conſtructed a Table of this kind. M. de la CAILLE changed it into a more convenient form, by putting the areas for the times; that which we have here given was computed by M. de LAMBRE.

646. Draw SY perpendicular to the tangent; then $SP : SY :: SY : SA$, therefore $\sqrt{SP} : \sqrt{SA} :: SP : SY ::$ rad. : cof. PSY, or $\frac{1}{2}PSA$ the true anomaly; or $SP : SA ::$ $\overline{\text{rad.}}^2 : \overline{\text{cof. } \frac{1}{2}\text{ true anom.}}^2$ Hence if $SA = 1$, and $a+x = \frac{1}{2}PSA$, $a-x = \frac{1}{2}pSA$, then $1 : \sqrt{SP} ::$ cof. $\overline{a+x}$: rad. and $\sqrt{Sp} : 1 ::$ rad. : cof. $\overline{a-x}$, hence $\sqrt{Sp} : \sqrt{SP} ::$ cof. $\overline{a+x}$: cof. $\overline{a-x}$.

647. Hence

647. Hence $SP = \frac{SA}{\text{cof. } \frac{1}{2} \text{ true anom.}^2}$, radius being unity; therefore from log. SA ſubtract twice the log. cof. $\frac{1}{2}$ true anomaly, and the remainder is the log. of the diſtance of the comet from the ſun.

FIG. 162. 648. Erect BD perpendicular to AB, take $BC = AB$, produce AC to E, and draw EDF perpendicular to AE, meeting AF parallel to BD in F, join AD, and draw DG, CH parallel to AB. Then, as $EAF = 45°$, $EA = EF$; alſo $FG = GD = AB$; hence $AF = BD + BA$, and $GH = BD - BA$; alſo, by ſimilar triangles, AF or $BD + BA : CD = GH$ or $BD - BA :: EF$ or $EA : ED ::$ rad. : tan. DAE; but $AB : BD ::$ rad. : tan. BAD, from which ſubſtract $45°$, and we have $BD + BA : BD - BA ::$ rad. : tan. of that difference. If $BD = \sqrt{SP}$, and $BA = \sqrt{Sp}$, then $\sqrt{SP} : \sqrt{Sp} ::$ rad. : tan. $BAD = \sqrt{\frac{Sp}{SP}}$; hence, to get that angle, take half the difference of the logarithms of SP and Sp, and add 10 to the index (becauſe in the log. tangents, the index of log. tan. of $45°$, or log. of rad. $= 1$, is 10, inſtead of 0,) and it gives the log. tangent of the angle; from which take $45°$, and we have $\sqrt{SP} + \sqrt{Sp} : \sqrt{SP} - \sqrt{Sp} ::$ rad. : tang. of that difference.

FIG. 160. 649. Hence if we know two radii SP, Sp, and the angle PSp between, we can find the two anomalies. For let a be $\frac{1}{4}$ of $ASP + ASp$, and x be $\frac{1}{4}$ of $ASP - ASp$, then $\frac{1}{2} ASP = a + x$, and $\frac{1}{2} ASp = a - x$; hence (646) $\sqrt{Sp} : \sqrt{SP} ::$ cof. $\overline{a+x}$: cof. $a - x ::$ (by plane Trig.) cof. $a \times$ cof. $x -$ ſin. $a \times$ ſin. x : cof. $a \times$ cof. $x +$ ſin. $a \times$ ſin. x, therefore $\sqrt{SP} + \sqrt{Sp} : \sqrt{SP} - \sqrt{Sp} ::$ cof. $a \times$ cof. $x :$ ſin. $a \times$ ſin. $x :: \frac{\text{cof. } a}{\text{ſin. } a} : \frac{\text{ſin. } x}{\text{cof. } x} ::$ cot. $a :$ tan. x. Now the ratio of the two firſt terms is found from the laſt Article, and as the angle PSp is given, the value of x will be given, hence we find a, and conſequently we know the ſum and difference of ASP, ASp, therefore we know the angles themſelves. If p lie on the other ſide of A, then we know a, to find x.

FIG. 163. 650. Given two diſtances SP, Sp from the focus to the curve of a parabola, and the angle between them, to find the parabola. With the centers P and p, and radii PS, pS, deſcribe two circular arcs rwt, mvn, to which draw the tangent $avwb$; draw ST perpendicular to ab, and biſect it in A, and it will be the vertex of the parabola; hence we may deſcribe the parabola.

Given the Elements of the Orbit of a Comet, to compute it's Place at any Time.

651. The elements of the orbit of a comet are, 1. The time when the comet paſſes the perihelion.—2. The place of the perihelion.—3. The diſtance of the perihelion from the ſun.—4. The place of the aſcending node.—5. The inclination of the orbit to the ecliptic. From theſe elements, the place at any time may be computed; and, for example, we ſhall take that given by M. de la CAILLE in his Aſtronomy. The comet in 1739, which was retrograde, paſſed it's perihelion on June 17, at 10*h*. 9′. 30″ mean time; the place of the perihelion was in 3ˢ. 12°. 38′. 40″; the perihelion diſtance was 0,67358, the mean diſtance of the earth from the ſun being unity; the aſcending node was in 0ˢ. 27°. 25′. 14″, and the inclination of the orbit 55°. 42′. 44″; to compute the place ſeen from the earth on Auguſt 17, at 14′. 20″ mean time.

Let *WAV* be the parabolic orbit of the comet, *N* the aſcending node, *P* the place of the comet, *T* the correſponding place of the earth, and draw *Pv* perpendicular to the ecliptic; produce *SN*, *Sv*, *SP*, *ST* to *n*, *u*, *p* and *t* the ſphere of the fixed ſtars, and deſcribe the great circles *np*, *nuϒt* and *pu*. FIG. 164.

I. The interval of time from the perihelion to the given time is 61*d*. 4*h*. 10′. 30″ = 61,174, whoſe log. is 1,786567; alſo the log. of ,67358 is 9,828388, $\frac{3}{2}$ of which log. (from the nature of logarithms) is 9,742582, which ſubtracted from 1,786567 leaves 2,043985, the log. of 110,6587 days, which, by the Table, anſwers to 3ˢ. 0°. 21′. 38″ the true anomaly *PSA* at the given time.

II. Subtract 3ˢ. 0°. 21′. 38″ from 3ˢ. 12°. 38′. 40″ the place of the perihelion, becauſe the comet was retrograde and had paſſed the perihelion, and it leaves 12°. 17′. 1″ for the heliocentric place *p* of the comet in it's orbit.

III. The longitude of *n* is 27°. 25′. 14″, alſo *pn* = 27°. 25′. 14″ − 12°. 17′. 1″ = 15°. 8′. 13″; hence rad. : coſ. *pnu* = 55°. 42′. 44″ :: tan. *pn* = 15°. 8′. 13″ : tan. *un* = 8°. 39′. 53″ the diſtance of the comet from the aſcending node, meaſured upon the ecliptic.

IV. Subtract this value of *un*, from the place of the node and there remains 18°. 45′. 21″ = ϒ*u* the true heliocentric place of the comet reduced to the ecliptic

V. As rad. : ſin. *pn* = 15°. 8′. 13″ :: ſin. *pnu* = 55°. 42′. 44″ : ſin. *pu* = 12°. 27′. 34″ the latitude ſeen from the ſun, which is ſouth.

VI. The true place *T* of the earth at the ſame time is 10ˢ. 24°. 34′. 36″; hence *TS*ϒ = 35°. 25′. 24″; therefore *TS*ϒ + ϒ*Su* = *TSv* = 1ˢ. 24°. 10′. 45″. Alſo *TS* = 1,0115.

VII. By Art. 646. $\overline{\text{coſ. } 45°.\ 10'.\ 49''}^{2}$: $\overline{\text{rad.}}^{2}$ :: ,67358 : *SP* = 1,3557.

VIII. As

VIII. As rad. : cof. $PSv = 12°.\ 27'.\ 34''$ :: $SP = 1{,}3557$: $Sv = 1{,}32377$.

IX. In the triangle $\mathcal{T}Sv$, we know $\mathcal{T}S$, Sv and the included angle $\mathcal{T}Sv$; hence, by plain Trigonometry, we find the angle $S\mathcal{T}v = 77°.\ 33'.\ 38''\frac{1}{2}$, which ſubtracted from $4^s.\ 24°.\ 34'.\ 36''$, the place of the ſun, leaves $2^s.\ 7°.\ 0'.\ 57''\frac{1}{2}$ for the comet's true *geocentric longitude*.

X. By Art. 278, as ſin. $54°.\ 10'.\ 45''$: ſin. $77°.\ 33'.\ 38''\frac{1}{2}$ :: tan. $PSv = 12°.\ 27'.\ 34''$: tan. $P\mathcal{T}v = 14°.\ 54'.\ 4''$ the comet's true *geocentric latitude*.

To determine the Orbit of a Comet from Obſervation.

652. Sir I. NEWTON firſt reſolved this problem, which he called *Problema longe difficillimum*. The orbit of a comet may be computed from three obſervations; but although that data be ſufficient, the direct ſolution of the problem is impractible. Aſtronomers therefore have ſolved this problem by indirect methods, firſt finding an orbit very near to the truth by mechanical and graphical operations, and then, by computation, correcting it, until ſuch a parabola was found as would ſatisfy the obſervations. We ſhall therefore begin, by ſhowing the methods by which the orbit may be nearly determined; and then explain the manner in which it may be corrected by calculation.

653. M. de la LANDE propoſes the following mechanical method of finding the orbit nearly. Divide the diſtance of the earth from the ſun into ten equal parts, and deſcribe ten parabolas whoſe perihelion diſtances are, 1, 2, 3, &c. of theſe parts, and divide theſe parabolas into days from the perihelion, anſwering to the motion of a body in each. Let S be the ſun, a, b, c, the places of the earth at the times of three obſervations of the comet. Then take three geocentric latitudes and longitudes of the comet, and ſet off the elongations $Sa\alpha$, $Sb\beta$, $Sc\gamma$ in longitude. From a, b, c, extend three fine threads am, an, ap, vertical to $a\alpha$, $b\beta$, $c\gamma$, making angles with them equal to the geocentric latitudes reſpectively. Then take any one of the parabolas, and placing it's focus in S apply the edge to the threads, and obſerve whether you can make it touch them all, and whether the intervals of time cut off by the threads upon the parabola be equal to the reſpective intervals of the obſervations, or very nearly ſo; and if theſe circumſtances take place, you have then gotten the true parabola, or very nearly the true one. But if the parabola do not agree, try others, till you find one which does agree, or very nearly ſo, and you will then have got very nearly the true parabola, whoſe inclination, place of the node, and perihelion are to be determined as accurately as poſſible from menſuration; alſo the projection upon the ecliptic.

FIG. 165.

If

If none of theſe parabolas ſhould nearly anſwer, it ſhows, that the perihelion diſtance muſt be greater than the diſtance of the earth from the ſun, in which caſe, other parabolas muſt be conſtructed; but this does not very often happen. This method will determine the elements very nearly; but it would be extremely troubleſome to conſtruct and divide ſo many parabolas, if we only wanted to compute the elements of one comet; for thoſe who purpoſe to make many computations of this kind, it might be worth while to have a ſet of parabolas thus divided. To avoid this trouble therefore, I propoſe to do it in the following manner by means of one parabola, without dividing it.

654. Take a firm board perfectly plane, and fix on paper for the projection; let a groove be cut near the edge, and five perpendiculars be moveable in it, ſo that they may be fixed at any diſtances. Let S repreſent the ſun, and deſcribe any number of circles about it. Compute five geocentric latitudes and longitudes of the comet, from which you will have the five elongations of the comet at the times of the reſpective obſervations. Draw SA, SB, SC, SD, SE, making the angles ASB, BSC, CSD, DSE, equal to the ſun's motion in the intervals of the obſervations; and on any one of the circles, make the angles $Sa\alpha$, $Sb\beta$, $Sc\gamma$, $Sd\delta$, See equal to the reſpective elongations in longitude, and fix the five perpendiculars, ſo that the edge of each may coincide with α, β, γ, δ, ε. From the points a, b, c, d, e, extend threads to the reſpective perpendiculars, making angles * with the plane equal to the geocentric latitudes of the comet; then fix the focus of the parabola in S, and apply it's edge to the threads, and if it can be made to touch them all, it will be the parabola required, correſponding to the mean diſtance Sa of the earth, which we here ſuppoſe to revolve in a circle, as it will be ſufficiently accurate for our purpoſe. If the parabola cannot be made to touch all the threads, change the points a, b, c, d, e, to ſuch of the other circles as you may judge, from your preſent trial, will be moſt likely to ſucceed, and try again; and by a few repetitions you will get ſuch a diſtance for the earth, that the parabola ſhall touch all the threads, in which poſition, find the inclination, obſerve the place of the node, and meaſure the perihelion diſtance, compared with the earth's diſtance, and you will get very nearly the elements of the orbit.

FIG. 166.

655. The next method of approximating to the orbit of a comet, which we ſhall explain, is that given by BOSCOVICH. Let S be the ſun, XZ the orbit of the earth, ſuppoſed to be a circle; T the place of the earth at the firſt obſervation

FIG. 167.

* The eaſieſt and moſt correct method to ſet off theſe angles, is, for inſtance, to meaſure $a\alpha$, and then compute the perpendicular from the angle; and the ſame for the reſt

tion, and t at the third; draw $\mathcal{T}C$, tc to represent the observed longitudes of the comet; and let L, l, λ be the longitudes at the first, second and third observations; m and n the geocentric latitudes of the comet at the first and third observations; and t, $\mathcal{T}$, the intervals of time between the first and second, second and third observations. Assume C for the place of the comet, at the first observation, reduced to the ecliptic; then to determine the point at the third observation, say, $\mathcal{T} \times \text{sin. } \overline{\lambda - l} : t \times \text{sin. } \overline{l - L} :: \mathcal{T}C : tc$, and c will be nearly * the place required; join Cc, and it will represent the path of the comet on the ecliptic, upon this assumption. Perpendicular to the ecliptic draw CK, ck, taking $CK : \mathcal{T}C ::$ tan. m : radius, and $ck : tc ::$ tan. n : radius; join Kk, and it will represent the orbit of the comet, if the first assumption be true. Bisect Cc in x, and draw xy parallel to CK, and y will bisect Kk; join yS. Let $SX = 1$; then if v be the mean velocity of the earth in it's orbit, the velocity of the comet at $y = \frac{\sqrt{2} \times v}{\sqrt{Sy}}$; taking therefore $v = \mathcal{T}t$, compute $\frac{\sqrt{2} \times v}{\sqrt{Sy}}$, and if this be equal to Kk, measured by the scale, the assumed point C was the true point. But if these quantities be not equal, assume a new point for C, in doing which, the error of the first assumption will direct you which way, from the first assumed point, it must be taken, and about how far from it; if, for instance, the computed value of Kk be greater than the true value, and the lines CK, ck are diverging from each other, and receding from the sun, the point C must be taken further from $\mathcal{T}$, and how much further we must conjecture from the value of the error, and also from hence, that the velocity of the comet diminishes as it recedes from the sun. These considerations will lead us to make a second assumption near to the truth. Having thus determined the true points C, c, very nearly, produce cC, kK to meet at N, join NS, and it will be the line of the nodes. Draw Cr, cz perpendicular to SN, and the angles KrC, kzc will measure the inclination of the orbit. From the two distances SC, Sc, and the angle between, the parabola may be (650) constructed, and applied as in the last method, from which the time of passing the perihelion may be found.

656. Another method by which we may readily get the orbit very nearly, is this. Let S be the sun, $\mathcal{T}$, t, τ three places of the earth at the times of the three observations; extend three threads $\mathcal{T}p$, tn, τm in the directions of the comet, as directed in Article 654. Assume a point y for the place of the comet at the second observation, and measure Sy; then if $S\mathcal{T} = 1$, and the velocity

FIG. 168.

* For the proof of this, see the Author's paper upon the subject in his *Opuscula*, Vol. iii. or Sir H. ENGLEFIELD's very valuable Work upon Comets, page 27.

velocity of the earth be v, the velocity of the comet at y will be $\frac{\sqrt{2} \times v}{\sqrt{Sy}}$; let v be represented by $Tt, t\tau$; and upon any ſtraight edge $P\mathcal{Q}$, ſet off $ce = \frac{\sqrt{2} \times Tt}{\sqrt{sy}}$, and $ed = \frac{\sqrt{2} \times t\tau}{\sqrt{Sy}}$; then apply the point e to y, and, by turning about the edge, try whether you can make the point c fall in Tp, and the point d in τm; if you find this can not be done, the error will direct you to aſſume another diſtance; and by a very few trials you will find the point y where the points c and d will fall in Tp, τm. This method is very eaſy in practice, and ſufficiently accurate to obtain a diſtance Sy from which you may begin to compute, in order to find the orbit more correctly, when the comet is not too near to the ſun, as I have found by experience.

657. Having determined the parabola nearly, we firſt aſſume ſome one quantity as known at the firſt and ſecond obſervations, and thence compute the place of the comet at thoſe times, and alſo the time between; and if that time agree with the obſerved interval, you have got a parabola which agrees with the two firſt obſervations; if the times do not agree, alter one of the aſſumed quantities, and ſee how it then agrees; and then, by the rule of falſe, you may correct the ſuppoſition which was altered, and get a parabola which will agree with the two firſt obſervations. In like manner, by altering the other aſſumed quantity, you get another parabola agreeing with the two firſt obſervations. Then ſee how they agree with the third obſervation, and if they do not, a correction muſt be made by proportion, and the three obſervations will be anſwered. But this will be beſt explained by an example; we ſhall therefore take that which is given by M. de la CAILLE in his Aſtronomy, and explain the method of computation, and the reaſons of the whole operation.

658. In the year 1739, M. ZANOTTI, at Bologna, made the following obſervations on a comet, where the mean time is reduced to the meridian of Paris. Both the comet and it's tail were moſt vivid about the middle of June, and therefore it muſt have been in it's perihelion about that time.

			Longitude	Latitude
May	28, at	8h. 48′	. . . 7°. 6′ ☌	27°. 9′ N.
July	25, —	13. 59	. . . 16. 1 ♊ . . .	2. 27 N.
——	27, —	14. 8	. . . 15. 26 ♊	1. 3 N.
Auguſt	2, —	13. 5	. . . . 13. 34 ♊	2. 48 S.
——	4, —	13. 12	. . . . 12. 53½ ♊	4. 13 S.
——	10, —	12. 49	. . . 10 33½ ♊ . . .	8. 53 S.
——	17, —	14 20	. . . . 7. 1 ♊ . . .	14. 57 S.

659. From theſe obſervations, let the orbit of the comet be determined and projected upon the ecliptic, as nearly as poſſible, by one of the methods already explained, and let *XZ* be the projection, and *S* the ſun; then the projection ſhows the comet to be retrograde. The method uſed by M. de la CAILLE, is that which was firſt propoſed by Sir I. NEWTON in his Algebra, by cutting four right lines by another right line, ſo that the parts intercepted may be equal; but this problem is unlimited. In conſequence of his uſing this method, he was obliged to interpolate and get the latitude and longitude at four equidiſtant times, one of which was Auguſt 6, at midnight, at which time the comet's geocentric longitude was found to be 2ˢ. 12°. 17′, the ſun's longitude 4ˢ. 13°. 55′, it's diſtance from the earth 10136, and the comet's weſtern elongation from the ſun was 61°. 38′. This time is one which he aſſumes; but by our methods, one of the times of the obſervations might have been uſed inſtead.

FIG. 169.

660. The comet paſſing from north to ſouth latitude, or through the deſcending node, between July 27, and Auguſt 2, interpolate the obſervations on July 25, 27, and Auguſt 2, to find the time and place when the comet had no latitude; this is found to be on July 29, at 8*h*. 48′ mean time, in 14°. 54′ ♊; at which time, the ſun's place was 4ˢ. 6°. 7′. 10″, which gives the comet's elongation weſt 51°. 13′. 10″; and it's diſtance from the earth was 10148,5.

661. On May 28, at 8*h*. 48′ mean time, the ſun's place was in 2ˢ. 6°. 56′. 10″, and therefore the comet's elongation was to the eaſt of the ſun 30°. 9′. 50″, and the ſun's diſtance from the earth was 10142. The interval of time between this and when the comet was in it's node is 62 days.

662. Now to find a parabola which anſwers to theſe two times on May 28, and July 29, let *a* be the place of the earth on Auguſt 6, at midnight, make the angles *aSL*, *aSK* equal to the motion of the earth from May 28, at 8*h*. 48′, and July 29, at 8*h*. 48′, to Aug. 6, at midnight, and take *SL* = 10148,5, and *SK* = 10142, then will *L* and *K* be the reſpective places of the earth at the two former times; and make the angles *SLN*, *SKM* equal to 51°. 13′. 10″, and 30°. 9′. 50″, the reſpective weſtern and eaſtern elongations of the comet, and *N* and

N and M will be the corresponding places of the comet on the ecliptic, and N the node. Measure, upon the scale, the lines SM and SN, and suppose them to be found 5500 and 10700.

663. *First Supposition.* Let $SM = 5500$, $SN = 10700$. Now in the triangle SKM, we have $SK = 10142$, $SM = 5500$, and the angle $SKM = 30°.\ 9'.\ 50''$; hence the angle $KSM = 81°.\ 54'.\ 8''$; which subtracted from the longitude of $K = 8^s.\ 6°.\ 56'.\ 10''$, leaves $5^s.\ 15°.\ 2'.\ 2''$ for the heliocentric longitude of the comet.

664. By Art. 278. sin. ang. $SKM = 30°.\ 9'.\ 50''$: sin. ang. $KSM = 81°.\ 54'.\ 8''$ :: tang. geoc. lat. $27°.\ 9'$: tang. hel. lat. $= 45°.\ 17'.\ 49''\frac{1}{2}$. And (plain Trig.) cos. hel. lat. $= 45°.\ 17'.\ 49''\frac{1}{2}$: rad. :: curt. dist. $SM = 5500$: 7818,84 the true distance of the comet from the sun in the plane of it's orbit.

665. For the position on July 29, in the triangle SLN, we have $SL =$ 10148,5, $SN = 10700$, and the angle $SLN = 51°.\ 13'.\ 10''$; hence the angle $LSN = 81°.\ 6'.\ 7''\frac{1}{2}$; which added to the longitude of $L = 10^s.\ 6°.\ 7'.\ 10''$, gives $0^s.\ 27°,\ 13'.\ 17''\frac{1}{2}$ for the heliocentric longitude of the comet. Now as the comet is in it's node at N, LN is the true distance of the comet from the sun, and it's latitude is nothing.

666. As the difference of the heliocentric longitudes of the comet is $137°.\ 48'.\ 44''\frac{1}{2}$, take an arc MN equal to that quantity, and make a spherical triangle, right angled at M, whose perpendicular $Mm = 45°.\ 17'.\ 49''\frac{1}{2}$, the comet's heliocentric latitude on May 28; and Nm will measure the angle described by the comet about the sun in the interval of these two times; now rad. : cos. $MN = 137°.\ 48'.\ 44''\frac{1}{2}$ :: cos. $Mm = 45°.\ 17'.\ 49''\frac{1}{2}$: cos. $Nm = 121°.\ 24'.\ 48''$.* FIG. 170.

667. Hence if WAZ be the true parabola which the comet describes about the sun S, we have found $SN = 10700$, $Sm = 7818{,}84$, and the angle mSN, $= 121°.\ 24'.\ 48''$, to determine the angles ASm, ASN, and the perihelion distance AS. FIG. 171.

668. By Art. 648. take half the difference of the logarithms of SN and Sm, which is 0,068121, add 10 to the index, and it becomes 10,068121, which is the log. tangent of $49°.\ 28'.\ 31''$, from which subtract 45°, and there remains $4°.\ 28'.\ 31''$; hence (649), rad. : tang. $4°.\ 28'.\ 31''$ :: cot. $\frac{1}{4}\overline{NSA + mSA} = 30°.\ 21'.\ 12''$: tan. $7°.\ 36'.\ 45''\frac{1}{2}$ which is $\frac{1}{4}$ of $\overline{NSA - mSA}$; hence, knowing the

* If the comet had not been in it's node N at one of the times, but at m', and $m'M'$ be perpendicular to NM, the heliocentric latitude $M'm'$ must have been calculated, and then from knowing two heliocentric latitudes Mm, $M'm'$, and difference MM' of longitudes, we can (281, or 282) find the place of the node N, and the inclination of the orbit; and then calculating Nm, Nm', we get mm' the angle which the comet has described about the sun; and from the curtate distance at M', and the heliocentric latitude, we can find the distance of the comet from the sun, as at the other time.

the ſum and difference of ASN, ASm, we find $ASN = 75°.\ 55'.\ 55''$, and $mSA = 45°.\ 28'.\ 53''$, the true anomalies.

669. To find SA, we have (646), $\overline{\text{rad.}}^2 : \overline{\text{col. } \frac{1}{2}\ ASm}^2 = 22°.\ 44'.\ 26''\frac{1}{2} :: 7818{,}84 : AS = 6650{,}5$, the mean diſtance of the earth from the ſun being 10000; but if we call that diſtance unity, then $AS = 0{,}66505$.

670. Now (Table III.) the number of days correſponding to the anomalies $45°.\ 28'.\ 53''$ and $75°.\ 55'\ 55''$, are 36,4781 and 77,1725, whoſe ſum is 113,6506; hence (645) if to $\frac{3}{2}$ log. of 0,66505 we add the log. of 113,6506, we have the log. of 61,638 days for the time from m to N, which ſhould have been 62 days. Hence we muſt make a new ſuppoſition.

671. *Second Suppoſition.* Let $SM = 5600$, and $SN = 10700$ as before. Then, proceeding as before, the heliocentric longitudes will be found to be $5^s.\ 12°.\ 38'$ and $0^s.\ 57°.\ 13'.\ 17''\frac{1}{2}$; the latitude on May 28, $= 45°.\ 26'.\ 32''\frac{1}{2}$; the log. of $Sm = 3{,}902083$; the angle $mSA = 45°.\ 32'.\ 40''\frac{1}{2}$, $NSA = 74°.\ 26'.\ 1''\frac{1}{2}$, the days (Table III.) correſponding to which are 36,541 and 74,448; the logarithm of SA is 9,831591, calling the mean diſtance of the earth unity; hence we have the time from m to $N = 62{,}039$ days.

672. Hence by increaſing SM 100 parts, the time has been increaſed 0,401 days; therefore, by the rule of falſe, 0,401 : 100 :: 0,362 of a day (the number wanting in the firſt ſuppoſition) : 90,5; increaſe therefore SM in the firſt ſuppoſition by 90,5, inſtead of 100.

673. *Third Suppoſition.* Let $SM = 5590{,}5$, $SN = 10700$. Then, by a like proceeding, the heliocentric longitudes will be found to be $5^s.\ 12°.\ 50'.\ 53''$ and $0^s.\ 27°.\ 13'.\ 17''\frac{1}{2}$; the latitude on May 28, $= 45°.\ 25'.\ 54''$; the log. of $Sm = 3{,}901264$; the angle $mSA = 45°.\ 32'.\ 8''$, $NSA = 74°.\ 34'.\ 16''$, the days (Table III.) correſponding to which are 36,531 and 74,693; the log. of SA is 9,830802; hence we have the time from m to $N = 62{,}001$ days, which anſwers very accurately to the obſerved time. A parabola therefore being found which agrees with two obſervations, we muſt ſee how it will agree with ſome third obſervation, for inſtance, that on Auguſt 17. To do this, we muſt firſt find all the other elements of the parabola.

674. Now the deſcending node N is in $0^s.\ 27°.\ 13'.\ 17''$; add to this the angle $NSA = 74°.\ 34'.\ 16''$, and it gives $3^s.\ 21°.\ 47'.\ 33''$, for the longitude of the perihelion on it's orbit. The time in the Table correſponding to the angle NSA is 74,693; hence (645) to $\frac{3}{2}$ log. of SA add the log. of 74,693, and it gives the log. of 41,6374 days, the time of deſcribing the angle ASN; ſubtract this from July 29, $8h.\ 48'$, the time when the comet was in it's node, and it gives June 17, $17h.\ 30'$ the time when the comet was in it's perihelion. Alſo as $NM = 120°.\ 6'.\ 24''$, and $Mm = 45°.25'.\ 54''$, the angle $MNm = 55°.\ 26'.\ 16''$, the inclination of the orbit.

675. From theſe elements, calculate (651) the geocentric longitude of the comet at ſome other time at which it was obſerved, for inſtance, on Auguſt 17, at 14*h*. 20′, and it will be found to be in ♊ 6°. 55′. 37″, differing 5′. 23″ from obſervation; the parabola therefore does not ſatisfy this third obſervation We muſt therefore make another ſuppoſition.

676. *Fourth Suppoſition.* Let $SM=5500$ and $SN=10800$. Then, proceeding as before, the heliocentric longitudes are 5ˢ. 15°. 2′. 2″ and 0ˢ. 27°. 47′. 4″ for May 28 and July 29, the latitude on May 28, $=47°.\ 17'.\ 49''\frac{1}{2}$; the logarithm of $Sm=3{,}893142$; the angle $mSA=44°.\ 49'.\ 57''$, $NSA=76°.\ 16'.\ 5''$; the correſponding days (Table III.) are 35,8365 and 77,8022; and the logarithm of $SA=9{,}824898$; hence the time from *m* to *N* = 62,068 days.

677. Hence this ſuppoſition, compared with the firſt, ſhows, that by increaſing *SN* by 100, the time has been increaſed 0,43 days; hence 0,43 : 100 :: 0,362 (the defect of the time from the firſt ſuppoſition) : 84 the quantity by which *SN* ſhould have been increaſed in the firſt ſuppoſition, to have made the time 62 days. Hence

678. *Fifth Suppoſition.* Let $SM=5500$ and $SN=10784$. Then, as before, the reſpective heliocentric longitudes will be found to be 5ˢ. 15°. 2′. 2″ and 0ˢ. 27°. 42′. 25″; the latitude $=45°.\ 17'.\ 49''\frac{1}{2}$; the logarithm of $Sm=3{,}893142$; the angle $mSA=44°.\ 55'.\ 54''$, $NSA=76°.\ 12'.\ 43''$; the correſponding days (Table III.) are 35,934 and 76,697; the logarithm of $SA=9{,}824588$; hence the time from *m* to *N* = 61,999 days, anſwering extremely nearly.

679. Hence, determine (671) the other elements of the orbit, and we ſhall find the deſcending node in 0ˢ. 27°. 42′. 25″; the perihelion in 3ˢ. 13°. 55′. 8″; the time of the paſſage through the perihelion June 16, at 23*h*. 23′; and the inclination 56°. 8′. 44″.

680. With theſe elements, calculate (651) the geocentric longitude of the comet on Auguſt 17, at 14*h*. 20′, and it will be found in ♊ 7°. 8′. 42″, which exceeds that by obſervation by 7′. 42″.

681. Now as the corrections made to the diſtances *SM*, *SN* are very nearly in proportion to the differences between the calculations and the obſervations, we have, as 5′. 23″ + 7′. 42″ (the ſum of the errors, or differences between the calculations and obſervations) : 5′. 23″ (the error from the third ſuppoſition) :: 90,5 and 84 (the corrections made to *SM*, *SN*) : 36,75 and 34 the corrections neceſſary to be made to ſatisfy all the conditions. If the two errors had been of the ſame kind, the firſt term muſt have been their difference.

682. To apply theſe corrections, it is manifeſt, from comparing the computations with the obſervations, that *SM* is too much in the third ſuppoſition, and too little in the fifth; therefore ſubtract 36,75 from 5590,5, and it gives

5553,75

5553,75 for SM; and, that the time may remain the ſame, add 34 to SN in the third ſuppoſition, and it gives $SN = 10734$. With theſe values make a

683. *Sixth Suppoſition.* Let $SM = 5553{,}75$, $SN = 10734$. Compute as before, and the heliocentric longitudes will be found to be $5^s.\ 13^\circ.\ 42'.\ 15''$, and $0^s.\ 27^\circ.\ 25'.\ 14''$; the latitude $= 45^\circ.\ 23'.\ 9''$ on May 28; the logarithm of $Sm = 3{,}898045$; the angle $mSA = 45^\circ.\ 16'.\ 54''$, $NSA = 75^\circ.\ 13'.\ 26''$, the days (Table III.) correſponding to which are 36,28 and 75,872; the logarithm of $SA = 9{,}828388$, and the time from m to N is 62 days. Hence, from theſe correct elements, as before explained, the place of the deſcending node is found to be in $0^s.\ 27^\circ.\ 54'.\ 14''$; the place of the perihelion in $3^s.\ 12^\circ.\ 38'.\ 40''$; the paſſage through the perihelion June 17, at $10h.\ 9'.\ 30''$ mean time; the inclination of the orbit $= 55^\circ.\ 42'.\ 44''$; and the true place of the comet (651) on Auguſt 17, at $14h.\ 20'$ is in $7^\circ.\ 0'.\ 57''\frac{1}{2}$ ♊, with $14^\circ.\ 54'.\ 4''$ ſouth latitude, agreeing, very nearly, with the obſervations.

684. When we compute to ſee how the aſſumptions agree with the obſervations on Auguſt 17, we might have compared the latitudes inſtead of the longitudes; and it will be beſt to do ſo, when the latitudes vary faſter than the longitudes, which will happen when the inclination of the orbit is very conſiderable, and the comet near the node.

685. If it ſo happen that the node does not lie ſo near the obſervations that it's place can be found by interpolation, after finding the orbit, compute two heliocentric latitudes and longitudes near to the node, and then (281, or 282) the place of the node, and the inclination of the orbit may be found at the ſame time. In this caſe, one of the firſt aſſumed times does not give the projected point N that of the node; but this makes no difference in the operation, except that we muſt then compute the heliocentric latitude (as at the other time), in order to get the diſtance of the comet from the ſun, and the angle deſcribed about the ſun, as explained in the *firſt* ſuppoſition

686. As the comets do not move in parabolas, but in very excentric ellipſes, it is impoſſible to find a parabola agreeing accurately to all the *data*; it will be ſufficient therefore when it agrees very nearly. If after aſſuming SM, SN, you find the reſult to differ very conſiderably from obſervation, the proportion will not give you their values ſufficiently near; in that caſe, you muſt compute again the error, and repeat the operation till the reſult from the computation does agree with the obſervations.

687. In Art. 266. it is proved, that if you increaſe any one quantity in the *data* from which a calculation is to be made, by a very ſmall quantity, the reſult will vary in proportion to that increaſe. Hence, the reaſon of the whole operation will be manifeſt from this Table.

$SM =$

	1st Sup.	2d Sup.	3d Sup.	4th Sup.	5th Sup.	3d Sup.	5th Sup.	6th Sup.
$SM=$	A	$A+a$	$A+\alpha$	A	A	$A+\alpha$	A	$A+\alpha+\alpha'$
$SN=$	B	B	B	$B+b$	$B+\beta$	B	$B+\beta$	$B+\beta'$
	m	n	o	r	o	x	y	o
	Error in the interval of *Time*.					Error in *Long*. or *Lat*.		

688. The ſuppoſitions A and B produce an error m; alſo $A+a$ and B produce an error n; hence, from what we have juſt now explained, as the difference of the reſults $m \pm n$ (according as the errors are of different or the ſame affections) muſt vary as a varies, $m \pm n : m :: a : \alpha$ the quantity by which we muſt alter A, in order to deſtroy the error m, or to make the error $=$ o. In like manner, the ſuppoſitions A and $B+b$ produce an error r; therefore $m \pm r : m :: b : \beta$ the quantity by which B muſt be altered in order to deſtroy the error m. Hence we have got two ſuppoſitions, or two parabolas, which will anſwer the firſt condition, that is, of the time. Now for the ſecond condition, the third and fifth ſuppoſitions will produce an error of x and y reſpectively; one of theſe ſuppoſitions therefore muſt be corrected, ſo that no error may remain in either condition; let therefore $A+\alpha+\alpha'$ and $B+\beta'$ be the values of SM and SN to ſatisfy both. Now A altered by α produces an effect m, for it corrects the whole error; hence $\alpha : \alpha' :: m : \frac{m\alpha'}{\alpha}$ the error that would be made by altering A by α'; and as B altered by β produces an effect m, we have $\beta : \beta' :: m : \frac{m\beta'}{\beta}$ the effect that would ariſe by altering B by β'; hence, that no error may be produced in the time in the third ſuppoſition, by adding α' to $A+\alpha$, and β' to B, theſe two effects thus produced muſt deſtroy each other, or $\frac{m\alpha'}{\alpha} + \frac{m\beta'}{\beta} = 0$, or $\frac{\alpha'}{\alpha} = -\frac{\beta'}{\beta}$. Hence, that no error may be made in the time in the third ſuppoſition, by altering the values SM and SN, the increments or decrements muſt be in the ratio of $\alpha : \beta$; this was done in Art. 681. and therefore the firſt condition will remain fulfilled. Now the changing of $A+\alpha$ to A, and of B to $B+\beta$, together produce an effect $x \pm y$, according as they are of different, or the ſame affections; hence, to produce the effect x, or to deſtroy that error, as the effect is in proportion to the variations of A and B,

$x \pm y : x :: \alpha$ and $\beta : \acute{\alpha}$ and β', the corrections to be applied to $A+\alpha$ and B to fulfil the ſecond condition, or make the error $=0$; and this alſo took place in Art. 681. Hence, by aſſuming $SM=A+\alpha+\acute{\alpha}$, and $SN=B+\beta'$, both errors are deſtroyed. Although a, b, α, β, $\acute{\alpha}$, β', are annexed to A and B by the ſign $+$, yet it muſt be underſtood that the ſign muſt be $+$ or $-$, according to circumſtances.

689. When great accuracy is required, we muſt take into conſideration, the effect of aberration and parallax; the former may be computed by Art. 532. and the latter, by taking the horizontal parallax : that of the ſun $=8'',75$ :: the diſtance of the ſun : the diſtance of the comet, and then finding the parallax in latitude and longitude, as for the planets.

Ex. On Auguſt 21, 1769, the diurnal motion of a comet was $63'$ in longitude, and $25'$ in latitude, and it's diſtance from the earth 0,667. Hence (532) the aberration in longitude $=14''$, and in latitude $=6''$, both to be added. Now the apparent longitude was $47°.\ 1'.\ 31''$, and latitude $5°.\ 53'.\ 48''$; hence the apparent longitude corrected for aberration was $47°.\ 1'.\ 45''$, and latitude $5°.\ 53'.\ 54''$. Alſo 0,667 : 1 :: $8'',75$: $13''$ the horizontal parallax. Hence, the parallax in longitude is found to be $4''$, to be added to the true, to give the apparent longitude, and as the true longitude (by computation) was $47°.\ 2'.\ 3''$, the apparent ought to have been $47°.\ 2'.\ 7''$; hence the error in longitude was $22''$. Alſo the parallax in latitude was $10''$, to be added to the true, to give the apparent latitude, and as the true latitude (by computation) was $5°.\ 54'.\ 16''$, the apparent ought to have been, $5°.\ 54'.\ 26''$; hence the error in latitude was $32''$.

690. It is extremely difficult to determine, from computation, the elliptic orbit of a comet, to any degree of accuracy; for when the orbit is very excentric, a very ſmall error in the obſervation will change the computed orbit into a parabola, or hyperbola. Now, from the thickneſs and inequality of the atmoſphere with which the comet is ſurrounded, it is impoſſible to determine, with any great preciſion, when either the limb or center of the comet paſs the wire at the time of obſervation. And this uncertainty in the obſervations will ſubject the computed orbit to a great error. Hence it happened, that M. Bouguer determined the orbit of the comet in 1729 to be an hyperbola. M. Euler firſt determined the ſame for the comet in 1744; but having received more accurate obſervations, he found it to be an ellipſe. The period of the comet in 1680 appears, from obſervation, to be 575 years, which M. Euler, by his computation, determined to be $166\frac{1}{2}$ years. The only ſafe way to get the period of comets, is to compare the elements of all thoſe which have been computed,

puted, and where you find they agree very well, you may conclude that they are elements of the *ſame* comet, it being ſo extremely improbable that the orbits of two different comets ſhould have the ſame inclination, the ſame perihelion diſtance, and the places of the perihelion and node the ſame. Thus, knowing the periodic time, we get the major axis of the ellipſe; and the perihelion diſtance being known, the minor axis will be known. When the elements of the orbits agree, the comets may be the ſame, although the periodic times ſhould vary a little; as that may ariſe from the attraction of the bodies in our ſyſtem, and which may alſo alter all the other elements a little. We have already obſerved, that the comet which appeared in 1759, had it's periodic time increaſed conſiderably by the attraction of *Jupiter* and *Saturn*. This comet was ſeen in 1682, 1607 and 1531, all the elements agreeing, except a little variation of the periodic time. Dr. HALLEY ſuſpected the comet in 1680, to have been the ſame which appeared in 1106, 531, and 44 years before CHRIST. He alſo conjectured, that the comet obſerved by APIAN in 1532, was the ſame as that obſerved by HEVELIUS in 1661; if ſo, it ought to have returned in 1790, but it has never been obſerved. But M. MECHAIN having collected all the obſervations in 1532, and calculated the orbit again, found it to be ſenſibly different from that determined by Dr. HALLEY, which renders it very doubtful whether this was the comet which appeared in 1661; and this doubt is increaſed, by it's not appearing in 1790. The comet in 1770, whoſe periodic time M. LEXELL computed to be 5 years and 7 months, has not been obſerved ſince. There can be no doubt but that the path of this comet, for the time it was obſerved, belonged to an orbit whoſe periodic time was that found by M. LEXELL, as the computations for ſuch an orbit agreed ſo very well with the obſervations. But the revolution was probably longer before 1770; for as the comet paſſed very near to Jupiter in 1767, it's periodic time might be ſenſibly increaſed by the action of that planet; and as it has not been obſerved ſince, we may conjecture, with M. LEXELL, that having paſſed in 1772 again into the ſphere of ſenſible attraction of Jupiter, a new diſturbing force might probably take place and deſtroy the effect of the other. According to the above elements, the comet would be in conjunction with Jupiter on Auguſt 23, 1779, and it's diſtance from Jupiter would be only $\frac{1}{491}$ of it's diſtance from the ſun, conſequently the ſun's action would be only $\frac{1}{224}$ times that of Jupiter. What a change muſt this make in the orbit! If the comet returned to it's perihelion in March 1776, it would then not be viſible. See M. LEXELL's account in the *Phil. Tranſ.* 1779. The elements of the orbits of the comets in 1264 and 1556 were ſo nearly the ſame, that it is very probable it was the ſame comet; if ſo, it ought to appear again about the year 1848.

On the Nature and Tails of Comets.

691. Comets are not viſible till they come into the planetary regions. They are ſurrounded with a very denſe atmoſphere, and from the ſide oppoſite to the ſun they ſend forth a tail, which increaſes as the comet approaches it's perihelion, immediately after which it is longeſt and moſt luminous, and then it is generally a little bent and convex towards thoſe parts to which the comet is moving; the tail then decreaſes, and at laſt it vaniſhes. Sometimes the tail is obſerved to put on this figure towards it's extremity ~, as that did in 1769. The ſmalleſt ſtars are ſeen through the tail, notwithſtanding it's immenſe thickneſs, which proves that it's matter muſt be extremely rare. The opinion of the ancient philoſophers, and of ARISTOTLE himſelf, was, that the tail is a very thin fiery vapour ariſing from the comet. APIAN, CARDAN, TYCHO, and others, believed that the ſun's rays being propagated through the tranſparent head of the comet, were refracted, as in a lens. But the figure of the tail does not anſwer to this; and, moreover, there ſhould be ſome reflecting ſubſtance to render the rays viſible, in like manner as there muſt be duſt or ſmoke flying about in a dark room, in order that a ray of light entering it may be ſeen by a ſpectator ſtanding ſide-ways from it. KEPLER ſuppoſed, that the rays of the ſun carry away ſome of the groſs parts of the comet which reflects the ſun's rays, and gives the appearance of a tail. HEVELIUS thought, that the thinneſt parts of the atmoſphere of a comet are rarefied by the force of the heat, and driven from the fore part and each ſide of the comet towards the parts turned from the ſun. Sir I. NEWTON thinks, that the tail of a comet is a very thin vapour, which the head, or nucleus of the comet, ſends out by reaſon of it's heat. He ſuppoſes, that when a comet is deſcending to it's perihelion, the vapours behind the comet in reſpect to the ſun, being rarefied by the ſun's heat, aſcend, and take up with them the reflecting particles with which the tail is compoſed, as air rarefied by heat carries up the particles of ſmoke in a chimney. But as beyond the atmoſphere of the comet, the ætherial air *(auram ætheream)* is extremely rare, he attributes ſomething to the the ſun's rays carrying with them the particles of the atmoſphere of the comet. And when the tail is thus formed, it, like the nucleus, gravitates towards the ſun, and by the projectile force it received from the comet, it deſcribes an ellipſe about the ſun, and accompanies the comet. It conduces alſo to the aſcent of theſe vapours, that they revolve about the ſun, and therefore endeavour to recede from it; whilſt the atmoſphere of the ſun is either at reſt or moves

with

with ſuch a ſlow motion as it can acquire from the rotation of the ſun about it's axis. Theſe are the cauſes of the aſcent of the tails in the neighbourhood of the ſun, where the orbit has a greater curvature, and the comet moves in a denſer atmoſphere of the ſun. The tail of the comet therefore being formed from the heat of the ſun, will increaſe till it comes to it's perihelion, and decreaſe afterwards. The atmoſphere of the comet is diminiſhed as the tail increaſes, and is leaſt immediately after the comet has paſſed it's perihelion, where it ſometimes appears covered with a thick black ſmoke. As the vapour receives two motions when it leaves the comet, it goes on with the compound motion, and therefore the tail will not be turned directly from the ſun, but decline from it towards thoſe parts which are left by the comet; and meeting with a ſmall reſiſtance from the æther, will be a little curved. When the ſpectator therefore is in the plane of the comet's orbit, the curvature will not appear. The vapour thus rarefied and dilated, may be at laſt ſcattered through the heavens, and be gathered up by the planets, to ſupply the place of thoſe fluids which are ſpent in vegetation and converted into earth. This is the ſubſtance of Sir I. NEWTON's account of the tails of comets. Againſt this opinion, Dr. HAMILTON, in his *Philoſophical Eſſays*, obſerves, that we have no proof of the exiſtence of a ſolar atmoſphere; and if we had, that when the comet is moving in it's perihelion in a direction at right angles to the direction of it's tail, the vapours which then ariſe, partaking of the great velocity of the comet, and being alſo ſpecifically lighter than the medium in which they move, muſt ſuffer a much greater reſiſtance than the denſe body of the comet does, and therefore ought to be left behind, and would not appear oppoſite to the ſun; and afterwards they ought to appear towards the ſun. Alſo, if the ſplendor of the tails be owing to the reflection and refraction of the ſun's rays, it ought to diminiſh the luſtre of the ſtars ſeen through it, which would have their light reflected and refracted in like manner, and conſequently their brightneſs would be diminiſhed. Dr. HALLEY, in his deſcription of the *Aurora Borealis* in 1716, ſays, "the ſtreams of light ſo much reſembled the long tails of comets, that at firſt ſight they might well be taken for ſuch." And afterwards, "this light ſeems to have a great affinity to that which the effluvia of electric bodies emit in the dark." *Phil. Tranſ.* N°. 347. D. de MAIRAN alſo calls the tail of a comet, the aurora borealis of the comet. This opinion Dr. HAMILTON ſupports by the following arguments. A ſpectator, at a diſtance from the earth, would ſee the aurora borealis in the form of a tail oppoſite to the ſun, as the tail of a comet lies. The aurora borealis has no effect upon the ſtars ſeen through it, nor has the tail of a comet. The atmoſphere is known to abound with electric matter, and the appearance of the electric matter in vacuo is exactly like the appearance of the aurora borealis, which, from it's great altitude, may be conſidered to be

be in as perfect a vacuum as we can make. The electric matter in vacuo ſuffers the rays of light to paſs through, without being affected by them. The tail of a comet does not expand itſelf ſideways, nor does the electric matter. Hence, he ſuppoſes the tails of comets, the aurora borealis, and the electric fluid, to be matter of the ſame kind. We may add, as a further confirmation of this opinion, that the comet in 1607 appeared to ſhoot out the end of it's tail. Le P. CYSAT remarked the undulations of the tail of the comet in 1618. HEVELIUS obſerved the ſame in the tails of the comets in 1652 and 1661. M. PINGRE' took notice of the ſame appearance in the comet of 1769. Theſe are circumſtances exactly ſimilar to the aurora borealis. Dr. HAMILTON conjectures, that the uſe of the comets may be to bring the electric matter, which continually eſcapes from the planets, back into the planetary regions. The arguments are certainly ſtrongly in favour of this hypotheſis; and if this be true, we may further add, that the tails are hollow; for if the electric fluid only proceed in it's firſt direction, and do not diverge ſideways, the parts directly behind the comet will not be filled with it; and this thinneſs of the tails will account for the appearance of the ſtars through them.

FIG. 172. 692. The length of a comet's tail may be thus found. Let *S* be the ſun, *E* the earth, *C* the comet, *CL* the tail when directed from the ſun; then knowing the place of the comet, we know the angle *ECL*, *EC*, and the angle *CEL*, the angle under which the tail appears; hence we find *CL* the length of the tail. If the tail deviate by any angle *LCM*, found from obſervation, then we ſhall know the angle *ECM*, with *CE*, and the angle *CEM*, to find *CM*. The tail of the comet in 1680 appeared under an angle of 70°, according to Sir I. NEWTON, and very brilliant; that of 1618, under an angle of 104°, according to LONGOMONTANUS; that of 1759, under an angle of 90°, according to M. PINGRE', but the light was very faint.

FIG. 173. 693. The limit of a comet's diſtance may be very eaſily aſcertained from it's tail, it being ſuppoſed to be directed from the ſun. For let *S* be the ſun, *E* the earth, *ET* the line in which the head of the comet appears, *EW* the line in which the extremity of the tail is obſerved, and draw *ST* parallel to *EW*; then the comet is within the diſtance *ET*; for if the comet were at *T*, the tail would be directed in a line parallel to *EW*, and therefore it never could appear in that line. Now we know *TEW* by obſervation, and conſequently it's equal *ETS*, together with *TES* the angular diſtance of the comet from the ſun, and *ES*, to find *ST* the limit. For example, on December 21, 1680, the diſtance of the comet from the ſun was 32°. 24′, and length of the tail 70°; hence, *ST* : *SE* :: ſin. 32°. 24′ : ſin. 70° :: 4 : 7 nearly, therefore the comet's diſtance from the ſun was leſs than $\frac{4}{7}$ of the earth's diſtance from the ſun.

Hence

Hence Sir I. NEWTON deduced this concluſion, that all comets, whilſt they are viſible, are not further diſtant from the ſun than three times the earth's diſtance from the ſun. This however muſt depend upon the goodneſs of the teleſcope, and magnitude of the comet.

694. In reſpect to the nature of comets, Sir I. NEWTON obſerves, that they muſt be ſolid bodies like the planets. For if they were nothing but vapours, they muſt be diſſipated when they come near the ſun. For the comet in 1680, when it was in it's perihelion, was leſs diſtant from the ſun than one ſixth of the ſun's diameter, conſequently the heat of the comet at that time was to the heat of the ſummer ſun as 28000 to 1. But the heat of boiling water is about three times greater than the heat which dry earth acquires from the ſummer ſun; and the heat of red hot iron about three or four times greater than the heat of boiling water. Therefore the heat of dry earth at the comet, when in it's perihelion, was about 2000 times greater than red hot iron. By ſuch heat, all vapours would be immediately diſſipated.

695. This heat of the comet muſt be retained a very long time. For a red hot globe of iron of an inch diameter, expoſed to the open air, ſcarce looſes all it's heat in an hour; but a greater globe would retain it's heat longer, in proportion to it's diameter, becauſe the ſurface, at which it grows cold, varies in that proportion leſs than the quantity of hot matter. Therefore a globe of red hot iron, as big as our earth, would ſcarcely cool in 50000 years.

696. The comet in 1680 coming ſo near to the ſun, muſt have been conſiderably retarded by the ſun's atmoſphere, and therefore being attracted nearer at every revolution, it will at laſt fall into the ſun, and be a freſh ſupply of fewel for what the ſun loſes by it's conſtant emiſſion of light. In like manner, fixed ſtars which have been gradually waſted, may be ſupplied with freſh fewel, and acquire new ſplendor, and paſs for new ſtars. Of this kind are thoſe fixed ſtars which appear on a ſudden, and ſhine with a wonderful brightneſs at firſt, and afterwards vaniſh by degrees. Such is the conjecture of Sir I. NEWTON.

697. From the beginning of our æra to this time, it is probable, according to the beſt accounts, that there have appeared about 500 comets. Before that time about 100 others are recorded to have been ſeen, but it is probable that not above half of them were comets. And when we conſider, that many others may not have been perceived, from being too near the ſun — from appearing in moon-light — from being in the other hemiſphere — from being too ſmall to be perceived, or which may not have been recorded, we might imagine the whole number to be conſiderably greater; but it is likely, that of the comets which are recorded to have been ſeen, the ſame may have appeared ſeveral times, and therefore the number may be leſs than is here

ſtated. The comet in 1786, which firſt appeared on Auguſt 1, was diſcovered by Miſs CAROLINE HERSCHEL, a ſiſter of Dr. HERSCHEL; ſince that time, ſhe has diſcovered three others. As the plan of this Work does not permit us to give all the different methods by which the orbits of comets may be computed, and all the various opinions reſpecting them, if the reader wiſh to ſee any thing further upon the ſubject, I refer him to a Treatiſe entitled, *Cométographie ou Traité Hiſtorique et Théorique des Comètes, par M. PINGRÉ, II. Tom. quarto, Paris*, 1784; or Sir H. ENGLEFIELD's *Determination of the Orbits of Comets*, a very valuable work, in which the ingenious Author has explained, with great clearneſs and accuracy, the manner of computing the orbits of comets, according to the methods of BOSCOVICH, and M. de la PLACE.

TABLES

TABLES

FOR

CONVERTING TIME INTO DECIMALS OF A DAY;

AND OF THE

PARABOLIC MOTION OF COMETS;

WITH

RULES FOR THE USE OF THE TABLES.

TABLE I.

FOR CONVERTING TIME INTO DECIMALS OF A DAY.

HOURS.	DECIMALS.	MINUTES.	DECIMALS.	MINUTES.	DECIMALS.
1	0,04166	1	0,000694	31	0,021527
2	0,08333	2	0,001388	32	0,022222
3	0,12500	3	0,002083	33	0,022916
4	0,16666	4	0,002777	34	0,023611
5	0,20833	5	0,003472	35	0,024305
6	0,25000	6	0,004166	36	0,025000
7	0,29166	7	0,004861	37	0,025694
8	0,33333	8	0,005555	38	0,026388
9	0,37500	9	0,006250	39	0,027083
10	0,41666	10	0,006944	40	0,027777
11	0,45833	11	0,007638	41	0,028472
12	0,50000	12	0,008333	42	0,029166
13	0,54166	13	0,009027	43	0,029861
14	0,58333	14	0,009722	44	0,030555
15	0,62500	15	0,010416	45	0,031250
16	0,66666	16	0,011111	46	0,031944
17	0,70833	17	0,011805	47	0,032638
18	0,75000	18	0,012500	48	0,033333
19	0,79166	19	0,013194	49	0,034027
20	0,83333	20	0,013888	50	0,034722
21	0,87500	21	0,014583	51	0,035416
22	0,91666	22	0,015277	52	0,036111
23	0,95833	23	0,015972	53	0,036805
24	1,00000	24	0,016666	54	0,037500
		25	0,017361	55	0,038194
		26	0,018055	56	0,038888
		27	0,018750	57	0,039583
		28	0,019444	58	0,040277
		29	0,020138	59	0,040972
		30	0,020833	60	0,041666

THE FIRST TABLE CONTINUED.

SECONDS.	DECIMALS.	SECONDS.	DECIMALS.
1	0,00001157	31	0,00035880
2	0,00002315	32	0,00037037
3	0,00003472	33	0,00038194
4	0,00004630	34	0,00039352
5	0,00005787	35	0,00040509
6	0,00006944	36	0,00041666
7	0,00008102	37	0,00042824
8	0,00009259	38	0,00043981
9	0,00010416	39	0,00045138
10	0,00011574	40	0,00046296
11	0,00012731	41	0,00047454
12	0,00013888	42	0,00048611
13	0,00015046	43	0,00049769
14	0,00016204	44	0,00050926
15	0,00017361	45	0,00052083
16	0,00018518	46	0,00053241
17	0,00019676	47	0,00054398
18	0,00020833	48	0,00055555
19	0,00021991	49	0,00056713
20	0,00023148	50	0,00057870
21	0,00024305	51	0,00059027
22	0,00025463	52	0,00060185
23	0,00026620	53	0,00061343
24	0,00027777	54	0,00062500
25	0,00028935	55	0,00063657
26	0,00030093	56	0,00064815
27	0,00031250	57	0,00065972
28	0,00032407	58	0,00067130
29	0,00033565	59	0,00068287
30	0,00034722	60	0,00069444

TABLE II.

FOR CONVERTING DECIMALS OF A DAY INTO TIME.

DEC.	H. M. S.	DEC.	H. M. S.	DEC.	M. S.	DEC.	M. S.	DEC.	S.
,1	2. 24. 0	,01	0. 14. 24	,001	1. 26,4	,0001	0. 8,64	,00001	0,864
,2	4. 48. 0	,02	0. 28. 48	,002	2. 52,8	,0002	0. 17,28	,00002	1,728
,3	7. 12. 0	,03	0. 43. 12	,003	4. 19,2	,0003	0. 25,92	,00003	2,592
,4	9. 36. 0	,04	0. 57. 36	,004	5. 45,6	,0004	0. 34,56	,00004	3,456
,5	12. 0. 0	,05	1. 12. 0	,005	7. 12,0	,0005	0. 43,20	,00005	4,320
,6	14. 24. 0	,06	1. 26. 24	,006	8. 38,4	,0006	0. 51,84	,00006	5,184
,7	16. 48. 0	,07	1. 40. 48	,007	10. 4,8	,0007	1. 0,48	,00007	6,048
,8	19. 12. 0	,08	1. 55. 12	,008	11. 31,2	,0008	1. 9,12	,00008	6,912
,9	21. 36. 0	,09	2. 9. 36	,009	12. 57,6	,0009	1. 17,76	,00009	7,776

TABLE III.

GENERAL TABLE OF THE PARABOLA. By M. De Lambre.

Days.	Anomaly. D. M. S.	Differ. M. S.	Days.	Anomaly. D. M. S.	Differ. M. S.	Days.	Anomaly. D. M. S.	Differ. M. S.
0,00	0. 0. 0,0	20. 54,5	8,00	11. 4. 53,0	20. 30,4	16,00	21. 45. 53,4	19. 25,4
25	0. 20. 54,5	54,4	25	11. 25. 23,4	29,0	25	22. 5. 18,8	22,8
50	0. 41. 48,9	54,3	50	11. 45. 52,4	27,5	50	22. 24. 41,6	20,2
0,75	1. 2. 43,2	20. 54,2	8,75	12. 6. 19,9	20. 26,0	16,75	22. 44. 1,8	19. 17,6
1,00	1. 23. 37,4	54,0	9,00	12. 26. 45,9	24,4	17,00	23. 3. 19,4	14,9
25	1. 44. 31,4	53,8	25	12. 47. 10,3	22,7	25	23. 22. 34,3	12,3
50	2. 5. 25,2	53,5	50	13. 7. 33,0	21,1	50	23. 41. 46,6	9,6
1,75	2. 26. 18,7	20. 53,2	9,75	13. 27. 54,1	20. 19,3	17,75	24. 0. 56,2	19. 6,9
2,00	2. 47. 11,9	52,8	10,00	13. 48. 13,4	17,7	18,00	24. 20. 3,1	4,1
25	3. 8. 4,7	52,4	25	14. 8. 31,1	15,8	25	24. 39. 7,2	19. 1,3
50	3. 28. 57,1	51,9	50	14. 28. 46.9	14,0	50	24. 58. 8,5	18. 58,6
2,75	3. 49. 49,0	20. 51,4	10,75	14. 49. 0,9	20. 12,2	18,75	25. 17. 7,1	55,8
3,00	4. 10. 40,4	50,8	11,00	15. 9. 13,1	10,3	19,00	25. 36. 2,9	52,9
25	4. 31. 31,2	50,3	25	15. 29. 23,4	8,4	25	25. 54. 55,8	50,0
50	4. 52. 21,5	49,6	50	15. 49. 31,8	6,4	50	26. 13. 45,8	47,2
3,75	5. 13. 11,1	20. 49,0	11,75	16. 9. 38,2	20. 4,3	19,75	26. 32. 33,0	18. 44,3
4,00	5. 34. 0,1	48,2	12,00	16. 29. 42,5	2,4	20,00	26. 51. 17,3	41,4
25	5. 54. 48,3	47,4	25	16. 49. 44,9	20. 0,3	25	27. 9. 58,7	38,4
50	6. 15. 35,7	46,6	50	17. 9. 45,2	19. 58,2	50	27. 28. 37,1	35,5
4,75	6. 36. 22,3	20. 45,7	12,75	17. 29. 43,4	56,0	20,75	27. 47. 12,6	18. 32,5
5,00	6. 57. 8,0	44,8	13,00	17. 49. 39,4	53,8	21,00	28. 5. 45,1	29,5
25	7. 17. 52,8	43,9	25	18. 9. 33,2	51,7	25	28. 24. 14,6	26,5
50	7. 38. 36,7	42,8	50	18. 29. 24,9	49,4	50	28. 42. 41,1	23,5
5,75	7. 59. 19,5	20. 41,8	13,75	18. 49. 14,3	19. 47,2	21,75	29. 1. 4,6	18. 20,4
6,00	8. 20. 1,3	40,7	14,00	19. 9. 1,5	44,9	22,00	29. 19. 25,0	17,3
25	8. 40. 42,0	39,6	25	19. 28. 46,4	42,5	25	29. 37. 42,3	14,3
50	9. 1. 21,6	38,4	50	19. 48. 28,9	40,1	50	29. 55. 56,6	11,2
6,75	9. 22. 0,0	20. 37,2	14,75	20. 8. 9,0	19. 37,8	22,75	30. 14. 7,8	18. 8,0
7,00	9. 42. 37,2	35,9	15,00	20. 27. 40,8	35,4	23,00	30. 32. 15,8	5,0
25	10. 3. 13,1	34,7	25	20. 47. 22,2	32,9	25	30. 50. 20,8	18. 1,8
50	10. 23. 47,8	33,3	50	21. 6. 55,1	30,4	50	31. 8. 22,6	17. 58,6
7,75	10. 44. 21,1	20. 31,9	15,75	21. 26. 25.5	19. 27,9	23,75	31. 26. 21,2	55,5

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D.	M.	S.	Differ. M.	S.	Days.	Anomaly. D.	M.	S.	Differ. M.	S.	Days.	Anomaly. D.	M.	S.	Differ. M.	S.
24,00	31.	44.	16,7	17.	52,3	32,00	40.	49.	2,9	16.	6,3	40,00	48.	56.	34,5	14.	19,3
25	32.	2.	9,0		49,1	25	41.	5.	9,2		2,8	25	49.	10.	53,8		16,0
50	32.	19.	58,1		45,9	50	41.	21.	12,0	15.	59,5	50	49.	25.	9,8		12,7
24,75	32.	37.	44,0			32,75	41.	37.	11,5			40,75	49.	39.	22,5		
				17.	42,7						56,1					14.	9,5
25,00	32.	55.	26,7			33,00	41.	53.	7,6		52,8	41,00	49.	53.	32,0		6,3
25	33.	13.	6,2		39,5	25	42.	9.	0,4		49,3	25	50.	7.	38,3	14.	3,0
50	33.	30.	42,4		36,2	50	42.	24.	49,7		45,9	50	50	21.	41,3	13.	59,8
25,75	33.	48.	15,4		33,0	33,75	42.	40.	35,6			41,75	50.	35.	41,1		
				17.	29,8					15.	42.6					13.	56,5
26,00	34.	5.	45,2		26,4	34,00	42.	56.	18,2		39,2	42,00	50.	49.	37,6		53,4
25	34.	23.	11,6		23,2	25	43.	11.	57,4		35,9	25	51.	3.	31,0		50,1
50	34.	40.	34,8		19,9	50	43.	27.	33,3		32,4	50	51.	17.	21,1		47,0
26,75	34.	57.	54,7			34,75	43.	43.	5,7			42,75	51.	31.	8,1		
				17.	16,7					15.	29,1					13.	43,7
27,00	35.	15.	11,4		13,3	35,00	43.	58.	34,8		25,7	43,00	51.	44.	51,8		40,6
25	35.	32.	24,7		10,0	25	44.	14.	0,5		22,4	25	51.	58.	32,4		37,4
50	35.	49.	34,7		6,7	50	44.	29.	22,9		19,0	50	52.	12.	9,8		34,3
27,75	36.	6.	41,4			35,75	44.	44.	41,9			43,75	52.	25.	44,1		
				17.	3,4					15.	15,6					13.	31,1
28,00	36.	23.	44,8	17.	0,0	36,00	44.	59.	57,5		12,3	44,00	52.	39.	15,2		27,9
25	36.	40.	44,8	16.	56,7	25	45.	15.	9,8		9,0	25	52.	52.	43,1		24,9
50	36.	57.	41,5		53,4	50	45.	30.	18,8		5,6	50	53	6.	8,0		21,7
28,75	37.	14.	34,9			36,75	45.	45.	24,4			44,75	53.	19.	29,7		
				16.	50,0					15.	2,2					13.	18,5
29,00	37.	31.	24,9		46,7	37,00	46.	0.	26,6	14.	58,9	45,00	53.	32.	48,2		15,5
25	37.	48.	11,6		43,3	25	46.	15.	25,5		55,6	25	53.	46.	3,7		12,4
50	38.	4.	54,9		40,0	50	46.	30.	21,1		52,2	50	53.	59.	16,1		9,3
29,75	38.	21.	34,9			37,75	46.	45.	13,3			45,75	54.	12.	25,4		
				16.	36,6					14.	48,9					13.	6,2
30,00	38.	38.	11,5		33,2	38,00	47.	0.	2,2		45,6	46,00	54.	25.	31,6		3,1
25	38.	54.	44,7		29,9	25	47.	14.	47,8		42,3	25	54.	38.	34,7	13.	0,1
50	39.	11.	14,6		26,5	50	47.	29.	30,1		39,0	50	54.	51.	34,8	12.	57,0
30,75	39.	27.	41,1			38,75	47.	44.	9,1			46,75	55	4.	31,8		54,0
				16.	23,1					14.	35,7						
31,00	39.	44.	4,2		19,7	39,00	47.	53.	44,8		32,3	47,00	55	17.	25,8		51,0
25	40.	0.	23,9		16,4	25	48.	13.	17,1		29,1	25	55.	30.	16,8		47,9
50	40.	16.	40,3		13,0	50	48.	27.	46,2		25,8	50	55.	43.	4,7		44,9
31,75	40.	32.	53,3			39,75	48.	42.	12,0			47,75	55.	55.	49,6		
				16.	9,0					14.	22,5					12.	41,9

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D. M. S.	Differ. M. S.	Days.	Anomaly. D. M. S.	Differ. M. S.	Days.	Anomaly. D. M. S.	Differ. M. S.
48,00	56. 8. 31,5	12. 39,0	56,00	62. 29. 32,1	11. 8,9	64,0c	68. 5. 26,9	9. 50,2
25	56. 21. 10,5	35,9	25	62. 40. 41,0	6,3	25	68. 15. 17,1	47,9
50	56. 33. 46,4	33,0	50	62. 51. 47,3	3,6	5c	68. 25. 5,c	45,6
48,75	56. 46. 19,4	12. 30,0	56,75	63. 2. 50,9	11. 1,1	64,75	68. 34. 50,6	9. 43,4
49,00	56. 58. 49,4	27,1	57,0c	63. 13. 52,0	10. 58,4	65,0c	68. 44. 34,0	41,1
25	57. 11. 7,5	24,1	25	63. 24. 50,4	55,9	25	68. 54. 15,1	38,9
50	57. 23. 40,6	21,2	50	63. 35. 46,3	53,3	5c	69. 3. 54,c	36,7
49,75	57. 36. 1,8	12. 18,3	57,75	63. 46. 39,6	10. 50,7	65,75	69. 13. 30,7	9. 34,4
50,00	57. 48. 20,1	15,4	58,00	63. 57. 30,3	48,1	66,00	69. 23. 5,1	32,3
25	58. 0. 35,5	12,5	25	64. 8 18,4	45,7	25	69. 32. 37,4	30,0
50	58. 12. 48,0	9,6	50	64. 19. 4,1	43,1	50	69. 42. 7,4	27,9
50,75	58. 24. 57,6	12. 6,7	58,75	64. 29. 47,2	10. 40,5	66,75	69. 51. 35,3	9. 25,7
51,00	58. 37. 4,3	3,9	59,00	64. 40. 27,7	38,1	67,00	70. 1. 1,0	23,5
25	58. 49. 8,2	12. 1,0	25	64. 51. 5,8	35,5	25	70. 10. 24,5	21,3
50	59. 1. 9,2	11. 58,2	50	65. 1. 41,3	33,1	50	70. 19. 45,8	19,2
51,75	59. 13. 7,4	55,3	59,75	65. 12. 14,4	10. 30,6	67,75	70. 29. 5,0	9. 17,1
52,00	59. 25. 2,7	52,5	60,00	65. 22. 45,0	28,1	68,00	70. 38. 22,1	15,0
25	59. 36. 55,2	49,7	25	65. 33. 13,1	25,7	25	70. 47. 37,1	12,9
50	59. 48. 44,9	46,9	50	65. 43. 38,8	23,2	50	70. 56. 50,0	10,7
52,75	60. 0. 31,8	11. 44,1	60,75	65. 54. 2,0	10. 20,8	68,75	71. 6. 0,7	9. 8,6
53,00	60. 12. 15,9	41,4	61,00	66. 4. 22,8	18,4	69,00	71. 15. 9,3	6,6
25	60. 23. 57,3	38,6	25	66. 14. 41,2	16,0	25	71. 24. 15,9	4,5
50	60. 35. 35,9	35,8	50	66. 24. 57,2	13,6	50	71. 33. 20,4	2,4
53,75	60. 47. 11,7	11. 33,1	61,75	66. 35. 10,8	10. 11,2	69,75	71. 42. 22,8	9. 0,4
54,00	60. 58. 44,8	30,4	62,00	66. 45. 22,0	8,8	70,00	71. 51. 23,2	8. 58,3
25	61. 10. 15,2	27,6	25	66. 55. 30,8	6,4	25	72. 0. 21,5	56,3
50	61. 21. 42,8	24,9	50	67. 5. 37,2	4,1	50	72. 9. 17,8	54,3
54,75	61. 33. 7,7	11. 22,2	62,75	67. 15. 41,3	10. 1,8	70,75	72. 18. 12,1	8. 52,3
55,0c	61. 44. 29,9	19,6	63,00	67. 25. 43,1	9. 59,4	71,00	72. 27. 4,4	50,2
25	61. 55. 49,5	16,8	25	67. 35. 42,5	57,1	25	72. 35. 54,6	48,2
50	62. 7. 6,3	14,2	50	67. 45. 39,6	54,8	50	72. 44. 42,8	46,3
55,75	62. 18. 20,5	11. 11,6	63,75	67. 55. 34,4	9. 52,5	71,75	72. 53. 29,1	8. 44,3

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D. M. S.	Differ. M. S.
72,00	73. 2. 13,4	8. 42,3
25	73. 10. 55,7	40,4
50	73. 19. 36,1	38,4
72,75	73. 28. 14,5	
		8. 36,5
73,00	73. 36. 51,0	
25	73. 45. 25,6	34,6
50	73. 53. 58,2	32,6
73,75	74. 2. 29,0	30,8
		8. 28,8
74,00	74. 10. 57,8	
25	74. 19. 24,7	26,9
50	74. 27. 49,8	25,1
74,75	74. 36. 13,0	23,2
		8. 21,3
75,00	74. 44. 34,3	
25	74. 52. 53,8	19,5
50	75. 1. 11,4	17,6
75,75	75. 9. 27,2	15,8
		8. 14,0
76,00	75. 17. 41,2	
25	75. 25. 53,3	12,1
50	75. 34. 3,6	10,3
76,75	75. 42. 12,1	8,5
		8. 6,8
77,00	75. 50. 18,9	
25	75. 58. 23,9	5,0
50	76. 6. 27,0	3,1
77,75	76. 14. 28,4	8. 1,4
		7. 59,7
78,00	76. 22. 28,1	
25	76. 30. 26,0	57,9
50	76. 38. 22,2	56,2
78,75	76. 46. 16,6	54,4
		7. 52,7
79,00	76. 54. 9,3	
25	77. 2. 0,3	51,0
50	77. 9. 49,6	49,3
79,75	77. 17. 37,2	47,6
		7. 45,9

Days.	Anomaly. D. M. S.	Differ. M. S.
80,00	77. 25. 23,1	7. 44,3
25	77. 33. 7,4	42,5
50	77. 40. 49,9	40,9
80,75	77. 48. 30,8	
		7. 39,3
81,00	77. 56. 10,1	37,6
25	78. 3. 47,7	35,9
50	78. 11. 23,6	34,3
81,75	78. 18. 57,9	
		7. 32,7
82,00	78. 26. 30,6	31,1
25	78. 34. 1,7	29,5
50	78. 41. 31,2	27,8
82,75	78. 48. 59,0	
		7. 26,3
83,00	78. 56. 25,3	24,7
25	79. 3. 50,0	23,1
50	79. 11. 13,1	21,6
83,75	79. 18. 34,7	
		7. 19,9
84,00	79. 25. 54,6	18,5
25	79. 33. 13,1	16,9
50	79. 40. 30,0	15,3
84,75	79. 47. 45,3	
		7. 13,8
85,00	79. 54. 59,1	12,3
25	80. 2. 11,4	10,7
50	80. 9. 22,1	9,3
85,75	80. 16. 31,4	
		7. 7,7
86,00	80. 23. 39,1	6,4
25	80. 30. 45,5	4,8
50	80. 37. 50,3	3,3
86,75	80. 44. 53,6	
		7. 1,8
87,00	80. 51. 55,4	
25	80. 58. 55,8	7. 0,4
50	81. 5. 54,7	6. 58,9
87,75	81. 12. 52,1	57,4
		6. 56,0

Days.	Anomaly. D. M. S.	Differ. M. S.
88,00	81. 19. 48,1	
25	81. 26. 42,7	6. 54,6
50	81. 33. 35,9	53,2
88,75	81. 40. 27,6	51,7
		6. 50,3
89,00	81. 47. 17,9	
25	81. 54. 6,8	48,9
50	82. 0. 54,3	47,5
89,75	82. 7. 40,4	46,1
		6. 44,8
90,00	82. 14. 25,2	
25	82. 21. 8,5	43,3
50	82. 27. 50,5	42,0
90,75	82. 34. 31,0	40,5
		6. 39,3
91,00	82. 41. 10,3	
25	82. 47. 48,1	37,8
50	82. 54. 24,7	36,6
91,75	83. 0. 59,9	35,2
		6. 33,8
92,00	83. 7. 33,7	
25	83. 14. 6,3	32,6
50	83. 20. 37,5	31,2
92,75	83. 27. 7,4	29,9
		6. 28,5
93,00	83. 33. 35,9	
25	83. 40. 3,2	27,3
50	83. 46. 29,1	25,9
93,75	83. 52. 53,8	24,7
		6. 23,4
94,00	83. 59. 17,2	
25	84. 5. 39,3	22,1
50	84. 12. 0,1	20,8
94,75	84. 18. 19,7	19,6
		6. 18,3
95,00	84. 24. 38,0	
25	84. 30. 55,1	17,1
50	84. 37. 10,9	15,8
95,75	84. 43. 25,5	14,6
		6. 13,3

THE THIRD TABLE CONTINUED.

Days.	Anomaly D. M. S.	Differ. M. S.	Days.	Anomaly. D. M. S.	Differ. M. S.	Days.	Anomaly. D. M. S.	Differ. M. S.
96,00	84. 49. 38,8	6. 12,1	104,00	87. 58. 26,5	5. 35,7	112,00	90. 49. 8,7	5. 4,2
25	84. 55. 50,9	10,9	25	88. 4. 2,2	34,6	25	90. 54. 12,9	3,4
50	85. 2. 1,8	9,6	50	88. 9. 36,8	33,6	50	90. 59. 16,3	2,4
96,75	85. 8. 11,4		104,75	88. 15. 10,4		112,75	91. 4. 18,7	
		6. 8,4			5. 32,5			5. 1,6
97,00	85. 14. 19,8		105,00	88. 20. 42,9	31,5	113,00	91. 9. 20,3	5. 0,6
25	85. 20. 27,1	7,3	25	88. 26. 14,4	30,4	25	91. 14. 20,9	4. 59,8
50	85. 26. 33,2	6,1	50	88. 31. 44,8	29,4	50	91. 19. 20,7	58,9
97,75	85. 32. 38,0	4,8	105,75	88. 37. 14,2		113,75	91. 24. 19,6	
		6. 3,6			5. 28,4			4. 58,0
98,00	85. 38. 41,6	2,5	106,00	88. 42. 42,6	27,4	114,00	91. 29. 17,6	57,1
25	85. 44. 44,1	1,3	25	88. 48. 10.0	26,3	25	91. 34. 14,7	56,2
50	85. 50. 45,4	6. 0,1	50	88. 53. 36,3	25,4	50	91. 39. 10,9	55,3
98,75	85. 56. 45,5		106,75	88. 59. 1,7		114,75	91. 44. 6,2	
		5. 58,9			5. 24,3			4. 54,5
99,00	86. 2. 44,4	57,8	107,00	89. 4. 26,0	23,3	115,00	91. 49. 0,7	53,6
25	86. 8. 42,2	56,7	25	89 9. 49,3	22,4	25	91. 53. 54,3	52,7
50	86. 14. 38,9	55,4	50	89. 15. 11,7	21,3	50	91. 58. 47,0	52,0
99,75	86. 20. 34,3		107,75	89. 20. 33,0		115,75	92. 3. 39,0	
		5. 54,4			5. 20,4			4. 51,0
100,00	86. 26. 28,7	53,2	108,00	89. 25. 53,4	19,4	116,00	92. 8. 30,0	50,1
25	86. 32. 21,9	52,0	25	89. 31. 12,8	18,4	25	92. 13. 20,1	49,4
50	86. 38. 13,9	51,0	50	89. 36. 31,2	17,4	50	92. 18. 9,5	48,5
100,75	86. 44. 4,9		108,75	89. 41. 48,6		116,75	92. 22. 58,0	
		5 49,8			5. 16,5			4. 47,7
101,00	86. 49. 54,7	48,7	109,00	89. 47. 5,1	15,5	117,00	92. 27. 45,7	46,8
25	86. 55. 43,4	47,5	25	89. 52. 20,6	14,5	25	92. 32. 32,5	46,0
50	87. 1. 30,9	46,5	50	89. 57. 35,1	13,6	50	92. 37. 18,5	45,2
101,75	87. 7. 17,4		109,75	90. 2. 48,7		117,75	92. 42. 3,7	
		5. 45,4			5. 12,6			4. 44,3
102,00	87. 13. 2,8	44,2	110,00	90. 8 1,3	11,7	118,00	92. 46. 48,0	43,6
25	87. 18. 47,0	43,2	25	90. 13. 13,0	10,8	25	92. 51. 31,6	42,7
50	87. 24. 30,2	42,1	50	90. 18. 23,8	9,8	50	92. 56. 14,3	41,9
102,75	87. 30. 12,3		110,75	90. 23. 33,6		118,75	93. 0. 56,2	
		5. 41,0			5. 8,8			4. 41,1
103,00	87. 35. 53,3	39,9	111,00	90. 28. 42,4	8,0	119,00	93. 5. 37,3	40,2
25	87. 41. 33,2	38,8	25	90. 33. 50,4	7,0	25	93. 10. 17,5	39,5
50	87. 47. 12,0	37,8	50	90. 38. 57,4	6,1	50	93. 14. 57,0	38,6
103,75	87. 52. 49,8		111,75	90. 44. 3,5		119,75	93. 19. 35,6	
		5. 36,7			5. 5,2			4. 38,0

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D.	M.	S.	Differ. M.	S.	Days.	Anomaly. D.	M.	S.	Differ. M.	S.	Days.	Anomaly. D.	M.	S.	Differ. M.	S.
120,00	93.	24.	13,6			128,00	95	45.	45,4			136,00	97.	55.	28,6		
25	93.	28.	50,7	4.	37,1	25	95.	49.	58,9	4.	13,5	25	97.	59.	21,4	3.	53,8
5c	93.	33.	27,0		36,3	5o	95.	54.	11,7		12,8	5o	98.	3.	13,6		52,2
120,75	93.	38.	2,5		35,5	128,75	95.	58.	23,7		12,0	136,75	98.	7.	5,2		51,6
				4.	34,7					4.	11,5					3.	51,0
121,00	93.	42.	37,2			129,00	96.	2.	35,2			137,00	98.	10.	56,2		
25	93.	47.	11,1		33,9	25	96.	6.	45,9		10,7	25	98.	14.	46,6		50,4
5o	93.	51.	44,3		33,2	5o	96.	10.	55,9		10,0	5o	98.	18.	36,4		49,8
121,75	93.	56.	16,7		32,4	129,75	96.	15.	5,3		9,4	137,75	98.	22.	25,7		49,3
				4.	31,7					4.	8,7					3.	48,6
122,00	94.	0.	48,4			130,00	96.	19.	14,0			138,00	98.	26.	14,3		
25	94.	5.	19,2		30,8	25	96.	23.	22,0		8,0	25	98.	30.	2,4		48,1
5c	94.	9.	49,3		30,1	5o	96.	27.	29,4		7,4	5o	98.	33.	49,8		47,4
122,75	94.	14.	18,7		29,4	130,75	96.	31.	36,1		6,7	138,75	98.	37.	36,7		46,9
				4.	28,4					4.	6,1					3.	46,3
123,00	94.	18.	47,3			131,00	96.	35.	42,2			139,00	98.	41.	23,0		
25	94.	23.	15,1		27,8	25	96.	39.	47,5		5,3	25	98.	45.	8,8		45,8
5o	94.	27.	42,2		27,1	5o	96.	43.	52,3		4,8	5o	98.	48.	53,9		45,1
123,75	94.	32.	8,6		26,4	131,75	96.	47.	56,4		4,1	139,75	98.	52.	38,5		44,6
				4.	25,6					4.	3,4					3.	44,0
124,00	94.	36.	34,2			132,00	96.	51.	59,8			140,00	98.	56.	22,5		
25	94.	40.	59,0		24,8	25	96.	56.	2,6		2,8	25	99.	0.	5,9		43,4
5c	94.	45.	23,1		24,1	5o	97.	0.	4,7		2,1	5o	99.	3.	48,8		42,9
124,75	94.	49.	46,5		23,4	132,75	97.	4.	6,2		1,5	140,75	99.	7.	31,1		42,3
				4.	22,7					4.	0,9					3.	41,8
125,00	94.	54.	9,2			133,00	97.	8.	7,1	4.	0,2	141,00	99.	11.	12,9		
25	94.	58.	31,2		22,0	25	97.	12.	7,3	3.	59,6	25	99.	14.	54,1		41,2
5o	95.	2.	52,4		21,2	5c	97.	16.	6,9		59,0	5o	99.	18.	34,7		40,6
125,75	95.	7.	12,9		20,5	133,75	97.	20.	5,9			141,75	99.	22.	14,8		40,1
				4.	19,8					3.	58,3					3.	39,5
126,00	95.	11.	32,7			134,00	97.	24.	4,2			142,00	99.	25.	54,3		
25	95.	15.	51,7		19,0	25	97.	28.	1,9		57,7	25	99.	29.	33,3		39,0
5o	95.	20.	10,1		18,4	5c	97.	31.	59,0		57,1	5o	99.	33.	11,7		38,4
126,75	95.	24.	27,7		17,6	134,75	97.	35.	55,5		56,5	142,75	99.	36.	49,6		37,9
				4	17,0					3.	55,8					3.	37,3
127,00	95.	28.	44,7			135,00	97.	39.	51,3			143,00	99.	40.	26,9		
25	95.	33.	0,9		16,2	25	97.	43.	46,6		55,3	25	99.	44.	3,7		36,8
5o	95.	37.	16,4		15,5	5c	97.	47.	41,2		54,6	5o	99.	47.	40,0		36,3
127,75	95.	41.	31,3		14,9	135,75	97.	51.	35,2		54,0	143,75	99.	51.	15,7		35,7
				4.	14,1					3.	53,4					3.	35,2

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D.	M.	S.	Differ. M.	S.	Days.	Anomaly. D.	M.	S.	Differ. M.	S.	Days.	Anomaly. D.	M.	S.	Differ. M.	S.
144,00	99.	54.	50,9			152,00	101.	45.	6,5			160,00	103.	27.	18,5		
25	99.	58.	25,6	3.	34,7	25	101.	48.	25,1	3.	18,6	25	103.	30.	23,0	3.	4,5
50	100.	1.	59,7		34,1	50	101.	51.	43,3		18,2	50	103.	33.	27,0		4,0
144,75	100.	5.	33,3		33,6	152,75	101.	55.	1,0		17,7	160,75	103.	36.	30,6		3,6
				3.	33,1					3.	17,2					3.	3,2
145,00	100.	9.	6,4			153,00	101.	58.	18,2		16,8	161,00	103.	39.	33,8		2,8
25	100.	12.	38,9		32,5	25	102.	1.	35,0		16,3	25	103.	42.	36,6		2,3
50	100.	16.	10,9		32,0	50	102.	4.	51,3		15,9	50	103.	45.	38,9		2,0
145,75	100.	19.	42,4		31,5	153,75	102.	8.	7,2			161,75	103.	48.	40,9		
				3.	31,0					3.	15,4					3.	1,5
146,00	100.	23.	13,4			154,00	102.	11.	22,6		14,9	162,00	103.	51.	42,4		1,2
25	100.	26.	43,9		30,5	25	102.	14.	37,5		14,5	25	103.	54.	43,6		0,7
50	100.	30.	13,9		30,0	50	102.	17.	52,0		14,0	50	103.	57.	44,3		0,3
146,75	100.	33.	43,3		29,4	154,75	102.	21.	6,0			162,75	104.	0.	44,6		
				3.	28,9					3.	13,6					3.	0,0
147,00	100.	37.	12,2			155,00	102.	24.	19,6		13,1	163,00	104.	3.	44,6		
25	100.	40.	40,6		28,4	25	102.	27.	32,7		12,7	25	104.	6.	44,1	2.	59,5
50	100.	44.	8,5		27,9	50	102.	30.	45,4		12,2	50	104.	9.	43,2		59,1
147,75	100.	47.	35,9		27,4	155,75	102.	33.	57,6			163,75	104.	12.	42,0		58,8
				3.	26,9					3.	11,8					2.	58,3
148,00	100.	51.	2,8			156,00	102.	37.	9,4		11,3	164,00	104.	15.	40,3		
25	100.	54.	29,2		26,4	25	102.	40.	20,7		10,9	25	104.	18.	38,2		57,9
50	100.	57.	55,1		25,9	50	102.	43.	31,6		10,5	50	104.	21.	35,8		57,6
148,75	101.	1.	20,5		25,4	156,75	102.	46.	42,1			164,75	104.	24.	32,9		57,1
				3.	24,9					3.	10,0					2.	56,8
149,00	101.	4.	45,4			157,00	102.	49.	52,1		9,6	165,00	104.	27.	29,7		
25	101.	8.	9,9		24,5	25	102.	53.	1,7		9,1	25	104.	30.	26,0		56,3
50	101.	11.	33,8		23,9	50	102.	56.	10,8		8,7	50	104.	33.	22,0		56,0
149,75	101.	14.	57,2		23,4	157,75	102.	59.	19,5			165,75	104.	36.	17,6		55,6
				3.	22,9					3.	8,3					2.	55,2
150,00	101.	18.	20,1			158,00	103.	2.	27,8		7,8	166,00	104.	39.	12,8		
25	101.	21.	42,6		22,5	25	103.	5.	35,6		7,4	25	104.	42.	7,6		54,8
50	101.	25.	4,6		22,0	50	103.	8.	43,0		7,0	50	104.	45.	2,1		54,5
150,75	101.	28.	26,1		21,5	158,75	103.	11.	50,0			166,75	104.	47.	56,1		54,0
				3.	21,0					3.	6,6					2.	53,7
151,00	101.	31.	47,1			159,00	103.	14.	56,6		6,1	167,00	104.	50.	49,8		
25	101.	35.	7,7		20,6	25	103.	18.	2,7		5,7	25	104.	53.	43,1		53,3
50	101.	38.	27,8		20,1	50	103.	21.	8,4		5,3	50	104.	56.	36,0		52,9
151,75	101.	41.	47,3		19,5	159,75	103.	24.	13,7			167,75	104.	59.	28,6		52,6
				3.	19,2					3.	4,8					2.	52,1

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
168,00	105.	2.	20,7	2.	51,8
25	105.	5.	12,5		51,4
50	105.	8.	3,9		51,1
168,75	105.	10.	55,0		
				2.	50,6
169,00	105.	13.	45,6		50,3
25	105.	16.	35,9		50,0
50	105.	19.	25,9		49,6
169,75	105.	22.	15,5		
				2.	49,2
170,00	105.	25.	4,7		48,8
25	105.	27.	53,5		48,5
50	105.	30.	42,0		48,1
170,75	105.	33.	30,1		
				2.	47,8
171,00	105.	36.	17,9		47,4
25	105.	39.	5,3		47,0
50	105.	41.	52,3		46,7
171,75	105.	44.	39,0		
				2.	46,4
172,00	105.	47.	25,4		46,0
25	105.	50.	11,4		45,6
50	105.	52.	57,0		45,3
172,75	105.	55.	42,3		
				2.	44,9
173,00	105.	58.	27,2		44,6
25	106.	1.	11,8		44,2
50	106.	3.	56,0		43,9
173,75	106.	6.	39,9		
				2.	43,5
174,00	106.	9.	23,4		43,2
25	106.	12.	6,6		42,9
50	106.	14.	49,5		42,5
174,75	106.	17.	32,0		
				2.	42,2
175,00	106.	20.	14,2		41,8
25	106.	22.	56,0		41,5
50	106.	25.	37,5		41,1
175,75	106.	28.	18,6		
				2.	40,8

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
176,00	106.	30.	59,4	2.	40,5
25	106.	33.	39,9		40,2
50	106.	36.	20,1		39,8
176,75	106.	38.	59,9		
				2.	39,5
177,00	106.	41.	39,4		39,2
25	106.	44.	18,6		38,8
50	106.	46.	57,4		38,5
177,75	106.	49.	35,9		
				2.	38,2
178,00	106.	52.	14,1		37,8
25	106.	54.	51,9		37,5
50	106.	57.	29,4		37,2
178,75	107.	0.	6,6		
				2.	36,9
179,00	107.	2.	43,5		36,5
25	107.	5.	20,0		36,2
50	107.	7.	56,2		35,9
179,75	107.	10.	32,1		
				2.	35,6
180,00	107.	13.	7,7		35,3
25	107.	15.	43,0		35,0
50	107.	18.	18,0		34,6
180,75	107.	20.	52,6		
				2.	34,3
181,00	107.	23.	26,9		34,0
25	107.	26.	0,9		33,7
50	107.	28.	34,6		33,4
181,75	107.	31.	8,0		
				2.	33,1
182,00	107.	33.	41,1		32,8
25	107.	36.	13,9		32,4
50	107.	38.	46,3		32,1
182,75	107.	41.	18,4		
				2.	31,8
183,00	107.	43.	50,2		31,6
25	107.	46.	21,8		31,2
50	107.	48.	53,0		31,0
183,75	107.	51.	24,0		
				2.	30,6

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
184,00	107.	53.	54,6	2.	30,3
25	107.	56.	24,9		30,0
50	107.	58.	54,9		29,8
184,75	108.	1.	24,7		
				2.	29,4
185,00	108.	3.	54,1		29,1
25	108.	6.	23,2		28,8
50	108.	8.	52,0		28,5
185,75	108.	11.	20,5		
				2.	28,3
186,00	108.	13.	48,8		27,9
25	108.	16.	16,7		27,7
50	108.	18.	44,4		27,3
186,75	108.	21.	11,7		
				2.	27,1
187,00	108.	23.	38,8		26,8
25	108.	26.	5,6		26,5
50	108.	28.	32,1		26,2
187,75	108.	30.	58,3		
				2.	25,9
188,00	108.	33.	24,2		25,6
25	108.	35.	49,8		25,3
50	108.	38.	15,1		25,1
188,75	108.	40.	40,2		
				2.	24,8
189,00	108.	43.	5,0		24,5
25	108.	45.	29,5		24,2
50	108.	47.	53,7		23,9
189,75	108.	50.	17,6		
				2.	23,7
190,00	108.	52.	41,3		23,3
25	108.	55.	4,6		23,1
50	108.	57.	27,7		22,8
190,75	108.	59.	50,5		
				2.	22,6
191,00	109.	2.	13,1		22,2
25	109.	4.	35,3		22,0
50	109.	6.	57,3		21,7
191,75	109.	9.	19,0		
				2.	21,4

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
192,00	109.	11.	40,4		
25	109.	14.	1,6	2.	21,2
50	109.	16.	22,5		20,9
192,75	109.	18.	43,1		20,6
				2.	20,4
193,00	109.	21.	3,5		20,0
25	109.	23.	23,5		19,8
50	109.	25.	43,3		19,6
193,75	109.	28.	2,9	2.	19,3
194,00	109.	30.	22,2		19,0
25	109.	32.	41,2		18,7
50	109.	34.	59,9		18,5
194,75	109.	37.	18,4	2.	18,3
195,00	109.	39.	36,7		17,9
25	109.	41.	54,6		17,7
50	109.	44.	12,3		17,5
195,75	109.	46.	29,8	2.	17,1
196,00	109.	48.	46,9		17,0
25	109.	51.	3,9		16,7
50	109.	53.	20,6		16,3
196,75	109.	55.	36,9	2.	16,2
197,00	109.	57.	53,1		15,9
25	110.	0.	9,0		15,6
50	110.	2.	24,6		15,4
197,75	110.	4.	40,0	2.	15,1
198,00	110.	6.	55,1		14,9
25	110.	9.	10,0		14,6
50	110.	11.	24,6		14,4
198,75	110.	13.	39,0	2.	14,1
199,00	110.	15.	53,1		13,9
25	110.	18.	7,0		13,6
50	110.	20.	20,6		13,4
199,75	110.	22.	34,0	2.	13,1

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
200,00	110.	24.	47,1		
5	110.	29.	12,6	4.	25,5
201,0	110.	33.	37,1		24,5
5	110.	38.	0,7		23,6
				4.	22,5
202,0	110.	42.	23,2		21,6
5	110.	46.	44,8		20,7
203,0	110.	51.	5,5		19,7
5	110.	55.	25,2	4.	18,7
204,0	110.	59.	43,9		17,8
5	111.	4.	1,7		16,9
205,0	111.	8.	18,6		16,0
5	111.	12.	34,6	4.	15,0
206,0	111.	16.	49,6		14,1
5	111.	21.	3,7		13,2
207,0	111.	25.	16,9		12,3
5	111.	29.	29,2	4.	11,3
208,0	111.	33.	40,5		10,5
5	111.	37.	51,0		9,6
209,0	111.	42.	0,6		8,7
5	111.	46.	9,3	4.	7,8
210,0	111.	50.	17,1		7,0
5	111.	54.	24,1		6,1
211,0	111.	58.	30,2		5,2
5	112.	2.	35,4	4.	4,3
212,0	112.	6.	39,7		3,5
5	112.	10.	43,2		2,6
213,0	112.	14.	45,8		1,8
5	112.	18.	47,6	4.	0,9
214,0	112.	22.	48,5	4.	0,1
5	112.	26.	48,6	3.	59,3
215,0	112.	30.	47,9		58,5
5	112.	34.	46,4	3.	57,6

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
216,0	112.	38.	44,0		
5	112.	42.	40,8	3.	56,8
217,0	112.	46.	36,8		56,0
5	112.	50.	31,9		55,1
218,0	112.	54.	26,3	3.	54,4
5	112.	58.	19,9		53,6
219,0	113.	2.	12,7		52,8
5	113.	6.	4,7		52,0
220,0	113.	9.	55,9	3.	51,2
5	113.	13.	46,3		50,4
221,0	113.	17.	36,0		49,7
5	113.	21.	24,8		48,8
222,0	113.	25.	12,9	3.	48,1
5	113.	29.	0,2		47,3
223,0	113.	32.	46,8		46,6
5	113.	36.	32,7		45,9
224,0	113.	40.	17,7	3.	45,0
5	113.	44.	2,0		44,3
225,0	113.	47.	45,5		43,5
5	113.	51.	28,3		42,8
226,0	113.	55.	10,4	3.	42,1
5	113.	58.	51,8		41,4
227,0	114.	2.	32,4		40,6
5	114.	6.	12,3		39,9
228,0	114.	9.	51,5	3.	39,2
5	114.	13.	30,0		38,5
229,0	114.	17.	7,7		37,7
5	114.	20.	44,7		37,0
230,0	114.	24.	21,0	3.	36,3
5	114.	27.	56,7		35,7
231,0	114.	31.	31,6		34,9
5	114.	35.	5,9		34,3
				3.	33,5

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
232,0	114.	38.	39,4		
5	114.	42.	12,3	3.	32,9
233,0	114.	45.	44,4		32,1
5	114.	48.	15,9		31,5
				3.	30,8
234,0	114.	52.	46,7		
5	114.	56.	16,9		30,2
235,0	114.	59.	46,4		29,5
5	115.	3.	15,2		28,8
				3.	28,1
236,0	115.	6.	43,3		
5	115.	10.	10,8		27,5
237,0	115.	13.	37,7		26,9
5	115.	17.	3,9		26,2
				3.	25,5
238,0	115.	20.	29,4		
5	115.	23.	54,3		24,9
239,0	115.	27.	18,5		24,2
5	115.	30.	42,1		23,6
				3.	23,0
240,0	115.	34.	5,1		
5	115.	37.	27,5		22,4
241,0	115.	40.	49,2		21,7
5	115.	44.	10,3		21,1
				3.	20,4
242,0	115.	47.	30,7		
5	115.	50.	50,6		19,9
243,0	115.	54.	9,8		19,2
5	115.	57.	28,4		18,6
				3.	18,0
244,0	116.	0.	46,4		
5	116.	4.	3,8		17,4
245,0	116.	7.	20,6		16,8
5	116.	10.	36,8		16,2
				3.	15,6
246,0	116.	13.	52,4		
5	116.	17.	7,4		15,0
247,0	116.	20.	21,8		14,4
5	116.	23.	35,6		13,8
				3.	13,3

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
248,0	116.	26.	48,9		
5	116.	30.	1,5	3,	12,6
249,0	116.	33.	13,6		12,1
5	116.	36.	25,1		11,5
				3.	10,9
250,0	116.	39.	36,0		
5	116.	42.	46,4		10,4
251,0	116.	45.	56,2		9,8
5	116.	49.	5,4		9,2
				3.	8,6
252,0	116.	52.	14,0		
5	116.	55.	22,1		8,1
253,0	116.	58.	29,7		7,6
5	117.	1.	36,7		7,0
				3.	6,4
254,0	117.	4.	43,1		
5	117.	7.	49,0		5,9
255,0	117.	10.	54,3		5,3
5	117.	13.	59,1		4,8
				3.	4,3
256,0	117.	17.	3,4		
5	117.	20.	7,1		3,7
257,0	117.	23.	10,3		3,2
5	117.	26.	12,9		2,6
				3.	2,1
258,0	117.	29.	15,0		
5	117.	32.	16,6		1,6
259,0	117.	35.	17,7		1,1
5	117.	38.	18,2		0,5
				3.	0,1
260,0	117.	41.	18,3		
5	117.	44.	17,8	2.	59,5
261,0	117.	47.	16,7		58,9
5	117.	50.	15,2		58,5
				2.	58,0
262,0	117.	53.	13,2		
5	117.	56.	10,6		57,4
263,0	117.	59.	7,6		57,0
5	118.	2.	4,0		56,4
				2.	56,0

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
264,0	118.	5.	0,0		
5	118.	7.	55,4	2.	55,4
265,0	118.	10.	50,4		55,0
5	118.	13.	44,8		54,4
				2.	54,0
266,0	118.	16.	38,8		
5	118.	19.	32,3		53,5
267,0	118.	22.	25,3		53,0
5	118.	25.	17,8		52,5
				2.	52,0
268,0	118.	28.	9,8		
5	118.	31.	1,3		51,5
269,0	118.	33.	52,4		51,1
5	118.	36.	42,9		50,5
				2.	50,1
270,0	118.	39.	33,0		
5	118.	42.	22,7		49,7
271,0	118.	45.	11,8		49,1
5	118.	48.	0,5		48,7
				2.	48,2
272,0	118.	50.	48,7		
5	118.	53.	36,5		47,8
273,0	118.	56.	23,8		47,3
5	118.	59.	10,7		46,9
				2.	46,3
274,0	119.	1.	57,0		
5	119.	4.	43,0		46,0
275,0	119.	7.	28,5		45,5
5	119.	10.	13,5		45,0
				2.	44,6
276,0	119.	12.	58,1		
5	119.	15.	42,2		44,1
277,0	119.	18.	25,9		43,7
5	119.	21.	9,1		43,2
				2.	42,8
278,0	119.	23.	51,9		
5	119.	26.	34,3		42,4
279,0	119.	29.	16,2		41,9
5	119.	31.	57,7		41,5
				2.	41,1

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D. M. S.	Differ. M. S.
280,0	119. 34. 38,8	2. 40,7
5	119. 37. 19,5	40,2
281,0	119. 39. 59,7	39,7
5	119. 42. 39,4	2. 39,4
282,0	119. 45. 18,8	38,9
5	119. 47. 57,7	38,5
283,0	119. 50. 36,2	38,1
5	119. 53. 14,3	2. 37,7
284,0	119. 55. 52,0	37,2
5	119. 58. 29,2	36,9
285,0	120. 1. 6,1	36,4
5	120. 3. 42,5	2. 36,0
286,0	120. 6. 18,5	35,6
5	120. 8. 54,1	35,3
287,0	120. 11. 29,4	34,7
5	120. 14. 4,1	2. 34,4
288,0	120 16. 38,5	34,0
5	120. 19. 12,5	33,6
289,0	120. 21. 46,1	33,2
5	120. 24. 19,3	2 32,8
290,0	120. 26. 52,1	32,4
5	120. 29. 24,5	32,1
291,0	120. 31. 56,6	31,6
5	120. 34. 28,2	2. 31,2
292,0	120. 36. 59,4	30,9
5	120. 39. 30,3	30,4
293,0	120. 42. 0,7	30,1
5	120. 44. 30,8	2. 29,7
294,0	120. 47. 0,5	29,3
5	120. 49. 29,8	28,9
295,0	120. 51. 58,7	28,6
5	120. 54. 27,3	2. 28,2

Days.	Anomaly. D. M. S.	Differ. M. S.
296,0	120. 56. 55,5	2. 27,8
5	120. 59. 23,3	27,4
297,0	121. 1. 50,7	27,0
5	121. 4. 17,7	2. 26,7
298,0	121. 6. 44,4	26,3
5	121. 9. 10,7	26,0
299,0	121. 11. 36,7	25,6
5	121. 14. 2,3	2. 25,2
300,0	121. 16. 27,5	24,8
5	121. 18. 52,3	24,5
301,0	121. 21. 16,8	24,2
5	121. 23. 41,0	2. 23,7
302,0	121. 26. 4,7	23,5
5	121. 28. 28,2	23,0
303,0	121. 30. 51,2	22,7
5	121. 33. 13,9	2. 22,5
304,0	121. 35. 36,4	22,0
5	121. 37. 58,4	21,6
305,0	121. 40. 20,0	21,3
5	121. 42. 41,3	2. 20,9
306,0	121. 45. 2,2	20,6
5	121. 47. 22,8	20,3
307,0	121. 49. 43,1	20,0
5	121. 52. 3,1	2. 19,6
308,0	121. 54. 22,7	19,2
5	121. 56. 41,9	18,9
309,0	121. 59. 0,8	18,6
5	122. 1. 19,4	2. 18,3
310,0	122. 3. 37,7	17,9
5	122. 5. 55,6	17,5
311,0	122. 8. 13,1	17,3
5	122. 10. 30,4	2. 16,9

Days.	Anomaly. D. M. S.	Differ. M. S.
312,0	122. 12. 47,3	2. 16,6
5	122. 15. 3,9	16,3
313,0	122. 17. 20,2	15,9
5	122. 19. 36,1	2. 15,6
314,0	122. 21. 51,7	15,3
5	122. 24. 7,0	15,0
315,0	122. 26. 22,0	14,6
5	122. 28. 36,6	2. 14,3
316,0	122. 30. 50,9	14,0
5	122. 33. 4,9	13,7
317,0	122. 35. 18,6	13,4
5	122. 37. 32,0	2. 13,1
318,0	122. 39. 45,1	12,7
5	122. 41. 57,8	12,5
319,0	122. 44. 10,3	12,1
5	122. 46. 22,4	2. 11,9
320,0	122. 48. 34,3	11,5
5	122. 50. 45,8	11,2
321,0	122. 52. 57,0	10,9
5	122. 55. 7,9	2. 10,5
322,0	122. 57. 18,4	10,3
5	122. 59. 28,7	10,0
323,0	123. 1. 38,7	9,7
5	123. 3. 48,4	2. 9,4
324,0	123. 5. 57,8	9,1
5	123. 8. 6,9	8,7
325,0	123. 10. 15,6	8,5
5	123. 12. 24,1	2. 8,2
326,0	123. 14. 32,3	7,9
5	123. 16. 40,2	7,6
327,0	123. 18. 47,8	7,3
5	123. 20. 55,1	2. 7,0

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D	M.	S.	Differ. M.	S.	Days.	Anomaly. D.	M.	S.	Differ. M.	S.	Days.	Anomaly. D.	M.	S.	Differ. M.	S.
328,0	123.	23.	2,1	2.	6,8	344,0	124.	28.	19,5	1.	58,0	360,0	125.	29.	13,3	1.	50,3
5	123.	25.	8,9		6,4	5	124.	30.	17,5		57,8	5	125.	31.	3,6		50,0
329,0	123.	27.	15,3		6,2	345,0	124.	32.	15,3		57,4	361,0	125.	32.	53,6		49,8
5	123.	29.	21,5			5	124.	34.	12,7			5	125.	34.	43,4		
				2.	5,8					1.	57,3					1.	49,6
330,0	123.	31.	27,3			346,0	124.	36.	10,0			362,0	125.	36.	33,0		
5	123.	33.	32,9		5,6	5	124.	38.	7,0		57,0	5	125.	38.	22,3		49,3
331,0	123.	35.	38,2		5,3	347,0	124.	40.	3,7		56,7	363,0	125.	40.	11,5		49,2
5	123.	37.	43,2		5,0	5	124.	42.	0,2		56,5	5	125.	42.	0,4		48,9
				2.	4,8					1.	56,2					1.	48,6
332,0	123.	39.	48,0			348,0	124.	43.	56,4			364,0	125.	43.	49,0		
5	123.	41.	52,4		4,4	5	124.	45.	52,4		56,0	5	125.	45.	37,5		48,5
333,0	123.	43.	56,6		4,2	349,0	124.	47.	48,1		55,7	365,0	125.	47.	25,7		48,2
5	123.	46.	0,5		3,9	5	124.	49.	43,6		55,5	5	125	49.	13,8		48,1
				2.	3,6					1.	55,3					1.	47,8
334,0	123.	48.	4,1			350,0	124.	51.	38,9			366,0	125.	51.	1,6		
5	123.	50.	7,4		3,3	5	124.	53.	33,9		55,0	5	125.	52.	49,1		47,5
335,0	123.	52.	10,5		3,1	351,0	124.	55	28,7		54,8	367,0	125.	54.	36,5		47,4
5	123.	54.	13,3		2,8	5	124.	57.	23,2		54,5	5	125.	56.	23,6		47,1
				2.	2,5					1.	54,2					1.	46,9
336,0	123.	56.	15,8			352,0	124.	59.	17,4			368,0	125.	58.	10,5		
5	123.	58.	18,0		2,2	5	125.	1.	11,5		54,1	5	125.	59.	57,2		46,7
337,0	124.	0.	20,0		2,0	353,0	125.	3.	5,3		53,8	369,0	126.	1.	43,7		46,5
5	124.	2.	21,7		1,7	5	125.	4.	58,8		53,5	5	126.	3.	30,0		46,3
				2.	1,4					1.	53,3					1.	46,0
338,0	124.	4.	23,1			354,0	125.	6.	52,1			370,0	126.	5.	16,0		
5	124.	6.	24,3		1,2	5	125.	8.	45,2		53,1	5	126.	7.	1,8		45,8
339,0	124.	8.	25,2		0,9	355,0	125.	10.	38,0		52,8	371,0	126.	8.	47,5		45,7
5	124.	10.	25,8		0,6	5	125.	12.	30,6		52,6	5	126.	10.	32,9		45,4
				2.	0,3					1.	52,3					1.	45,1
340,0	124.	12.	26,1			356,0	125.	14.	22,9			372,0	126.	12.	18,0		
5	124.	14.	26,2	2.	0,1	5	125.	16.	15,0		52,1	5	126.	14.	3,0		45,0
341,0	124.	16.	26,0	1.	59,8	357,0	125.	18.	6,9		51,9	373,0	126.	15.	47,8		44,8
5	124.	18.	25,6		59,6	5	125.	19.	58,6		51,7	5	126.	17.	32,4		44,6
				1.	59,3					1.	51,4					1.	44,4
342,0	124.	20.	24,9			358,0	125.	21.	50,0			374,0	126.	19.	16,8		
5	124.	22.	23,9		59,0	5	125.	23.	41,2		51,2	5	126.	21.	0,9		44,1
343,0	124.	24.	22,7		58,8	359,0	125.	25.	32,1		50,9	375,0	126.	22.	44,8		43,9
5	124.	26.	21,2		58,5	5	125.	27.	22,8		50,7	5	126.	24	28,6		43,8
				1.	58,3					1.	50,5					1.	43,5

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D. M. S.	Differ. M. S.	Days.	Anomaly. D. M. S.	Differ. M. S.	Days.	Anomaly. D. M. S.	Differ. M. S.
376,0	126. 26. 12,1		392,0	127. 19. 40,1		416	128. 34. 2,2	
		1. 43,3			1. 37,1			2. 57,6
5	126. 27. 55,4		5	127. 21. 17,2		417	128. 36. 59,8	
		43,1			36,9			56,9
377,0	126. 29. 38,5		393,0	127. 22. 54,1		418	128. 39. 56,7	
		43,0			36,7			56,3
5	126. 31. 21,5		5	127. 24. 30,8		419	128. 42. 53,0	
		1. 42,7			1. 36,6			2. 55,7
378,0	126. 33. 4,2		394,0	127. 26. 7,4		420	128. 45. 48,7	
		42,5			36,3			55,1
5	126. 34. 46,7		5	127. 27. 43,7		421	128. 48. 43,8	
		42,3			36,2			54,4
379,0	126. 36. 29,0		395,0	127. 29. 19,9		422	128. 51. 38,2	
		42,1			36,0			53,8
5	126. 38. 11,1		5	127. 30. 55,9		423	128. 54. 32,0	
		1. 41,9			1. 35,8			2. 53,3
380,0	126. 39. 53,0		396,0	127. 32. 31,7		424	128. 57. 25,3	
		41,7			35,7			52,6
5	126. 41. 34,7		5	127. 34. 7,4		425	129. 0. 17,9	
		41,5			35,4			52,0
381,0	126. 43. 16,2		397,0	127. 35. 42,8		426	129. 3. 9,9	
		41,3			35,3			51,4
5	126. 44. 57,5		5	127. 37. 18,1		427	129. 6. 1,3	
		1. 41,1			1. 35,1			2. 50,8
382,0	126. 46. 38,6		398,0	127. 38. 53,2		428	129. 8. 52,1	
		40,9			34,9			50,2
5	126. 48. 19,5		5	127. 40. 28,1		429	129. 11. 42,3	
		40,7			34,7			49,6
383,0	126. 50. 0,2		399,0	127. 42. 2,8		430	129. 14. 31,9	
		40,5			34,6			49,0
5	126. 51. 40,7		5	127. 43. 37,4		431	129. 17. 20,9	
		1. 40,4			1. 34,4			2. 48,5
384,0	126. 53. 21,1		400	127. 45. 11,8		432	129. 20. 9,4	
		40,1			3. 8,2			47,9
5	126. 55. 1,2		401	127. 48. 20,0		433	129. 22. 57,3	
		39,9			7,6			47,3
385,0	126. 56. 41,1		402	127. 51. 27,6		434	129. 25. 44,6	
		39,8			6,8			46,7
5	126. 58. 20,9		403	127. 54. 34,4		435	129. 28. 31,3	
		1. 39,5			3. 6,2			2. 46,2
386,0	127. 0. 0,4		404	127. 57. 40,6		436	129. 31. 17,5	
		39,4			5,5			45,6
5	127. 1. 39,8		405	128. 0. 46,1		437	129. 34. 3,1	
		39,1			4,7			45,0
387,0	127. 3. 18,9		406	128. 3. 50,8		438	129. 36. 48,1	
		39,0			4,2			44,5
5	127. 4. 57,9		407	128. 6. 55,0		439	129. 39. 32,6	
		1. 38,8			3. 3,4			2. 43,9
388,0	127. 6. 36,7		408	128. 9. 58,4		440	129. 42. 16,5	
		38,6			2,8			43,4
5	127. 8. 15,3		409	128. 13. 1,2		441	129. 44. 59,9	
		38,4			2,1			42,8
389,0	127. 9. 53,7		410	128. 16. 3,3		442	129. 47. 42,7	
		38,2			1,4			42,3
5	127. 11. 31,9		411	128. 19. 4,7		443	129. 50. 25,0	
		1. 38,0			3. 0,8			2. 41,7
390,0	127. 13. 9,9		412	128. 22. 5,5		444	129. 53. 6,7	
		37,8			3. 0,2			41,2
5	127. 14. 47,7		413	128. 25. 5,7		445	129. 55. 47,9	
		37,7			2. 59,5			40,6
391,0	127. 16. 25,4		414	128. 28. 5,2		446	129. 58. 28,5	
		37,4			58,8			40,1
5	127. 18. 2,8		415	128. 31. 4,0		447	130. 1. 8,6	
		1. 37,3			2. 58,2			2. 39,6

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D. M. S.	Differ. M. S.	Days.	Anomaly. D. M. S.	Differ. M. S.	Days.	Anomaly. D. M. S.	Differ. M. S.
448	130. 3. 48,2	2. 39,1	480	131. 24. 31,0	2. 23,6	512	132. 37. 37,3	2. 10,5
449	130. 6. 27,3	38,5	481	131. 26. 54,6	23,2	513	132. 39. 47,8	10,2
450	130. 9. 5,8	38,0	482	131. 29. 17,8	22,7	514	132. 41. 58,0	9,7
451	130. 11. 43,8		483	131. 31. 40,5		515	132. 44. 7,7	
		2. 37,5			2. 22,2			2. 9,4
452	130. 14. 21,3	37,0	484	131. 34. 2,7	21,8	516	132. 46. 17,1	9,0
453	130. 16. 58,3	36,4	485	131. 36. 24,5	21,4	517	132. 48. 26,1	8,7
454	130. 19. 34,7	36,0	486	131. 38. 45,9	21,0	518	132. 50. 34,8	8,2
455	130. 22. 10,7		487	131. 41. 6,9		519	132. 52. 43,0	
		2. 35,4			2. 20,5			2. 7,9
456	130. 24. 46,1	35,0	488	131. 43. 27,4	20,1	520	132. 54. 50,9	7,6
457	130. 27. 21,1	34,4	489	131. 45. 47,5	19,7	521	132. 56. 58,5	7,2
458	130. 29. 55,5	33,9	490	131. 48. 7,2	19,3	522	132. 59. 5,7	6,8
459	103. 32. 29,4		491	131. 50. 26,5		523	133. 1. 12,5	
		2. 33,5			2. 18,8			2. 6,5
460	130. 35. 2,9	32,9	492	131. 52. 45,3	18,5	524	133. 3. 19,0	6,1
461	130. 37. 35,8	32,5	493	131. 55. 3,8	18,0	525	133. 5. 25,1	5,8
462	130. 40. 8,3	31,9	494	131. 57. 21,8	17,5	526	133. 7. 30,9	5,4
463	130. 42. 40,2		495	131. 59. 39,3		527	133. 9. 36,3	
		2. 31,5			2. 17,2			2. 5,0
464	130. 45. 11,7	31,0	496	132. 1. 56,5	16,8	528	133. 11. 41,3	4,7
465	130. 47. 42,7	30,5	497	132. 4. 13,3	16,4	529	133. 13. 46,0	4,4
466	130. 50. 13,2	30,0	498	132. 6. 29,7	16,0	530	133. 15. 50,4	4,0
467	130. 52. 43,2		499	132. 8. 45,7		531	133. 17. 54,4	
		2. 29,6			2. 15,5			2. 3,7
468	130. 55. 12,8	29,0	500	132. 11. 1,2	15,2	532	133. 19. 58,1	3,3
469	130. 57. 41,8	28,6	501	132. 13. 16,4	14,7	533	133. 22. 1,4	3,0
470	131. 0. 10,4	28,1	502	132. 15. 31,1	14,4	534	133. 24. 4,4	2,7
471	131. 2. 38,5		503	132. 17. 45,5		535	133. 26. 7,1	
		2. 27,7			2. 14,0			2. 2,3
472	131. 5. 6,2	27,2	504	132. 19. 59,5	13,6	536	133. 28. 9,4	1,9
473	131. 7. 33,4	26,7	505	132. 22. 13,1	13,2	537	133. 30. 11,3	1,7
474	131. 10. 0,1	26,3	506	132. 24. 26,3	12,8	538	133. 32. 13,0	1,3
475	131. 12. 26,4		507	132. 26. 39,1		539	133. 34. 14,3	
		2. 25,8			2. 12,4			2. 1,0
476	131. 14. 52,2	25,4	508	132. 28. 51,5	12,0	540	133. 36. 15,3	0,6
477	131. 17. 17,6	24,9	509	132. 31. 3,5	11,7	541	133. 38. 15,9	0,3
478	131. 19. 42,5	24,5	510	132. 33. 15,2	11,2	542	133. 40. 16,2	2. 0,0
479	131. 22. 7,0		511	132. 35. 26,4		543	133. 42. 16,2	
		2. 24,0			2. 10,9			1. 59,7

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D.	M.	S.	Differ. M.	S.	Days.	Anomaly. D.	M.	S.	Differ. M.	S.	Days.	Anomaly. D.	M.	S.	Differ. M.	S.
544	133.	44.	15,9			576	134.	45.	21,7			608	135.	41.	39,4		
				1.	59,3					1.	49,7					1.	41,4
545	133.	46.	15,2			577	134.	47.	11,4			609	135.	43.	20,8		
					59,0						49,5						41,1
546	133.	48.	14,2			578	134.	49.	0,9			610	135.	45.	1,9		
					58,7						49,1						40,8
547	133.	50.	12,9			579	134.	50.	50,0			611	135.	46.	42,7		
				1.	58,4					1.	48,8					1.	40,6
548	133.	52.	11,3			580	134.	52.	38,8			612	135.	48.	23,3		
					58,1						48,6						40,3
549	133.	54.	9,4			581	134.	54.	27,4			613	135.	50.	3,6		
					57,7						48,3						40,1
550	133.	56.	7,1			582	134.	56.	15,7			614	135.	51.	43,7		
					57,5						48,1						39,9
551	133.	58.	4,6			583	134.	58.	3,8			615	135.	53.	23,6		
				1.	57,1					1.	47,8					1.	39,6
552	134.	0.	1,7			584	134.	59.	51,6			616	135.	55.	3,2		
					56,8						47,5						39,4
553	134.	1.	58,5			585	135.	1.	39,1			617	135.	56.	42,6		
					56,4						47,3						39,2
554	134.	3.	54,9			586	135.	3.	26,4			618	135.	58.	21,8		
					56,2						47,0						38,9
555	134.	5.	51,1			587	135.	5.	13,4			619	136.	0.	0,7		
				1.	55,9					1.	46,7					1.	38,7
556	134.	7.	47,0			588	135.	7.	0,1			620	136.	1.	39,4		
					55,5						46,4						38,5
557	134.	9.	42,5			589	135.	8.	46,5			621	136.	3.	17,9		
					55,3						46,1						38,2
558	134.	11.	37,8			590	135.	10.	32,6			622	136.	4.	56,1		
					54,9						45,9						38,0
559	134.	13.	32,7			591	135.	12.	18,5			623	136.	6.	34,1		
				1.	54,7					1.	45,6					1.	37,8
560	134.	15.	27,4			592	135.	14.	4,1			624	136.	8.	11,9		
					54,3						45,4						37,5
561	134.	17.	21,7			593	135.	15.	49,5			625	136.	9.	49,4		
					54,1						45,1						37,3
562	134.	19.	15,8			594	135.	17	34,6			626	136.	11.	26,7		
					53,7						44,9						37,1
563	134.	21.	9,5			595	135.	19.	19,5			627	136.	13.	3,8		
				1.	53,4					1.	44,6					1.	36,9
564	134.	23.	2,9			596	135.	21.	4,1			628	136.	14.	40,7		
					53,2						44,3						36,6
565	134.	24.	56,1			597	135.	22.	48,4			629	136.	16.	17,3		
					52,9						44,1						36,4
566	134.	26.	49,0			598	135.	24.	32,5			630	136.	17.	53,7		
					52,5						43,8						36,2
567	134.	28.	41,5			599	135.	26.	16,3			631	136.	19.	29,9		
				1.	52,3					1.	43,6					1.	36,0
568	134.	30.	33,8			600	135.	27.	59,9			632	136.	21.	5,9		
					52,0						43,3						35,7
569	134.	32.	25,8			601	135.	29.	43,2			633	136.	22.	41,6		
					51,7						43,1						35,5
570	134.	34.	17,5			602	135.	31.	26,3			634	136.	24.	17,1		
					51,4						42,8						35,3
571	134.	36.	8,9			603	135.	33.	9,1			635	136.	25.	52,4		
				1.	51,2					1.	42,6					1.	35,1
572	134.	38.	0,1			604	135.	34.	51,7			636	136.	27.	27,5		
					50,8						42,3						34,8
573	134.	39.	50,9			605	135.	36.	34,0			637	136.	29.	2,3		
					50,5						42,0						34,7
574	134.	41.	41,4			606	135.	38.	16,0			638	136.	30.	37,0		
					50,3						41,8						34,4
575	134.	43.	31,7			607	135.	39.	57,8			639	136.	32.	11,4		
				1.	50,0					1.	41,6					1.	34,2

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D. M. S.	Differ. M. S.	Days.	Anomaly. D. M. S.	Differ. M. S.	Days.	Anomaly. D. M. S.	Differ. M. S.
640	136. 33. 45,6		672	137. 22. 10,7		710,0	138. 15. 28,2	
641	136. 35. 19,6	1. 34,0	673	137. 23. 38,2	1. 27,5	712,5	138. 18. 49,8	3. 21,6
642	136. 36. 53,4	33,8	674	137. 25. 5,5	27,3	715,0	138. 22. 10,3	20,5
643	136. 38. 26,9	33,5	675	137. 26. 32,6	27,1	717,5	138. 25. 29,8	19,5
		1. 33,4			1. 27,0			3. 18,6
644	136. 40. 0,3		676	137. 27. 59,6	26,7	720,0	138. 28. 48,4	17,5
645	136. 41. 33,4	33,1	677	137. 29. 26,3	26,6	722,5	138. 32. 5,9	16,5
646	136. 43. 6,3	32,9	678	137. 30. 52,9	26,4	725,0	138. 35. 22,4	15,5
647	136. 44. 39,0	32,7	679	137. 32. 19,3		727,5	138. 38. 37,9	
		1. 32,5			1. 26,2			3. 14,6
648	136. 46. 11,5		680	137. 33. 45,5	26,0	730,0	138. 41. 52,5	13,6
649	136. 47. 43,8	32,3	681	137. 35. 11,5	25,8	732,5	138. 45. 6,1	12,6
650	136. 49. 15,9	32,1	682	137. 36. 37,3	25,6	735,0	138. 48. 18,7	11,6
651	136. 50. 47,8	31,9	683	137. 38. 2,9		737,5	138. 51. 30,3	
		1. 31,7			1. 25,5			3. 10,8
652	136. 52. 19,5		684	137. 39. 28,4	25,3	740,0	138. 54. 41,1	9,8
653	136. 53. 50,9	31,4	685	137. 40. 53,7	25,1	742,5	138. 57. 50,9	8,8
654	136. 55. 22,2	31,3	686	137. 42. 18,8	24,9	745,0	139. 0. 59,7	8,0
655	136. 56. 53,2	31,0	687	137. 43. 43,7		747,5	139. 4. 7,7	
		1. 30,9			1. 24,7			3. 7,0
656	136. 58. 24,1		688	137. 45. 8,4	24,6	750,0	139. 7. 14,7	6,2
657	136. 59. 54,7	30,6	689	137. 46. 33,0	24,3	752,5	139. 10. 20,9	5,2
658	137. 1. 25,2	30,5	690	137. 47. 57,3	24,2	755,0	139. 13. 26,1	4,3
659	137. 2. 55,4	30,2	691	137. 49. 21,5		757,5	139. 16. 30,4	
		1. 30,1			1. 24,1			3. 3,5
660	137. 4. 25,5		692	137. 50. 45,6	23,8	760,0	139. 19. 33,9	2,5
661	137. 5. 55,3	29,8	693	137. 52. 9,4	23,7	762,5	139. 22. 36,4	1,7
662	137. 7. 25,0	29,7	694	137. 53. 33,1	23,5	765,0	139. 25. 38,1	0,9
663	137. 8. 54,4	29,4	695	137. 54. 56,6		767,5	139. 28. 39,0	
		1. 29,3			1. 23,3			3. 0,0
664	137. 10. 23,7		696	137. 56. 19,9	23,1	770,0	139. 31. 39,0	2. 59,1
665	137. 11. 52,7	29,0	697	137. 57. 43,0	23,0	772,5	139. 34. 38,1	58,3
666	137. 13. 21,6	28,9	698	137. 59. 6,0	22,8	775,0	139. 37. 36,4	57,5
667	137. 14. 50,2	28,6	699	138. 0. 28,8		777,5	139. 40. 33,9	
		1. 28,5			1. 22,6			2. 56,6
668	137 16. 18,7		700	138. 1. 51,4	3. 25,8	780,0	139. 43. 30,5	55,9
669	137. 17. 47,0	28,3	702,5	138. 5. 17,2	24,7	782,5	139. 46. 26,4	55,0
670	137. 19. 15,1	28,1	705,0	138. 8. 41,9	23,7	785,0	139. 49. 21,4	54,2
671	137. 20. 43,0	27,9	707,5	138. 12. 5,6		787,5	139. 52. 15,6	
		1. 27,7			3. 22,6			2. 53,4

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D. M. S.	Differ. M. S.	Days.	Anomaly. D. M. S.	Differ. M S.	Days.	Anomaly. D. M. S.	Differ. M. S.
790,0	139. 55. 9,0		870,0	141. 21. 9,0		950,0	142. 36 24,4	
		2. 52,6			2. 30,1			2. 12,2
792,5	139. 58. 1,6		872,5	141. 23. 39,1		952,5	142. 38. 36,6	
		51,8			29,5			11,7
795,0	140. 0. 53,4		875,0	141. 26. 8,6		955,0	142. 40. 48,3	
		51,0			28,8			11,3
797,5	140. 3. 44,4		877,5	141. 28. 37,4		957,5	142. 42. 59,6	
		2. 50,3			2. 28,3			2. 10,7
800,0	140. 6. 34,7		880,0	141. 31. 5,7		960,0	142. 45. 10,3	
		49,4			27,6			10,2
802,5	140. 9. 24,1		882,5	141. 33. 33,3		962,5	142. 47. 20,5	
		48,7			27,1			9,8
805,0	140. 12. 12,8		885,0	141. 36. 0,4		965,0	142. 49. 30,3	
		48,0			26,4			9,3
807,5	140. 15. 0,8		887,5	141. 38. 26,8		967,5	142. 31. 39,6	
		2. 47,2			2. 25,9			2. 8,7
810,0	140. 17. 48,0		890,0	141. 40. 52,7		970,0	142. 53. 48,3	
		46,4			25,2			8,3
812,5	140. 20. 34,4		892,5	141. 43. 17,9		972,5	142. 55. 56,6	
		45,7			24,7			7,9
815,0	140. 23. 20,1		895,0	141. 45. 42,6		975,0	142. 58. 4,5	
		45,0			24,0			7,3
817,5	140. 26. 5,1		897,5	141. 48. 6,6		977,5	143. 0. 11,8	
		2. 44,3			2. 23,6			2. 6,9
820,0	140. 28. 49,4		900,0	141. 50. 30,2		980,0	143. 2. 18,7	
		43,5			22,9			6,5
822,5	140. 31. 32,9		902,5	141. 52. 53,1		982,5	143. 4. 25,2	
		42,8			22,3			5,9
825,0	140. 34. 15,7		905,0	141. 55. 15,4		985,0	143 6. 31,1	
		42,1			21,7			5,5
827,5	140. 36. 57,8		907,5	141. 57. 37,1		987,5	143. 8. 36,6	
		2. 41,4			2. 21,3			2. 5,1
830,0	140. 39. 39,2		910,0	141. 59. 58,4		990,0	143. 10. 41,7	
		40,6			20,7			4,6
832,5	140. 42. 19,8		912,5	142. 2. 19,1		992,5	143. 12. 46,3	
		40,0			20,1			4,1
835,0	140. 44. 59,8		915,0	142. 4. 39,2		995,0	143. 14. 50,4	
		39,3			19,5			3,7
837,5	140. 47. 39,1		917,5	142. 6. 58,7		997,5	143. 16. 54,1	
		2. 38,6			2. 19,0			2. 3,3
840,0	140. 50. 17,7		920,0	142. 9. 17,7		1000,0	143. 18. 57,4	
		37,9			18,5			2,8
842,5	140. 52. 55,6		922,5	142. 11. 36,2		1002,5	143. 21. 0,2	
		37,2			17,9			2,3
845,0	140. 55. 32,8		925,0	142. 13. 54,1		1005,0	143. 23. 2,5	
		36,6			17,4			2,0
847,5	140. 58. 9,4		927,5	142. 16. 11,5		1007,5	143. 25. 4,5	
		2. 35,9			2. 16,9			2. 1,5
850,0	141. 0 45,3		930,0	142. 18. 28,4		1010,0	143. 27. 6,0	
		35,2			16,3			1,0
852,5	141. 3. 20,5		932,5	142. 20. 44,7		1012,5	143. 29. 7,0	
		34,6			15,8			0,6
855,0	141. 5. 55,1		935,0	142. 23. 0,5		1015,0	143 31. 7,6	
		33,9			15,3			2. 0,2
857,5	141. 8. 29,0		937,5	142. 25. 15,8		1017,5	143. 33. 7,8	
		2. 33,3			2 14,8			1. 59,8
860,0	141. 11. 2,3		940,0	142. 27. 30,6		1020,0	143. 35. 7,6	
		32,6			14,2			59,4
862,5	141. 13. 34,9		942,5	142. 29. 44,8		1022,5	143. 37. 7,0	
		32,0			13,7			58,9
865,0	141. 16. 6,9		945,0	142. 31. 58,5		1025,0	143. 39. 5,9	
		31,4			13,2			58,5
867,5	141. 18. 38,3		947,5	142. 34. 11,7		1027,5	143. 41. 4,4	
		2. 30,7			2. 12,7			1. 58,1

THE THIRD TABLE CONTINUED.

Days	Anomaly D. M. S.	Differ. M. S.	Days.	Anomaly. D. M. S.	Differ. M. S.	Days.	Anomaly. D. M. S.	Differ. M. S.
1030,0	143. 43. 2,5		1110,0	144. 42. 37,4		1190,0	145. 36. 20,9	
1032,5	143. 45. 0,2	I. 57,7	1112,5	144. 44. 23,1	I. 45,7	1192,5	145. 37. 56,6	I. 35,7
1035,0	143. 46. 57,5	57,3	1115,0	144. 46. 8,5	45,4	1195,0	145. 39. 32,0	35,4
1037,5	143. 48. 54,4	56,9	1117,5	144. 47. 53,5	45,0	1197,5	145. 41. 7,2	35,2
		I. 56,5			I. 44,6			I. 34,9
1040,0	143. 50. 50,9		1120,0	144. 49. 38,1		1200	145. 42. 42,1	
1042,5	143. 52. 47,0	56,1	1122,5	144. 51. 22,6	44,5	1205	145. 45. 50,9	3. 8,8
1045,0	143. 54. 42,6	55,6	1125,0	144. 53. 6,7	44,1	1210	145. 48. 58,7	7,8
1047,5	143. 56. 37,9	55,3	1127,5	144. 54. 50,4	43,7	1215	145. 52. 5,3	6,6
		I. 54,9			I. 43,4			3. 5,6
1050,0	143. 58. 32,8		1130,0	144. 56. 33,8		1220	145. 55. 10,9	
1052,5	144. 0. 27,3	54,5	1132,5	144. 58. 16,9	43,1	1225	145. 58. 15,4	4,5
1055,0	144. 2. 21,4	54,1	1135,0	144. 59. 59,6	42,7	1230	146. 1. 18,8	3,4
1057,5	144. 4. 15,1	53,7	1137,5	145. 1. 42,0	42,4	1235	146. 4. 21,1	2,3
		I. 53,4			I. 42,1			3. 1,3
1060,0	144. 6. 8,5		1140,0	145. 3. 24,1		1240	146. 7. 22,4	
1062,5	144. 8. 1,4	52,9	1142,5	145 5. 5,9	41,8	1245	146. 10. 22,7	0,3
1065,0	144. 9. 54,0	52,6	1145,0	145. 6. 47,3	41,4	1250	146. 13. 21,9	2. 59,2
1067,5	144. 11. 46,2	52,2	1147,5	145. 8. 28,5	41,2	1255	146. 16. 20,1	58,2
		I. 51,8			I. 40,8			2. 57,2
1070,0	144. 13. 38,0		1150,0	145. 10. 9,3		1260	146. 19. 17,3	
1072,5	144. 15. 29,4	51,4	1152,5	145. 11. 49,8	40,5	1265	146. 22. 13,5	56,2
1075,0	144. 17. 20,5	51,1	1155,0	145. 13. 30,0	40,2	1270	146. 25. 8,7	55,2
1077,5	144. 19. 11,2	50,7	1157,5	145. 15. 9,8	39,8	1275	146. 28. 2,9	54,2
		I. 50,3			I. 39,6			2. 53,2
1080,0	144. 21. 1,5		1160,0	145. 16. 49,4		1280	146. 30. 56,1	
1082,5	144. 22. 51,5	50,0	1162,5	145. 18. 28,7	39,3	1285	146. 33. 48,4	52,3
1085,0	144. 24. 41,0	49,5	1165,0	145. 20. 7,7	39,0	1290	146. 36. 39,8	51,4
1087,5	144. 26. 30,2	49,2	1167,5	145. 21. 46,3	38,6	1295	146. 39. 30,2	50,4
		I. 48,9			I. 38,4			2. 49,4
1090,0	144. 28. 19,1		1170,0	145. 23. 24,7		1300	146. 42. 19,6	
1092,5	144. 30. 7,6	48,5	1172,5	145. 25. 2,8	38,1	1305	146. 45. 8,1	48,5
1095,0	144. 31. 55,8	48,2	1175,0	145. 26. 40,5	37,7	1310	146. 47. 55,7	47,6
1097,5	144. 33. 43,6	47,8	1177,5	145. 28. 18,0	37,5	1315	146. 50. 42,4	46,7
		I. 47,4			I. 37,1			2. 45,8
1100,0	144. 35. 31,0		1180,0	145. 29. 55,1		1320	146. 53. 28,2	
1102,5	144. 37. 18,1	47,1	1182,5	145. 31. 32,0	36,9	1325	146. 56. 13,1	44,9
1105,0	144. 39. 4,9	46,8	1185,0	145. 33. 8,6	36,6	1330	146. 58. 57,2	44,1
1107,5	144. 40. 51,3	46,4	1187,5	145. 34. 44,9	36,3	1335	147. 1. 40,3	43,1
		I. 46,1			I. 36,0			2. 42,3

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D. M. S.	Differ. M. S.	Days.	Anomaly. D. M. S.	Differ. M. S.	Days.	Anomaly. D. M. S.	Differ. M. S.
1340	147. 4. 22,6		1500	148. 24. 0,8		1660	149. 32. 26,5	
		2. 41,4			2. 17,6			1. 59,2
1345	147. 7. 4,0		1505	148. 26. 18,4		1665	149. 34. 25,7	
		40,6			16,9			58,7
1350	147. 9. 44,6		1510	148. 28. 35,3		1670	149. 36. 24,4	
		39,7			16,3			58,2
1355	147. 12. 24,3		1515	148. 30. 51,6		1675	149. 38. 22,6	
		2. 38,9			2. 15,7			1. 57,7
1360	147. 15. 3,2		1520	148. 33. 7,3		1680	149. 40. 20,3	
		38,1			15,0			57,3
1365	147. 17. 41,3		1525	148. 35. 22,3		1685	149. 42. 17,6	
		37,2			14,4			56,7
1370	147. 20 18,5		1530	148. 37. 36,7		1690	149. 44. 14,3	
		36,5			13,8			56,2
1375	147. 22. 55,0		1535	148. 39. 50,5		1695	149. 46. 10,5	
		2. 35,6			2. 13,1			1. 55,8
1380	147. 25. 30,6		1540	148. 42. 3,6		1700	149. 48. 6,3	
		34,8			12,6			55,3
1385	147. 28. 5,4		1545	148. 44. 16,2		1705	149. 50. 1,6	
		34,1			11,9			54,8
1390	147. 30. 39,5		1550	148. 46. 28,1		1710	149. 51. 56,4	
		33,2			11,3			54,3
1395	147. 33. 12,7		1555	148. 48 39,4		1715	149. 53. 50,7	
		2. 32,5			2. 10,8			1. 53,9
1400	147. 35. 45,2		1560	148. 50. 50,2		1720	149. 55. 44,6	
		31,7			10,1			53,4
1405	147. 38. 16,9		1565	148. 53. 0,3		1725	149. 57. 38,0	
		30,9			9,6			52,9
1410	147 40. 47,8		1570	148. 55. 9,9		1730	149. 59. 30,9	
		30,1			8,9			52,5
1415	147. 43. 17,9		1575	148. 57. 18,8		1735	150. 1, 23,4	
		2. 29,5			2. 8,5			1. 52,0
1420	147. 45. 47,4		1580	148. 59. 27,3		1740	150. 3. 15,4	
		28,6			7,8			51,6
1425	147. 48. 16,0		1585	149. 1. 35,1		1745	150. 5. 7,0	
		28,0			7,2			51,1
1430	147. 50. 44,0		1590	149. 3. 42,3		1750	150. 6. 58,1	
		27,2			6,7			50,6
1435	147. 53. 11,2		1595	149. 5. 49,0		1755	150. 8. 48,7	
		2. 26,5			2 6,2			1. 50,2
1440	147. 55. 37,7		1600	149. 7. 55,2		1760	150. 10. 38,9	
		25,7			5,5			49,8
1445	147. 58. 3,4		1605	149. 10. 0,7		1765	150. 12. 28,7	
		25,0			5,0			49,4
1450	148. 0. 28,4		1610	149. 12. 5,7		1770	150. 14. 18,1	
		24,4			4,5			48,9
1455	148. 2. 52,8		1615	149. 14. 10,2		1775	150. 16. 7,0	
		2. 23,6			2. 4,0			1. 48,5
1460	148. 5. 16,4		1620	149. 16. 14,2		1780	150. 17. 55,5	
		22,9			3,4			48,0
1465	148. 7. 39,3		1625	149. 18. 17,6		1785	150. 19. 43,5	
		22,3			2,8			47,7
1470	148. 10. 1,6		1630	149. 20. 20,4		1790	150. 21. 31,2	
		21,5			2,3			47,2
1475	148. 12. 23,1		1635	149. 22. 22,7		1795	150. 23. 18,4	
		2. 20,9			2. 1,8			1. 46,8
1480	148. 14. 44,0		1640	149. 24. 24,5		1800	150. 25. 5,2	
		20,2			1,3			3. 32,3
1485	148. 17. 4,2		1645	149. 26. 25,8		1810	150. 28. 37,5	
		19,6			0,7			30,6
1490	148. 19. 23,8		1650	149. 28. 26,5		1820	150. 32. 8,1	
		18,8			0,3			29,1
1495	148. 21. 42,6		1655	149. 30. 26,8		1830	150. 35. 37,2	
		2. 18,2			1. 59,7			3. 27,4

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D. M. S.	Differ. M. S.	Days.	Anomaly. D. M. S.	Differ. M S.	Days.	Anomaly. D. M. S.	Differ. M. S.
1840	150. 39. 4,6		2160	152. 17. 27,5		2480	153. 37. 17,3	
1850	150. 42. 30,5	3. 25,8	2170	152. 20. 11,9	2. 44,4	2490	153. 39. 32,9	2. 15,6
1860	150. 45. 54,8	24,3	2180	152. 22. 55,3	43,4	2500	153. 41. 47,8	14,9
1870	150. 49. 17,6	22,8	2190	152. 25. 37,7	42,4	2510	153. 44. 1,9	14,1
		3. 21,2			2. 41,3			2. 13,4
1880	150. 52. 38,8		2200	152. 28. 19,0		2520	153. 46. 15,3	
1890	150. 55. 58,6	19,8	2210	152. 30. 59,3	40,3	2530	153. 48. 27,9	12,6
1900	150. 59. 16,8	18,2	2220	152. 33. 38,5	39,2	2540	153. 50. 39,8	11,9
1910	150. 2. 33,6	16,8	2230	152. 36. 16,8	38,3	2550	153. 52. 51,0	11,2
		3. 15,4			2. 37,3			2. 10,4
1920	151. 5. 49,0		2240	152. 38. 54,1		2560	153. 55. 1,4	
1930	151. 9. 2,9	13,9	2250	152. 41. 30,4	36,3	2570	153. 57. 11,1	9,7
1940	151. 12. 15,4	12,5	2260	152. 44. 5,7	35,3	2580	153. 59. 20,2	9,1
1950	151. 15. 26,5	11,1	2270	152. 46. 40,1	34,4	2590	154. 1. 28,6	8,4
		3. 9,8			2. 33,4			2. 7,7
1960	151. 18. 36,3		2280	152. 49. 13,5		2600	154. 3. 36,3	
1970	151. 21. 44,7	8,4	2290	152. 51. 46,0	32,5	2610	154. 5. 43,3	7,0
1980	151. 24. 51,8	7,1	2300	152. 54. 17,5	31,5	2620	154. 7. 49,6	6,3
1990	151. 27. 57,5	5,7	2310	152. 56. 48,1	30,6	2630	154. 9. 55,2	5,6
		3. 4,5			2. 29,7			2. 5,0
2000	151. 31. 2,0		2320	152. 59. 17,8		2640	154. 12. 0,2	
2010	151. 34. 5,1	3,1	2330	153. 1. 46,7	28,9	2650	154. 14. 4,5	4,3
2020	151. 37. 7,0	1,9	2340	153. 4. 14,7	28,0	2660	154. 16. 8,2	3,7
2030	151. 40. 7,6	0,6	2350	153. 6. 41,8	27,1	2670	154. 18. 11,2	3,0
		2. 59,3			2. 26,2			2. 2,4
2040	151. 43. 6,9		2360	153. 9. 8,0		2680	154. 20. 13,6	
2050	151. 46. 5,1	58,2	2370	153. 11. 33,3	25,3	2690	154. 22. 15,3	1,7
2060	151. 49. 2,0	56,9	2380	153. 13. 57,7	24,4	2700	154. 24. 16,4	1,1
2070	151. 51. 57,8	55,8	2390	153. 16. 21,3	23,6	2710	154. 26. 16,9	0,5
		2. 54,5			2. 22,8			1. 59,9
2080	151. 54. 52,3		2400	153. 18. 44,1		2720	154. 28. 16,8	
2090	151. 57. 45,7	53,4	2410	153. 21. 6,1	22,0	2730	154. 30. 16,1	59,3
2100	152. 0. 37,9	52,2	2420	153. 23. 27,3	21,2	2740	154. 32. 14,7	58,6
2110	152. 3. 28,9	51,0	2430	153. 25. 47,6	20,3	2750	154. 34. 12,8	58,1
		2. 49,9			2. 19,5			1. 57,5
2120	152. 6. 18,8		2440	153. 28. 7,1		2760	154. 36. 10,3	
2130	152. 9. 7,7	48,9	2450	153. 30. 25,8	18,7	2770	154. 38. 7,1	56,8
2140	152. 11. 55,4	47,7	2460	153. 32. 43,7	17,9	2780	154. 40. 3,4	56,3
2150	152. 14. 42,0	46,6	2470	153. 35. 0,9	17,2	2790	154. 41. 59,1	55,7
		2. 45,5			2. 16,4			1. 55,1

THE THIRD TABLE CONTINUED.

Days.	Anomaly D. M. S.	Differ. M. S.	Days.	Anomaly. D. M. S.	Differ. M. S.	Days.	Anomaly. D. M. S.	Differ. M. S.
2800	154. 43. 54,2		3150,0	155. 45. 36,2		3550,0	156. 45. 24,9	
		1. 54,6			2. 1,5			1. 43,0
2810	154. 45. 48,8		3162,5	155. 47. 37,7		3562,5	156. 47. 7,9	
		54,0			0,9			42,5
2820	154. 47. 42,8		3175,0	155. 49. 38,6		3575,0	156. 48. 50,4	
		53,4			0,3			42,1
2830	154. 49. 36,2		3187,5	155. 51. 38,9		3587,5	156. 50. 32,5	
		1. 52,9			1. 59,6			1. 41,5
2840	154. 51. 29,1		3200,0	155. 53. 38,5		3600,0	156. 52. 14,0	
		52,3			58,9			41,1
2850	154. 53. 21,4		3212,5	155. 55. 37,4		3612,5	156. 53. 55,1	
		51,8			58,3			40,6
2860	154. 55. 13,2		3225,0	155. 57. 35,7		3625,0	156. 55. 35,7	
		51,3			57,7			40,1
2870	154. 57. 4,5		3237,5	155. 59. 33,4		3637,5	156. 57. 15,8	
		1. 50,7			1. 57,0			1. 39,6
2880	154. 58. 55,2		3250,0	156. 1. 30,4		3650,0	156. 58. 55,4	
		50,1			56,4			39,2
2890	155. 0. 45,3		3262,5	156. 3. 26,8		3662,5	157. 0. 34,6	
		49,7			55,8			38,7
2900	155. 2. 35,0		3275,0	156. 5. 22,6		3675,0	157. 2. 13,3	
		49,1			55,2			38,2
2910	155. 4. 24,1		3287,5	156. 7. 17,8		3687,5	157. 3. 51,5	
		1. 48,6			1. 54,6			1. 37,8
2920	155. 6. 12,7		3300,0	156. 9. 12,4		3700,0	157. 5. 29,3	
		48,0			54,0			37,3
2930	155. 8. 0,7		3312,5	156. 11. 6,4		3712,5	157. 7. 6,6	
		47,6			53,3			36,9
2940	155. 9. 48,3		3325,0	156. 12. 59,7		3725,0	157. 8. 43,5	
		47,1			52,8			36,4
2950	155. 11. 35,4		3337,5	156. 14. 52,5		3737,5	157. 10. 19,9	
		1. 46,6			1. 52,2			1. 36,0
2960	155. 13. 22,0		3350,0	156. 16. 44,7		3750,0	157. 11. 55,9	
		46,0			51,7			35,6
2970	155. 15. 8,0		3362,5	156. 18. 36,4		3762,5	157. 13. 31,5	
		45,6			51,0			35,1
2980	155. 16. 53,6		3375,0	156. 20. 27,4		3775,0	157. 15. 6,6	
		45,1			50,5			34,6
2990	155. 18. 38,7		3387,5	156. 22. 17,9		3787,5	157. 16. 41,2	
		1. 44,5			1. 50,0			1. 34,2
3000,	155. 20. 23,2		3400,0	156. 24. 7,9		3800,0	157. 18. 15,4	
		2. 10,1			49,4			33,8
3012,5	155. 22. 33,3		3412,5	156. 25. 57,3		3812,5	157. 19 49,2	
		9,3			48,8			33,4
3025,0	155. 24. 42,6		3425,0	156. 27. 46,1		3825,0	157. 21. 22,6	
		8,6			48,2			33,0
3037,5	155. 26. 51,2		3437,5	156. 29. 34,3		3837,5	157. 22. 55,6	
		2. 7,8			1. 47,7			1. 32,6
3050,0	155. 28. 59,0		3450,0	156. 31. 22,0		3850,0	157. 24. 28,2	
		7,1			47,2			32,1
3062,5	155. 31. 6,1		3462,5	156. 33. 9,2		3862,5	157. 26. 0,3	
		6,4			46,7			31,7
3075,0	155. 33. 12,5		3475,0	156. 34. 55,9		3875,0	157. 27. 32,0	
		5,7			46,1			31,3
3087,5	155. 35. 18,2		3487,5	156. 36. 42,0		3887,5	157. 29. 3,3	
		2. 5,0			1. 45,6			1. 31,0
3100,0	155. 37. 23,2		3500,0	156. 38. 27,6		3900,0	157. 30. 34,3	
		4,3			45,1			30,5
3112,5	155. 39. 27,5		3512,5	156. 40. 12,7		3912,5	157. 32. 4,8	
		3,6			44,6			30,1
3125,0	155. 41. 31,1		3525,0	156. 41. 57,3		3925,0	157. 33. 34,9	
		2,9			44,0			29,7
3137,5	155. 43. 34,0		3537,5	156. 43. 41,3		3937,5	157. 35. 4,6	
		2. 2,2			1. 43,6			1. 29,3

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D.	M.	S.	Differ. M.	S.	Days.	Anomaly. D.	M.	S.	Differ. M.	S.	Days.	Anomaly. D.	M.	S.	Differ. M.	S.
3950,0	157.	36.	33,9			4700	158.	55.	32,7			5500	160.	2.	39,3		
3962,5	157.	38.	2,8	1.	28,9	4725	158.	57.	52,5	2.	19,8	5525	160	4.	32,0	1.	52,7
3975,0	157.	39.	31,4		28,6	4750	159.	0.	11,3		18,8	5550	160.	6.	24,0		52,0
3987,5	157.	40.	59,5		28,1	4775	159.	2.	29,1		17,8	5575	160.	8.	15,3		51,3
				1.	27,8					2.	16,8					1.	50,7
4000	157.	42.	27,3			4800	159.	4.	45,9			5600	160.	10.	6,0		
4025	157.	45.	21,8	2.	54,5	4825	159.	7.	1,7		15,8	5625	160.	11.	56,0		50,0
4050	157.	48.	14,7		52,9	4850	159.	9.	16,5		14,8	5650	160.	13.	45,3		49,3
4075	157.	51.	6,2		51,5	4875	159.	11.	30,4		13,9	5675	160.	15.	33,9		48,6
				2.	50,1					2.	13,0					1.	48,0
4100	157.	53.	56,3			4900	159.	13.	43,4			5700	160.	17.	21,9		
4125	157.	56.	44,9		48,6	4925	159.	15.	55,4		12,0	5725	160.	19.	9,3		47,4
4150	157.	59.	32,1		47,2	4950	159.	18.	6,5		11,1	5750	160.	20.	56,0		46,7
4175	158.	2.	17,9		45,8	4975	159.	20.	16,7		10,2	5775	160.	22.	42,0		46,0
				2.	44,5					2.	9,3					1.	45,5
4200	158.	5.	2,4			5000	159.	22.	26,0			5800	160.	24.	27,5		
4225	158.	7.	45,5		43,1	5025	159.	24.	34,4		8,4	5825	160.	26.	12,3		44,8
4250	158.	10.	27,3		41,8	5050	159.	26.	42,0		7,6	5850	160.	27.	56,5		44,2
4275	158.	13.	7,8		40,5	5075	159.	28.	48,7		6,7	5875	160.	29.	40,1		43,6
				2.	39,2					2.	5,8					1.	43,0
4300	158.	15.	47,0			5100	159.	30.	54,5			5900	160.	31.	23,1		
4325	158.	18.	24,9		37,9	5125	159.	32.	59,5		5,0	5925	160.	33.	5,5		42,4
4350	158.	21.	1,6		36,7	5150	159.	35.	3,7		4,2	5950	160.	34.	47,3		41,8
4375	158.	23.	37,1		35,5	5175	159.	37.	7,0		3,3	5975	160.	36.	28,6		41,3
				2.	34,2					2.	2,5					1.	40,6
4400	158.	26.	11,3			5200	159.	39.	9,5			6000	160.	38.	9,2		
4425	158.	28.	44,3		33,0	5225	159	41.	11,2		1,7	6025	160.	39.	49,3		40,1
4450	158.	31.	16,2		31,9	5250	159.	43.	12,1		0,9	6050	160.	41.	28,8		39,5
4475	158.	33.	46,9		30,7	5275	159.	45.	12,2		0,1	6075	160.	43.	7,8		39,0
				2.	29,5					1.	59,4					1.	38,4
4500	158.	36.	16,4			5300	159.	47.	11,6			6100	160.	44.	46,2		
4525	158.	38.	44,7		28,3	5325	159.	49.	10,2		58,6	6125	160.	46.	24,0		37,8
4550	158.	41.	12,0		27,3	5350	159.	51.	8,0		57,8	6150	160.	48.	1,3		37,3
4775	158.	43.	38,1		26,1	5375	159.	53.	5,0		57,0	6175	160.	49.	38,1		36,8
				2.	25,1					1.	56,3					1.	36,2
4600	158.	46.	3,2			5400	159.	55.	1,3			6200	160.	51.	14,3		
4625	158.	48.	27,2		24,0	5425	159.	56.	56,9		55,6	6225	160.	52.	50,0		35,7
4650	158.	50.	50,1		22,9	5450	159.	58.	51,8		54,9	6250	160.	54.	25,2		35,2
4675	158.	53.	11,9		21,8	5475	160.	0.	45,9		54,1	6275	160.	55.	59,8		34,6
				2.	20,8					1.	53,4					1.	34,2

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D. M. S.	Differ. M. S.	Days.	Anomaly. D. M. S.	Differ. M. S.	Days.	Anomaly. D. M. S.	Differ. M. S.
6300	160. 57. 34,0		7200	161. 48. 56,9		8800	163. 1. 36,6	
6325	160. 59. 7,6	1. 33,6	7250	161. 51. 32,6	2. 35,7	8850	163. 3. 35,2	1. 58,6
6350	161. 0. 40,7	33,1	7300	161. 54. 6,9	34,3	8900	163. 5. 32,8	57,6
6375	161. 2. 13,3	32,6	7350	161. 56. 39,7	32,8	8950	163. 7. 29,6	56,8
		1. 32,2			2. 31,4			1. 55,8
6400	161. 3. 45,5		7400	161. 59. 11,1		9000	163. 9. 25,4	
6425	161. 5. 17,1	31,6	7450	162. 1. 41,1	30,0	9050	163. 11. 20,4	55,0
6450	161. 6. 48,3	31,2	7500	162. 4. 9,8	28,7	9100	163. 13. 14,5	54,1
6475	161. 8. 18,9	30,6	7550	162. 6. 37,1	27,3	9150	163. 15. 7,8	53,3
		1. 30,2			2. 25,9			1. 52,5
6500	161. 9. 49,1		7600	162. 9. 3,0		9200	163. 17. 0,3	
6525	161. 11. 18,8	29,7	7650	162. 11. 27,7	24,7	9250	163. 18. 51,9	51,6
6550	161. 12. 48,1	29,3	7700	162. 13. 51,1	23,4	9300	163. 20. 42,7	50,8
6575	161. 14. 16,9	28,8	7750	162. 16. 13,2	22,1	9350	163. 22. 32,7	50,0
		1. 28,3			2. 20,9			1. 49,1
6600	161. 15. 45,2		7800	162. 18. 34,1		9400	163. 24. 21,8	
6625	161. 17. 13,1	27,9	7850	162. 20. 53,7	19,6	9450	163. 26. 10,2	48,4
6650	161. 18. 40,5	27,4	7900	162. 23. 12,2	18,5	9500	163. 27. 57,9	47,7
6675	161. 20. 7,5	27,0	7950	162. 25. 29,4	17,2	9550	163. 29. 44,7	46,8
		1. 26,5			2. 16,1			1. 46,1
6700	161. 21. 34,0		8000	162. 27. 45,5		9600	163. 31. 30,8	
6725	161. 23. 0,1	26,1	8050	162. 30. 0,4	14,9	9650	163. 33. 16,2	45,4
6750	161. 24. 25,7	25,6	8100	162. 32. 14,2	13,8	9700	163. 35. 0,8	44,6
6775	161. 25. 50,9	25,2	8150	162. 34. 26,9	12,7	9750	163. 36. 44,6	43,8
		1. 24,8			2 11,6			1. 43,2
6800	161. 27. 15,7		8200	162. 36. 38,5		9800	163. 38. 27,8	
6825	161. 28. 40,1	24,4	8250	162. 38. 49,0	10,5	9850	163. 40. 10,2	42,4
6850	161. 30. 4,0	23,9	8300	162. 40. 58,4	9,4	9900	163. 41. 52,0	41,8
6875	161. 31. 27,6	23,6	8350	162. 43. 6,7	8,3	9950	163. 43. 33,0	41,0
		1. 23,1			2. 7,3			1. 40,4
6900	161. 32. 50,7		8400	162. 45. 14,0		10000	163. 45. 13,4	
6925	161. 34. 13,4	22,7	8450	162. 47. 20,3	6,3	10050	163. 46. 53,0	39,6
6950	161. 35. 35,7	22,3	8500	162. 49. 25,6	5,3	10100	163. 48. 32,0	39,0
6975	161. 36. 57,6	21,9	8550	162. 51. 29,8	4,2	10150	163. 50. 10,4	38,4
		1. 21,4			2. 3,3			1. 37,7
7000	161. 38. 19,0		8600	162. 53. 33,1		10200	163. 51. 48,1	
7050	161. 41. 0,8	2. 41,8	8650	162. 55. 35,4	2,3	10250	163. 53. 25,1	37,0
7100	161. 44. 41,1	40,3	8700	162. 57. 36,8	1,4	10300	163. 55. 1,5	36,4
7150	161. 46. 19,8	38,7	8750	162. 59. 37,2	0,4	10350	163. 56. 37,2	35,7
		2. 37,1			1. 59,4			1. 35,2

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D. M. S.	Differ. M. S.	Days.	Anomaly. D. M. S.	Differ. M. S.	Days.	Anomaly. D. M. S.	Differ. M. S.
10400	163. 58. 12,4		12000	164. 44. 3,8		15200	165. 54. 53,4	
		1. 34,5			2. 35,3			1. 52,9
10450	163. 59. 46,9		12100	164. 46. 39,1		15300	165. 56. 46,3	
		33,9			33,6			51,9
10500	164. 1. 20,8		12200	164. 49. 12,7		15400	165. 58. 38,2	
		33,3			31,9			51,0
10550	164. 2. 54,1		12300	164. 51. 44,6		15500	166. 0. 29,2	
		1. 32,7			2. 30,2			1. 49,9
10600	164. 4. 26,8		12400	164. 54. 14,8		15600	166. 2. 19,1	
		32,1			28,6			49,0
10650	164. 5. 58,9		12500	164. 56. 43,4		15700	166. 4. 8,1	
		31,5			26,9			48,1
10700	164. 7. 30,4		12600	164. 59. 10,3		15800	166. 5. 56,2	
		31,0			25,4			47,2
10750	164. 9. 1,4		12700	165. 1. 35,7		15900	166. 7. 43,4	
		1. 30,3			2. 23,9			1. 46,2
10800	164. 10. 31,7		12800	165. 3. 59,6		16000	166. 9. 29,6	
		29,9			22,3			45,4
10850	164. 12. 1,5		12900	165. 6. 21,9		16100	166. 11. 15,0	
		29,3			20,9			44,5
10900	164. 13. 30,8		13000	165. 8. 42,8		16200	166. 12. 59,5	
		28,7			19,4			43,6
10950	164. 14. 59,5		13100	165. 11. 2,2		16300	166. 14. 43,1	
		1. 28,1			2. 18,0			1. 42,7
11000	164. 16. 27,6		13200	165. 13. 20,2		16400	166. 16. 25,8	
		27,6			16,5			41,9
11050	164. 17. 55,2		13300	165. 15. 36,7		16500	166. 18. 7,7	
		27,1			15,2			41,1
11100	164. 19. 22,3		13400	165. 17. 51,9		16600	166. 19. 48,8	
		26,5			13,8			40,2
11150	164. 20. 48,8		13500	165. 20. 5,7		16700	166. 21 29,0	
		1. 26,0			2. 12,5			1. 39,5
11200	164. 22. 14,8		13600	165. 22. 18,2		16800	166. 23. 8,5	
		25,5			11,2			38,6
11250	164. 23. 40,3		13700	165. 24. 29,4		16900	166. 24. 47,1	
		25,0			9,8			37,9
11300	164. 25. 5,3		13800	165. 26. 39,2		17000	166. 26. 25,0	
		24,4			8,6			37,1
11350	164. 26. 29,7		13900	165. 28. 47,8		17100	166. 28. 2,1	
		1. 24,0			2. 7,4			1. 36,3
11400	164. 27. 53,7		14000	165. 30. 55,2		17200	166. 29. 38,4	
		23,5			6,1			35,6
11450	164. 29. 17,2		14100	165. 33. 1,3		17300	166. 31. 14,0	
		22,9			5,0			34,8
11500	164. 30. 40,1		14200	165. 35. 6,3		17400	166. 32. 48,8	
		22,5			3,7			34,1
11550	164. 32. 2,6		14300	165. 37. 10,0		17500	166. 34. 22,9	
		1. 22,0			2. 2,6			1. 33,4
11600	164. 33. 24,6		14400	165. 39. 12,6		17600	166. 35. 56,3	
		21,5			1,4			32,7
11650	164. 34. 46,1		14500	165. 41. 14,0		17700	166. 37. 29,0	
		21,1			0,3			31,9
11700	164. 36. 7,2		14600	165. 43. 14,3		17800	166. 39. 0,9	
		20,6			1. 59,2			31,3
11750	164. 37. 27,8		14700	165. 45. 13,5		17900	166. 40. 32,2	
		1. 20,1			1. 58,1			1. 30,6
11800	164. 38. 47,9		14800	165. 47. 11,6		18000	166. 42. 2,8	
		19,6			57,0			29,8
11850	164. 40 7,5		14900	165. 49. 8,6		18100	166. 43. 32,6	
		19,2			56,0			29,2
11900	164. 41. 26,7		15000	165. 51. 4,6		18200	166. 45. 1,8	
		18,8			54,9			28,6
11950	164. 42. 45,5		15100	165. 52. 59,5		18300	166. 46. 30,4	
		1. 18,3			1. 53,9			1. 27,9

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
18400	166.	47.	58,3		
				1.	27,3
18500	166.	49.	25,6		
					26,6
18600	166.	50.	52,2		
					26,0
18700	166.	52.	18,2		
				1.	25,4
18800	166.	53.	43,6		
					24,7
18900	166.	55.	8,3		
					24,2
19000	166.	56.	32,5		
					23,5
19100	166.	57.	56,0		
				1.	23,0
19200	166.	59.	19,0		
					22,4
19300	167.	0.	41,4		
					21,8
19400	167.	2.	3,2		
					21,3
19500	167.	3.	24,5		
				1.	20,7
19600	167.	4.	45,2		
					20,1
19700	167.	6.	5,3		
					19,6
19800	167.	7.	24,9		
					19,0
19900	167.	8.	43,9		
				1.	18,5
20000	167.	10.	2,4		
					18,0
20100	167.	11.	20,4		
					17,5
20200	167.	12.	37,9		
					17,0
20300	167.	13.	54,9		
				1.	16,4
20400	167.	15.	11,3		
					16,0
20500	167.	16.	27,3		
					15,5
20600	167.	17.	42,8		
					14,9
20700	167.	18.	57,7		
				1.	14,5
20800	167.	20.	12,2		
					14,0
20900	167.	21.	26,2		
					13,5
21000	167.	22.	39,7		
					13,0
21100	167.	23.	52,7		
				1.	12,6
21200	167.	25.	5,3		
					12,1
21300	167.	26.	17,4		
					11,7
21400	167.	27.	29,1		
					11,2
21500	167.	28.	40,3		
				1.	10,8

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
21600	167.	29.	51,1		
				1.	10,3
21700	167.	31.	1,4		
					9,9
21800	167.	32.	11,3		
					9,4
21900	167.	33.	20,7		
				1.	9,0
22000	167.	34.	29,7		
					8,6
22100	167.	35.	38,3		
					8,2
22200	167.	36.	46,5		
					7,8
22300	167.	37.	54,3		
				1.	7,4
22400	167.	39.	1,7		
					6,9
22500	167.	40.	8,6		
					6,6
22600	167.	41.	15,2		
					6,2
22700	167.	42.	21,4		
				1.	5,7
22800	167.	43.	27,1		
					5,4
22900	167.	44.	32,5		
					5,0
23000	167.	45.	37,5		
					4,6
23100	167.	46.	42,1		
				1.	4,3
23200	167.	47.	46,4		
					3,9
23300	167.	48.	50,3		
					3,5
23400	167.	49.	53,8		
					3,1
23500	167.	50.	56,9		
				1.	2,8
23600	167.	51.	59,7		
					2,4
23700	167.	53.	2,1		
					2,1
23800	167.	54.	4,2		
					1,7
23900	167.	55.	5,9		
				1.	1,3
24000	167.	56.	7,2		
				2.	1,8
24200	167.	58.	9,0		
					0,4
24400	168.	0.	9,4		
				1.	59,0
24600	168.	2.	8,4		
				1.	57,8
24800	168.	4.	6,2		
					56,5
25000	168.	6.	2,7		
					55,2
25200	168.	7.	57,9		
					54,0
25400	168.	9.	51,9		
				1.	52,8

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
25600	168.	11.	44,7		
				1.	51,7
25800	168.	13.	36,4		
					50,5
26000	168.	15.	26,9		
					49,3
26200	168.	17.	16,2		
				1.	48,2
26400	168.	19.	4,4		
					47,1
26600	168.	20.	51,5		
					46,0
26800	168.	22.	37,5		
					45,0
27000	168.	24.	22,5		
				1.	43,9
27200	168.	26.	6,4		
					42,9
27400	168.	27.	49,3		
					41,9
27600	168.	29.	31,2		
					40,9
27800	168.	31.	12,1		
				1.	40,0
28000	168.	32.	52,1		
					39,0
28200	168.	34.	31,1		
					38,0
28400	168.	36.	9,1		
					37,1
28600	168.	37.	46,2		
				1.	36,2
28800	168.	39.	22,4		
					35,3
29000	168.	40.	57,7		
					34,4
29200	168.	42.	32,1		
					33,5
29400	168.	44.	5,6		
				1.	32,7
29600	168.	45.	38,3		
					31,9
29800	168.	47.	10,2		
					31,1
30000	168.	48.	41,3		
					30,2
30200	168.	50.	11,5		
				1.	29,4
30400	168.	51.	40,9		
					28,6
30600	168.	53	9,5		
					27,8
30800	168.	54.	37,3		
					27,1
31000	168.	56.	4,4		
				1.	26,4
31200	168.	57.	30,8		
					25,6
31400	168.	58.	56,4		
					24,9
31600	169.	0.	21,3		
					24,1
31800	169.	1.	45,4		
				1.	23,5

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D.	M.	S.	Differ. M.	S.	Days.	Anomaly. D.	M.	S.	Differ. M.	S.	Days.	Anomaly. D.	M.	S.	Differ. M.	S.
32000	169.	3.	8,9			38400	169.	42.	18,2			46000	170.	18.	44,5		
32200	169.	4.	31,6	1.	22,7	38600	169.	43.	23,0	1	4,8	46250	170.	19.	48,0	1.	3,5
32400	169.	5.	53,7		22,1	38800	169.	44.	27,3		4,3	46500	170.	20.	51,0		3,0
32600	169.	7.	15,0		21,3	39000	169.	45.	31,2		3,9	46750	170.	21.	53,6		2,6
				1.	20,7					1.	3,5					1.	2,2
32800	169.	8.	35,7		20,1	39200	169.	46.	34,7		3,1	47000	170.	22.	55,8		1,7
33000	169.	9.	55,8		19,4	39400	169.	47.	37,8		2,6	47250	170.	23.	57,5		1,3
33200	169.	11.	15,2		18,7	39600	169.	48.	40,4		2,2	47500	170.	24.	58,8		0,9
33400	169.	12.	33,9			39800	169.	49.	42,6			47750	170.	25.	59,7		
				1.	18,2					1.	1,7					1.	0,4
33600	169.	13.	52,1		17,5	40000	169.	50.	44,3		16,6	48000	170.	27.	0,1		59,6
33800	169.	15.	9,6		16,9	40250	169.	52.	0,9		16,0	48500	170.	28.	59,7		58,0
34000	169.	16.	26,5		16,2	40500	169.	53.	16,9		15,4	49000	170.	30.	57,7		56,4
34200	169.	17.	42,7			40750	169.	54.	32,3			49500	170.	32.	54,1		
				1.	15,7					1.	14,8					1.	54,8
34400	169.	18.	58,4		15,1	41000	169.	55.	47,1		14,1	50000	170.	34.	48,9		53,3
34600	169.	20.	13,5		14,5	41250	169.	57.	1,2		13,6	50500	170.	36.	42,2		51,8
34800	169.	21.	28,0		14,0	41500	169.	58.	14,8		12,9	51000	170.	38.	34,0		50,3
35000	169.	22.	42,0			41750	169.	59.	27,7			51500	170.	40.	24,3		
				1.	13,4					1.	12,4					1.	48,9
35200	169.	23.	55,4		12,8	42000	170.	0.	40,1		11,8	52000	170.	42.	13,2		47,6
35400	169.	25.	8,2		12,3	42250	170.	1.	51,9		11,2	52500	170.	44.	0,8		46,1
35600	169.	26.	20,5		11,7	42500	170.	3.	3,1		10,7	53000	170.	45.	46,9		44,8
35800	169.	27.	32,2			42750	170.	4.	13,8			53500	170.	47.	31,7		
				1.	11,1					1.	10,1					1.	43,5
36000	169.	28.	43,3		10,7	43000	170.	5.	23,9		9,6	54000	170.	49.	15,2		42,2
36200	169.	29.	54,0		10,1	43250	170.	6.	33,5		9,0	54500	170.	50.	57,4		41,0
36400	169.	31.	4,1		9,7	43500	170.	7.	42,5		8,5	55000	170.	52.	38,4		39,7
36600	169.	32.	13,8			43750	170.	8.	51,0			55500	170.	54.	18,1		
				1.	9,1					1.	7,9					1.	38,6
36800	169.	33.	22,9		8,6	44000	170.	9.	58,9		7,4	56000	170.	55.	56,7		37,4
37000	169.	34.	31,5		8,1	44250	170.	11.	6,3		6,9	56500	170.	57.	34,1		36,2
37200	169.	35.	39,6		7,6	44500	170.	12.	13,2		6,5	57000	170.	59.	10,3		35,1
37400	169.	36.	47,2			44750	170.	13.	19,7			57500	171.	0.	45,4		
				1.	7,1					1.	5,9					1.	34,0
37600	169.	37.	54,3		6,7	45000	170.	14.	25,6		5,5	58000	171.	2.	19,4		32,9
37800	169.	39.	1,0		6,2	45250	170.	15.	31,1		4,9	58500	171.	3.	52,3		31,9
38000	169.	40.	7,2		5,7	45500	170.	16.	36,0		4,5	59000	171.	5.	24,2		30,8
38200	169.	41.	12,9			45750	170.	17.	40,5			59500	171.	6.	55,0		
				1.	5,3					1.	4,0					1.	29,8

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D.	M.	S.	Differ. M.	S.	Days.	Anomaly. D.	M.	S.	Differ. M.	S.	Days.	Anomaly. D.	M.	S.	Differ. M.	S.
60000	171	8.	24,8			76000	171.	48.	58,5			92000	172.	19.	27,5		
				1.	28,8					1.	4,8					0.	50,2
60500	171.	9.	53,6			76500	171.	50.	3,3			92500	172.	20.	17,7		
					27,8						4,2						49,8
61000	171.	11.	21,4			77000	171.	51.	7,5			93000	172.	21.	7,5		
					26,9						3,6						49,4
61500	171.	12.	48,3			77500	171.	52.	11,1			93500	172.	21.	56,9		
				1.	25,9					1.	3,1					0.	49,1
62000	171.	14.	14,2			78000	171.	53.	14,2			94000	172.	22.	46,0		
					25,0						2,5						48,7
62500	171.	15.	39,2			78500	171.	54.	16,7			94500	172.	23.	34,7		
					24,1						2,1						48,5
63000	171.	17.	3,3			79000	171.	55.	18,8			95000	172.	24.	23,2		
					23,2						1,5						48,0
63500	171.	18.	26,5			79500	171.	56.	20,3			95500	172.	25.	11,2		
				1.	22,3					1.	1,0					0.	47,7
64000	171.	19.	48,8			80000	171.	57.	21,3			96000	172.	25.	58,9		
					21,5						0,4						47,4
64500	171.	21.	10,3			80500	171.	58.	21,7			96500	172.	26.	46,3		
					20,6						0,0						47,1
65000	171.	22.	30,9			81000	171.	59.	21,7			97000	172.	27.	33,4		
					19,8					0.	59,5						46,8
65500	171.	23.	50,7			81500	172.	0.	21,2			97500	172.	28.	20,2		
				1.	19,0					0.	59,0					0.	46,4
66009	171.	25.	9,7			82000	172.	1.	20,2			98000	172.	29.	6,6		
					18,2						58,5						46,1
66500	171.	26.	27,9			82500	172.	2.	18,7			98500	172.	29.	52,7		
					17,4						58,0						45,8
67000	171.	27.	45,3			83000	172.	3.	16,7			99000	172.	30.	38,5		
					16,6						57,6						45,5
67500	171.	29.	1,9			83500	172.	4.	14,3			99500	172.	31.	24,0		
				1.	15,9					0.	57,1					0.	45,2
68000	171.	30.	17,8			84000	172.	5.	11,4			100000	172.	32.	9,2		
					15,1						56,7					1.	29,5
68500	171.	31.	32,9			84500	172.	6.	8,1			101000	172.	33.	38,7		
					14,4						56,2						29,1
69000	171.	32.	47,3			85000	172.	7.	4,3			102000	172	35.	7,8		
					13,7						55,8						26,3
69500	171.	34.	1,0			85500	172.	8.	0,1			103000	172.	36.	34,1		
				1.	13,0					0.	55,3					1.	26,0
70000	171.	35.	14,0			86000	172.	8.	55,4			104000	172.	38.	0,1		
					12,3						54,9						25,0
70500	171.	36.	26,3			86500	172.	9.	50,3			105000	172.	39.	25,1		
					11,6						54,5						23,8
71000	171.	37.	37,9			87000	172.	10.	44,8			106000	172.	40.	48,9		
					10,9						54,1						22,8
71500	171.	38.	48,8			87500	172.	11.	38,9			107000	172.	42.	11,7		
				1.	10,3					0.	53,6					1.	21,8
72000	171.	39.	59,1			88000	172.	12.	32,5			108000	172.	43.	33,5		
					9,6						53,2						20,7
72500	171.	41.	8,7			88500	172.	13.	25,7			109000	172.	44.	54,2		
					8,9						52,9						19,8
73000	171.	42.	17,6			89000	172.	14.	18,6			110000	172.	46.	14,0		
					8,3						52,4						18,8
73500	171.	43.	25,9			89500	172.	15.	11,0			111000	172.	47.	32,8		
				1.	7,8					0.	52,1					1.	17,8
74000	171.	44.	33,7			90000	172.	16.	3,1			112000	172.	48.	50,6		
					7,1						51,6						17,0
74500	171.	45.	40,8			90500	172.	16.	54,7			113000	172.	50.	7,6		
					6,5						51,3						16,0
75000	171.	46.	47,3			91000	172.	17.	46,0			114000	172.	51.	23,6		
					5,9						50,9						15,1
75500	171.	47.	53,2			91500	172.	18.	36,9			115000	172.	52.	38,7		
				1.	5,3					0.	50,6					1.	14,3

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D. M. S.	Differ. M. S.	Days.	Anomaly. D. M. S.	Differ. M. S.	Days.	Anomaly. D. M. S.	Differ. M. S.
116000	172. 53. 53,0		144000	173. 23. 39,0		172000	173. 46. 33,5	
		1. 13,4			0. 55,1			0. 43,5
117000	172. 55. 6,4		145000	173. 24. 34,1		173000	173. 47. 17,0	
		12,6			54,5			43,1
118000	172. 56. 19,0		146000	173. 25. 28,6		174000	173. 48. 0,1	
		11,8			54,0			42,8
119000	172. 57. 30,8		147000	173. 26. 22,6		175000	173. 48. 42,9	
		1. 10,9			0. 53,6			0. 42,4
120000	172. 58. 41,7		148000	173. 27. 16,2		176000	173. 49. 25,3	
		10,2			53,1			42,0
121000	172. 59. 51,9		149000	173. 28. 9,3		177000	173. 50. 7,3	
		9,4			52,5			41,7
122000	173. 1. 1,3		150000	173. 29. 1,8		178000	173. 50. 49,0	
		8,6			52,2			41,3
123000	173. 2. 9,9		151000	173. 29. 54,0		179000	173. 51. 30,3	
		1. 7,9			0. 51,8			0. 41,0
124000	173. 3. 17,8		152000	173. 30. 45,8		180000	173. 52. 11,3	
		7,3			51,0			40,7
125000	173. 4. 25,1		153000	173. 31. 36,8		181000	173. 52. 52,0	
		6,4			50,8			40,4
126000	173. 5. 31,5		154000	173. 32. 27,6		182000	173. 53. 32,4	
		5,8			50,3			40,1
127000	173. 6. 37,3		155000	173. 33. 17,9		183000	173. 54. 12,5	
		1. 5,1			0. 49,9			0. 39,8
128000	173. 7. 42,4		156000	173. 34. 7,8		184000	173. 54. 52,3	
		4,3			49,5			39,6
129000	173. 8. 46,7		157000	173. 34. 57,3		185000	173. 55. 31,9	
		3,8			49,0			39,3
130000	173. 9. 50,5		158000	173. 35. 46,3		186000	173. 56. 11,2	
		3,1			48,6			39,1
131000	173. 10. 53,6		159000	173. 36. 34,9		187000	173. 56. 50,3	
		1. 2,4			0. 48,3			0. 38,9
132000	173. 11. 56,0		160000	173. 37. 23,2		188000	173. 57. 29,2	
		1,9			47,9			38,5
133000	173. 12. 57,9		161000	173. 38. 11,1		189000	173. 58. 7,7	
		1,1			47,6			38,3
134000	173. 13. 59,0		162000	173. 38. 58,7		190000	173. 58. 46,0	
		0,5			47,2			38,1
135000	173. 14. 59,5		163000	173. 39. 45,9		191000	173. 59. 24,1	
		1. 0,1			0. 46,7			0. 37,8
136000	173. 15. 59,6		164000	173. 40. 32,6		192000	174. 0. 1,9	
		0. 59,4			46,4			37,5
137000	173. 16. 59,0		165000	173. 41. 19,0		193000	174. 0. 39,4	
		58,8			46,0			37,2
138000	173. 17. 57,8		166000	173. 42. 5,0		194000	174. 1. 16,6	
		58,2			45,6			37,0
139000	173. 18. 56,0		167000	173. 42. 50,6		195000	174. 1. 53,6	
		0. 57,7			0. 45,3			0. 36,7
140000	173. 19. 53,7		168000	173. 43. 35,9		196000	174. 2. 30,3	
		57,1			44,9			36,5
141000	173. 20. 50,8		169000	173. 44. 20,8		197000	174. 3. 6,8	
		56,6			44,6			36,3
142000	173. 21. 47,4		170000	173. 45. 5,4		198000	174. 3. 43,1	
		56,1			44,2			36,0
143000	173. 22. 43,5		171000	173. 45. 49,6		199000	174. 4. 19,1	
		0. 55,5			0. 43,9			0. 35,7
						200000	174. 4. 54,8	

TABLE IV.

CONTAINING THE

ABSCISSAS AND CORRESPONDING ORDINATES OF A PARABOLA;

USEFUL FOR CONSTRUCTING THAT CURVE.

Abſciſſæ.	Ordinates.	Abſciſſæ.	Ordinates.	Abſciſſæ.	Ordinates.
0,125	0,70710	18	8,4853	48	13,8564
0,25	1,00000	19	8,7178	49	14,0000
0,50	1,41421	20	8,9444	50	14,1422
0,75	1,73205	21	9,1651	52	14,4222
1,00	2,00000	22	9,3808	56	14,9666
1,5	2,44950	23	9,5916	60	15,4920
2,0	2,82843	24	9,7979	64	16,0000
2,5	3,16225	25	10,0000	68	16,4924
3,0	3,46410	26	10,1980	72	16,9706
3,5	3,74165	27	10,3923	76	17,4356
4,0	4,00000	28	10,5830	80	17,8888
4,5	4,24265	29	10,7703	81	18,0000
5,0	4,47210	30	10,9545	84	18,3302
5,5	4,69040	31	11,1355	88	18,7616
6,0	4,89900	32	11,3138	92	19,1832
6,5	5,09900	33	11,4891	96	19,5958
7,0	5,29150	34	11,6619	100	20,0000
7,5	5,47725	35	11,8322	104	20,3960
8,0	5,65690	36	12,0000	108	20,7846
8,5	5,83095	37	12,1655	112	21,1660
9,0	6,00000	38	12,3288	116	21,5406
9,5	6,16400	39	12,4900	120	21,9090
10,0	6,3245	40	12,6490	121	22,0000
11	6,6332	41	12,8062	124	22,2710
12	6,9282	42	12,9615	128	22,6276
13	7,2111	43	13,1149	132	22,9782
14	7,4833	44	13,2664	136	23,3238
15	7,7460	45	13,4164	140	23,6644
16	8,0000	46	13,5647	144	24,0000
17	8,2462	47	13,7113	...	.....

EXPLANATION

EXPLANATION AND USE OF THE TABLES.

TABLE I, is for the reduction of hours, minutes and ſeconds of time, into decimal parts of a day.

RULE. Find, in the column of time, the hours, minutes and ſeconds given, and oppoſite to each is the correſponding decimal, the ſum of which is the decimal fraction required.

Ex. Required the decimal of a day of 17*h*. 27′. 44″.

17 hours	0,708333
27 minutes	0,018750
44 ſeconds	0,000509
Sum. Decimal of 17*h*. 27′. 44″	0,727592

TABLE II, is for the reduction of decimal parts of a day, into hours, minutes and ſeconds.

RULE. Enter the Table with the firſt figure to the left hand of the given decimal, and take out it's value in hours, &c. Repeat the ſame operation with the ſecond, third, and the reſt of the figures; and the ſum of the times, ſo taken from the Table, is the value of the decimal required.

Ex. Required the value in time of 0,727592 of a day.

	H.	M.	S.
0,7	16.	48.	0
0,02	0.	28.	48
0,007	0.	10.	4,8
0,0005	0.	0.	43,2
0,00009	0.	0.	7,776
0,000002	0.	0.	0,173
Sum. Value of 0,727592 . . .	17.	27.	43,949

TABLE III, of the motion of comets in a parabolic orbit, was firſt publiſhed by Dr. HALLEY, and ſince augmented by M. de la CAILLE, M. de la LANDE and SCHULZE of Berlin. M. PINGRE' recomputed and extended the whole, ſo as to make it much more complete than any before publiſhed. And lately, M. de LAMBRE, whoſe abilities as a Calculator are well known, has recomputed the whole Table to decimals of ſeconds, and ſtill farther enlarged it.

The perihelion diſtance of any comet, and the time of it's paſſage through the perihelion being given, to find it's true anomaly, or angular diſtance from the perihelion, for any given time before or after the perihelion.

RULE. To the logarithm of the perihelion diſtance of the comet, add it's half; ſubtract the ſum from the logarithm of the time elapſed (expreſſed in days) between the given time and the arrival of the comet at it's perihelion, and the remainder will be the logarithm of a number of days; find this number in the Table of the parabola, and oppoſite to it is the anomaly ſought. If the given number be not in the Table, a ſimple proportion will give the anomaly.

If the characteriſtic of the logarithm of the perihelion diſtance be 9, 8, or 7, in taking it's half, it muſt be ſuppoſed 19, 18, or 17.

Ex. The logarithm of the perihelion diſtance of the comet in 1769 was 9,0886320, according to EULER. What was it's anomaly at 50 days before or after it's perihelion?

Log. of perihelion diſtance	9,0886320
It's half	9,5443160
Their ſum	8,6329480
Log. of 50 days	1,6985700
Remainder	3,0656220

Which is the logarithm of 1164,185 days. Seeking this number in the Table of the parabola, it is not found there; but for 1160 days the anomaly is 145°. 16′. 49″, and for 1165 days the anomaly is 145°. 20′. 7″; hence the

difference for 5 days is 3′. 18″; therefore, as 5 days : 4,185 days :: 3′. 18″ : 2′. 46″, which muſt be added to 145°. 16′. 49″, the anomaly for 1160 days, and the ſum 145°. 19′. 35″ is the true anomaly for 1164,185 days in the Table, or for 50 actual days before or after the paſſage of the comet through the perihelion.

If the true anomaly of a comet, for any inſtant, be given, and the time elapſed between that and the paſſage through the perihelion be required, it may be found from the ſame Table.

RULE. Seek in the Table the given anomaly, and find the time correſponding to it, taking, if neceſſary, proportional parts. To the logarithm of the perihelion diſtance, add it's half, and the logarithm of the days found in the Table; their ſum is the logarithm of the time elapſed between the comet's paſſing the perihelion, and it's arrival at the given anomaly.

EX. Given the anomaly of Dr. HALLEY's comet of 1759, 64°. 36′. 37″; to find the time it took to deſcribe that angle from the perihelion; the logarithm of it's perihelion diſtance being 9,766033, according to M. de la CAILLE.

The given anomaly is not in the Table; the two neareſt are 64°. 29′. 47″, and 64°. 40′. 28″. The firſt anſwers to 58,75 days, and the other to 59,0 days. The difference of the given anomaly from the firſt of theſe two tabular ones is 6′. 50″, or 410″; the difference of the tabular anomalies is 10′. 41″, or 601″, and the difference of times is 0,25 days; hence, 641 : 410 :: 0,25 : 0,15991, which muſt therefore be added to the tabular time 58,75 days, anſwering to the anomaly 64°. 29′. 47″, and the ſum 58,90991 days, will be the tabular time anſwering to the given anomaly. Now the

Logarithm of 58,90991	1,7701883
Logarithm of perihelion diſtance	9,7660330
Half logarithm of perihelion diſtance . . .	9,8830165
Sum	1,4192378

Whoſe number is 26,25656, the number of days that the comet will employ in deſcribing the angle of 64°. 36′. 37″ on either ſide of the perihelion.

This

This general Table will be ſufficient in all caſes to determine the true anomaly from the time given; but it will not be equally accurate for finding the time from the anomaly; for at conſiderable diſtances from the perihelion, errors will ariſe. The following little Table ſhows how far the Table may be uſed without incurring an error greater than 30 ſeconds of time.

Perihelion diſtance.	Anomalies. D.
0,25	130
0,50	118
0,80	100
1,00	90
1,20	80
1,50	65
2,00	50

Beyond theſe anomalies, comets of the reſpective perihelion diſtances are ſeldom viſible, and for comets of a leſs perihelion diſtance, the limits extend proportionably further. Indeed, except when extreme accuracy is required, this Table may be uſed far beyond the limits here preſcribed; and if the utmoſt preciſion be neceſſary, the following Rule will give the time free from error, in all caſes. The demonſtration will be found in PINGRE', Vol. ii. page 339.

RULE. To the log. tangent of half the given anomaly, add the conſtant logarithm 1,9149328; and to triple the log. tangent of half the anomaly, add the conſtant logarithm 1,4378116; find the numbers to theſe logarithms, and add them together. To the logarithm of the ſum, add $\frac{3}{2}$ of the logarithm of the perihelion diſtance of the comet, and the ſum will be the logarithm of the days from the perihelion.

EX. Required the time from the perihelion, anſwering to 144°. 38'. 28 of anomaly for the comet of 1769; it's perihelion diſtance being 9,0886320.

Tang.

	Logarithms.	Numbers.
Tang. ½ anomaly	0,4965560	
Conſtant	1,9149328	
Sum	2,4114888	. . . 257,92225
Triple tang. ½ anomaly	1,4896680	
Conſtant	1,4378116	
Sum	2,9274796	. . . 846,21275
Log. of ſum of numbers	3,0430222	1104,13500
Perihelion diſtance ½	8,6322480	
Log. of days from the perihelion . .	1,6759702	

Hence the time from the perihelion is 47,421 days.

THE

THE ELEMENTS OF SEVENTY-EIGHT COMETS,

WHICH HAVE BEEN OBSERVED AND CALCULATED TO 1790.

Order of the Comets.	Years of appearance.	Paſſage through the perihelion, mean time at Greenwich.	Longitude of the Aſcending Node.	Inclination of the Orbit.	Place of the Perihelion.	Perihelion diſtance, that of the Sun being 1.	Motion.	Authors who have calculated the Orbits.
		DAYS. H. M. S	S. D. M. S.	D. M. S.	S. D. M. S.			
I.	837	1 March . . .	6. 26. 33. 0	10, or 12°	9. 19. 3. 0	0,58	Retrograde.	PINGRE'.
II.	1231	30 January . . 7. 12. 40	0. 13. 30. 0	6. 5. 0	4. 14. 48. 0	0,9478	Direct.	PINGRE'.
III.	1264	6 July 7. 50. 40	5. 19. 0. 0	36. 30. 0	9. 21. 0. 0	0,445	Direct.	DUNTHORN.
		17 July 6 0. 40	5. 28. 45. 0	30. 25. 0	9. 5. 45. 0	0,41081	Direct.	PINGRE'.
IV.	1299	31 March . . . 7. 28. 40	3. 17. 8. 0	68. 57. 0	0. 3. 20. 0	0,3179	Retrograde.	PINGRE'.
V.	1301	22 Oct. (about)	0. 15 (about)	70 (about)	9, or 10[s]	0,457	Retrograde.	PINGRE'.
VI.	1337	2 June 6. 24. 40	2. 24. 21. 0	32. 11. 0	1. 7. 59. 0	0,40666	Retrograde	HALLEY.
		1 June 0. 30. 40	2. 6. 22. 0	32. 11. c	0. 20. 0. 0	0,6445		PINGRE'.
49.	1456	8 June 22. 0. 40	1. 18. 30. 0	17. 56. 0	10. 1. 0. 0	0,5855	Retrograde.	PINGRE'.
VII.	1472	28 February . 22. 22. 40	9. 11. 46. 20	5. 20. 0	1. 15. 33. 30	0,54273	Retrograde	HALLEY.
49.	1531	24 Auguſt . . 21. 17. 40	1. 19. 25. 0	17. 56. 0	10. 1. 39. 0	0,56700	Retrograde.	HALLEY.
19.	1532	19 October . . 22. 11. 40	2. 20 27. 0	32. 36 0	3. 21. 7. 0	0,50910	Direct.	HALLEY.
VIII.	1533	16 June 19. 29. 40	4. 5. 44. 0	35. 49. 0	4. 27. 16. 0	0,2028	Retrograde	DOUWES.
3.	1556	21 April . . . 20. 2. 40	5. 25. 42. 0	32. 6. 30	9. 8 50. 0	0,46390	Direct	HALL.
IX.	1577	26 October . . 18 44. 40	0. 25. 52. 0	74. 32. 45	4. 9. 22. 0	0,18342	Regrograde.	HALLEY.
X.	1580	28 November 13. 44. 40	0. 19. 7. 37	64. 51. 50	3. 19. 11. 55	0,59553	Direct.	PINGRE'.
XI.	1582	7 May	7. 5, or 21	59, or 61	8. 5, or 9. 11	0,23 or 0,04	Retrograde.	PINGRE'.
XII.	1585	7 Oct. N. S. 19. 19. 40	1. 7. 42. 30	6. 4. 0	0. 8. 51. 0	1,09358	Direct.	HALLEY.
XIII.	1590	8 Feb. N. S. 3. 44. 40	5. 15. 30. 0	29. 40. 40	7. 6. 54. 30	0,57661	Retrograde.	HALLEY.
XIV.	1593	18 July N. S. 13. 38. 40	5. 14. 15. 0	87. 58. 0	5. 26. 19. 0	0,08911	Direct.	De la CAILLE.
XV.	1596	8 Auguſt . . . 15. 33. 40	10. 15. 36. 50	52. 9. 45	7. 28. 30. 50	0,549415	Retrograde.	PINGRE'.
49.	1607	26 October . . 3. 49. 40	1. 20. 21. 0	17. 2. 0	10. 2. 16. 0	0,58680	Retrograde.	HALLEY.
XVI.	1618	17 Auguſt . . . 3. 2. 40	9. 23. 25. 0	21. 28. 0	10. 18. 20. 0	0,51298	Direct.	PINGRE'.
XVII.	1618	8 November 12. 22. 40	2. 16. 1. 0	37. 34. 0	0. 2. 14. 0	0,37975	Direct.	HALLEY.
XVIII.	1652	12 November 15. 39. 40	2. 28. 10. 0	79. 28. 0	0. 28. 18. 40	0,84750	Direct.	HALLEY.
XIX.	1661	26 January . 23. 40. 40	2. 22. 30. 30	32. 35. 50	3. 25. 58. 40	0,44851	Direct.	HALLEY.
XX.	1664	4 December 11. 51. 40	2. 21. 14. 0	21. 18. 30	4. 10. 41. 25	1,025755	Retrograde.	HALLEY.
XXI.	1665	24 April 5. 14. 40	7. 18. 2. 0	76. 5 0	2. 11 54. 30	0,10649	Retrograde	HALLEY.
XXII.	1672	1 March . . . 8. 36. 40	9. 27. 30 30	83. 22. 10	1. 16. 59. 30	0,69739	Direct.	HALLEY.
XXIII.	1677	6 May 0. 36. 40	7. 26. 49. 10	79 3. 15	4. 17. 37 5	0,28059	Retrograde.	HALLEY.
XXIV.	1678	26 Auguſt . . 14. 2. 40	5. 11 40. 0	3. 4. 20	10. 27. 46. 0	1,23801	Direct.	DOUWES.
XXV.	1680	18 December . 0. 1. 2	9. 1. 57. 13	61. 22. 55	8. 22. 40. 10	0,006030	Direct	PINGRE'.
49.	1682	14 September . 7. 38. 40	1. 21. 16. 30	17. 56. 0	10. 2. 52. 45	0,58328	Retrograde.	HALLEY.
XXVI.	1683	13 July 2. 49. 40	5. 23 23. 0	83. 11. 0	2. 25. 29. 30	0,56020	Retrograde	HALLEY.
XXVII.	1684	8 June 10. 15. 40	8. 28. 15. 0	65. 48. 40	7. 28. 5 . 0	0,96015	Direct	HALLEY.
XXVIII.	1686	16 September 14. 32. 40	11. 20. 34. 40	31. 21. 40	2. 17 0. 30	0,32500	Direct.	HALLEY.
XXIX.	1689	1 December 14. 55 40	10. 23. 45. 20	69. 17. 0	8. 23. 44. 45	0,016889	Retrograde	PINGRE'.
XXX.	1698	18 October . 16. 56. 40	8. 27. 44. 15	11. 46. 0	9 0. 51 15	0,69129	Retrograde	HALLEY.
XXXI.	1699	13 January . . . 8. 22. 40	10. 21. 45. 35	69. 20. 0	7. 2. 31. 6	0,75435	Retrograde	De la CAILLE.
XXXII.	1702	13 March . . . 14. 12. 40	6. 9. 25. 15	4. 30. 0	4 18 41. 3	0,64590	Direct.	De la CAILLE.
XXXIII.	1706	30 January . . 4. 55. 40	0. 13. 11. 23	55. 14. 5	2. 12. 36. 25	0,426865	Direct.	STRUYCK.
XXXIV.	1707	11 December 23. 43 27	1. 22. 50. 29	88. 37 40	2. 19. 58. 9	0,85904	Direct.	STRUYCK.
XXXV.	1718	15 January . . 1. 15. 16	4 7. 55. 20	31. 12. 53	4. 1. 26. 36	1,02565	Retrograde.	DOUWES.
XXXVI.	1723	27 September 16. 10. 40	0. 14. 16. 0	49. 59. 0	1. 12. 52. 20	0,99865	Retrograde.	BRADLEY.
XXXVII.	1729	25 June 11. 6. 40	10 10. 32. 37	76. 58. 4	10. 22. 40. 0	4,26140	Direct.	De la CAILLE.
		23 June 6. 36. 2	10. 10. 35. 15	77. 1. 58	10. 22. 16. 53	4,0698		DOUWES
XXXVIII.	1737	30 January . . 8. 20. 40	7. 16. 22. 0	18. 20. 45	10. 25. 55. 0	0,22282	Direct.	BRADLEY.
XXXIX.	1739	17 June 9. 59. 40	6. 27. 25. 14	55. 42. 44	3. 12. 38. 40	0,67358	Retrograde.	De la CAILLE.

THE ELEMENTS OF SEVENTY-EIGHT COMETS,

WHICH HAVE BEEN OBSERVED AND CALCULATED TO 1790.

Order of the Comets.	Years of appearance.	Paſſage through the perihelion, mean time at Greenwich. DAYS. H. M. S.	Longitude of the Aſcending Node. S. D. M. S	Inclination of the Orbit. D. M. S.	Place of the Perihelion. S. D. M. S.	Perihelion diſtance, that of the Sun being 1.	Motion.	Authors who have calculated the Orbits.
XL	1742	8 February . 4. 38. 40	6. 5. 38. 29	66. 59. 14	7. 7. 35. 13	0,76568	Retrograde.	De la Caille.
		8 February . 4. 21. 10	6. 5. 34 45	67. 4. 11	7. 7. 33. 14	0,765555	Retrograde.	Struyck.
XLI.	1743	10 January . 20. 25. 40	2. 8. 21. 15	2. 19. 33	3. 2. 41. 45	0,83501	Direct.	De la Caille.
		10 January . . 21. 15. 37	2. 8. 10. 48	2. 15. 50	3. 2. 58. 4	0,838115		Struyck.
XLII.	1743	20 September 21. 16. 40	0. 5. 16. 25	45. 48. 20	8. 6. 33. 52	0,52157	Retrograde.	Klinkenberg.
XLIII.	1744	1 March . . . 8. 17. 0	1. 15. 45. 20	47. 8. 36	6. 17. 12. 55	0,22206	Direct.	Bliss.
XLIV.	1746	3 Mar. 1747, 7. 10. 40	4. 27. 18. 50	79. 6. 20	9. 7. 2. 0	1,19851	Retrograde.	De la Caille.
XLV.	1748	28 April . . . 19. 35. 25	7. 22. 52. 16	85. 26 57	7. 5. 0. 50	0,84067	Retrograde.	Maraldi.
XLVI.	1748	18 June 1. 23. 40	1. 4. 39. 43	56. 59. 3	9. 6. 9. 24	0,65525	Direct.	Struyck.
XLVII.	1757	21 October . . 9. 46. 40	7. 4. 4. 0	12. 48. 0	4. 2. 49. 0	0,3380	Direct.	Pingre'.
XLVIII.	1758	11 June 3. 17. 40	7. 20. 50. 0	68. 19. 0	8. 27. 38. 0	0,21535	Direct.	Pingre'.
XLIX.	1759	12 March . . . 13. 31. 40	1. 23. 49. 0	17. 39 0	10. 3. 16. 0	0,58349	Retrograde.	De la Caille.
		12 March . . . 13. 50. 4	1. 23. 45. 35	17. 40. 14	10. 3. 8. 10	0,58490		De la Lande.
		12 March . . 12. 48. 16	1. 23. 49. 21	17. 35. 20	10. 3. 16. 20	0,58360		Maraldi.
L.	1760	27 Nov. 1759, 0. 2. 37	4. 19. 39. 41	79. 6. 38	1. 23. 34. 19	0,80139	Direct.	Pingre'.
LI.	1760	16 Dec. 1759, 21. 3. 40	2. 19. 50. 45	4. 51. 32	4. 18. 24. 35	0,96599	Retrograde.	De la Caille.
LII.	1762	28 May 15. 17. 40	11. 19, 20. 0	84. 45. 0	3. 15. 15. 0	1,0124	Direct	De la Lande.
		28 May 6. 51. 29	11. 19. 2. 22	85. 3. 2	3. 14. 29. 46	1,009856		Struyck.
		29 May 0. 18. 28	11. 18. 55. 31	85. 22. 21	3. 15. 22. 23	1,01415		Maraldi.
LIII.	1763	1 November 19. 43. 18	11. 26. 23. 26	72. 40. 40	2. 24. 51. 54	0,49876	Direct.	Pingre'.
LIV.	1764	12 February . 13. 42. 16	4. 0. 4. 33	52. 53. 31	0. 15. 14. 52	0,55522	Retrograde.	Pingre'.
LV.	1766	17 February . . 8. 40. 40	8. 4. 10. 50	40. 50. 20	4. 23. 15. 25	0,50533	Retrograde.	Pingre'.
LVI.	1766	22 April . . . 20. 46. 20	2. 14. 22. 50	11. 8. 4	8. 2. 17. 53	0,33274	Direct.	Pingre'.
LVII.	1769	7 October . 12. 20. 40	5. 25. 0. 43	40. 37. 33	4. 24. 5. 54	0,12376	Direct.	De la Lande.
		7 October . . 13. 36. 53	5. 25. 6. 33	40. 48. 49	4. 24. 11. 7	0,12272		Prosperin.
LVIII.	1770	14 Auguſt . . . 0. 4. 4	4. 12. 17. 3	1. 34. 30	11. 26. 26. 13	0,676893	Direct.	Pingre'.
		13 Auguſt . . 12. 55. 40	4. 12. 0. 0	1. 33. 40	11. 26. 16. 26	0,674381	Direct.	Lexell.
LIX.	1771	22 Nov. 1770, 5. 38. 40	3. 18. 42. 10	31. 25. 55	6. 28. 22. 44	0,52824	Retrograde.	Pingre'.
LX.	1771	18 April . . . 22. 5. 7	0. 27. 51. 0	11. 15. 20	3. 13. 28. 13	0,90576	Direct.	Pingre'.
LXI.	1772	18 February . 20. 41. 15	8. 12. 43. 5	18. 59. 40	3. 18. 6. 22	1,01815	Direct.	De la Lande.
LXII.	1773	5 September 11. 9. 25	4. 1. 15. 37	61. 25. 21	2. 15. 35. 43	1,1339	Direct.	Pingre'.
LXIII.	1774	15 Auguſt . . . 10. 46. 15	6. 0. 49. 48	83. 0. 25	10. 17. 22. 4	1,4286	Direct.	Mechain.
LXIV.	1779	4 January . . 2. 2. 40	0. 25. 5. 51	32. 24. 0	2. 27. 13. 11	0,71312	Direct.	Mechain.
		4 January . . . 2. 15. 10	0. 25. 3. 57	32. 25. 30	2. 27. 13. 40	0,7132		Chev. d'Angos
LXV.	1780	30 September 18. 3. 30	4. 4. 9. 19	53. 48. 5	8. 6. 21. 18	0,09925	Retrograde.	Mechain.
LXVI.	1781	7 July 4. 32. 0	2. 23. 0. 38	81. 43. 26	7. 29. 11. 25	0,775861	Direct.	Mechain.
LXVII.	1781	29 November 12. 32. 26	2. 17. 22. 52	27. 13. 8	0. 16. 3. 28	0,96101	Retrograde.	Mechain.
LXVIII.	1783	15 November 5. 44. 3	1. 24. 13. 50	53. 9. 9	1. 15. 24 46	1,5653	Direct.	Mechain.
LXIX.	1784	21 January . . 4. 47. 40	1. 26. 49. 21	51. 9. 12	2. 20. 44. 24	0,70786	Retrograde.	Mechain.
LXX.	1784	9 April . . . 21. 7. 26	2. 26. 52. 9	47. 55. 8	10. 28. 54. 57	0,650531	Retrograde.	Chev. d'Angos
LXXI.	1785	27 January . . 7. 48. 44	8. 24. 12. 15	70. 14. 12	3. 19. 51. 56	1,143398	Direct.	Mechain.
LXXII.	1785	8 April 8. 58. 52	2. 4. 33. 36	87. 31. 54	9. 27. 29. 33	0,427300	Retrograde.	Mechain.
LXXIII.	1786	7 July 21. 50. 52	6. 14. 22. 40	50. 54. 28	5. 9. 25. 36	0,41010	Direct.	Mechain.
LXXIV.	1787	10 May 19. 48. 40	3. 16. 51. 36	48. 15. 51	0. 7. 44. 9	0,34891	Retrograde.	P. de Saron.
LXXV.	1788	10 November . 7. 25. 40	5. 7. 10. 38	12. 28. 20	3. 9. 8. 27	1,06301	Retrograde.	Mechain.
LXXVI.	1788	20 November . 9. 4. 25	11. 21. 42. 15	64. 52. 32	0. 23. 12. 22	0,766911	Direct.	Mechain.
LXXVII.	1790	17 January . . .	5. 22. 0. 0	29. 31. 0	1. 28. 0. 0	0,75	Retrograde	De S . . . .
LXXVIII.	1790	28 January . . 7. 36. 13	8. 27. 8. 37	56. 58. 13	3. 21. 44. 37	1,063286	Direct.	Mechain.

The numbers, denoted by the *Figures* in the firſt column, ſhow the ſame comets with thoſe of the ſame number marked with the *Numerals*. Thus 49, which ſtands againſt the year 1456, denotes the ſame comet as that againſt which XLIX ſtands. Some of the comets are put down, as calculated by two or three Authors.

CHAP.

CHAP. XXVII.

ON THE FIXED STARS.

Art. 698. ALL the heavenly bodies beyond our ſyſtem are called *Fixed Stars*, becauſe they do not appear to have any proper motion of their own, except ſome few, which will be mentioned hereafter. From their immenſe diſtance, as appears by Article 524, they muſt be bodies of very great magnitude, otherwiſe they could not be viſible; and when we conſider the weakneſs of reflected light, there can be no doubt but that they ſhine with their own light. They are eaſily known from the planets, by their twinkling. The number of ſtars viſible at once to the naked eye is about 1000; but Dr. HERSCHEL, by his improvements of the reflecting teleſcope, has diſcovered that the whole number is great, beyond all conception. In that bright tract of the heavens called the *Milky Way*, which, when examined by good teleſcopes, appears to be an immenſe collection of ſtars which gives that whitiſh appearance to the naked eye, he has, in a quarter of an hour, ſeen 116000 ſtars paſs through the field of view of a teleſcope of only 15′ aperture. Every improvement of his teleſcopes has diſcovered ſtars not ſeen before, ſo that there appears to be no bounds to their number, or to the extent of the univerſe. Theſe ſtars, which can be of no uſe to us, are probably ſuns to other ſyſtems of planets.

699. From an attentive examination of the ſtars with good teleſcopes, many which appear only ſingle to the naked eye, are found to conſiſt of two, three, or more ſtars. Dr. MASKELYNE had obſerved α *Herculis*, to be a double ſtar; Dr. HORNSBY had found π *Bootis* to be double; M. CASSINI, Mr. MAYER, Mr. PIGOTT, and many other Aſtronomers had made diſcoveries of the like kind. But Dr. HERSCHEL, by his improved teleſcopes, has found about 700, of which, not above 42 had been obſerved before. We ſhall here give an account of a few of the moſt remarkable.

α *Herculis*, FLAM. 64, a beautiful double ſtar; the two ſtars very unequal, the largeſt is red, and the ſmalleſt blue, inclining to green.

δ *Lyræ*, FLAM. 12, double, very unequal, the largeſt red, and ſmalleſt duſky; not eaſily to be ſeen with a magnifying power of 227.

α *Geminorum*,

α *Geminorum*, FLAM. 66, double, a little unequal, both white; with a power of 146, their diſtance appears equal to the diameter of the ſmalleſt.

ε *Lyræ*, FLAM. 4 and 5, a double-double ſtar; at firſt ſight it appears double at a conſiderable diſtance, and by a little attention each will appear double; one ſet are equal, and both white; the other unequal, the largeſt white, and the ſmalleſt inclined to red. The interval of the ſtars, of the unequal ſet, is one diameter of the largeſt, with a power of 227.

γ *Andromedæ*, FLAM. 57, double, very unequal, the largeſt reddiſh white, the ſmalleſt a fine bright ſky-blue, inclining to green. A very beautiful object.

α *Urſæ minoris*, FLAM. 1, double, very unequal, the largeſt white, the ſmalleſt red.

β *Lyræ*, FLAM. 10, quadruple, unequal, white, but three of them a little inclined to red.

α *Leonis*, FLAM. 32, double, very unequal, largeſt white, ſmalleſt duſky.

ε *Bootis*, FLAM. 36, double, very unequal, largeſt reddiſh, ſmalleſt blue, or rather a faint lilac; very beautiful.

b Draconis, FLAM. 39, a very ſmall double ſtar, very unequal, the largeſt white, ſmalleſt inclining to red.

λ *Orionis*, FLAM. 39, quadruple, or rather a double ſtar, and has two more at a ſmall diſtance, the double ſtar conſiderably unequal, the largeſt white, ſmalleſt pale roſe colour.

ξ *Libræ*, FLAM. *ultima*, double-double, one ſet very unequal, the largeſt a very fine white.

μ *Cygni*, FLAM. 78, double, conſiderably unequal, the largeſt white, the ſmalleſt blueiſh.

μ *Herculis*, FLAM. 86, double, very unequal, the ſmall ſtar is not viſible with a power of 278, but is ſeen very well with one of 460; the largeſt is inclined to a pale red, ſmalleſt duſkiſh.

α *Capricorni*, FLAM. 5, double, very unequal, the largeſt white, ſmalleſt duſky.

ν *Lyræ*, FLAM. 8, treble, very unequal, the largeſt white, ſmalleſt both duſky.

α *Lyræ*, FLAM. 3, double, very unequal, the largeſt a fine brilliant white, the ſmalleſt duſky; it appears with a power of 227. Dr. HERSCHEL meaſured the diameter of this fine ſtar, and found it to be 0″,3553.

700. Theſe are a few of the principal double, treble, &c. ſtars mentioned by Dr. HERSCHEL in his catalogues which he has given us in the *Phil. Tranſ.* 1782 and 1785. The examination of double ſtars with a teleſcope is a very excellent and ready method of proving it's powers. Dr. HERSCHEL recommends

mends the following method. The teleſcope and the obſerver having been ſome time in the open air, adjuſt the focus of the teleſcope to ſome ſingle ſtar of nearly the ſame magnitude, altitude and colour of the ſtar to be examined; attend to all the phænomena of the adjuſting ſtar as it paſſes through the field of view—whether it be perfectly round and well defined, or affected with little appendages playing about the edge, or any other circumſtances of the like kind. Such deceptions may be detected by turning the object glaſs a little in it's cell, when theſe appendages will turn the ſame way. Thus you will detect the imperfections of the inſtrument, and therefore will not be deceived when you come to examine the double ſtar.

701. If $ABCD$ be the earth's orbit, and it's diameter AC bear a ſenſible proportion to the diſtance As of a near fixed ſtar s, this ſtar will appear in different ſituations in the heavens when the earth is at A and C; and it will, in the courſe of a year, appear to deſcribe a circle $abcd$, or an ellipſe, according as the plane of $abcd$ is perpendicular or oblique to the axis Ssm, or according as the ſtar is in or out of the pole of the ecliptic. The angle AsC is called the *Annual Parallax* of the ſtar. FIG. 174.

702. Dr. HERSCHEL propoſes to find the annual parallax of the fixed ſtars, by obſerving how the angle between two ſtars, very near to each other, vary in oppoſite parts of the year. This method was ſuggeſted by GALILEO in his *Syſtem. Coſ. Dial.* 3. The theory is true, if you admit his poſtulata, which is, that the ſtars are all of the ſame magnitude, and that a ſtar of the ſecond magnitude is double the diſtance of one of the firſt, and ſo on. But we have no reaſon whatever for making the former ſuppoſition, and if we reaſon from the bodies in our own ſyſtem, analogy will be againſt it; and in reſpect to the magnitudes, the arrangement of that is merely arbitrary. We will however explain the method in the moſt ſimple caſe. Let x and y be two ſtars ſituated in a line with the earth at A, and perpendicular to the diameter AB of the earth's orbit, and when the earth is at B, obſerve the angle xBy. Let P = the angle AxB, or the annual parallax of x, p = the angle xBy found from obſervation, M and m the angles under which the diameters of x and y appear, and draw zx perpendicular to Bx. Then $p : P :: xz : AB :: xy : Ay$ :: (becauſe $M : m :: Ay : Ax$) $M - m : M$, hence $P = \frac{p \times M}{M - m}$ the parallax of x. If x be a ſtar of the firſt magnitude, and y one of the third, and $p = 1''$, then $P = 1''\frac{1}{2}$ on theſe ſuppoſitions. See the *Phil. Tranſ.* 1782. FIG. 175.

703. Several ſtars mentioned by ancient Aſtronomers are not now to be found, and ſeveral are now obſerved, which do not appear in their catalogues. The moſt ancient obſervation of a new ſtar is that by HIPPARCHUS, about 120 years before J. C. which occaſioned his making a catalogue of the fixed

fixed ſtars, in order that future Aſtronomers might ſee what alterations had taken place ſince his time. We have no account where this new ſtar appeared. A new ſtar is alſo ſaid to have appeared in the year 130; another in 389; another in the ninth century, in 15° of *Scorpio*; a fifth in 945; and a ſixth in 1264; but the accounts we have of all theſe are very imperfect.

704. The firſt new ſtar we have any accurate account of, is that which was diſcovered by CORNELIUS GEMMA, on November 8, 1572, in the *Chair of Caſſiopea*. It exceeded *Sirius* in brightneſs, and was ſeen at mid-day. It firſt appeared bigger than *Jupiter*, but it gradually decayed, and after ſixteen months it entirely diſappeared. It was obſerved by TYCHO BRAHE, who found that it had no ſenſible parallax; and he concluded that it was a fixed ſtar. Some have ſuppoſed that this is the ſame which appeared in 945, and 1264, the ſituation of it's place favouring this opinion.

705. On Auguſt 13, 1596, DAVID FABRICIUS obſerved a new ſtar in the *Neck of the Whale*, in 25°. 45′ of Aries, with 15°. 54′ ſouth latitude. It diſappeared after October in the ſame year. PHOCYLLIDES HOLWARDA diſcovered it again in 1637, not knowing that it had ever been ſeen before; and after having diſappeared for nine months, he ſaw it come into view again. BULLIALDUS determined the periodic time between it's greateſt brightneſs to be 333 days. It's greateſt brightneſs is that of a ſtar of the ſecond magnitude, and it's leaſt, that of a ſtar of the ſixth. It's greateſt degree of brightneſs however is not always the ſame, nor are the ſame phaſes always at the ſame interval.

706. In the year 1600, WILLIAM JANSENIUS diſcovered a changeable ſtar in the *Neck of the Swan*. It was ſeen by KEPLER, who wrote a treatiſe upon it, and determined it's place, to be 16°. 18′ ♒, with 55°. 30′ or 32′ north latitude. RICCIOLUS ſaw it in 1616, 1621, 1624 and 1629. He is poſitive that it was inviſible in the laſt years from 1640 to 1650. M. CASSINI ſaw it again in 1655; it increaſed till 1660, and then grew leſs, and at the end of 1661 it diſappeared. In November 1665, it appeared again, and diſappeared in 1681. In 1715 it appeared of the ſixth magnitude, as it does at preſent.

707. On June 20, 1670, another changeable ſtar was diſcovered near the *Swan's Head*, by P. ANTHELME. It diſappeared in October, and was ſeen again on March 17, 1671. On September 11, it diſappeared. It appeared again in March 1672, and diſappeared in the ſame month, and has never ſince been ſeen. It's longitude was 1°. 52′. 26″ of ♒, and it's latitude 47°. 25′. 22″ N. The days are here put down for the new ſtyle.

708. In 1686, KIRCHIUS obſerved χ in the *Swan* to be a changeable ſtar; and, from 20 years obſervations, the period of the return of the ſame phaſes was

found to be 405 days; the variations of it's magnitude however were ſubject to ſome irregularity.

709. In the year 1604, at the beginning of October, KEPLER diſcovered a new ſtar near the heel of the right foot of *Serpentarius*, ſo very brilliant, that it exceeded every fixed ſtar, and even *Jupiter* in magnitude. It was obſerved to be every moment changing into ſome of the colours of the rainbow, except when it was near the horizon, when it was generally white. It gradually diminiſhed, and diſappeared about October 1605, when it came too near the ſun to be viſible, and was never ſeen after. It's longitude was 17°. 40′ of ♐, with 1°. 56′ north latitude, and was found to have no parallax.

710. MONTANARI diſcovered two ſtars in the conſtellation of the *Ship*, marked β and γ by BAYER, to be wanting. He ſaw them in 1664, but loſt them in 1668. The ſtar θ in the tail of the *Serpent*, reckoned by TYCHO of the third, was found, by him, of the fifth magnitude. The ſtar ρ in *Serpentarius* did not appear, from the time it was obſerved by him, till 1695. The ſtar ψ in the *Lion*, after diſappearing, was ſeen by him in 1667. He obſerved alſo that β in *Meduſa's Head* varied in it's magnitude.

711. M. CASSINI diſcovered *one* new ſtar of the fourth, and *two* of the fifth magnitude in *Caſſiopea*; alſo *five* new ſtars in the ſame conſtellation, of which three have diſappeared; *two* new ones in the beginning of the conſtellation *Eridamus*, of the fourth and fifth magnitude; and *four* new ones of the fifth or ſixth magnitude, near the north pole. He further obſerved, that the ſtar, placed by BAYER near ε of the *Little Bear*, is no longer viſible; that the ſtar A of *Andromeda*, which had diſappeared, had come into view again in 1695; that in the ſame conſtellation, inſtead of one in the *Knee*, marked υ, there are two others come more northerly; and that ξ is diminiſhed; that the ſtar placed by TYCHO at the end of the *Chain of Andromeda*, as of the fourth magnitude, could then ſcarcely be ſeen; and that the ſtar which, in TYCHO's catalogue, is the twentieth of *Piſces*, was no longer viſible.

712. M. MARALDI obſerved, that the ſtar ϰ in the left leg of *Sagittarius*, marked by BAYER of the third magnitude, appeared of the ſixth, in 1671; in 1676 it was found by Dr. HALLEY to be of the third; in 1692 it could hardly be perceived, but in 1693 and 1694 it was of the fourth magnitude. In 1704 he diſcovered a ſtar in *Hydra* to be periodical; it's poſition is in a right line with thoſe in the tail marked π and γ. The time between it's greateſt luſtre, of the fourth magnitude, was about two years; in the intermediate time it diſappeared. In 1666, HEVELIUS ſays he could not find a ſtar of the fourth magnitude in the eaſtern ſcale of *Libra*, obſerved by TYCHO and BAYER; but MARALDI in 1709, ſays, that it had then been ſeen for 15 years, ſmaller than one of the fourth. See *Elem. d'Aſtron.* page 57.

713. J. GOODRICKE,

713. J. GOODRICKE, Esq. has determined the periodic variation of *Algol*, or β *Persei* (observed by MONTANARI to be variable) to be about 2*d*. 21*h*. It's greatest brightness is of the second magnitude, and least, of the fourth. It changes from the second to the fourth in about three hours and a half, and back again in the same time, and retains it's greatest brightness for the other part of the time. See the *Phil. Trans.* 1783. In the *Connoissance des Temps*, for 1792, M. de la LANDE has given the following Tables to find the time when the brightness is the least. I have reduced the epochs to the meridian of Greenwich.

TABLES OF THE VARIATION OF *ALGOL*.

EPOCHS.

YEARS.	D.	H.	M.
1796 *B*	2.	7.	38
1797	1.	11.	25
1798	0.	15.	12
1799	2.	15.	49
1800 *C*	1.	19.	36
1801	0.	23.	23
1802	0.	3.	10
1803	2.	3.	47
1804 *B*	0.	7.	34

In leap-year, we muſt add a day to the calculation, in January and February.

MEAN MOTION FOR MONTHS.

MONTHS.	D.	H.	M.
January	0.	0.	0
February	0.	12.	59
March	1.	5.	10
April	1.	18.	9
May	0.	10.	19
June	0.	23.	19
July	2.	12.	18
Auguſt	0.	4.	28
September	0.	17.	28
October	2.	6.	27
November	2.	19.	27
December	1.	11.	37

MEAN MOTION FOR YEARS.

YEARS.	D.	H.	M.
1	2.	0.	36
2	1.	4.	23
3	0.	8.	11
4 *B*	1.	8.	47
5	0.	12.	34
6	2.	13.	10
7	1.	16.	58
8 *B*	2.	17.	34

REVOLUTIONS.

	D.	H.	M.	S.
1	2.	20.	49.	2
2	5.	17.	38.	4
3	8.	14.	27.	6
4	11.	11.	16.	8
5	14.	8.	5.	10
6	17.	4.	54.	12
7	20.	1.	43.	14
8	22.	22.	32.	16
9	25.	19.	21.	18
10	28.	16.	10.	20

714. Mr. GOODRICKE alſo diſcovered, that β *Lyræ* was ſubject to a periodic variation. The following is the reſult of his obſervations. It completes all it's phaſes in 12 days 19 hours, during which time, it undergoes the following changes.—1. It is of the third magnitude for about two days.—2. It diminiſhes in about 1 ¼ days.—3. It is between the fourth and fifth magnitude for leſs than a day.—4. It increaſes in about two days.—5. It is of the third magnitude for about three days.—6. It diminiſhes in about one day.—7. It is ſomething larger than the fourth magnitude for a little leſs than a day.—8. It increaſes in about one day and three quarters to the firſt point, and ſo completes a whole period. See the *Phil. Tranſ.* 1785. He has alſo found, that δ *Cephei* is ſubject to a periodic variation of 5*d*. 8*h*. 37′½, during which time it undergoes the following changes. 1. It is at it's greateſt brightneſs about 1 day 13 hours.—2. It's diminution is performed in about 1 day 18 hours.—3. It is at it's greateſt obſcuration about 1 day 12 hours.—4. It increaſes in about 13 hours. It's greateſt and leaſt brightneſs is that between the third and fourth, and between the fourth and fifth magnitudes.

715. E. PIGOTT, Eſq. has diſcovered η *Antinoi* to be a variable ſtar, with a period of 7 days 4 hours 38 minutes. The changes happen as follows. 1. It is at it's greateſt brightneſs 44 ± hours.—2. It decreaſes 62 ± hours.—3. It is at it's leaſt brightneſs 30 ± hours.—4. It increaſes 36 ± hours. When moſt bright it is of the third or fourth magnitude, and when leaſt, of the fourth or fifth. See the *Phil. Tranſ.* 1785.

716. In the *Phil. Tranſ.* 1796, Dr. HERSCHEL has propoſed a method of obſerving the changes that may happen to the fixed ſtars; with a catalogue of their comparative brightneſs, in order to aſcertain the permanency of their luſtre.

717. Dr. HERSCHEL, in a Paper in the *Phil. Tranſ.* 1783, upon the proper motion of the ſolar ſyſtem, has given a large collection of ſtars which were formerly ſeen, but are now loſt; alſo a catalogue of variable ſtars, and of new ſtars; and very juſtly obſerves, that it is not eaſy to prove that a ſtar was never ſeen before; for though it ſhould not be contained in any catalogue whatever, yet the argument for it's former non-appearance, which is taken from it's not having been obſerved before, is only ſo far to be regarded, as it can be made probable, or almoſt certain, that a ſtar would have been obſerved, had it been viſible.

718. There have been various conjectures to account for the appearances of the changeable ſtars. M. MAUPERTUIS ſuppoſes, that they may have ſo quick a motion about their axes, that the centrifugal force may reduce them to flat oblate ſpheroids, not much unlike a mill ſtone; that it's plane may be

inclined to the plane of the orbits of it's planets, by whoſe attraction the poſition of the body may be altered, ſo that when it's plane paſſes through the earth, it may be almoſt or entirely inviſible, and then become again viſible as it's broad ſide is turned towards us. Others have conjectured, that conſiderable parts of their ſurfaces are covered with dark ſpots, ſo that when, by the rotation of the ſtar, theſe ſpots are preſented to us, the ſtars become almoſt or entirely inviſible. Others have ſuppoſed, that theſe ſtars have very large opaque bodies revolving about and near to them, ſo as to obſcure them when they come in conjunction with us. The irregularity of the phaſes of ſome of them, ſhows the cauſe to be variable, and therefore may perhaps be beſt accounted for, by ſuppoſing that a great part of the body of the ſtar is covered with ſpots, which appear and diſappear like thoſe on the ſun's ſurface. The total diſappearance of a ſtar may probably be the deſtruction of it's ſyſtem; and the appearance of a new ſtar, the creation of a new ſyſtem of planets.

719. The fixed ſtars are not all evenly ſpread through the heavens, but the greater part of them are collected into cluſters, of which it requires a large magnifying power, with a great quantity of light, to be able to diſtinguiſh the ſtars ſeparately. With a ſmall magnifying power and quantity of light, they only appear ſmall whitiſh ſpots, ſomething like a ſmall light cloud, and from thence they were called *Nebulæ*. There are ſome nebulæ, however, which do not receive their light from ſtars. In the year 1656, HUYGENS diſcovered a nebula in the middle of *Orion's Sword*; it contains only ſeven ſtars, and the other part is a bright ſpot upon a dark ground, and appears like an opening into brighter regions beyond. See Fig. 176. In 1612, SIMON MARIUS diſcovered a nebula in the *Girdle of Andromeda*. Dr. HALLEY, when he was obſerving the ſouthern ſtars, diſcovered one in the *Centaur*, but this is never viſible in England. In 1714, he found another in *Hercules*, nearly in a line with ζ and η of BAYER. This ſhows itſelf to the naked eye, when the ſky is clear and the moon abſent. M. CASSINI diſcovered one between the *Great Dog* and the *Ship*, which he deſcribes as very full of ſtars, and very beautiful, when viewed with a good teleſcope. There are two whitiſh ſpots near the ſouth pole, called, by ſailors, the *Magellanic Clouds*, which, to the naked eye, reſemble the milky way, but through teleſcopes, they appear to be compoſed of ſtars. M. de la CAILLE, in his catalogue of fixed ſtars obſerved at the Cape of Good Hope, has remarked 42 nebulæ which he obſerved, and which he divided into three claſſes; fourteen, in which he could not diſcover the ſtars; fourteen, in which he could ſee a diſtinct maſs of ſtars, and fourteen, in which the ſtars appeared of the ſixth magnitude, or below, accompanied with white ſpots, and nebulæ of the firſt and third kind. In the *Connoiſſance des Temps*, for 1783, and 1784, there is a catalogue of 103 nebulæ, obſerved by MESSIER and

FIG. 176.

MECHAIN, ſome of which they could reſolve, and others they could not. But Dr. HERSCHEL has given us a catalogue of 2000 nebulæ and cluſter of ſtars, which he himſelf has diſcovered. Some of them form a round compact ſyſtem, others are more irregular, of various forms, and ſome are long and narrow. The globular ſyſtems of ſtars appear thicker in the middle than they would do if the ſtars were all at equal diſtances from each other; they are therefore condenſed towards the center. That the ſtars ſhould be thus accidentally diſpoſed, is too improbable a ſuppoſition to be admitted; he ſuppoſes therefore, that they are thus brought together by their mutual attractions, and that the gradual condenſation towards the center, is a proof of a central power of ſuch a kind. He further obſerves, that there are ſome additional circumſtances in the appearance of extended cluſters and nebulæ, that very much favour the idea of a power lodged in the brighteſt part. For although the form of them be not globular, it is plainly to be ſeen that there is a tendency towards ſphericity, by the ſwell of the dimenſions as they draw near the moſt luminous place, denoting, as it were, a courſe, or tide of ſtars, ſetting towards a center. As the ſtars in the ſame nebula muſt be very nearly all at the ſame relative diſtances from us, and they appear nearly of the ſame ſize, their real magnitudes muſt be nearly equal. Granting therefore that theſe nebulæ and cluſters of ſtars are formed by their mutual attraction, Dr. HERSCHEL concludes that we may judge of their relative age by the diſpoſition of their component parts, thoſe being the oldeſt which are moſt compreſſed. He ſuppoſes the milky way to be a nebula, of which our ſun is one of it's component ſtars. See the *Phil. Tranſ.* 1786 and 1789.

720. Dr. HERSCHEL has diſcovered other phænomena in the heavens which he calls *Nebulous Stars*, that is, ſtars ſurrounded with a faint luminous atmoſphere, of a conſiderable extent. Cloudy or nebulous ſtars, he obſerves, have been mentioned by ſeveral Aſtronomers; but this name ought not to be applied to the objects which they have pointed out as ſuch; for, on examination, they proved to be either cluſters of ſtars, or ſuch appearances as may reaſonably be ſuppoſed to be occaſioned by a multitude of ſtars at a vaſt diſtance. He has given an account of ſeventeen of theſe ſtars, one of which he has thus deſcribed. "November 13, 1790. A moſt ſingular phænomenon! A ſtar of the eighth magnitude, with a faint luminous atmoſphere, of a circular form, and of about 3′ diameter. The ſtar is perfectly in the center, and the atmoſphere is ſo diluted, faint and equal throughout, that there can be no ſurmiſe of it's conſiſting of ſtars; nor can there be a doubt of the evident connection between the atmoſphere and the ſtar. Another ſtar not much leſs in brightneſs, and in the ſame field of view with the above, was perfectly free from any ſuch appearance." Hence he draws the following conſequences. Granting the connec-

connection between the ſtar and the ſurrounding nebuloſity, if it conſiſt of ſtars very remote which gives the nebulous appearance, the central ſtar, which is viſible, muſt be immenſely greater than the reſt; or if the central ſtar be no bigger than common, how extremely ſmall and compreſſed muſt be thoſe other luminous points which occaſion the nebuloſity? As, by the former ſuppoſition, the luminous central point muſt far exceed the ſtandard of what we call a ſtar, ſo, in the latter, the ſhining matter about the center will be much too ſmall to come under the ſame denomination; we therefore either have a central body which is not a ſtar, or a ſtar which is involved in a ſhining fluid, of a nature totally unknown to us. This laſt opinion Dr. HERSCHEL adopts. The exiſtence of this ſhining matter, he ſays, does not ſeem to be ſo eſſentially connected with the central points, that it might not exiſt without them. The great reſemblance there is between the chevelure of theſe ſtars, and the diffuſed nebuloſity there is about the conſtellation of *Orion*, which takes up a ſpace of more than 60 ſquare degrees, renders it highly probable that they are of the ſame nature. If this be admitted, the ſeparate exiſtence of the luminous matter is fully proved. Light reflected from the ſtar could not be ſeen at this diſtance. And beſides, the outward parts are nearly as bright as thoſe near the ſtar. In further confirmation of this, he obſerves, that a cluſter of ſtars will not ſo completely account for the milkineſs, or ſoft tint of the light of theſe nebulæ, as a ſelf luminous fluid. This luminous matter ſeems more fit to produce a ſtar by it's condenſation, than to depend on the ſtar for it's exiſtence. There is a teleſcopic milky way extending in right aſcenſion from $5^h.\ 15'.\ 8''$ to $5^h.\ 39'.\ 1''$, and in polar diſtance from $87°.\ 46'$ to $98°.\ 10'$. This, Dr. HERSCHEL thinks, is better accounted for, by a luminous matter, than from a collection of ſtars. He obſerves, that perhaps ſome may account for theſe nebulous ſtars, by ſuppoſing that the nebuloſity may be formed by a collection of ſtars at an immenſe diſtance, and that the central ſtar may be a near ſtar accidentally ſo placed; the appearance however of the luminous part does not, in his opinion, at all favour the ſuppoſition that it is produced by a great number of ſtars; on the other hand, it muſt be granted that it is extremely difficult to admit the other ſuppoſition, when we know nothing but a ſolid body that is ſelf-luminous, or, at leaſt, that a fixed luminary muſt immediately depend upon ſuch, as the flame of a candle upon the candle itſelf. See the *Phil. Tranſ.* 1791, for Dr. HERSCHEL's account.

On the Conſtellations.

721. As ſoon as Aſtronomy began to be ſtudied, it muſt have been found neceſſary to divide the heavens into ſeparate parts, and to give ſome repreſentations to them, in order that Aſtronomers might deſcribe and ſpeak of the ſtars, ſo as to be underſtood. Accordingly we find that theſe circumſtances took place very early. The ancients divided the heavens into *Conſtellations*, or collections of ſtars, and repreſented them by animals, and other figures, according to the ideas which the diſpoſitions of the ſtars ſuggeſted. We find ſome of them mentioned by JOB, and although it has been diſputed, whether our tranſlation has ſometimes given the true meaning to the Hebrew words, yet it is agreed, that they ſignify the conſtellations. Some of them are mentioned by HOMER and HESIOD, but ARATUS profeſſedly treats of all the ancient ones, except three which were invented after his time. The number of the ancient conſtellations was 48, but the preſent number upon a globe is about 70; by rectifying which (as will be afterwards explained), and ſetting it to correſpond with the ſtars in the heavens, you may, by comparing them, very eaſily get a knowledge of the different conſtellations and ſtars. Thoſe ſtars which do not come into any of the conſtellations, are called *uninformed ſtars*. The ſtars viſible to the naked eye are divided into ſix claſſes, according to their magnitudes; the largeſt are called of the firſt magnitude, the next of the ſecond, and ſo on. Thoſe which cannot be ſeen without teleſcopes, are called *Teleſcopic Stars*. The ſtars are now generally marked upon maps and globes with BAYER's letters; the firſt letter in the Greek alphabet being put to the greateſt ſtar of each conſtellation; the ſecond letter to the next greateſt, and ſo on, and when any more letters are wanted, the italic characters are generally uſed; this ſerves as a name to the ſtar, by which it may be pointed out. Twelve of theſe conſtellations lie upon the ecliptic, including a ſpace about 15° broad, called the *Zodiac*, within which all the planets move. The conſtellation *Aries*, or the *Ram*, about 2000 years ago, lay in the *firſt* ſign of the ecliptic; but, on account of the preceſſion of the equinox, it now lies in the *ſecond*. The following are the names of the conſtellations, and the number of the ſtars obſerved in them by different Aſtronomers. *Antinous* was made out of the uninformed ſtars near *Aquila*; and *Coma Berenices* out of the uninformed ſtars near the *Lyon's Tail*. They are both mentioned by PTOLEMY, but as uninformed ſtars. The conſtellations as far as the Triangle, with Coma Berenices, are *northern*; thoſe after Piſces, are *ſouthern*.

THE

THE ANCIENT CONSTELLATIONS.

		Ptolemy.	Tycho.	Hevelius.	Flamstead.
Urſa Minor	The Little Bear	8	7	12	24
Urſa Major	The Great Bear	35	29	73	87
Draco	The Dragon	31	32	40	80
Cæpheus	Cæpheus	13	4	51	35
Bootes	Bootes	23	18	52	54
Corona Borealis	The Northern Crown	8	8	8	21
Hercules	Hercules kneeling	29	28	45	113
Lyra	The Harp	10	11	17	21
Cygnus	The Swan	19	18	47	81
Caſſiopea	The Lady in her Chair	13	26	37	55
Perſeus	Perſeus	29	29	46	59
Auriga	The Waggoner	14	9	40	66
Serpentarius	Serpentarius	29	15	40	74
Serpens	The Serpent	18	13	22	64
Sagitta	The Arrow	5	5	5	18
Aquila	The Eagle	15	12	23	71
Antinous	Antinous		3	19	
Delphinus	The Dolphin	10	10	14	18
Equulus	The Horſe's Head	4	4	6	10
Pegaſus	The Flying Horſe	20	19	38	89
Andromeda	Andromeda	23	23	47	66
Triangulum	The Triangle	4	4	12	16
Aries	The Ram	18	21	27	66
Taurus	The Bull	44	43	51	141
Gemini	The Twins	25	25	38	85
Cancer	The Crab	23	15	29	83
Leo	The Lion	35	30	49	95
Coma Berenices	Berenice's Hair		14	21	43
Virgo	The Virgin	32	33	50	110
Libra	The Scales	17	10	20	51
Scorpius	The Scorpion	24	10	20	44
Sagittarius	The Archer	31	14	22	69
Capricornus	The Goat	28	28	29	51
Aquarius	The Water-bearer	45	41	47	108
Piſces	The Fiſhes	38	36	39	113

Cetus

THE ANCIENT CONSTELLATIONS CONTINUED.

		Ptolemy.	Tycho.	Hevelius.	Flamstead.
Cetus	The Whale	22	21	45	97
Orion	Orion	38	42	62	78
Eridanus	Eridanus,	34	10	27	84
Lepus	The Hare	12	13	16	19
Canis Major	The Great Dog	29	13	21	31
Canis Minor	The Little Dog	2	2	13	14
Argo	The Ship	45	3	4	64
Hydra	The Hydra	27	19	31	60
Crater	The Cup	7	3	10	31
Corvus	The Crow	7	4		9
Centaurus	The Centaur	37			35
Lupus	The Wolf	19			24
Ara	The Altar	7			9
Corona Auſtralis	The Southern Crown	13			12
Piſcis Auſtralis	The Southern Fiſh	18			24

THE NEW SOUTHERN CONSTELLATIONS.

Columba Naochi	Noah's Dove	10
Robur Carolinum	The Royal Oak	12
Grus	The Crane	13
Phœnix	The Phœnix	13
Indus	The Indian	12
Pavo	The Peacock	14
Apus, *Avis Indica*	The Bird of Paradiſe	11
Apis, *Muſca*	The Bee or Fly	4
Chamæleon	The Chameleon	10
Triangulum Auſtralis	The South Triangle	5
Piſcis volans, *Paſſer*	The Flying Fiſh	8
Dorado, *Xiphias*,	The Sword Fiſh	6
Toucan	The American Gooſe	9
Hydrus	The Water Snake	10

HEVELIUS's

HEVELIUS's CONSTELLATIONS

Made out of the uninformed Stars.

		Hevelius.	Flamstead.
Lynx	The Lynx	19	44
Leo Minor	The Little Lion		53
Aſteron and Chara	The Greyhounds	23	25
Cerberus	Cerberus	4	
Vulpecula and Anſer	The Fox and Gooſe	27	35
Scutum Sobieſki	Sobieſki's Shield	7	
Lacerta	The Lizard		16
Camelopardalus	The Camelopard	32	58
Monoceros	The Unicorn	19	31
Sextans	The Sextant	11	41

Beſides the letters which are prefixed to the ſtars, many of them have names, as *Regulus*, *Syrius*, *Arcturus*, &c.

722. Kepler, who was afterwards in this conjecture followed by Dr. Halley, has made a very ingenious obſervation upon the magnitudes and diſtances of the fixed ſtars. He obſerves, that there can be only 13 points upon the ſurface of a ſphere as far diſtant from each other as from the center; and ſuppoſing the neareſt fixed ſtars to be as far from each other as from the ſun, he concludes that there can be only 13 ſtars of the firſt magnitude. Hence, at twice that diſtance from the ſun, there may be placed four times as many, or 52; at three times that diſtance, nine times as many, or 117; and ſo on. Theſe numbers will give pretty nearly the number of ſtars of the firſt, ſecond, third, &c. magnitudes. Dr. Halley further remarks, that if the number of ſtars be finite, and occupy only a part of ſpace, the outward ſtars would be continually attracted towards thoſe which are within, and in proceſs of time they would coaleſce and unite into one. But if the number be infinite, and they occupy an infinite ſpace, all the parts would be nearly in equilibrio, and conſequently each fixed ſtar being drawn in oppoſite directions would keep it's place, or move on till it had found an equilibrium. *Phil. Tranſ.* N°. 364.

On the Catalogues of the Fixed Stars.

723. At the time of HIPPARCHUS of Rhodes, about 120 years before J. C. a new ſtar appeared, upon which he ſet about numbering the fixed ſtars and reducing them to a *Catalogue*, that poſterity might know whether any changes had taken place in the heavens. PTOLEMY however mentions that TYMOCHARIUS and ARISTYLLUS left ſeveral obſervations made 180 years before. The catalogue of HIPPARCHUS contained 1022 ſtars, with their latitudes and longitudes, which PTOLEMY publiſhed, with the addition of four more. Theſe Aſtronomers made their obſervations with an armillary ſphere, placing the armilla, or hoop repreſenting the ecliptic, to coincide with the ecliptic in the heavens by means of the ſun in the day-time, and then they determined the place of the moon in reſpect to the ſun by a moveable circle of latitude. The next night, by the help of the moon (whoſe place before found they corrected by allowing for it's motion in the interval of time) they placed the hoop in ſuch a ſituation as was agreeable to the preſent moment of time, and then compared, in like manner, the places of the ſtars with the moon. Thus they found the latitudes and longitudes of the ſtars; it could not however be done with ſuch an inſtrument to any very great degree of accuracy. PTOLEMY adapted his catalogue to the year 137 after J. C.; but ſuppoſing, with HIPPARCHUS who made the diſcovery, the preceſſion of the equinoxes to be 1° in 100 years, inſtead of about 72 years, he only added 2°. 40′ to the numbers in HIPPARCHUS for 265 years (the difference of the epochs) inſtead of 3°. 42′. 22″ according to Dr. MASKELYNE's Tables. To compare his Tables therefore with the preſent, we muſt firſt increaſe his numbers by 1°. 2′. 22″, and then allow for the preceſſion from that time to this. The next Aſtronomer who obſerved the fixed ſtars a-new, was ULUGH BEIGH, the Grandſon of TAMERLANE the Great; he made a catalogue of 1022 ſtars, reduced to the year 1437. WILLIAM, the moſt illuſtrious Landgrave of Heſſe, made a catalogue of 400 ſtars which he obſerved; he computed their latitudes and longitudes from their obſerved right aſcenſions and declinations. In the year 1610, TYCHO BRAHE's catalogue of 777 ſtars was publiſhed from his own obſervations, made with great care and diligence. It was afterwards, in 1627, copied into the *Rudolphine Tables*, and increaſed by 223 ſtars from other obſervations of TYCHO. Inſtead of a *zodiacal* armilla, TYCHO ſubſtituted the *equatorial* armilla, by which he obſerved the difference of right aſcenſions, and the declinations, out of the meridian, the meridian altitude being always made uſe of to confirm the others. From thence he computed the latitudes and

longitudes.

longitudes. TYCHO compared *Venus* with the ſun, and then the other ſtars with Venus, allowing for it's parallax and refraction; and having thus aſcertained the places of a few ſtars, he ſettled the reſt from them; and although his inſtrument was very large, and conſtructed with great accuracy, yet not having pendulum clocks to meaſure his time, his obſervations cannot be very accurate. The next catalogue was that of R. P. RICCIOLUS, which was taken from TYCHO's, except 101 ſtars which he himſelf had obſerved. HEVELIUS of Dantzick in 1690 publiſhed a catalogue of 1930 ſtars, of which 950 were known to the ancients; 603 he calls his own, becauſe they had not been accurately obſerved by any one before himſelf; and 377 of Dr. HALLEY which were inviſible to his hemiſphere. Their places were fixed for the year 1660. The *Britiſh Catalogue*, which was publiſhed by Mr. FLAMSTEAD, contains 3000 ſtars, rectified for the year 1689. They are diſtinguiſhed into ſeven degrees of magnitude (of which the ſeventh degree are teleſcopic) in their proper conſtellations. This catalogue is more correct than any of the others, the obſervations having been made with better inſtruments. He alſo publiſhed an *Atlas Cæleſtis*, or maps of the ſtars, in which each ſtar is laid down in it's true place, and delineated of it's own magnitude. Each ſtar is marked with a letter, beginning with the firſt letter α of the Greek alphabet for the largeſt ſtar of each conſtellation, and ſo on according to their magnitudes, following, in this reſpect, the charts of the ſame kind which were publiſhed by J. BAYER, a German, in 1603. In the year 1757, M. de la CAILLE publiſhed his *Fundamenta Aſtronomiæ*, in which their is a catalogue of 397 ſtars; and in 1763, he publiſhed a catalogue of 1942 ſouthern ſtars, from the tropic of Capricorn to the ſouth pole, with their right aſcenſions and declinations for 1750. He alſo publiſhed a catalogue of zodiacal ſtars in the *Ephemerides* from 1765 to 1774. Mr. MAYER alſo publiſhed a catalogue of 600 zodiacal ſtars. In the *Nautical Almanac* for 1773, there is publiſhed a catalogue of 380 ſtars obſerved by Dr. BRADLEY, with their longitudes and latitudes. In the year 1782, J. E. BODE, Aſtronomer at Berlin, publiſhed a ſet of *Celeſtial Charts*, containing a greater number of ſtars than in thoſe of Mr. FLAMSTEAD, with many of the double ſtars and nebulæ. He alſo publiſhed, in the ſame work, a catalogue of ſtars, that of FLAMSTEAD being the foundation, omitting ſome ſtars, whoſe poſitions were left incomplete, and altering the numbers of others; to which he has added ſtars from HEVELIUS, M. de la CAILLE, MAYER and others. In the year 1776, there was publiſhed at Berlin, a work entitled, *Recuiel de Tables Aſtronomiques*, in which is contained a very large catalogue of ſtars from HEVELIUS, FLAMSTEAD, M. de la CAILLE, and Dr BRADLEY, with their latitudes and

longitudes for the beginning of 1800; with a catalogue of the ſouthern ſtars of M. de la CAILLE; —of double ſtars; —of changeable ſtars, and of nebulous ſtars. This is a very uſeful Work for the Practical Aſtronomer. But the moſt complete catalogue is that publiſhed by the Rev. Mr. WOLLASTON, F. R. S. in 1789, entitled, *A Specimen of a General Aſtronomical Catalogue, arranged in Zones of North Polar Diſtance, and adapted to January* 1, 1790; *containing a Comparative View of the Mean Poſitions of Stars, Nebulæ, and Cluſters of Stars, as they come out upon Calculation from the Tables of ſeveral principal Obſervers.* By arranging the ſtars into zones parallel to the equator, an obſerver, with his teleſcope on an equatorial ſtand, will have the ſtars paſs through in the order in which he finds them in the catalogue, by which he will more readily find out what he wants, being prepared for it's appearance. The firſt Table contains a catalogue of the mean right aſcenſions of 36 principal ſtars for January 1, 1790, as ſettled by Dr. MASKELYNE, with their annual preceſſions, and proper motions. The ſecond Table contains the general catalogue of all the ſtars whoſe places have been well aſcertained, together with thoſe nebulæ and cluſters of ſtars which can eaſily be ſeen by a good common teleſcope, with their right aſcenſions and north polar diſtances, and their annual preceſſions; alſo their magnitudes, and the number, name or character of the object, and by whom it was obſerved. The third Table contains an index to the ſtars in the Britiſh Catalogue; referring to the zone of north polar diſtance in which each is to be found. The fourth Table contains an index of thoſe ſtars in M. de la CAILLE's fundamental catalogue, which are not in FLAMSTEAD's. The fifth Table contains FLAMSTEAD's Britiſh Catalogue, and M. de la CAILLE's ſouthern catalogue, with about eighty circumpolar ſtars from HEVELIUS which had been omitted by FLAMSTEAD, arranged in their order of right aſcenſions in time for January 1, 1790. The ſixth Table contains a catalogue of the zodiacal ſtars for 9° of latitude, arranged in their order of longitude for January 1, 1790. The whole concludes with a plan for examining the heavens, propoſing that different perſons ſhould undertake different zones and examine them very minutely; recommending a ſyſtem of wires in a teleſcope which he has found very convenient for that purpoſe. The Practical Aſtronomer is under very great obligations to Mr. WOLLASTON for ſo uſeful and complete a Work.

On the Proper Motion *of the Fixed Stars.*

724. Dr. MASKELYNE, in the explanation and uſe of his Tables which he publiſhed with the firſt Volume of his *Obſervations*, obſerves, that many, if not all the fixed ſtars, have ſmall motions among themſelves, which are called their *Proper Motions*; the cauſe and laws of which are hid for the preſent in almoſt equal obſcurity. From comparing his own obſervations at that time with thoſe of Dr. BRADLEY, Mr. FLAMSTEAD, and M. ROËMER, he then found the annual proper motion of the following ſtars in right aſcenſion to be, of *Sirius* – 0″,63, of *Caſtor* – 0″,28, of *Procyon* – 0″,8, of *Pollux* – 0″,93, of *Regulus* – 0″,41, of *Arcturus* – 1″,4, and of *α Aquilæ* + 0″,57; and of *Sirius* in north polar diſtance 1″,20, and of *Arcturus* 2″,01 both ſouthwards. But ſince that time he had continued his obſervations, and from a catalogue of the mean right aſcenſions of 36 principal ſtars (which he communicated to Mr. WOLLASTON, and is found in his Work), it appears that 35 of them have a *proper motion* in right aſcenſion.

725. In the year 1756, M. MAYER obſerved 80 ſtars, and compared them with the obſervations of ROËMER in 1706. M. MAYER is of opinion, that (from the goodneſs of the inſtruments with which the obſervations were made) where the diſagreement is at leaſt 10″ or 15″, it is a very probable indication of a proper motion of ſuch a ſtar. He further adds, that when the diſagreement is ſo great as he has found it in ſome of the ſtars, amongſt which is *Fomahand*, where the difference was 21″ in 50 years, he has no doubt of a proper motion. Dr. HERSCHEL, following MAYER's judgment of his own and ROËMER's obſervations, has compared the obſervations, and leaving out of his account all thoſe ſtars which did not ſhow a diſagreement amounting to 10″, he found that 56 of them had a proper motion. From thence he attempts to deduce the motion of the ſolar ſyſtem in the following manner.

726. If the ſun be firſt at *S*, and then move from *S* to *C* in the line *AB*, a ſtar at *s* would appear to move from *a* to *b*; hence if we ſuppoſe *BKAL* to be the ecliptic, any ſtar in the ſemicircle *BKA*, ſuppoſing that to be the order of the ſigns, will have it's longitude, reckoned from the point to which the ſun is moving, increaſed; but a ſtar in the other ſemicircle will have it's longitude, ſo reckoned, diminiſhed. Thoſe ſtars which do not lie in the ecliptic would have their latitudes altered; thoſe would be increaſed, towards which the ſun was moving, and thoſe diminiſhed, from which it was receding. The effect will be leſs in proportion as the diſtance of the ſtar is greater, and as it is nearer to *A* and *B* in angular diſtance. Theſe would be the appearances, if the ſtars FIG. 177.

them-

themſelves were at reſt; but if any of them be in motion, theſe effects will be altered according to their motion compared with the motion of our ſun. Some of them therefore from their own proper motions might deſtroy, or more than counteract the effects ariſing from the motion of the ſun, and appear to have motions contrary to what is here deſcribed. Like effects will be produced, if our ſyſtem move in any direction out of the ecliptic. Hence, in whatever direction our ſyſtem ſhould move, it would be very eaſy to find what effect of latitude and longitude would have taken place upon any ſtar by means of a celeſtial globe, by conceiving the ſun to move from the center upon any radius directed to the point to which the ſun is moving. Dr. HERSCHEL deſcribes the effect thus. Let an arc of 90° be applied to the ſurface of a globe, and always paſſing through that point to which the motion of the ſyſtem is directed. Then whilſt one end moves along the equator, the other will deſcribe a curve paſſing through it's pole and returning into itſelf; and the ſtars in the northern hemiſphere, within this curve, will appear to move to the north; and the the reſt will go to the ſouth. A ſimilar curve may be deſcribed in the ſouthern hemiſphere, and like appearances will take place.

727. Now Dr. HERSCHEL firſt takes the ſeven ſtars before mentioned, whoſe proper motions had been determined by Dr. MASKELYNE, and he finds, that if the point *A* be aſſumed about the 77° of right aſcenſion, and the ſun to move from *S* to *C*, that it will account for all the motions in right aſcenſion. And if inſtead of ſuppoſing the ſun to move in the plane of the equator, it ſhould aſcend to a point near to λ *Herculis*, it will account for the obſerved change of declination of *Sirius* and *Arcturus*. In reſpect to the *quantity* of motion of each, that muſt depend upon their unknown relative diſtances; he only ſpeaks here of the *directions* of the motions.

728. He next takes twelve ſtars from the catalogue of 56, whoſe proper motions have been determined from a compariſion of the obſervations of ROËMER and MAYER, and adds to them *Regulus* and *Caſtor*; theſe have all a proper motion in right aſcenſion and declination, except Regulus, which has none in declination. Of theſe 27 motions, the above ſuppoſed motion of the ſolar ſyſtem will ſatisfy 22. There are alſo ſome remarkable circumſtances in the *quantities* of theſe motions. *Arcturus* and *Sirius* being the largeſt, and therefore probably the neareſt, ought to have the greateſt apparent motion, and ſo we find they have. Alſo Arcturus is better ſituated to have a motion in right aſcenſion, and it has the greateſt motion. Several other agreements of the ſame kind are found alſo to take place. But there is a very remarkable circumſtance in reſpect to Caſtor. Caſtor is a double ſtar; now how extraordinary muſt appear the concurrence, that two ſuch ſtars ſhould both have a proper motion ſo exactly alike, that they have never been found to vary a

ſingle

ſingle ſecond! This ſeems to point out the common cauſe, the motion of the ſolar ſyſtem.

729. Dr. HERSCHEL next takes 32 more of the ſame catalogue of 56 ſtars, and ſhows that their motions agree very well with his ſuppoſed motion of the ſolar ſyſtem. But the motions of the other 12 ſtars cannot be accounted for upon this hypotheſis. In theſe therefore he ſuppoſes the effect of the ſolar motion has been deſtroyed and counteracted by their own proper motions. The ſame may be ſaid of 19 ſtars out of the 32, which only agrees with the ſolar motion one way, and are, as to ſenſe, at reſt the other. According to the rules of philoſophizing therefore, which direct us to refer all phænomena to as few and ſimple principles as are ſufficient to explain them, Dr. HERSCHEL thinks we ought to admit the motion of the ſolar ſyſtem. Perphaps, however, this argument cannot be properly applied here, becauſe, there is no new cauſe or principle introduced by ſuppoſing each ſtar to have a proper motion. Admitting the doctrine of univerſal gravitation, the fixed ſtars ought to move as well as the ſun. But the ſun's motion, as here eſtimated, cannot be owing to the action of a body upon it which might give it a rotatory motion at the ſame time, as M. de la LANDE conjectures; becauſe a body acting on the ſun to give it it's rotation about it's axis, would not, at the ſame time, give it that progreſſive motion. See Dr. HERSCHEL's Account in the *Phil. Tranſ.* 1783.

730. But it will be proper to conſider how far this motion of the ſolar ſyſtem agrees with the proper motion of the 35 ſtars determined by Dr. MASKELYNE. Now upon ſuppoſition that the ſun moves, as conjectured by Dr. HERSCHEL, that motion will account for the motion of 20 of them, ſo far as regards their direction; but the motion of the other 15 is contrary to that which ought to ariſe from this ſuppoſition. As ſome of the ſtars muſt have a proper motion of their own, even upon the hypotheſis of a ſolar motion, and which probably ariſes from their mutual attraction, it is very probable that they have all a proper motion from the ſame cauſe, but moſt of them ſo very ſmall as not yet to have been diſcovered. And it might alſo happen, that ſuch a motion might be the ſame as that which would ariſe from the motion of the ſolar ſyſtem. Yet it muſt be confeſſed, that the circumſtance of *Caſtor*, and the motions both in right aſcenſion and declination of many of the ſtars being ſuch as ariſe from this hypotheſis, with the apparent motion being greateſt of thoſe ſtars which are probably neareſt, form a ſtrong argument in it's favour.

On the Zodiacal Light.

731. The *Zodiacal Light* is a pyramid of light which ſometimes appears in the morning before ſun riſe. It has the ſun for it's baſis, and in appearance reſembles the *Aurora Borealis*. It's ſides are not ſtraight, but a little curved, it's figure reſembling a lens ſeen edge ways. It is generally ſeen here about October and March, that being the time of our ſhorteſt twilight; for it cannot be ſeen in the twilight; and when the twilight laſts a conſiderable time, it is withdrawn before the twilight ends. It was obſerved by M. CASSINI, in 1683, a little before the vernal equinox, in the evening, extending along the ecliptic from the ſun. He thinks however that it has appeared formerly, and afterwards diſappeared, from an obſervation of Mr. J. CHILDREY, in a book publiſhed in 1661, entitled, *Britannia Baconica*. He ſays, that "in the month of February, for ſeveral years, about ſix o'clock in the evening, after twilight, he ſaw a path of light tending from the twilight towards the *Pleiades*, as it were touching them. This is to be ſeen whenever the weather is clear, but beſt when the moon does not ſhine. I believe this phænomenon has been formerly, and will hereafter appear always at the abovementioned time of the year. But the cauſe and nature of it I cannot gueſs at, and therefore leave it to the enquiry of poſterity." From this deſcription, there can be no doubt but that this was the zodiacal light. He ſuſpects alſo, that this is what the ancients called *Trabes*, which word they uſed for a meteor, or impreſſion in the air like a beam. PLINY, lib. II. p. 26, ſays, *Emicant Trabes, quos docos vocant.* Des CARTES alſo ſpeaks of a phænomenon of the ſame kind. M. FATIO de DUILLIER obſerved it immediately after the diſcovery by M. CASSINI, and ſuſpected that it has always appeared. It was ſoon after obſerved by M. KIRCH and EIMMART in Germany. In the year 1707, on April 3, it was obſerved by Mr. DERHAM in Eſſex. It appeared in the weſtern part of the heavens, about a quarter of an hour after ſun ſet, in the form of a pyramid, perpendicular to the horizon. The baſe of this pyramid he judged to be the ſun. It's vertex reached 15° or 20° above the horizon. It was throughout of a duſky red colour, and at firſt appeared pretty vivid and ſtrong, but fainteſt at the top. It grew fainter by degrees, and vaniſhed about an hour after ſun ſet. This ſolar atmoſphere has alſo been ſeen about the ſun in a total ſolar eclipſe, a luminous ring appearing about the moon at the time when the eclipſe was total.

FIG. 178. 732. Let *HOR* be the horizon, *S* the ſun 18° below at the end of twilight, then will *AIO* repreſent the appearance and poſition of the zodiacal light ſeen at

at Paris on the laſt day of February, and *zge* will repreſent the ſame the next morning before the beginning of twilight, the ſun being at *S'*, as determined by M. de MAIRAN in his treatiſe *De l'Aurore Boreale*. The diſtance *SA* was then about 90°, and *IO* about 20°. The axis *AZ*, *az* coincide with the ſun's equator, and therefore makes an angle of about $7\frac{1}{3}°$ with the ecliptic. Therefore as the angle which the ecliptic makes with the horizon changes at different times of the day, the angle which the axis of this light makes with the horizon will alſo be variable. Hence, if we determine the angle which the ecliptic makes with the horizon at any time, it will give us the poſition. If we ſet a celeſtial globe to the hour, it will ſhow us it's poſition, and through what ſtars it will paſs, which will therefore direct us very accurately where to look for it. Hence it will be moſt viſible, cæteris paribus, when the ecliptic makes the greateſt angle with the horizon. On October 6, 1684, M. FATIO perceived the point *A* diſtinctly terminated, the angle of which was $26\frac{1}{2}°$. M. EIMMART obſerved the ſame on January 13, 1694. In 1683, when M. CASSINI firſt obſerved it, *SA* was 50° or 60°, and *IO* about 8° or 9°. In 1686 and 1687, *SA* extended from 90° to 103°, and *IO* was about 20°. On January 6, 1688, *SA* did not appear to be above 45°, but the horizon was then filled with fogs, and *Venus* ſhone very bright. The appearance therefore depends upon the ſtate of the atmoſphere, and ſituation of the planets, which may produce light enough partly to obſcure it. *IO* has ſometimes been extended to 30°. M. PINGRE', in the torrid zone, has obſerved *SA* to be 120°. The thickneſs *IO* ought to appear different at different times of the year, becauſe the earth will be in a different ſituation in reſpect to it's edge.

733. Let *ABC* be a ſection of the zodiacal light perpendicular to it's axis, *T* the earth, and *TA* a line drawn to the higheſt point above the horizon; now the angle *STA* having been obſerved greater than 90°, *ST* muſt be leſs than *SA*, or the light muſt extend to a diſtance from the ſun, greater than the earth's diſtance. Hence when the earth is about the nodes of this light, or the points where the plane *ABC* interſects the ecliptic, it will be immerſed in this zodiacal light, or, as it is alſo called, the ſolar atmoſphere. M. de MAIRAN thinks the *Aurora Borealis* depends upon this. FIG. 179.

734. M. FATIO conjectured, that this appearance ariſes from a collection of corpuſcles encompaſſing the ſun in the form of a lens, reflecting the light of the ſun. M. CASSINI ſuppoſed that it might ariſe from an infinite number of planets revolving about the ſun; ſo that this light might owe it's exiſtence to theſe bodies, as the milky way does to an innumerable number of fixed ſtars. It is now however generally ſuppoſed, that it is matter detached from the ſun by it's rotation about it's axis. The velocity of the equatorial parts of the ſun being the greateſt, would throw the matter to the greateſt diſtance,

and on account of the diminution of velocity towards it's poles, the height to which the matter would there riſe would be diminiſhed; and as it would probably ſpread a little ſideways, it would form an atmoſphere about the ſun ſomething in the form of a lens, whoſe ſection perpendicular to it's axis would coincide with the ſun's equator. And this agrees very well with obſervation. There is however a difficulty in thus accounting for this phænomenon. It is very well known, that the centrifugal force of a point of the ſun's equator is a great many times leſs than it's gravity. It does not appear therefore, how the ſun, from it's rotation, can detach any of it's groſs particles. If they be particles detached from the ſun, they muſt be ſent off by ſome other unknown force; and in that caſe they might be ſent off equally in all directions, which would not agree with the obſerved figure. The cauſe is probably owing to the ſun's rotation, although not immediately to the centrifugal force ariſing therefrom.

CHAP.

CHAP. XXVIII.

ON THE LONGITUDE OF PLACES UPON THE SURFACE OF THE EARTH.

Art. 735. THE ſituation of any place upon the earth's ſurface is determined from it's latitude and longitude. The latitude may be found from the meridian altitude of the ſun, or a known fixed ſtar; from two altitudes of the ſun, and the time between; and by a variety of other methods. Theſe operations are ſo eaſy in practice, and opportunities are ſo continually offering themſelves, that the latitude of a place may generally be determined, even under the moſt unfavourable circumſtances, to a degree of accuracy ſufficient for all nautical purpoſes. But the longitude cannot be ſo readily found. PHILIP III. King of Spain, was the firſt perſon who offered a reward for it's diſcovery; and the States of Holland ſoon after followed his example, they being at that time rivals to Spain, as a maritime power. During the minority of LEWIS XV. of France, the regent power promiſed a great reward to any perſon who ſhould diſcover the longitude at ſea. In the time of CHARLES II. about 1675, the Sieur de St. PIERRE, a Frenchman, propoſed a method of finding the longitude by the moon. Upon this, a commiſſion was granted to Lord Viſcount BROUNKER, preſident of the Royal Society, Mr. FLAMSTEAD, and ſeveral others, to receive his propoſals, and give their opinion reſpecting it. Mr. FLAMSTEAD gave his opinion, that if we had Tables of the places of the fixed ſtars, and of the moon's motions, we might find the longitude, but not by the method propoſed by the Sieur de St. PIERRE. Upon this, Mr. FLAMSTEAD was appointed Aſtronomer Royal, and an Obſervatory was built at Greenwich for him; and the inſtructions to him and his ſucceſſors were, "that they ſhould apply themſelves with the utmoſt care and diligence to rectify the Tables of the motions of the heavens, and the places of the fixed ſtars, in order to find out the ſo much deſired longitude at ſea, for the perfecting of the art of navigation."

736. In the year 1714, the Britiſh Parliament offered a reward for the diſcovery of the longitude; the ſum of £.10000, if the method determined the longitude to 1 degree of a great circle, or 60 geographical miles; of £.15000, if it determined it to 40 miles; and of £.20000, if it determined it to 30 miles, with this proviſo, that if any ſuch method extend no further than to 80 miles adjoining to the coaſt, the propoſer ſhall have no more than half ſuch

rewards.* The Act alſo appoints the Firſt Lord of the Admiralty, the Speaker of the Houſe of Commons, the Firſt Commiſſioner of Trade, the Admirals of the Red, White, and Blue Squadrons, the Maſter of the Trinity Houſe, the Preſident of the Royal Society, the Royal Aſtronomer at Greenwich, the two Savilian Profeſſors at Oxford, and the Lucaſian and Plumian Profeſſors at Cambridge, with ſeveral other perſons, as Commiſſioners for the Longitude at Sea. The Lowndian Profeſſor at Cambridge was afterwards added. After this Act of Parliament, ſeveral other Acts paſſed in the reigns of GEORGE II. and III. for the encouragement of finding the longitude. At laſt, in the year 1774, an Act paſſed, repealing all other Acts, and offering ſeparate rewards to any perſon who ſhall diſcover the longitude, either by the lunar method, or by a watch keeping true time, within certain limits, or by any other method. The Act propoſes as a reward for a time-keeper, the ſum of £.5000, if it determine the longitude to one degree, or 60 geographical miles; the ſum of £.7500, if it determine the ſame to 40 miles; and the ſum of £.10000, it it determine the ſame to 30 miles, after proper trials ſpecified in the Act. If the method be by improved ſolar and lunar Tables, conſtructed upon Sir I. NEWTON's theory of gravitation, the author ſhall be intitled to £.5000, if ſuch Tables ſhall ſhow the diſtance of the moon from the ſun and ſtars within fifteen ſeconds of a degree, anſwering to about ſeven minutes of longitude, after making an allowance of half a degree for the errors of obſervation. And for any other method, the ſame rewards are offered as thoſe for the time-keeper, provided it gives the longitude true within the ſame limits, and be practicable at ſea. The commiſſioners have alſo a power of giving ſmaller rewards, as they ſhall judge proper, to any one who ſhall make any diſcovery for finding the longitude at ſea, though not within the above limits. Provided however, that if ſuch perſon or perſons ſhall afterwards make any further diſcovery as to come within the above-mentioned limits, ſuch ſum or ſums as they may have received, ſhall be conſidered as part of ſuch greater reward, and deducted therefrom accordingly.

737. After the deceaſe of Mr. FLAMSTEAD, Dr. HALLEY, who was appointed to ſucceed him, made a ſeries of obſervations on the moon's tranſit over the meridian, for a complete revolution of the moon's apogee, which obſervations being compared with the places computed from the Tables then extant, he was enabled to correct the Tables of the moon's motion. And as Mr. HADLEY had then invented an inſtrument by which altitudes could be taken at ſea, and alſo the moon's diſtance from the ſun or a fixed ſtar, Dr. HALLEY

* See WHISTON's Account of the proceedings to obtain this Act, in the Preface to his *Longitude diſcovered by Jupiter's Planets.*

HALLEY ſtrongly recommended the method of finding the longitude from ſuch obſervations,* having found from experience the impracticability of all other methods, particularly at ſea.

738. To diſcover the longitude of any place from Greenwich, we muſt be able to aſcertain the time at that place, and compare it with the time at the ſame inſtant at Greenwich. The methods which have been propoſed to effect this are—By the moon's diſtance from the ſun or a fixed ſtar—By the moon's tranſit over the meridian compared with that of a fixed ſtar—By the occultation of a fixed ſtar by the moon—By a ſolar eclipſe—By a time-keeper—And by an eclipſe of the moon, or of *Jupiter's* ſatellites.

By the Moon's Diſtance from the Sun or a Fixed Star.

739. Dr. MASKELYNE, our preſent Aſtronomer Royal, in his two voyages, one to St. Helena, and the other to Barbadoes, proved the utility of this method of finding the longitude at ſea; and which he very fully explained in a Treatiſe, entitled, *The Britiſh Mariner's Guide*. But the great labour and nicety of the calculations ſeemed to be a material objection to it; particularly the calculation of the moon's latitude and longitude, which are neceſſary to compute it's diſtance from the ſun or a fixed ſtar. To facilitate this, and many other parts of the computation, Dr. MASKELYNE propoſed the publication of the *Nautical Almanac*, in which, amongſt a great many other things, the moon's true diſtance from the ſun or proper fixed ſtar is put down for every three hours; ſo that it's diſtance at any other time may be found by only one proportion. Another requiſite was, an eaſy practical rule for finding the true diſtance of the moon from the ſun or a ſtar from their apparent diſtance and altitudes. Dr. MASKELYNE gave a practical method of doing this, in the above-mentioned Work, and afterwards he improved it. The firſt *Nautical Almanac* was publiſhed in 1767, in which are given two other methods of finding the moon's true diſtance from the ſun or ſtar from their obſerved diſtance,

* The idea of finding the longitude by the moon, was firſt thought of by JOHN WERNER of Nuremberg, in 1514; it was afterwards recommended by PETER APIAN, in 1524; and by ORONCE FINE, and GEMMA FRISIUS; the latter of which propoſed to find the place of the moon at any time, by obſerving it's diſtance from a fixed ſtar, and then to calculate the time when the moon ought to be at that diſtance, by which you will have the difference of the meridians of the place of obſervation and the place for which the calculation was made. KEPLER alſo mentioned this as an excellent method of finding the longitude, and after him LONGOMONTANUS. But without correct Tables of the moon's motions, and proper inſtruments to meaſure it's diſtance from a fixed ſtar, this method could not be put in practice

diſtance, one by Mr. LYONS and the other by Mr. DUNTHORNE. In the Requiſite Tables theſe two methods are improved. Another method is alſo given by Mr. WITCHELL in that Work. Various other methods have been alſo given. For the ſame purpoſe, a ſet of Tables were publiſhed by order of the Board of Longitude, containing the corrections for refraction and parallax to every degree of the moon's diſtance from the ſun or a fixed ſtar, and for every degree of altitude of each, under the care of Dr. SHEPHERD, the late Plumian Profeſſor of Aſtronomy and Experimental Philoſophy, at Cambridge. They were computed by Mr. LYONS, Mr. PARKINSON, and Mr. WILLIAMS. The objection to the direct method of ſolving this problem was, partly from the length of the operation, and partly from the tedioufneſs of proportioning to find the logarithms to ſeconds. But ſince the publication of Mr. TAYLOR's Logarithms, this latter objection is taken away.

740. The ſteps by which we find the longitude by this method, are theſe;

From the obſerved altitudes of the moon and the ſun or a fixed ſtar, and their obſerved diſtance, compute the moon's true diſtance from the ſun or ſtar.

From the *Nautical Almanac* find the time at Greeenwich when the moon was at that diſtance.

From the altitude of the ſun or ſtar, find the time at the place of obſervation.

The difference of the times thus found, gives the difference of the longitudes.

741. To find the true diſtance of the moon from the ſun or ſtar by obſervation, let Z be the zenith, S the apparent place of the ſun or a ſtar, s the true place, M the apparent place of the moon, m it's true place; then in the triangle ZSM, we know SM the apparent diſtance, SZ, SM the complements of the apparent altitudes, to find the angle Z; and then in the triangle sZm, we know the angle Z, and sZ, mZ the complements of the true altitudes, to find sm the true diſtance. But the problem may be otherwiſe ſolved thus.

FIG. 180.

742. By ſpherical Trigonometry, ver. ſin. $\angle Z = r^2 \times \frac{\text{cof. } \overline{ZS - ZM} - \text{cof. } SM}{\text{ſin. } ZS \times \text{ſin. } ZM}$

$= r^2 \times \frac{\text{cof. } \overline{Zs - Zm} - \text{cof. } sm}{\text{ſin. } Zs \times \text{ſin. } Zm}$; but if $\frac{1}{2} SM + \frac{1}{2} . \overline{ZS - ZM} = A$, $\frac{1}{2} SM - \frac{1}{2} . \overline{ZS - ZM} = B$, then by plane Trigonometry, cof. $\overline{ZS - ZM}$ − cof. $SM =$

2 ſin.

$\frac{2 \text{ sin. } A \times \text{sin. } B}{r}$; hence, cos. $sm = \text{cos. } \overline{Zs - Zm} - \frac{2 \text{ sin. } A \times \text{sin. } B}{r} \times \frac{\text{sin. } Zm}{\text{sin. } ZM} \times \frac{\text{sin. } Zs}{\text{sin. } ZS}$. Now the ninth of the Requiſite Tables gives the arithmetic complement of the difference between the logarithmic ſines of ZM and Zm, increaſed by 120; for at all altitudes above 25°, this number is uniformly the difference between the logarithmic ſines of Zs and ZS for all celeſtial objects not affected by parallax. At altitudes leſs than 25° this uniformity ceaſes, and the difference between the ſines is leſs than 120 by the numbers in Table XI. for a ſtar. But for the ſun, which is ſenſibly affected by parallax, the difference between the ſines is leſs than 120 by the numbers in Table X. In theſe caſes therefore the logarithm in Table IX. muſt be diminiſhed by the numbers contained in Tables X, or XI. Hence we have the following Rule.

To log. 2 add the log. ſines of A and B, alſo the log. from the ninth of the Requiſite Tables, corrected if neceſſary by Table X, or XI. and reject 20 from the index, and find the natural number correſponding, the difference between which and the natural coſine of the difference of the true zenith diſtances, gives the natural coſine of the true diſtance required.

Ex. Suppoſe, on June 29, 1793, the ſun's apparent zenith diſtance ZS was obſerved to be 70°. 56′. 24″, the moon's apparent zenith diſtance ZM to be 48°. 53′. 58″, their apparent diſtance SM to be 103°. 29′. 27″, and the moon's horizontal parallax to be 58′. 35″; to find their true diſtance sm.

By Requiſite Table VIII. the correction Mm for the moon's parallax and refraction is 43′. 3″, and by Table I. and III. the correction Ss for the ſun's parallax and refraction is 2′. 36″; hence $Zm = 48°. 10′. 55″$, and $Zs = 70°. 59′$.

	$ZS - ZM$	= 22°. 2′. 26″
Therefore,	$\frac{1}{2}$. $ZS - ZM$ . .	= 11. 1. 13
	$\frac{1}{2}$ SM	= 51. 44. 43
Therefore,	A	= 62. 45. 56
	B	= 40. 43. 30

Alſo

Alſo Zs . . . . . = 70°. 59′. 0″
Zm . . . . . = 48. 10. 55

Therefore $Zs - Zm$ . . . . . = 22. 48. 5

Hence we have the following operation, according to the rule, to find the true diſtance of the ſun from the moon.

Log. 2			0,3010300
Log. ſin. A			9,9489708
Log. ſin. B			9,8145333
Log. from Req. Tab. IX . . .	9,995314	}	9,9953070
——— ——— ——— X. . . .	−7	}	
		10,059841	Nat. Numb. 1147741
Natural coſine 22°. 48′. 5″			921854
Natural coſine 103°. 3′. 18″ = sm			225887

The radius to the Table of natural ſines and coſines to ſix figures is 1000000, and the index to the log. for the radius in the Tables of log. ſines, coſines, &c. is 10; in this caſe therefore, an index 10 points out 7 places of whole numbers, and conſequently an index 9 points out 6 places, &c. When the natural coſine of $Zs - Zm$ is leſs than the natural number ſtanding above it, the difference gives the natural coſine of an arc above 90°, as in this caſe; otherwiſe the arc is below 90°. In this method there is no diſtinction of caſes, and it only requires three logarithms and one natural coſine to be taken correſponding to a given angle, one natural number correſponding to a logarithm, and an arc correſponding to a natural coſine; Mr. DUNTHORNE's method was by natural coſines, and required only the ſame number of quantities to be taken; but Dr. MASKELYNE has deduced from it the following method of computing by logarithms only.

743. By the laſt Article, coſ. ms = coſ. $\overline{Zs - Zm}$ − coſ. $\overline{\overline{ZS - ZM}}$ − coſ. SM × $\frac{\text{ſin. } Zm}{\text{ſin. } ZM} \times \frac{\text{ſin. } Zs}{\text{ſin. } ZS}$; put $H = Zs - Zm$, $h = ZS - ZM$, $\mathfrak{Q} = \frac{\text{ſin. } Zm}{\text{ſin. } ZM} \times \frac{\text{ſin. } Zs}{\text{ſin. } ZS}$, $A = \overline{\text{coſ. } h - \text{coſ. } SM} \times \mathfrak{Q}$; then coſ. ms = coſ. $H - A$. Now

coſ.

$\overline{\text{cos. } \frac{1}{2} ms}^2 = \frac{1}{2} + \frac{1}{2} \text{ cos. } ms = \frac{1}{2} + \frac{1}{2} \text{ cos. } H - \frac{1}{2} A$; let $\frac{1}{2} A$ be the ſquare of the ſine of an arc $\frac{1}{2} B$, then $\overline{\text{ſin. } \frac{1}{2} B}^2 = \frac{1}{2} . \overline{\text{cos. } h - \text{cos. } SM} \times \mathcal{Q} = \text{ſin. } \overline{\frac{1}{2} SM + \frac{1}{2} h} \times \text{ſin. } \overline{\frac{1}{2} SM - \frac{1}{2} h} \times \mathcal{Q}$, hence the arc B is known. But $\frac{1}{2} \text{ cos. } B = \frac{1}{2} - \overline{\text{ſin. } \frac{1}{2} B}^2 = \frac{1}{2} - \frac{1}{2} A$; hence $\overline{\text{cos. } \frac{1}{2} ms}^2 = \frac{1}{2} \text{ cos. } B + \frac{1}{2} \text{ cos. } H = \text{cos. } \overline{\frac{1}{2} B + \frac{1}{2} H} \times \text{cos. } \overline{\frac{1}{2} B - \frac{1}{2} H}$. Hence we have the following Rule.

Add together, log. ſine of $\overline{\frac{1}{2}$ obſ. diſt. $+ \frac{1}{2}$ diff. of app. alt^{s}. log. ſine of $\frac{1}{2}$ obſ. diſt. $- \frac{1}{2}$ diff. of app. alt$^{s}}$. and arith. comp. of $\mathcal{Q}$ taken from Requiſite Tables IX. and X. or XI. as the caſe may require, and ſubtract 10 from the index, divide the ſum by 2, and you have the ſine of $\frac{1}{2} B$.

Add log. coſ. $\overline{\frac{1}{2} B + \frac{1}{2}$ diff. of true alt^{s}. to log. coſ. $\frac{1}{2} B - \frac{1}{2}$ diff. of true alt$^{s}}$. take half this ſum, and you get the log. coſine of half the true diſtance.

To apply this to the laſt Example, we have,

$\frac{1}{2}$ Obſ. diſt.	62°. 45′. 56″$\frac{1}{2}$		
$\frac{1}{2}.\overline{ZS - ZM}$	11. 1. 13		
$\frac{1}{2}$ Sum .	62. 45. 56 $\frac{1}{2}$	log. ſine	9,9489713
$\frac{1}{2}$ Diff. .	40. 43. 40 $\frac{1}{2}$	log. ſine	9,8145346
	Log. from Tab. IX. & X.		9,9953070
			2)19,7588129
$\frac{1}{2} B =$ . .	49°. 14′. 52″$\frac{1}{2}$	log. ſin.	9,8794064
$\frac{1}{2}.\overline{Zs - Zm}$	11. 24. 2 $\frac{1}{2}$		
Sum . .	60. 38. 55	coſ.	9,6903418
Difference	37. 50. 50	coſ.	9,8974344
			2)19,5877762
$\frac{1}{2}.sm =$.	51. 31. 39	coſ.	9,7938881

Hence the true diſtance is 103°. 3′. 18″.

As we have now logarithmic Tables to every ſecond of the quadrant, this is a conſiderable improvement upon Mr. DUNTHORNE's rule. There is alſo no diſtinction of caſes in this, which there is in Mr. DUNTHORNE's method. As we deduce, by this rule, half the true diſtance, it is manifeſt, that any error in the ſeconds will be doubled in the true diſtance; upon that account we were obliged to take in the half ſeconds, for if we had not, the half diſtance would have come out 51°. 31′. 38″, and conſequently the true diſtance would have been found 103°. 3′. 16″. This is a circumſtance very neceſſary to be attended to in all the rules that firſt give half the true diſtance.

This laſt Rule may be applied without the Requiſite Tables, by conſidering, that the logarithms taken from Tables IX, X, or XI. in that Work, give the arithm. complem. of $\frac{\text{fin. } m}{\text{fin } ZM} \times \frac{\text{fin. } Zs}{\text{fin. } ZS}$, which quantity may be taken from the logarithmic Tables, by adding to the log. ſines of Zm, Zs, the arith. comp. of the log. ſines of ZM and ZS, and ſubtracting 10 from the index. If we apply this to the laſt Example, we have,

Zm = 48°. 10′. 55″ . . .	fin. 9,8723113	
ZM = 48. 53. 58	0,1228839	arith. comp. of ſine.
Zs = 70. 59. 0 . . .	fin. 9,9756265	
ZS = 70. 56. 24	0,0244868	arith. comp. of ſine.
	9,9953085	

This differs a little from the number taken from the Requiſite Tables, which gives only ſix figures. It would indeed lengthen the work a little to take this quantity from the logarithmic Tables, but it would add to the accuracy. But the moſt complete Rule is that given by Dr. MASKELYNE, in his Preface to the Tables of Logarithms by Mr. TAYLOR, in which thoſe Tables only are requiſite; and it is certainly beſt to uſe as few auxiliary Tables as poſſible, as, by that means, you ſubject the operation to fewer probable errors.

744. Dr. MASKELYNE's Rule for clearing the moon's apparent diſtance from a ſtar or the ſun from the effect of parallax and refraction.

I. To the log. ſine of the moon's horizontal parallax add the log. coſine of the moon's apparent altitude, uſing five decimal places of the logarithms; the ſum, abating 10 from the index, is the log. ſine of the moon's parallax in altitude, from which ſubtract the moon's refraction taken with the moon's apparent altitude out of Table I. (Requiſite Tables) and you will have the

the correction of the moon's altitude. Add this to the moon's apparent altitude, and you will have the moon's true altitude. Alſo with the ſtar's apparent altitude take the ſtar's refraction out of Table I. which ſubtract from the ſtar's apparent altitude, and you will have the ſtar's true altitude. But if the moon's diſtance was obſerved from the ſun, with the ſun's apparent altitude take the refraction out of Table I. and it's parallax out of Table III. and take the difference, and ſubtract it from the ſun's apparent altitude, and you will have the ſun's true altitude. Take the difference of the true altitudes of the moon and ſtar, or moon and ſun, and the difference of their apparent altitudes.

II. Take half the ſum and half the difference of the apparent diſtance and difference of the apparent altitudes.

III. To the log. ſines of the above half ſum and half difference add the log. coſines of the true altitudes, and the arithmetical complements of the log. coſines of the apparent altitudes; and take half the ſum.

IV. From this half ſum take the log. ſine of half the difference of the true altitudes, and look for the remainder among the tangents, and take out the correſponding log. coſine, without taking out the arc, which is unneceſſary.

V. Subtract the ſaid log. coſine from the log. ſine of half the difference of the true altitudes increaſed by 10 in the index; the remainder will be the log. ſine of half the true diſtance.

DEMONSTRATION. Put $ZS-ZM=X$, $Zs-Zm=x$, $D=SM$, $d=sm$; now by Art. 742. ſin. $ZS \times$ ſin. ZM : ſin. $Zs \times$ ſin. Zm :: coſ. $X-$ coſ. D : coſ. $x-$ coſ. d, or ver. ſin. $d-$ ver. ſin. x; but by plane Trigonometry, coſ. $X-$ coſ. $D=2 \times$ ſin. $\frac{D+X}{2} \times$ ſin. $\frac{D-X}{2}$, and ver. ſin. $d-$ ver. ſin. $x=2.\overline{\text{ſin. } \frac{1}{2}d}^2-2\times\overline{\text{ſin. } \frac{1}{2}x}^2$; hence ſin. $ZS \times$ ſin. ZM : ſin. $Zs \times$ ſin. Zm :: ſin. $\frac{D+X}{2} \times$ ſin. $\frac{D-X}{2}$: $\overline{\text{ſin. } \frac{1}{2}d}^2-\overline{\text{ſin. } \frac{1}{2}x}^2$, conſequently

$$\frac{\text{ſin. } Zs \times \text{ſin. } Zm \times \text{ſin. } \frac{D+X}{2} \times \text{ſin. } \frac{D-X}{2}}{\text{ſin. } ZS \times \text{ſin. } ZM \times \overline{\text{ſin. } \frac{1}{2}x}^2} = \frac{\overline{\text{ſin. } \frac{1}{2}d}^2}{\overline{\text{ſin. } \frac{1}{2}x}^2} - 1,$$ which put $=$

$\overline{\text{tan. } a}^2$; hence tan. $a = \frac{\sqrt{\text{ſin. } Zs \times \text{ſin. } Zm \times \text{ſin. } \frac{D+X}{2} \times \text{ſin. } \frac{D-X}{2}}}{\sqrt{\text{ſin. } ZS \times \text{ſin. } ZM} \times \text{ſin. } \frac{1}{2}x}$; but

ſin. $\overline{\frac{1}{2}d}^2=\overline{\text{ſin. } \frac{1}{2}x}^2 \times 1+\overline{\text{tan. } a}^2=\overline{\text{ſin. } \frac{1}{2}x}^2 \times \overline{\text{ſec. } a}^2$; conſequently ſin. $\frac{1}{2}d=$ ſin. $\frac{1}{2}x \times$ ſec. $a=\frac{\text{ſin. } \frac{1}{2}x}{\text{coſ. } a}$.

FIG. 180.

EXAMPLE I.

Suppose the apparent altitude of ✳ to be 24°. 48′, that of ☽ 's center 12°. 30′, and the apparent distance of ✳ from ☽ 's center 51°. 28′. 35″, and ☽ 's hor. par. 56′. 15″; required the true distance of ✳ from ☽ 's center, cleared of refraction and parallax.

☽ 's apparent altitude . .	12°. 30′. 0″	Log. cosine . .	9,98958
☽ 's horizontal parallax . .	56. 15	Log. sine . . .	8,21383
☽ 's parallax in altitude . .	54. 55	Log. sine . . .	8,20341
☽ 's refract. from Req. Tab. I.	− 4. 13		
Correction of ☽ 's altitude .	+ 50. 42		
☽ 's apparent altitude . .	12. 30. 0		
☽ 's true altitude	13. 29. 42		
✳ 's apparent altitude . .	24. 48. 0		
Refraction from Table I. .	− 2. 3		
✳ 's true altitude	24. 45. 57		
☽ 's true altitude	13. 20. 42		
Diff. of true alt^s. of ☽ and ✳	11. 25. 15		
✳ 's apparent altitude . .	24. 48. 0		
☽ 's apparent altitude . .	12. 30. 0		
Diff. of app. alt^s. of ☽ and ✳	12. 18. 0		
Apparent distance	51. 28. 35		
Sum	63. 46. 35		
Difference	39. 10. 35		
Half sum	31. 53. 17	Log. sine . . .	9,7228488
Half difference	19. 35. 17	Log. sine . . .	9,5253755
☽ 's apparent altitude . .	12. 30. 0	Co-ar. log. cosine .	0,0104185
☽ 's true altitude	13. 20. 42	Log. cosine . .	9,9881119
✳ 's apparent altitude . .	24. 48. 0	Co-ar. log. cosine .	0,0420206
✳ 's true altitude . . .	24. 45. 57	Log. cosine . .	9,9580990
			2)39,2468743
			19,6234371
Half differ. of true altitudes	5. 42. 37½	Log. sine . . .	8,9978159
		Log. tang. of arc	10,6256212
		Corresp. log. cosine	9,3625337
Half true distance . . .	25. 34. 54½	Log. sine . . .	9,6352822
	2		
True distance	51. 9. 49		

EXAMPLE II.

Let the apparent altitude of ☽'s center be 5°. 17′, *that of* ☉ 84°. 7′, *and their apparent diſtance* 90°. 21′. 13″, *and ☽'s horizontal parallax* 61′. 48″; *required the true diſtance of* ☉ *and ☽.*

☽'s horizontal parallax . .	1°. 1′. 48″	Log. ſine . . .	8,25469
☽'s apparent altitude . .	5. 17. 0	Log. coſine . .	9,99815
☽'s parallax in altitude . .	1. 1. 32	Log. ſine . . .	8,25284
☽'s refraction from Tab. I. .	− 9. 28		
Correct. of ☽'s altitude . .	+ 52. 4		
☽'s apparent altitude . . .	5. 17. 0		
☽'s true altitude	6. 9. 4		
☉'s apparent altitude . .	84. 7. 0		
Diff. of refraction and parallax	− 0. 5		
☉'s true altitude . . .	84. 6. 55		
☽'s true altitude	6. 9. 4		
Diff. of true alt^{s}. of ☉ and ☽	77. 57. 51		
☉'s apparent altitude . .	84. 7. 0		
☽'s apparent altitude . .	5. 17. 0		
Diff. of app. alt^{s}. of ☉ and ☽	78. 50. 0		
Apparent diſtance . . .	90. 21. 13		
Sum	169. 11. 13		
Difference	11. 31. 13		
Half ſum	84. 35. 36	Log. ſine . . .	9,9980635
Half difference	5. 45. 36	Log. ſine . . .	9,0015681
☽'s apparent altitude . . .	5. 17. 0	Co-ar. log. coſine .	0,0018490
☽'s true altitude	6. 9. 4	Log. coſine . .	9,9974924
☉'s apparent altitude . .	84. 7. 0	Co-ar. log. coſine .	0,9892626
☉'s true altitude	84. 6. 55	Log. coſine . .	9,0108395
			2)38,9990751
			19,4995375
½ Diff. of true alt^{s}. of ☽ and ☉	38. 58. 55	Log. ſine . . .	9,7987027
		Log. tang. of arc .	9,7008348
		Correſp. log. coſine	9,9511707
Half true diſtance	44. 44. 36½	Log. ſine . . .	9,8475320
	2		
True diſtance	89. 29. 13		

745. The true distance of the moon from the sun or star being thus found, we are next to find the time at Greenwich. For this purpose, the sun or such fixed stars are chosen, as lie in or very near the moon's way, so that looking upon the moon's motion to be uniform for a small time, the moon may be considered as approaching to, or receding from the sun or star uniformly. To determine therefore the time at Greenwich corresponding to any given true distance of the moon from the sun or star, the true distance is computed in the *Nautical Almanac* for every three hours for the meridian of Greenwich. Hence, considering that distance as varying uniformly, the time corresponding to any other distance may be thus computed. Look into the *Nautical Almanac* and take out two distances, one next greater and the other next less than the true distance deduced from observation, and the difference D of these distances gives the access of the moon to, or recess from the sun or star in three hours; then take the difference d between the moon's distance at the beginning of that interval and the true distance deduced from observation; and then say, $D : d ::$ 3 hours : the time the moon is acceding to, or receding from the sun or star by the quantity d; which added to the time at the beginning of the interval, gives the apparent time at Greenwich, corresponding to the given true distance of the moon from the sun or star. To find the fourth term in the above proportion, there is, in the Requisite Tables, a Table of proportional logarithms, where the log. of 3 hours is made $=0$, and therefore the log. of the fourth term is found by subtraction only. The same Table will serve, if one of the terms be three degrees instead of three hours.

Ex. On June 29, 1793, in latitude 52°. 12′. 35″ the sun's altitude in the morning was by observation 19°. 3′. 36″, the moon's altitude was observed to be 41°. 6′. 2″, the sun's declination at that time was 23°. 14′. 4″, and the moon's horizontal parallax 58°. 35′; to find the apparent time at Greenwich.

True dist. of ☾ from ☉ by Art. 742. . . .	103°.	3′.	18″		
——— by *Naut. Alm.* on June 29, at 3*h*. .	103.	4.	58		
——— at 6*h*. .	101.	26.	42		
	0.	1.	40	pr. log.	2,0334
	1.	38.	16	pr. log.	0,2629
Time of approaching 0°. 1′. 40″	0.	3.	3	pr. log.	1,7705
Beginning of the interval	3^h.	0.	0		
Apparent time at Greenwich, June 29, . . .	3.	3.	3		

746. Now

746. Now to find the apparent time at the place of obſervation, we have the ſun's declination 23°. 14′. 4″, it's altitude 19°. 3′. 36″, it's refraction 2′. 44″, and parallax 8″; hence it's true altitude was 19°. 1′, and therefore it's true zenith diſtance was 70°. 59′; alſo the complement of declination was 66°. 45′. 56″; hence by Art. 92.

66°. 45′. 56″	. . . . .	arith. comp. ſin. 0,0367325
37. 47. 25	. . . . .	arith. comp. ſin. 0,2127004
70. 56. 24		
175. 29. 45		
87. 44. 52	. . . . .	ſin. 9,9996644
70. 56. 24		
16. 48. 28	. . . . .	ſin. 9,4601408
		2)19,7092381
		9,8546190

the coſine of 44°. 18′. 52″, which doubled gives 88°. 37′. 44″ the hour angle from apparent noon, which in time gives 5*h*. 54′. 31″ the time before apparent noon, or 18*h*. 5′. 29″ on June 28. Hence,

Apparent time at place of obſerv.	June 28,	.	18ʰ. 5′. 29″
——— at Greenwich	June 29,	.	3. 3. 3
Difference of meridians in time		. . . . .	8 57. 34

Which converted into degrees gives 123°. 50′. 16″ the longitude of the place of obſervation weſt of Greenwich.

If a ſtar be obſerved, find the time by Art. 106. The ſun's declination is firſt taken from the *Nautical Almanac*, and then corrected by Req. Tab. VI. If a ſtar be obſerved, take it's declination from Requſite Table VII. The longitude being nearly known by account, will be ſufficiently exact to enter Table VI. with.

747. In order to apply this method of finding the longitude, three obſervers are convenient, two to take the altitudes of the moon and ſun or a ſtar, and one to take their diſtance; the latter muſt be taken with great care, as the determination of the true diſtance depends principally upon that, a ſmall error in

in the altitudes not ſenſibly affecting it. If a ſingle obſerver ſhould want to apply this method, he may do it with a very conſiderable degree of accuracy in the following manner. Let him firſt take the altitude of the moon and then of the ſun or ſtar, his aſſiſtant noting the times; then let him take ſeveral diſtances of the moon from the ſun or ſtar at one or two minutes diſtance of time from each other, and note the times; and laſtly, let him again take the altitude of the moon and then of the ſun or ſtar, noting the times. Then taking the mean of all the diſtances, and the mean of the times when they were taken, he will have the moon's diſtance from the ſun or ſtar at that mean time. Take the difference of the moon's altitudes at the two obſervations, and the difference of the times, and then ſay, as that difference of times, is to the difference between the time of the firſt obſervation of it's altitude and the mean of the times at which the diſtances were taken, ſo is the variation of the moon's altitude between the firſt and ſecond obſervations, to the variation of it's altitude from the time of the firſt obſervation to the above mean time, which added to or ſubtracted from it's firſt obſerved altitude, according as the moon aſcends or deſcends, gives it's altitude at that mean time. In the ſame manner he may get the ſun's or ſtar's altitude at the ſame time. Thus he may get the two altitudes and the correſponding diſtance.

748. In general, the altitudes of the ſtars at ſea are too uncertain for finding the time; they may do in a fine ſummer's night, or in twilight; and if the ſun be uſed, it may be ſo near the meridian, or the horizon may be ſo hazy and ill-defined, that the altitude cannot be determined with ſufficient accuracy to deduce the time from it, although it may be ſufficiently exact to calculate their true diſtance. In this caſe, the obſerver muſt be careful to find the error of his watch by ſome altitude taken near to the time of obſervation, by which he may correct the time ſhown by the watch at the time of obſervation. But as, in this caſe, the watch ſhows the time at the meridian under which the altitude of the ſun or ſtar was taken in order to correct it, the longitude thus found is that under which the watch was regulated, and not that where the diſtance of the moon from the ſun or ſtar was obſerved. If the watch cannot be depended upon to keep time tolerably well for a ſmall interval, the error of the watch muſt be found at two obſervations, from which you get it's rate of going; by this means you may determine the time very accurately. If this be done at ſea, the altitude at the ſecond obſervation muſt be reduced to the altitude at that time at the place of the firſt obſervation; the method of doing which is as follows. Let T be the place on the earth to which the ſun was vertical at the firſt obſervation, Z the place of a ſpectator at the firſt obſervation, ZV or ZV' the diſtance run between the obſervations, then TV or TV' would have been the zenith diſtance at the firſt obſervation, if it had been made at the

FIG. 181.

place

place where the ſpectator was when he made the ſecond obſervation; draw VW, $V'W'$ perpendicular to ZT; and then, as the angle T is ſmall, TV is very nearly equal to TW, and TV' to TW', and therefore they may be conſidered as reſpectively equal; hence ZW, ZW' may be conſidered as the difference of the zenith diſtances, *increaſing* the diſtance in the former caſe, and *decreaſing* it in the latter. To find which, obſerve the angle VZT or $V'ZT$ between the ſhip's courſe and the ſun's bearing; and then in the right angled plane * triangles VZW, $V'ZW$,' we know all the angles, and the ſide ZV or ZV,' the obſerved diſtance run by the ſhip, to find ZW or ZW', which muſt be reduced into degrees at the rate of 69½ miles for a degree. Or the ſame thing may be done by the *Traverſe Table*, which is a Table ready calculated to take out theſe quantities at once.

749. The obſerver ſhould be furniſhed with a good HADLEY's *Quadrant* to obſerve the altitudes and diſtance. Great care muſt be taken to examine the error of adjuſtment as near to the time of obſervation as poſſible, as it is very liable to alter. Altitudes ſhould not be taken nearer the horizon than 5° or 6°, on account of the uncertainty of refraction at lower altitudes. The principal obſerver is he who takes the diſtance; and as ſoon as he has completed his obſervation he muſt give notice to the other two obſervers, who ought to complete their obſervations as ſoon as poſſible, at leaſt within a minute. Note the time alſo by the watch when the ſun's or ſtar's altitudes were taken, by which, and the eſtimated longitude at the place of obſervation, you will have nearly the time at Greenwich, which is neceſſary in order to get the ſun's declination at the time of obſervation, in order to compute the time. A full account of the adjuſtments and uſes of HADLEY's *Quadrant*, may be ſeen in my *Practical Aſtronomy*.

750. The accuracy of this method of finding the longitude was eſtabliſhed by Dr. MASKELYNE from his own experience in two voyages, one to St. Helena, and the other to Barbadoes, by the following irrefragable proofs. 1. On the near agreement of the longitude inferred by his obſervations, made within a few days or hours of making land, with the known longitude of ſuch land. 2. From the near agreement of the longitude of the ſhip from obſervations made on a great many different days near to one another, when connected together by the help of the common reckoning. 3. From the near agreement of the longitude of the ſhip, deduced from obſervations of ſtars on different ſides of the moon, taken on the ſame night. For here, all the moſt probable kinds of error, whether ariſing from a faulty diviſion of the limb of the inſtrument, a refraction of the ſpeculums or dark glaſſes, a wrong allowance for

* The triangles may be conſidered as plane, on account of the ſmall diſtance run by the ſhip.

for the error of adjuſtment, or from a bad habit of eſtimating the contact of the ſtar with the moon's limb, operating different ways, their effect, if any, muſt be ſenſible in the reſult. But in all the double longitudes thus determined, the difference was ſo ſmall as to warrant him to ſay, that by good inſtruments and careful obſervers, theſe errors may be ſo far reduced as to be of very little conſequence; and all the obſervations which have been made ſince, agree in confirming it; and ſhow that the longitude thus deduced may be determined to a very great degree of accuracy, and fully ſufficient for all nautical purpoſes.

751. At ſea, the moon and ſun or ſtar's altitude muſt be corrected for the dip of the horizon, by ſubtracting the dip; for the obſerver being on the deck of the ſhip muſt ſee below his own horizon, and the altitudes are taken above his viſible horizon. The following Table gives the dip correſponding to the obſerver's height.

Height	Dip	Height	Dip	Height	Dip
1	0′. 57″	13	3′. 26″	26	4′. 52″
2	1. 21	14	3. 34	28	5 3
3	1. 39	15	3. 42	30	5. 14
4	1. 55	16	3. 49	35	5. 39
5	2. 8	17	3. 56	40	6. 2
6	2. 20	18	4. 3	45	6. 24
7	2. 31	19	4. 10	50	6. 44
8	2. 42	20	4. 16	60	7. 23
9	2. 52	21	4. 22	70	7. 59
10	3. 1	22	4. 28	80	8. 32
11	3. 10	23	4. 34	90	9. 3
12	3. 18	24	4. 40	100	9. 33

752. The moon's true diſtance from the ſun or a fixed ſtar, as put down in the *Nautical Almanac*, is thus calculated. Let Z be the pole of the ecliptic, s the true place of the *ſtar*, m the true place of the *moon*; then Zs, Zm are the complements of latitudes, and the angle Z the difference of their longitudes; draw st perpendicular to Zm; and by ſpher. Trig. log. tan. Zt = log. coſ. Z + log. tan. Zs − 10, and log. coſ. sm = log. coſ. Zs + log. coſ. tm + arith. comp. log. coſ. Zt − 10. If s be the *Sun*, $Zs = 90°$; hence rad. × coſ. sm = coſ. Z × ſin. Zm, or log. coſ. Z + log. ſin. Zm − 10, = log. coſ. sm. If Z be the pole of the equator, then Zs, Zm will be the complements of declination, and the angle Z the difference of right aſcenſions.

Fig. 180.

Ex.

Ex. I. To find the moon's diſtance from the ſun, on Auguſt 24, 1796, at apparent noon at Greenwich.

The ſun's longitude at the given time is $5^s.\ 1°.\ 55'.\ 58''$, the moon's longitude is $1^s.\ 13°.\ 50'.\ 53''$, the difference of which is $108°.\ 5'.\ 5''$ the angle sZm; alſo the moon's latitude is $4°.\ 13'.\ 51''$, hence $Zm = 85°.\ 46'.\ 9''$.

Log. coſ. $Z = 108°.\ 5'.\ 5''$	. . . .	9,4919538
Log. ſin. $Zm = 85.\ 46.\ 9$	. . . .	9,9988149
Log. coſ. $sm = 108.\ 2.\ 1$	. . . .	9,4907687

Ex II. Let the right aſcenſion of a ſtar be $2^s.\ 7°.\ 2'.\ 25''$, and north declination $5°.\ 28'.\ 40''$, alſo the right aſcenſion of the moon $0^s.\ 11°.\ 54'.\ 4''$, and ſouth declination $3°.\ 22'.\ 32''$; to find the moon's diſtance from the ſtar.

In this caſe, Z repreſents the pole of the equator, and the difference of right aſcenſions is $55°.\ 8'.\ 21'' =$ the angle Z, alſo $Zs = 84°.\ 31'.\ 20''$, and $Zm = 93°.\ 22'.\ 32''$; hence

Log. coſ.	. . . . .	$55°.\ 8'.\ 21''$	. . 9,7570809
Log. tan.	. . . . .	$84.\ 31.\ 20$	. . 11,0181922
Log. tan. Zt	. . .	$=80.\ 28.\ 33$	. . 10,7752731
Zm	. . .	$=93.\ 22.\ 32$	
Log. coſ. tm	. . .	$=12.\ 53.\ 59$	. . 9,9888987
Log. coſ. Zs	. . .	$=84.\ 31.\ 20$	. . 8,9798200
Arith. comp. log. coſ. Zt		$=80.\ 28.\ 33$	. . 0,7812975
Log. coſ. sm	. . .	$=55.\ 46.\ 50$	. . 9,7500162

Thus the moon's true diſtance from the ſun or a fixed ſtar may be calculated for every three hours, as given in the *Nautical Almanac*.

To find the Difference of the Longitudes of two Places, by the obſerved Tranſits of the Moon and a fixed Star over the Meridian at each Place.

753. This method of finding the longitude was propoſed by Dr. MASKELYNE in the *Nautical Almanac* for 1769. It is extremely eaſy in practice, and capable of great accuracy. The Rule is thus inveſtigated. FIG. 182. Let P be the pole of the earth $R\mathcal{Q}$, PG a meridian of Greenwich paſſing through the moon at M, PD the meridian of any other place, and when it comes into the ſituation Pd let it paſs through the moon at m. At each tranſit, obſerve the differences MPS, mPS, between the right aſcenſions of the moon and a fixed ſtar S, the difference of which is the angle MPm, or the increaſe of the moon's right aſcenſion in the interval of the tranſits. From the *Nautical Almanac*, find the increaſe (A) of the moon's right aſcenſion in 12 hours apparent time, and reduce it into mean time thus; let a = the variation of the equation of time in 12 hours, then 12 hours apparent time is $12h. \pm a$ of mean time, where the ſign + is uſed if the equation be *increaſing* and *additive*, or *decreaſing* and *ſubtractive*; and the ſign −, when *increaſing* and *ſubtractive*, or *decreaſing* and *additive*. Now $A : Mpm :: 12h. \pm a : x$ the angle (expreſſed in mean time) deſcribed by a meridian of the earth in the time the moon deſcribes Mpm; hence $x \times \frac{366}{365} =$ the angle DPm of longitude deſcribed by a meridian in that time, becauſe in 12 hours mean time the earth revolves through $180° \times \frac{366}{365}$ of longitude, very nearly. Conſequently the difference DPG of the meridians $= x \times \frac{366}{365} - Mpm$. If the places do not differ much in longitude, $x \times \frac{366}{365} = x + \frac{x}{6 \times 60}$ ſufficiently near; in this caſe alſo, the apparent may be uſed for the mean time.

Ex.

Ex. On June 13, 1791, the following obſervations of the paſſage of the moon and α *Serpentis* were made at Greenwich and Dublin Obſervatories.

AT DUBLIN.

Right aſcenſion of ☽'s firſt limb		$15^h.\ 6'.\ 12'',49$
———	α Serpentis	15. 33. 36, 91
		27. 24, 42
Daily rate of clock, $-16'',88$		+ 0, 32
		27. 24, 74

AT GREENWICH.

A.R. ☽'s firſt limb $15^h.\ 5'.\ 3'',52$ at $9^h.\ 36'$ apparent time.
A.R. of α Serpentis 15. 33. 34, 70

	28. 31, 18
	27. 24, 74
Difference . . .	1. 6, 44 = 16'. 36'',6 in ſpace.

As the places do not differ much in longitude, it is unneceſſary to reduce apparent to mean time.

This difference 16'. 36'',6 is the increaſe of the moon's right aſcenſion in the interval of it's paſſages over the meridians at Greenwich and Dublin Obſervatories. By the *Nautical Almanac*, we find the following differences of the right aſcenſions of the ſame limb of the moon, and the ſtar, about the ſame time;

		Difference.
June 12, midnight . .	213°. 15′	
		7°. 23′
13, noon . . .	220. 38	
		7. 33
13, midnight . .	228. 11	
		7. 42
14, noon . . .	235. 53	
		7. 50.
14, midnight . .	243. 43	

If the places differ much in longitude, the motion in right aſcenſion ſhould be calculated to ſeconds.

The

The ſecond differences are always ſufficiently uniform, that we may take 7°. 37′,5, the middle of the firſt differences, for the rate of increaſe for 12 hours at the middle time. Hence 7°. 37′,5 : 16′. 36″,6 :: 12*h*. : $x = 126{,}1407$, and $x + \frac{x}{6 \times 60} = 26'.\ 12'',7$; conſequently the difference of the longitudes is 26′. 12″,73 − 1′. 6″,44 = 25′. 6″,29. Mr. BRINKLEY was ſo good as to favour me with this; and he further obſerves, that when the two places differ much in longitude, an allowance ought to be made for the change of the moon's ſemidiameter in the interval of the paſſages ariſing from it's change of diſtance, and alſo for the change of ſemidiameter in right aſcenſion from it's change of declination. He very ſtrongly recommends this method, as being extremely eaſy in practice, and capable of great accuracy, far beyond that from the eclipſes of *Jupiter's* ſatellites.

To find the Difference of the Longitudes of two Places, from the Occultation of a fixed Star by the Moon.

754. The principal part of the calculation is made by the following Rule, given by Dr. MASKELYNE, for finding the true longitude and latitude of the point of occultation in the moon's limb.

I. Find the angle between the parallels to the ecliptic and equator paſſing through the ſtar, by ſaying, cof. ſtar's latitude : cof. of it's right aſcenſion :: ſine of the obliquity of the ecliptic : ſine of the angle between the parallels. This may alſo be found by Table XXIX. and XXX. at the end of this Volume.

II. From 9 ſigns to 3 ſigns of the right aſcenſion of the ſtar, in a place of *north* latitude, the parallel to the ecliptic *aſcends above* the parallel to the equator; but from 3 ſigns to 9 ſigns, it *deſcends below* the ſame. The contrary for a place of ſouth latitude.

III. If the parallel to the ecliptic *aſcend above* the parallel to the equator, ſubtract the angle juſt found from 90°; but if it *deſcend below*, add it to 90°, and you will have the angle between the meridian paſſing through the ſtar and the parallel to the ecliptic.

IV. Subtract the right aſcenſion of the ſtar from that of the meridian of the place, or the right aſcenſion of the meridian from that of the ſtar, borrowing 360° if neceſſary, ſo that the remainder may be under 180°, and you will have

the horary angle of the ſtar, which will be eaſt or weſt, according as the right aſcenſion of the meridian was ſubtracted from that of the ſtar, or the right aſcenſion of the ſtar ſubtracted from that of the meridian.

V. With this angle, and the ſtar's declination and latitude of the place (corrected for the ſpheroidical figure of the earth), compute the ſtar's altitude, and the angle of poſition at the ſtar.

VI. If the ſtar be *eaſt* of the meridian, *add* the angle of poſition to the angle between the meridian and parallel to the ecliptic, but if the ſtar be to the *weſt* of the meridian, *ſubtract* the former from the latter, borrowing 360° if neceſſary, and you have the angle between the vertical circle and the parallel to the ecliptic.

VII. To the ſine of the moon's equatorial parallax (corrected for the ſpheroidical figure of the earth) add the coſine of the ſtar's altitude and the ſine of the angle between the vertical circle and the parallel to the ecliptic, and the ſum, rejecting 20 from the index, is the ſine of the principal part of the parallax in *latitude*. This muſt be *added* to the ſtar's latitude, if of the *ſame* denomination with the latitude of the place, but *ſubtracted*, if of a *contrary* denomination, unleſs the angle between the vertical circle and the parallel to the ecliptic is greater than 180°, when it muſt be applied in a contrary manner, to obtain the true latitude nearly of the point of the moon's limb at which the occultation happens. This is to be corrected by a ſmall quantity found hereafter.

VIII. To the ſine of the moon's equatorial parallax (corrected as before) add the coſine of the ſtar's altitude, the coſine of the angle between the vertical circle and the parallel to the ecliptic and the arithmetical complement of the coſine of the latitude of the true point of occultation, found nearly in the laſt Article, and the ſum, rejecting 20 from the index, is the ſine of the parallax in *longitude*.

IX. To the conſtant logarithm 4,7124 add twice the ſine of the parallax in longitude and the ſine of twice the true latitude of the point of occultation found nearly, and the ſum, rejecting 30 from the index, is the logarithm of a number of ſeconds, which ſubtracted from the true latitude of the point of occultation of the moon's limb found nearly by Art. VII. gives the true latitude of that point correctly.

X. If the angle between the vertical circle and the parallel to the ecliptic be *more* than 270° or *leſs* than 90°, *add* the parallax of longitude to the longitude of the ſtar; but if that angle be *more* than 90° and *leſs* than 270°, *ſubtract* the parallax in longitude from the longitude of the ſtar, and you will have the true longitude of the point of the moon's limb where the ſtar immerges or emerges.

DEMONSTRATION.

FIG. 183. DEMONSTRATION. Let ΥC be the ecliptic, ΥK the equator, S the ſtar, Z the zenith, P the pole of the equator, p the pole of the ecliptic, and draw the great circles PSA, PZB, pSv, ZSH, and Sm, Sn perpendicular to pS and PS reſpectively. Then by ſpherical Trigonometry,

$$\text{Cof. } \Upsilon zA : \text{ſin. } \Upsilon :: \text{cof. } \Upsilon A : \text{rad.}$$
$$\text{Cof. } vS : \text{rad.} :: \text{cof. } \Upsilon zA : \text{ſin. } zSv$$
$$\therefore \text{Cof. } vS : \text{ſin. } \Upsilon :: \text{cof. } \Upsilon A : \text{ſin. } zSv, \text{ or } nSm,$$

which proves the firſt proportion of the Rule; and by taking the ſtar in all poſſible ſituations, the ſecond and third articles are found to be true.

As $PSn = 90°$, $90° \pm mSn = PSm$, according to the caſes in the Rule.

Alſo $\Upsilon B \sim \Upsilon A = AB$ the meaſure of the angle ZPS.

With ZPS, PS and PZ, compute ZS, and ZSP which is here called the angle of poſition, but it is not the angle generally underſtood under this appellation, as defined in Art. 53.

Next, $PSm \pm ZSP = ZSm$, which will be found to agree with the Rule in the different caſes, as there ſtated.

Let p be the north or ſouth pole of the ecliptic, according as the place is in north or ſouth latitude (no matter whether p be elevated above the horizon or not), S the ſtar touching the moon's limb, Sx the parallax in altitude of that part of the moon's limb, then Spx is the parallax in *longitude*, and $pS - px$ the parallax in *latitude*. Draw xr a portion of a parallel to the ecliptic; and xs a portion of a great circle perpendicular to Sp; then the true latitude s (a point of the moon's limb) $= zr = zs - rs = zS \pm Ss - rs = zS \pm Sx \times \text{cof. } xSs - \frac{xs^2}{2 \text{ tan. } xp}$* = (becauſe $ZSm = 90° \pm xSs$) $zS \pm Sx \times \text{ſin. } ZSm - \frac{xs^2}{2 \text{ tan. } xp}$; if therefore from this we ſubtract zS, the apparent latitude of the point S of the moon's limb, we have the parallax in latitude $= \pm Sx \times \text{ſin. } ZSm - \frac{xs^2}{2 \text{ tan. } xp}$; but if h = the horizontal parallax of the moon, then (154) $Sx = h \times$ ſin. ſtar's app. zenith diſt.; therefore the firſt and principal part $= \pm h \times$ ſin. ſtar's app. zenith diſt. $\times$ ſin. ZSm, where the ſign is $+$ or $-$, according as the latitude of the place is north or ſouth, except ZSm is more than 180°, when it becomes of a contrary denomination, that is, $-$ in north latitude, and $+$ in

* See the Note in page 67.

+ in ſouth latitude. Having gotten the true latitude nearly, before we find the ſecond part of the parallax in latitude, we will find the parallax in longitude.

Rad. : ſin. ZS :: ſin. h : ſin. Sx (154)
Rad. : ſin. xSr (coſ. ZSm) :: ſin. Sx : ſin. xr
Coſ. rv : rad. :: ſin.. xr : ſin. xpS

$\therefore$ Rad. $\times$ coſ. rv : ſin. $ZS \times$ coſ. ZSm :: ſin. h : ſin. xpS,

hence ſin. $xpS = \frac{\text{ſin. } ZS \times \text{coſ. } ZSm \times \text{ſin. } h}{\text{rad.} \times \text{coſ. } rv}$ the ſine of the parallax in *longitude*, agreeing with the Rule. This parallax will be +, that is, eaſterly, or −, that is, weſterly, according as ZSm is acute or obtuſe, reckoning thoſe angles acute which are from 270° to 90°, and thoſe obtuſe which are from 90° to 270°.

Now to find the ſecond part of the parallax in latitude, we may further reduce the expreſſion thus; $xs = \frac{\text{ſin. } xpS \times \text{ſin. } xp}{\text{rad.}}$, and tan. $xp = \frac{\text{rad.} \times \text{ſin. } xp}{\text{coſ. } xp}$; hence this ſecond part $= -\frac{\overline{\text{ſin. } xpS}^2}{2 \times \text{rad.}^3} \times \text{ſin. } xp \times \text{coſ. } xp = -\frac{\overline{\text{ſin. } xpS}^2}{2 \times \text{rad.}^3} \times \text{ſin. } vr \times \text{coſ. } vr = -\frac{\overline{\text{ſin. } xpS}^2}{4 \times \text{rad.}^3} \times \text{ſin. } 2vr$; and to reduce this to ſeconds of a degree, we have 206264″,8 the ſeconds in an arc equal to radius; hence, rad. $= 1 : -\frac{\overline{\text{ſin. } xpS}^2}{4 \times \text{rad.}^3} \times \text{ſin. } 2vr :: 206265'' : -51566'' \times \frac{\overline{\text{ſin. } xpS}^2 \times \text{ſin. } 2vr}{\text{rad.}^3}$; but the log. of 51566 is 4,7124, and xpS is the parallax in longitude; this therefore proves the truth of the Rule for the ſecond part of the parallax in latitude. This is always of a contrary denomination to the moon's latitude.

755. This calculation being made, we very readily find the difference of the longitudes of two places by the following Rule.

I. From the meridian obſervations of the moon, compute it's true latitude and longitude, and compare it with the latitude and longitude computed from the Tables (which may be taken from the *Nautical Almanac*), and you get the error of the Tables in latitude and longitude.

II. Compute, by the above Rule, the true latitude and longitude of the point of the moon's limb where the occultation takes place.

FIG. 184.

III. Take the difference CP of the latitudes of the point of occultation and the moon's center, and knowing Cs the moon's ſemidiameter, we have log. $sP = \log. \sqrt{Cs^2 - CP^2} = \frac{1}{2} \times \log. \overline{Cs+CP} - \log. \overline{Cs-CP}$.

IV. Find the value of sP in longitude, by dividing it (108) by the coſine of the ſtar's latitude, and you get the true difference of longitudes of the moon and ſtar, which difference applied to the true longitude of the ſtar, gives the true longitude of the moon's center at the time of the occultation.

V. Find the ſame for any other place, and take the difference, and then ſay, as the moon's horary motion : that difference :: one hour : the time between the immerſions at the two places.

VI. Apply that time to the time of occultation at one place, and the difference between that reſult and the time of occultation at the other place gives the difference of longitudes.

EXAMPLE.

On March 27, 1792, the immerſion of *Aldebaran* at the moon's dark limb at Greenwich was at $8^h. 37'. 36'',8$ apparent time, and at the Obſervatory of Trinity College Dublin it was at $8^h. 4'. 51'',5$ apparent time; to find the difference of their longitudes.

As the immerſions only were obſerved, it is neceſſary to have the exact latitude of the moon at the immerſions, for finding which we have the following obſervations.

By obſervations at Greenwich on the day of occultation when the moon paſſed the meridian, the right aſcenſion of it's firſt limb was found to be $63°. 35'. 55'',8$, and the zenith diſtance of it's lower limb, corrected for refraction and the error of the line of collimation, was $35°. 41'. 0'',7$, and the apparent time of it's paſſage was $3^h. 46'. 10''$. Hence it's latitude by obſervation was $4°. 38'. 40''$ S. and it's longitude was $64°. 55'. 57'',2$. By the *Nautical Almanac*, it's latitude computed was $4°. 38'. 40''$, and it's longitude was $64°. 56'. 47'',6$; hence, on that day there was no error in the Tables of the moon's latitude, but an error of $+50'',4$ in it's longitude.

By the *Nautical Almanac*, the moon's latitude at the immerſion at Greenwich was $4°. 44'. 52'',7$ S., and at the Obſervatory at Dublin it was $4°. 44'. 43'',7$. The apparent longitude of Aldebaran was $66°. 52'. 59'',2$, and it's apparent latitude $5°. 29'. 5'',6$; it's right aſcenſion was 66°, and it's declination was $16'. 4'',4$; the obliquity of the ecliptic was $23°. 27'. 48''$; and the moon's horary motion in longitude was $30'. 9'',2$ by the *Nautical Almanac*.

Log. cof.	5°. 29′. 5″,6	. . .	arith. comp.	0,00200
cof.	66. 0. 0	. . . . .		9,60931
ſin.	23. 27. 48	. . . . .		9,60006
ſin.	9. 21. 48	. ∠ bet. ‖ ecl. and ‖ equat.		9,21137
	90. 0. 0			
	80. 38. 12	the angle between the meridian paſſing through		

the ſtar and a parallel to the ecliptic.

Calculation for the Obſervatory at Greenwich.

Right aſcen. of the mid-heaven at immerſion	9^h. 6′. 34″,5
——— ſtar	4. 24. 0
Horary angle 70°. 38′,6	4. 42. 34, 5

With this, and the declination 16°. 4′,4, and latitude reduced 51°. 14′,1, we find the angle of poſition = 40°. 29′,7, and the ſtar's altitude = 24°. 32′,3; hence, 80°. 38′,2 − 40°. 29′,7 = 40°. 8′,5 the angle made by a vertical circle and a parallel to the ecliptic.

The moon's equatorial parallax	= 54′. 45″,9	*(Naut. Alm.)*
Reduction (169)	− 8, 7	
Horizontal parallax reduced . .	54. 37, 2	

Sin.	54′. 37″,2	hor. par. red.		8,20106
Cof.	24°. 32′,3	ſtar's altitude		9,95889
Sin.	40. 8, 5	∠ bet. ver. circle and ‖ ecl. .		9,80934
Sin.	32′. 1″,9	parallax in *latitude* nearly . .		7,96929
	5°. 29. 5, 6	S. lat.		
	4. 57. 3, 7	true lat. of the point of occultation nearly.		

Sin. 54′. 37″,2	horizontal parallax	8,20106
Cof. 24°. 32′,3	ſtar's altitude	9,95889
Cof. 40. 8, 5	∠ bet. ver. circle and ‖ ecl. . .	9,88335
Cof. 4. 57	tr. lat. of point of occ. nearly - ar. com.	0,00162
Sin. 38′. 7″,5	parallax in *longitude*	8,04492

Conſtant logarithm	4,7124
2 × ſin. par. in long.	16,0898
Sin. twice true latitude	9,2353
Logarithm of 1″,1	0,0375

4°. 57′. 3, 7

4. 57. 2, 6 true latitude of the point of occultation.

Apparent longitude of *Aldebaran* . . .	2ˢ. 6°. 52′. 59″,2
Parallax	38. 7, 5
Longitude of the point of occultation . .	2. 7. 31. 6,7

Lat. of point *s* of moon – lat. of center $C = 4°.\ 57'.\ 2'',6 - 4°.\ 44'.\ 52'',7 = 12'.\ 9'',9 = CP$; alſo $Cs = 14'.\ 55'',4$;

Hence Cs = 14′. 55″,4
CP = 12. 9, 9

Sum = 27. 5, 3 = 1625″,3	. . . .	3,2109335
Diff. = 2. 45, 5 = 165, 3	. . . .	2,2187980
		2)5,4297315
		2,7148657
Log. cof. lat.	arith. comp.	0,0016228
Log. 520″,6 = 8′. 40″,6 = *sP*		2,7164885

2ˢ. 7°. 31. 6, 7 longitude of *s*

2. 7. 22. 26, 1 long. of ☽'s center at immer. at Greenwich.

Calculation

Calculation for the Obſervatory of Trinity College, Dublin.

Right aſcen. of the mid-heaven at immerſion	$8^h. 33'. 48'',5$
——————— ſtar	4. 24. 0
Horary angle 62°. 27′	4. 9. 48, 5

With this, and the declination 16°. 4′. 26″, and latitude reduced 53°. 9′, we find the angle of poſition = 37°. 32′, and the ſtar's altitude = 29°. 12′,9; hence 80°. 38′,2 − 37°. 32′ = 43°. 6′,2 the angle made by a vertical circle and a parallel to the ecliptic.

The moon's equatorial parallax	54′. 45″,9
Reduction	−9, 1
Horizontal parallax	54. 36, 8

Sin.	54′. 36″,8	hor. par. red.	8,20100
Coſ.	29°. 12′,9	ſtar's altitude	9,94091
Sin.	43. 6, 2	∠ bet. ver. circle and ‖ eclip. .	9,83462
Sin.	32′ 34″,2	parallax in *latitude* nearly . .	7,97653
5°. 29	5, 6	S. lat.	
4. 56.	31, 4	true lat. of the point of occultation nearly.	

Sin.	54′. 36″,8	hor par. red	8,20100
Coſ.	29°. 12′,9	ſtar's altitude	9,94091
Coſ.	43. 6, 2	∠ bet. vert. circle and ‖ ecl. . .	9,86340
Coſ.	4. 57	tr. lat. of point of occ. nearly - ar. com	0,00162
Sin.	34′. 55″,9	parallax in *longitude*	8,00693

Conſtant

Conſtant logarithm	4,7124
2 × ſin. par. in long.	16,0154
Sin. twice true latitude	9,2346
Logarithm of 0″,9	−1,9624
4°. 56′. 31, 4	
4. 56. 30, 5 true latitude of the point of occultation.	

Apparent longitude of *Aldebaran* . .	2ˢ. 6°. 52′. 59″,2	
Parallax	34. 55, 9	
Longitude of the point of occultation . .	2. 7. 27. 55, 1	

Lat. of the point *s* of moon − lat. of center $C = 4°.\ 56'.\ 30'',5 - 4°.\ 44'.\ 43'',7 = 11'.\ 46'',8 = CP$; alſo $Cs = 14'.\ 55'',4$;

Hence $Cs = 14'.\ 55'',4$
$CP = 11.\ 46,\ 8$

Sum	26. 42,.2 = 1602″,2	3,2047167
Diff.	3. 8, 6 = 188, 6	2,2755417
		2)5,4802584
		2,7401292
Log. coſ. lat.	arith. comp.	0,0016228
Log. 551″,8 = 9′. 11″,8 = *sP*		2,7417520
2ˢ. 7°. 27. 55, 1 longitude of *s*.		
2. 7. 18. 43, 3 long. of ☽'s cent. at immer. at Dublin Obſer.		
2. 7. 22. 26, 1 ——— Greenwich.		
Diff. 0. 0. 3. 42, 8		

Hence 30′. 9″,2 (☾'s hor. mot. in long.) : 3′ 42″,8 :: 1 hour : 7′. 23″,3 the time between the immerſions at Greenwich and the Obſervatory of Trinity College, Dublin.

Immerſion

Immerſion at Obſer. of Trin. Coll. Dublin .	8h.	4′. 51″,5	
		7. 23, 3	
Time at the Obſer. of Trin. Coll. Dublin, when the occult. happened at Greenwich	8.	12. 14, 8	
Time of occultation at Greenwich	8.	37. 36, 8	
Longitude of Obſer. Trin. Coll. Dublin . .		25. 22, 0	W.

For this computation I am indebted to Mr. BRINKLEY, who obſerves, that the accuracy in the reſult will not be affected by an error in the longitude of the ſtar, and that a ſmall error in it's latitude will not ſenſibly affect the reſult, when the places do not differ much in longitude and latitude.

To find the Difference of Longitudes of two Places from a Solar Eclipſe.

I. Find (164) the moon's parallax in latitude and longitude for the given time and place of obſervation.

II. Compute the moon's true latitude, and to it apply the error * of the Tables, and you get the true latitude correctly; to which apply the parallax in latitude, and you get the apparent latitude ME, M being the center of the moon, S of the ſun, SE the ecliptic, and ME perpendicular to it. FIG. 185.

III. Hence, for the beginning or end of the eclipſe, knowing SM the ſum of the ſemidiameters, or at any other time knowing the diſtance SM of their centers from obſervation, we get $SE = \sqrt{SM^2 - ME^2} = \sqrt{\overline{SM+ME} \times \overline{SM-ME}}$ the apparent difference of longitudes, to which apply the parallax in longitude and you get the true difference of longitudes of the centers.

IV. Then ſay, as the horary motion of the moon from the ſun : that difference :: 1 hour : the time between the obſervation and the time of the true conjunction, which applied to the time of obſervation gives the time of the true conjunction.

V. Find the ſame for any other place, and the difference of the times gives the difference of the longitudes.

* The error of latitude of the Tables is found by comparing the latitude deduced from obſervation with the computed latitude; in this Example, it is found from the obſervation of the middle of the eclipſe. In cannot here be found as it was at the occultation of a fixed ſtar by the moon, from a meridian obſervation of the moon, as ſuch an obſervation cannot be made at, or near to the time of the eclipſe, the moon being inviſible.

EXAMPLE

EXAMPLE.

On September 4, 1793, the beginning of a ſolar eclipſe at Greenwich was obſerved to be at 21*h*. 39′. 21″ apparent time; at the Obſervatory of Trinity College Dublin, the beginning was at 8*h*. 4′. 50″,2 ſidereal time, or 21*h*. 6′. 47″ apparent time, the middle at 9*h*. 36′. 12″ ſidereal time, or 22*h*. 37′. 54″,6 apparent, and the breadth of the lucid part at the middle, meaſured with a divided object glaſs micrometer, was 6′. 47″; to find the difference of the longitudes.

Calculation for the Obſervatory at Dublin.

The latitude is 53°. 23′,3, and the reduction (173) 14′,3, hence the latitude reduced is 53°. 9′. The obliquity of the ecliptic was 23°. 27′,7, the moon's horary motion in longitude was 29′. 37″,9, in latitude 2′. 42″,8, and it's horary motion from the ſun 27′. 12″,1.

Firſt, to find the error of the Tables in latitude from the obſervation of the middle. At the middle, the moon's latitude by the Tables was 37′. 43″, and longitude 162°. 51′; and the right aſcenſion of medium cœli was 9*h*. 36′. 12″ = 144°. 3′; hence the following calculation * to find the parallax in latitude and longitude.

Cof.	144°. 3′	right aſcenſion med. cœli	9,90823
Cof	53. 9	latitude reduced	9,77795
Cof arc I. = 119. 2,8			9,68618

Cot.

* Mr. Brinkley, who favoured me with this Calculation, has not computed the longitude of the nonageſimal by the ſame Rule as in Art. 164. his Rule is this; let *L* = the latitude reduced, *O* = the obliquity of the ecliptic, and *A* = the right aſcenſion of medium cœli; then cof. arc I. = cof. *A* + cof. *L*; cot. arc II. = cot. *L* + ſin. *A*; arc III. = arc II. ± *O*, — when Aries is paſt the meridian, and + when it is not paſt; cof. *Alt.* non. = ſin. arc I. + ſin. arc III; tang. *long.* non. = cof. arc III. + tan. arc I.; the nonageſimal is in the ſame quadrant as arc I. and arc I. is in the ſame quadrant as *A*,

Cot. 53°. 9′ lat. red.		9,87474
Sin. 144. 3 right aſcen, med. cœli		9,76870
Cot. arc II. = 66°. 15′		9,64344
Obliq. ecl. = 23. 27,7		
Sin. arc III. = 42. 47,3		9,83205
Sin. arc I. = 119. 2,8		9,94161
Coſ. *alt.* non. = 53°. 34′,2		9,77366
Coſ. arc III. = 42°. 47′,3		9,86561
Tan. arc I. = 119. 2, 8		10,25541
Tan. *long.* non. = 127°. 7′		10,12102
Moon's longitude = 162. 51		
Moon's diſt. from nonag.	35. 44	
The moon's equatorial parallax	54′. 12	
Reduction	− 8,9	
Moon's horizontal parallax	54. 3,1 = 3243″,1	
Sun's horizontal parallax		8, 6
Hor. par. ☽ from ⊙		3234, 5
Log. 3234,5		3,50981
Sin. 53°. 34′,2 alt. nonag.		9,90557
Coſ. 37′. 43″ ☾'s lat. by Tab.	arith. comp.	0,00003
		3,41541
Sin. 35°. 44′ ☾'s diſt. from nonag.		9,76642
Log. 1520″ = 25′. 20″ par. in long. nearly		3,18183
35°. 44		
Sin. 36. 9		9,77078
		3,41541
Log. 1535″,3 = 25′. 35″,3 paral. in *Longitude*		3,18619

Log. 3234,5		3,50981
Cof. 53°. 34′,2 alt. nonag.		9,77366
Cof. app. lat. ☾	arith. comp.	0,00000
Log. 1920″,8 = 32′. 0″,8	firſt part par. in lat.	3,28347
37. 43	☽'s lat. by Tab.	
5. 42,2	apparent latitude very nearly.	

Log. 3234,5		3,510
Sin. 53°. 34′,2 alt. nonag.		9,906
Sin. 5′. 42″,2 app. lat. of ☾		7,220
Cof. $35°.\ 44' + \frac{25'.\ 35''}{2}$		9,908
Log. 3″,5	ſecond part par. lat.	0,544
32′. 0, 8	firſt part	
31. 57, 3	parallax in *Latitudę*.	

Moon's horizontal ſemidiameter	14′. 46″
Inflex. light	−3
	14. 43
Augmen. for ☽'s alt. 35°	+9
Moon's ſemidiameter	14. 52

Lucid

Lucid part at the middle	6′. 47″
	− 6 *
Correction of lucid part	6. 41
The moon's ſemidiameter	14. 52
Sum	21. 33
The ſun's ſemidiameter	15. 53
	5. 40 the diſ-

tance of the center of the ſun from that of the moon; and as the apparent path of the moon makes an angle of only about 1°. 5′. 40″ with the ecliptic, it may be conſidered as the moon's apparent latitude at the middle of the eclipſe.

Apparent latitude	5′. 40″
Parallax in latitude	31. 57, 3
True latitude by obſervation	37. 37, 3
——————— by the Tables	37. 43
Error of the Tables in latitude	− 5, 7

To

* Mr. BRINKLEY obſerves, that imperfect or bad achromatic teleſcopes are found to give the ſun's diameter greater than it really is. MAYER's teleſcope, which was not achromatic, gave, according to the late obſervations of Dr. MASKELYNE, the ſun's diameter too great as ſet down in the *Nautical Almanac* by 6″. An achromatic with a divided object glaſs micrometer may be conſidered as an indifferent teleſcope, perhaps little better than a teleſcope which is not achromatic. I have therefore diminiſhed the meaſure 6″. I could give ſeveral reaſons why I fix it at 6″. The correction may be diſputed; but it is of little or no conſequence in the reſult, except when the true conjunction is determined ſeparately from the beginning and end, and not then of much, except the eclipſe be ſmall. Dr. MASKELYNE has found the ſun's diameter as put down in the *Nautical Almanac* 6″ too much; that correction therefore to the diameter in the *Nautical Almanac* is here applied.

To find the true Time of the Conjunction at Dublin Obſervatory.

Latitude of Dublin Obſervatory reduced = 53°. 9′.

Coſ.	121°. 12′,6	right aſcen. med. cœli . . .	9,71448
Coſ.	53. 9	lat. red.	9,77795
Coſ.	arc I. = 108°. 6′,5	. . . . .	9,49243
Cot.	53°. 9′	lat. red.	9,87474
Sin	121. 12,6	right aſcen. med. cœli	9,93211
Cot.	arc II. = 57°. 20′,5	. . . . .	9,80685
Obliq.	ecl. = 23. 27,7		
Sin.	arc. III. = 33. 52, 8	. . . . .	9,74620
Sin.	arc I. = 108. 6, 5	. . . . .	9,97793
Coſ.	*alt.* nonag. = 58°. 0′,5	. . . . .	9,72413
Coſ.	arc III. = 33°. 52′,8	. . . . .	9,91920
Tan.	arc I. = 108. 6, 5	. . . . .	10,48545
Tan.	*long.* nonag. = 111°. 29′,9	. . . . .	10,40465
Moon's	longitude = 162. 6	(*Naut. Alm.*)	
☾'s	diſt. à nonag. = 50. 36, 1		

Moon's

Moon's equatorial parallax	54′. 11″	
Reduction	−8,9	
Moon's horizontal parallax	54. 2,1	= 3242″,1
Sun's horizontal parallax		= 8, 6
Horizontal parallax of ☽ from ⊙		3233, 5

Moon's latitude by Tables	33′. 36″ N.	
Error of Tables		−5, 7
Moon's true latitude	33. 30, 3	

Log. 3233,5		3,50967
Sin. 58°. 0′,5 alt. nonag.		9,92846
Cof. 33′. 30″,3 ☾'s true lat. . . arith. comp.		0,00002
		3,43815
Sin. 50°. 36′,1 ☾'s diſt. from nonag.		9,88804
Log. 2119″,3 = 35′. 19″,3 par. in long. nearly .		3,32619
50°. 36′,1		
Sin. . 51. 11, 4		9,89166
		3,43815
Log. 2136″,8 = 35′. 36″,8 par. in *longitude* . . .		3,32981

Log. 3233,5			3,50967
Cof. 58°. 0′,5 alt. nonag.			9,72410
Log. 1713″,1 = 28′. 33″,1		firſt part par. in lat. .	3,23377
33. 30, 3		moon's true latitude.	
4. 57, 2		moon's apparent latitude nearly.	

Log.

Log. 3233,5	3,510
Sin. 58°. 0′,5 alt. nonag.	9,928
Sin. 4′. 57″,2 ☾'s app. latitude nearly	7,159
Cof. 50°. 36′,1 + $\frac{35'.\ 36'',8}{2}$	9,799
Log. 2″,5 ſecond part of par. in latitude . . .	0,396
28′. 33, 1 firſt part of par. in latitude.	
28. 30, 6 parallax in *latitude*.	

Moon's true latitude	33′. 30″,3
Parallax in latitude	28. 30, 6
ME=299″,7=	4. 59, 7

The moon's ſemidiameter	14′. 46″
Inflexion of light	− 3
	14. 43
Augmen. for ☽'s alt. 32°	+ 8,4
Moon's ſemidiameter	14. 51,4

The ſun's ſemidiameter reduced	15′. 53″
Moon's ſemidiameter	14. 51,4
SM=1844″,4=	30. 44,4

1844,4

	1844,4	
	299,7	
Sum	2144,1	log. 3,33125
Diff.	1544,7	log. 3,18884
		2)6,52009
Log. SE = 1819″,9 =	30′. 19″,9	3,26004
Parallax in long. =	35. 36, 8	
	1°. 5. 56, 7 true diff. long. ⊙ and ☾.	

Hence 27′. 12″,1 : 1°. 5′. 56″,7 :: 1*h*. : 2*h*. 25′. 28″ the interval from the beginning to the time of the true conjunction; consequently 21*h*. 6′. 47″ (beg.) + 2*h*. 25′. 28″ = 23*h*. 32′. 15″ for the time of the conjunction at the Obſervatory of Trinity College, Dublin.

To compute the ſame for Greenwich.

Beginning at	21^h. 39′. 21″ apparent time.
Sun's right aſcenſion . . .	10. 58. 3
Right aſcenſion med. cœli . .	8. 37. 24 = 129°. 21′.

Latitude of Greenwich reduced = 51°. 14′,1

Coſ. 129°. 21′ right aſcen. of med. cœli . .	9,80213
Coſ. 51. 14,1 latitude reduced	9,79668
Coſ. arc I. = 113°. 23′,5	9,59881

Cot.

Cot. 51°. 14′,1	latitude reduced	9,90475
Sin. 129. 21	right aſcen. of med. cœli . .	9,88834
Cot. arc II. = 58°. 9′,7	. . . . .	9,79309
Obliq. ecl. = 23. 27,7		
Sin. arc III. = 34. 42	. . . . .	9,75532
Sin. arc I. = 113. 23,5	. . . . .	9,96275
Cof. *alt.* nonag. = 58°. 30′	. . . . .	9,71807
Cof. arc III. = 34°. 42′	. . . . .	9,91495
Tan. arc I. = 113. 23,5	. . . . .	10,36394
Tan. *longitude* nonag. = 117°. 45′	. . . . .	10,27889
Moon's longitude = 162. 10 (*Naut. Alm.*)		
Moon's diſt. from non. 44. 25		

Moon's equatorial parallax	. . . . 54′. 11″	
Reduction	. . . . . − 7,1	
Moon's horizontal parallax	. . . . 54. 3,9 = 3243″,9	
Sun's horizontal parallax	. . . . . 8, 6	
Horizontal parallax ☽ from ⊙	. . . . .	3235, 3
Moon's latitude by the Tables	. . . . .	33′. 55″,9
Error of the Tables	. . . . .	− 5, 7
Moon's true latitude	. . . . .	33. 50, 2

Log.

Log 3235, 3	3,50991
Sin. 58°. 30′ alt. nonag.	9,93077
Cof. 33′. 50″ moon's true lat. . . arith. comp.	0,00002
	3,44070
Sin. 44°. 25′ moon's diſt. from nonag.	9,84502
Log. 1930″,7 = 32′. 10″,7 par. in long. nearly .	3,28572
44°. 25	
Sin. . . 44. 57. 10,7	9,84912
	3,44070
Log. 1949″,1 = 32′. 29″,1 par. in *Longitude* . .	3,28982

Log. 3235, 3	3,50991
Cof. 58°. 30′ alt. nonag.	9,71807
Cof. moon's apparent latitude . . arith. comp.	0,00000
Log. 1690″,4 = 28′. 10″,4 firſt part par. in lat. .	3,22798
33. 50, moon's true latitude.	
5. 40 apparent latitude nearly.	

Log. 3235, 3	3,510
Sin. 58°. 30′ alt. nonag.	9,931
Sin. 5′. 40″ apparent latitude of moon	7,217
Cof. 44°. 25′ + $\frac{32'.\ 29''}{2}$	9,852
Log. 3″,2 ſecond part of par. in latitude	0,510
28′. 10, 4 firſt part of par. in latitude.	
28. 7, 2 parallax in *Latitude*.	

Moon's true latitude	33′. 50″,2
Parallax in latitude	28. 7, 2
$ME = 343'' =$	5. 43

The moon's ſemidiameter	14′. 46″
Inflexion of light	− 3
	14. 43
Augmen. for ☽'s alt. 37°	+ 9,5
Moon's ſemidiameter	14. 52,5

The ſun's ſemidiameter reduced	15′. 53″
The moon's ſemidiameter	14. 52,5
$SM = 1845'',5$	30. 45,5

1845,5
343

Sum 2188,5	log.	3,34015
Diff. 1502,5	log.	3,17681
		2)6,51696
Log. $SE = 1813''3 = 30'. 13'',3$		3,25848
Parallax in longitude = 32. 29, 1		
1°. 2. 42, 4 true diff. of long. ☉ and ☾.		

Hence, 27′. 12″,1 : 1°. 2′. 42″,4 :: 1 hour : 2*h*. 18′. 18″,9 the interval from the beginning to the time of the true conjunction; conſequently 21*h*. 39′. 21″ (beg.) +2*h*. 18′. 18″,9 = 23*h*. 57′. 39″,9 the time of the true conjunction at Greenwich.

Time

Time of conjunction at Greenwich Obſerv.	$23^h. 57'. 39'',9$
——————— at Dublin Obſervatory	23. 32. 15
Difference of the meridians	25. 24, 9

To find the error of the Tables in longitude, we have

☉'s long. at time of conj. by MAYER's Tab.	$5^s. 13°. 17'. 6''$
☾'s long. ———————	5. 13. 17. 48
Error of the lunar Tables in longitude, ſuppoſing the ſolar Tables to be accurate	+ 42

Mr. BRINKLEY obſerves, that in an occultation, or eclipſe of the ſun, when the calculation is made for the difference of longitudes to be deduced from the beginnings or endings at two places, it will be ſufficient to uſe the equatorial parallax to the neareſt ſecond, and not to regard the inflexion and irradiation of light; but when the difference of longitudes is to be deduced from the beginning at one place and the ending at the other, theſe circumſtances ought to be ſtrictly attended to.

To find the Longitude by a Time-keeper.

756. Let the *Time-keeper* be well regulated, and ſet to the meridian of Greenwich; then if it neither gain nor loſe, it will always ſhow the time at Greenwich. Hence, to find the longitude of any other place, find the mean time, either by the ſun's altitude or that of a fixed ſtar by Art. 92, or 106. and obſerve, at the inſtant of taking the altitude, the time by the watch; and the difference of theſe times, converted into degrees, at the rate of 15° for an hour, gives the longitude from Greenwich. If, for example, the time by the watch when the altitude was taken, was $6h.$ $19'$, and the mean time deduced from that altitude was $9h.$ $23'$, the difference $3h.$ $4'$ converted into degrees gives 46° the longitude of the place *eaſt* from Greenwich, becauſe the time at the place of obſervation is *forwarder* than that at Greenwich. Thus the longitude could be very readily determined, if you could depend upon the watch. But as a watch

watch will always gain or loſe, before it is ſent out it's gaining or loſing every day for ſome time, a month for inſtance, is obſerved; this is called the *rate of going* of the watch, and from thence the *mean* rate of going is thus found.

757. Suppoſe, for inſtance, I examine the rate of a watch for 30 days; on ſome of thoſe days I find it has gained, and on ſome it has loſt; add together all the quantities which it has gained, and ſuppoſe they amount to 17″; add together all the quantities which it has loſt, and let the ſum be 13″; then the difference 4″ is the *mean* rate of gaining for 30 days, which divided by 30 gives 0,″133 for a *mean daily rate* of gaining. Or you may get the mean daily rate thus. Take the *difference* between what the watch was too faſt, or too ſlow on the firſt and laſt days of obſervation, if it be too faſt or too ſlow on each day; but take the *ſum*, if it be too faſt on one day and too ſlow on the other, and divide by the number of days between the obſervations *. And to find the time at the place of trial at any future period by this watch, you muſt put down, at the end of the above trial, how much the watch is too faſt or too ſlow; then ſubtract from the time ſhown by the watch, 0″,133 × number of days from the end of the trial, being the quantity which it has gained according to the above mean rate of gaining, and you are then ſuppoſed to get the true time affected with the error at the end of the trial. This would be all the error, if the watch had continued to gain according to the above rate; and although, from the different temperatures of the air to which the watch may be expoſed, and from the imperfection of the workmanſhip, this cannot be expected, yet by taking it into conſideration, the probable error of the time will be diminiſhed. In watches which are under trial at the Royal Obſervatory at Greenwich, as candidates for the rewards offered by Parliament for the diſcovery of the longitude, this allowance of a mean rate to be applied in order to get the time is not granted by the Act of Parliament, but it requires that the watch itſelf ſhould go within the limits aſſigned; the Commiſſioners however are ſo indulgent as to grant the application of a mean rate, which is undoubtedly favourable to the watches.

758. As the rate of going of a watch is ſubject to vary from ſo many circumſtances, the obſerver, whenever he goes aſhore and has ſufficient time, ſhould compare his watch for ſeveral days with the mean time deduced from the altitude of the ſun or a ſtar, by which he will be able to determine it's rate of going. And whenever he comes to a place whoſe longitude is known, he may correct his watch and ſet it to Greenwich time. For inſtance, if he go to a place known to be 30° eaſt longitude from Greenwich, his watch ſhould be

* For further information on this ſubject, ſee Mr. WALES's *Method of finding the Longitude at Sea.*

be two hours ſlower than the time at that place. Find therefore the time at that place by the altitude of the ſun or a fixed ſtar, and correct it by the equation of time, and compare the time ſo found with the time by the watch when the altitude was taken, and if the watch be two hours ſlower than the time deduced from obſervation, it is right; if not, correct it by the difference, and it again gives Greenwich time.

759. In long voyages, unleſs you have ſometimes the means of adjuſting the watch to Greenwich time, it's error will probably be very conſiderable, and conſequently the longitude deduced from it will be ſubject to a proportional error. In ſhort voyages a watch is undoubtedly very uſeful, and alſo in long ones, where you have the means of correcting it from time to time. It ſerves to carry on the longitude from one known place to another, ſuppoſing the interval of time not to be very long; or to keep the longitude from that which is deduced from a lunar obſervation, till you can get another obſervation. Thus the watch may be rendered of great ſervice in Navigation.

To find the Longitude by an Eclipſe of the Moon, and of Jupiter's Satellites.

760. By an eclipſe of the moon: This eclipſe begins when the umbra of the earth firſt touches the moon, and it ends when it leaves the moon. Having the times calculated when the eclipſe begins and ends at Greenwich, obſerve the times when it begins and ends at any other place; the difference of theſe times converted into degrees, gives the difference of longitudes. For as the phaſes of the moon in an eclipſe happen at the ſame inſtant to every obſerver, the difference of the times at different places when any phaſe is obſerved will give the difference of the longitudes. This would be a very ready and accurate method, if the time of the firſt and laſt contact could be accurately obſerved; but the darkneſs of the penumbra continues to increaſe till it comes to the umbra, ſo that until the umbra actually gets upon the moon, it is not diſcovered. The umbra itſelf is alſo very badly defined. The beginning and end of a lunar eclipſe cannot, in general, be determined nearer than 1′ of time; and very often not nearer than 2′ or 3′. Upon theſe accounts, the longitude, from the obſerved beginning and end of an eclipſe, is ſubject to a conſiderable degree of uncertainty. Aſtronomers therefore determine the difference of the longitudes of two places by correſponding obſervations of other phaſes, that is, when the umbra biſects any of the ſpots upon the moon's ſurface. And this can be determined with a greater

degree

degree of accuracy than the beginning and end; becauſe, when the umbra is gotten upon the moon's ſurface, the obſerver has leiſure to conſider and fix upon the proper line of termination, in which he will be aſſiſted by running his eye along the circumference of the umbra. Thus the coincidence of the umbra with the ſpots may be obſerved to a tolerable accuracy. The obſerver therefore ſhould have a good map of the moon at hand, that he may not miſtake. The teleſcope to obſerve a lunar eclipſe ſhould have but a ſmall magnifying power with a great deal of light. The ſhadow comes upon the moon on the eaſt ſide, and goes off on the weſt; but if the teleſcope invert, the appearance will be contrary.

761. The eclipſes of *Jupiter's* ſatellites afford the readieſt method of determining the longitude of places at land. It was alſo hoped that ſome method might be invented to obſerve them at ſea, and Mr. IRWIN made a chair to ſwing for that purpoſe, for the obſerver to ſit in; but Dr. MASKELYNE, in a voyage to Barbadoes, under the direction of the Commiſſioners of longitude, found it totally impracticable to derive any advantage from it; and he obſerves that "conſidering the great power requiſite in a teleſcope for making theſe obſervations well, and the violence as well as irregularities of the motion of a ſhip, I am afraid the complete management of a teleſcope on ſhip-board will always remain among the deſiderata. However, I would not be underſtood to mean to diſcourage any attempt, founded upon good principles, to get over this difficulty." The teleſcopes proper for making theſe obſervations are common refracting ones from 15 to 20 feet, reflecting ones of 18 inches or 2 feet, or the 46 inch achromatic with three object glaſſes which were firſt made by Mr. DOLLOND. On account of the uncertainty of the theory of the ſatellites, Dr. MASKELYNE adviſes the obſerver to be ſettled at his teleſcope three minutes before the expected time of an immerſion of the firſt ſatellite, ſix or eight before that of the ſecond or third, and at leaſt a quarter of an hour before that of the fourth. And if the longitude of the place be alſo uncertain, he muſt look out proportionably ſooner. Thus, if the longitude be uncertain to 2°, anſwering to eight minutes of time, he muſt begin to look out eight minutes ſooner than is mentioned above. However, when he has obſerved one eclipſe and found the error of the Tables, he may allow the ſame correction to the calculations of the Ephemeris for ſeveral months, which will advertiſe him very nearly of the time of expecting the eclipſes of the ſame ſatellite, and diſpenſe with his attending ſo long. Before the oppoſition of Jupiter to the ſun, the immerſions and emerſions happen on the weſt ſide of Jupiter, and after oppoſition, on the eaſt ſide; but if the teleſcope invert, the appearance will be the contrary. Before oppoſition, the immerſions only of the firſt ſatellite are viſible; and after

after oppoſition, the emerſions only. The ſame is generally the caſe with reſpect to the ſecond ſatellite; but both immerſion and emerſion are frequently obſerved in the two outer ſatellites. See Art. 456.

762. When the obſerver is waiting for an emerſion, as ſoon as he ſuſpects that he ſees it, he ſhould look at his watch and note the ſecond, or begin to count the beats of the clock, till he is ſure that it is the ſatellite, and then look at the clock and ſubtract the number of ſeconds which he has counted from the time then obſerved, and he will have the time of emerſion. If Jupiter be 8° above the horizon and the ſun as much below, an eclipſe will be viſible; this may be determined near enough by a common globe.

763. The immerſion or emerſion of a ſatellite being obſerved according to apparent time, the longitude of the place from Greenwich is found by taking the difference between that time and the time ſet down in the *Nautical Almanac*, which is calculated for apparent time.

Ex. Suppoſe the emerſion of a ſatellite to have been obſerved at the Cape of Good Hope, May 9, 1767, at 10*h*. 46′. 45″ apparent time; now the time in the *Nautical Almanac* is 9*h*. 33′. 12″; the difference of which times is 1*h*. 13′. 33″ the longitude of the Cape *eaſt* of Greenwich in time, or 18°. 23′. 15″.

764. But to find the longitude of a place from an obſervation of an eclipſe of a ſatellite, it is better to compare it with an obſervation made under ſome well known meridian, than with the calculations of the Ephemeris, becauſe of the imperfections of the theory; but where a correſponding obſervation cannot be obtained, find what correction the calculations of the Ephemeris require, by the neareſt obſervations to the given time that can be obtained; and this correction applied to the calculation of the given eclipſe in the Ephemeris, renders it almoſt equivalent to an actual obſervation. The obſerver muſt be careful to regulate his clock or watch by apparent time, or at leaſt to know the difference; this may be done either by equal altitudes of the ſun or proper ſtars; or the latitude being known, from one altitude at a diſtance from the meridian, by the methods already explained.

765. In order the better to determine the difference of longitudes of two places from correſponding obſervations, the obſervers ſhould be furniſhed with the ſame kind of teleſcopes. For at an immerſion, as the ſatellite enters the ſhadow it grows fainter and fainter, till at laſt the quantity of light is ſo ſmall that it becomes inviſible even before it is wholly immerſed in the ſhadow; the inſtant therefore that it becomes inviſible will depend upon the quantity of light which the teleſcope receives, and it's magnifying power. The inſtant

therefore

therefore of the diſappearance of a ſatellite will be later the better the teleſcope is, and the ſooner it will appear at it's emerſion. Now the immerſion is the inſtant the ſatellite is wholly gotten into the ſhadow, and the emerſion is the inſtant before it begins to emerge from the ſhadow; if therefore two teleſcopes ſhow the diſappearance or appearance of the ſatellite at the ſame diſtance of time from the immerſion or emerſion, the difference of the times will be the ſame as the difference of the true times of their immerſions or emerſions, and therefore will ſhow the difference of longitudes accurately. But if the obſerved time at one place be compared with the computed time at another, then we muſt allow for the difference between the apparent and true time of immerſion or emerſion in order to get the true time where the obſervation was made to compare with the true time from computation at the other place. This difference may be found by obſerving an eclipſe at any place whoſe longitude is known, and comparing it with the time by computation. Obſervers, therefore, ſhould ſettle the difference accurately by the mean of a great number of obſervations thus compared with the computations, by which means the longitude will be aſcertained to a much greater accuracy and certainty. After all this precaution, however, the different ſtates of the air at different times, and alſo the different ſtates of the eye, will introduce a ſmall degree of uncertainty; the latter caſe may perhaps, in a great meaſure, be obviated, if the obſerver will be careful to remove himſelf from all warmth and light for a little time before he makes the obſervation, that the eye may be reduced to a proper ſtate; which precaution the obſerver ſhould alſo attend to when he ſettles the difference between the apparent and true times of immerſion and emerſion. Perhaps alſo the difference ariſing from the different ſtates of the air might, by proper obſervations, be aſcertained to a conſiderable degree of accuracy; and as this method of determining the longitude is, of all others, the moſt ready, no means ought to be left untried to reduce it to the greateſt certainty. For further directions, ſee Art. 460.

CHAP.

CHAP. XXIX.

ON THE USE OF THE GLOBES.

Art. 766. THERE are two Globes, one called *Terreſtrial*, upon which the places on the earth are delineated, and the other called *Celeſtial*, upon which the principal fixed ſtars are put down in their proper places, and the figures of the conſtellations drawn. The terreſtrial globe is a perfect map of the earth, repreſenting accurately the relative ſituations of all the places upon it's ſurface. The celeſtial globe ſerves to explain all the phænomena ariſing from the diurnal motion of the earth about it's axis, and alſo the variation of ſeaſons ariſing from it's motion about the ſun, only ſuppoſing the ſun to move in the ecliptic inſtead of the earth, which will not alter any of the appearances. Fig. 186. repreſents the conſtruction of each globe. FIG. 186. *HR* is a flat circular frame of wood ſupported by ſemicircular pieces coming from the foot, the plane of which paſſes through the center of the globe. *P ♑ p E* is a braſs circle called the *Brazen Meridian*; it is ſupported at it's loweſt point upon a roller on which it turns in it's own plane, and paſſes through the horizon *HR* in two groves cut for that purpoſe. The globe itſelf is ſupported within this circle by an axis *Pp* on which it turns; this axis paſſes through the brazen meridian at *P* and carries an index round with it over a circular plate *hc*, which is divided into hours, &c. *E ♑* repreſents the equator, and *CL* the ecliptic; to each of which circles on the celeſtial globe ſecondaries are drawn to every 10 or 15 degrees, but on the terreſtrial, they are drawn only to the equator. From *C* and *L* are drawn the two tropical circles; and on the terreſtrial globe are drawn the parallels of latitude. There is alſo part of another circle *Za*, called a *Quadrant of Altitude*, which is occaſionally fixed to the brazen meridian; it is a thin plate of braſs, having a nut and a ſcrew at one end to faſten it to the meridian in it's zenith *Z*, and then the lower end is put between the globe and horizon, and can be turned round to any point; it is divided into degrees, &c. by which the altitude of any object above the horizon may be meaſured, and at the ſame time it refers the object to the horizon, by which it's azimuth may be determined. From one point *E* of the brazen meridian correſponding to the equator, the degrees begin and are continued up to 90° at each pole *P*, *p*; but for the other ſemicircle of the meridian, the degrees begin at the poles and are continued to 90° at the equator. On the horizon, the degrees begin at the eaſt and weſt points,

and are continued both ways to 90°, or to the north and ſouth points. The points of the compaſs are alſo generally put upon the horizon; and on two other circles drawn thereon are put the ſigns of the zodiac, and the months and days correſponding to the ſun's place, which ſerves as a calendar to ſhow the place of the ſun on any day; this however cannot be accurate, as the ſun is not always in the ſame point of the ecliptic on the ſame day. The ecliptic and equator begin their degrees at one of their interſections, called *Aries*, which are continued the ſame way all round up to 360°, and the former is divided into and marked with the twelve ſigns; the equator is alſo divided from the ſame point into 24 hours, which is therefore ſometimes made uſe of inſtead of the hour circle. Upon the foot of the globe there is often put a compaſs, by which the brazen meridian may be ſet north and ſouth. In the *Phil. Tranſ.* 1789, Mr. SMEATON has given a deſcription of an improved quadrant of altitude. Inſtead of a ſtrip of thin flexible braſs, he makes it of a more ſolid conſtruction. It is fixed to a braſs ſocket, and made to turn upon an upright ſteel ſpindle, fixed in the zenith, by which you meaſure altitudes and azimuths with as much accuracy as you do any other arcs. He approves of the common hour circle, and ſays, that one of four inches diameter may be divided into 720 diſtinguiſhable diviſions, anſwering to two minutes of time; and if inſtead of a *Pointer*, an *Index Stroke* is uſed in the ſame plane with that of the diviſions, half minutes may be eaſily diſtinguiſhed. He therefore thinks the hour circle ſhould rather be improved than omitted, as it is upon ſome globes.

ON THE USE OF THE TERRESTRIAL GLOBE.

To find the Latitude of a Place.

767. Bring the place under that ſemicircle of the brazen meridian where the diviſions begin at the equator, and obſerve what degree it is under, and it is the latitude required.

To rectify the Globe for the Latitude of a Place.

768. Elevate the pole above the horizon till it's altitude is equal to the latitude of the place, and it then ſtands right for the ſolution of all problems for that latitude.

To

To find the Longitude of a Place from any given Meridian.

769. Bring the place to the brazen meridian, and obſerve the point of the equator which lies under it, and the diſtance of that point from the point where the given meridian cuts the equator, is the longitude required.

Given the Latitude and Longitude of a Place, to find that Place.

770 Bring the given degree of longitude to the meridian, and then under the degree of latitude upon the meridian you have the place required.

When it is Noon at any Place A, *to find the Hour at any other Place* B.

771. Bring *A* to the meridian, and ſet the index to XII.; then turn the globe till *B* comes under the meridian, and the index will ſhow the hour at *B*. If it be not noon at *A*, ſet the index to the hour, and proceed as before, and you get the correſponding hour at *B*.

To find the Diſtance of A *from* B.

772. Bring *A* to the meridian, and ſcrew the quadrant of altitude over it, and carry it over *B*, and you get the number of degrees between *A* and *B*, which multiply by 69,2, the miles in one degree, and you have the diſtance.

To find the Bearing of B *from* A.

773. Rectify the globe for the latitude of *A*, and bring it to the meridian, and fix the quadrant of altitude to it; then direct the quadrant to *B*, and the point where it cuts the horizon ſhows the bearing required.

To find the Place A *to which the Sun is vertical at any Hour of the Day, at a given Place* B.

774. Find the ſun's place in the ecliptic, and bring it to the meridian, and mark the declination; then bring *B* to the meridian, ſet the index to the given hour, and turn the globe till the index comes to XII. at noon, and the place under the ſun's declination upon the meridian is that required.

To find on any Day and Hour, the Places where the Sun is riſing, ſetting, or on the Meridian; alſo thoſe Places which are enlightened, and where the Twilight is beginning and ending.

775. Find (774) the place to which the ſun is vertical at the given time, and bring the ſame to the meridian, and rectify the globe for a latitude equal to the ſun's declination; then to all thoſe places in the *weſtern* ſemicircle of the horizon, the ſun is *riſing*; to thoſe in the *eaſtern*, it is *ſetting*; and to thoſe under the *meridian*, it is *noon*. Alſo, all the places above the horizon are enlightened; and the altitude of the ſun above the horizon at any one place at that time, is equal to the diſtance of that place from the horizon, which may be meaſured by the quadrant of altitude. Laſtly, in all thoſe places 18° below the weſtern horizon the twilight is juſt beginning in the morning, and in thoſe on the eaſtern, it is juſt ending in the evening.

To find all the Places to which a Lunar Eclipſe is viſible at any inſtant.

776. Find the place to which the ſun is vertical at the given time, and bring that place to the zenith, and the eclipſe will be viſible to all the hemiſphere *under* the horizon, becauſe the moon is oppoſite to the ſun.

777. We cannot, by a globe only, determine the ſame for a *ſolar* eclipſe, becauſe that eclipſe does not happen to the whole hemiſphere of the earth next the ſun, nor does it happen at the ſame time to thoſe places where it is viſible.

778. The inhabitants of the earth are diſtinguiſhed by the different directions of their ſhadows. They who live in the torrid zone are called *Amphiſcii*, becauſe their ſhadows at noon are caſt ſometimes to the north and ſometimes

to

to the ſouth. But when the ſun is vertical to them at noon, they then caſt no ſhadows, and are called *Aſcii*. The inhabitants of the temperate zones are called *Heteroſcii*, becauſe they never caſt their meridian ſhadows but one way. They who inhabit the frigid zones are called *Periſcii*, becauſe the ſun is ſometimes above their horizon for a day, or for a longer time even to ſix months, ſo that their ſhadows turn all round them.

779. The inhabitants of the earth have alſo been diſtinguiſhed into three ſorts, in reſpect to their relative ſituations in latitude or longitude. They who live under oppoſite points of the ſame parallel to the equator are called, in reſpect to each other, *Periæci*. Theſe have the ſame ſeaſons at the ſame time, only they differ 12 hours in time, it being midnight to one when it is noon to the other. They who live in the ſame meridian, but on oppoſite ſides of the equator and equidiſtant from it, are called *Antæci*. Theſe have day and night at the ſame time, the hours being the ſame, but they have different ſeaſons, it being ſummer with one when it is winter with the other. They who live in oppoſite parallels to the equator, and in oppoſite meridians, or who live on oppoſite points of the globe, are called *Antipodes*. With theſe, it is day to one when it is night to the other, and ſummer to one when it is winter to the other.

ON THE USE OF THE CELESTIAL GLOBE.

To find the Sun's Right Aſcenſion and Declination.

780 Bring the ſun's place in the ecliptic to the meridian, and it points out upon the meridian, the declination; and the degree of the equator which is cut by the meridian is the right aſcenſion.

Given the Right Aſcenſion and Declination of an heavenly Body, to find it's Place.

781. Bring the degree of right aſcenſion on the equator to the meridian, and the point correſponding to the declination, is the place required.

Given

Given the Latitude of the Place, the Day and Hour, to find the Altitude and Amplitude of the heavenly Bodies.

782. Rectify the globe (768) to the latitude of the place, and bring the ſun's place in the ecliptic to the meridian, and ſet the index to XII.; then turn the globe till the index points to the given hour, and in that poſition the globe repreſents the proper ſituation of all the heavenly bodies upon it, in reſpect to the meridian and horizon. Then fix the quadrant of altitude to the zenith, and direct it's graduated edge to the place of the body, and it ſhows the altitude of the body, and the degree where it cuts the horizon ſhows it's amplitude. If the body be the moon or a planet, after having found it's place, put a very ſmall patch upon it to denote it's place.

Given as before, to ſet the Globe ſo that the Stars upon it may correſpond to their Situations in the Heavens.

783. The globe being fixed as in the laſt Article, let the meridian be ſet in the meridian of the place, with the north pole to the north; then will all the ſtars upon the globe correſpond to their places in the heavens, ſo that an eye at the center of the globe would refer every ſtar upon it's ſurface to the place of the ſtar in the heavens. By comparing therefore the ſtars in the heavens with their places on the globe, you will very eaſily get acquainted with all the ſtars.

To find the Time when any of the heavenly Bodies riſe, ſet, or come to the Meridian; alſo their Azimuth at riſing or ſetting.

784. Every thing remaining as in Art. 782, turn the globe till the given body comes to the eaſtern part of the horizon, and the index ſhows the time of it's riſing; bring it to the meridian, and the index ſhows the time of it's coming to the meridian; laſtly, bring the body to the weſtern horizon, and the index ſhows the time of it's ſetting. When the body is in the horizon, the arc upon the horizon between it and the north or ſouth will give it's azimuth. If you thus find the time of the ſun's riſing and ſetting, you get the length of the day. If you turn the globe about it's axis, all thoſe ſtars which do not deſcend below the horizon in a revolution, never ſet in that place; and thoſe which do not come above it, never riſe.

To

To explain, in general, the Alteration of the Length of the Days, and Difference of the Seaſons.

785. Put ſeveral patches upon the ecliptic from Aries both ways to the two tropics; and then the globe being rectified to the latitude of the place, turn it about, and you will ſee, for north latitudes, that as the patches approach the tropic of Cancer, the correſponding diurnal arcs will increaſe; and as the patches approach the tropic of Capricorn, the correſponding diurnal arcs will decreaſe; alſo, the former arcs are greater than a ſemicircle, and the latter leſs; and the patch in the equator will deſcribe a ſemicircle above the horizon. Therefore when the ſun is in the equator, days and nights are equal; as he advances towards the tropic of Cancer, the days increaſe and the nights decreaſe, till he comes to that tropic, where the days are longeſt and the nights ſhorteſt; then as he approaches the equator, the lengths of the days diminiſh and that of the nights increaſe, and when he comes to the equator, there will be again equal days and nights. Then as he advances towards the tropic of Capricorn, the days diminiſh and the nights increaſe, until he comes to that tropic, where the days are ſhorteſt and the nights are longeſt; and then as he approaches the equator, the days increaſe and the nights decreaſe, and when he comes to the equator, the length of the days and nights are equal. Whatever be the latitude of the place, when the ſun is in the equator the days and nights are equal in length. To an inhabitant at the pole, the ſun will appear to be half a year above the horizon, and half a year below. To an inhabitant at the equator, the days and nights will appear to be always equal; alſo, all the the heavenly bodies will be found to be as long above the horizon as below. At the arctic circle, the longeſt day will be found 24 hours, and the longeſt night of the ſame length; this appears, by rectifying the globe to that latitude, and putting the patch, firſt at the tropic of Cancer, and then of Capricorn. Laſtly, it will be found that all places enjoy equally the ſun in reſpect to time, and are equally deprived of it, the length of the days at one time of the year being exactly equal to that of the nights at the oppoſite ſeaſon. This will appear, by putting a patch upon the ecliptic at equal diſtances on each ſide of the equator.

To find the Latitude and Longitude of a given Star, alſo the Diſtance of two Stars.

786. Bring the ſolſtitial colure to the meridian, and fix the quadrant of altitude over the pole of the ecliptic; then turn the quadrant over the given ſtar, and the arc contained between the ſtar and the ecliptic will be the *latitude*, and the degree on the ecliptic cut by it will be the *longitude*. The diſtance of two ſtars may be found, by laying the quadrant of altitude over both, and counting the degrees between.

To explain the Phænomena of the Harveſt Moon.

787. Rectify the globe for any northern latitude, for inſtance, that of London; and as the moon's orbit makes but a ſmall angle with the ecliptic, let us ſuppoſe the ecliptic to repreſent the moon's orbit. Now in September, when the ſun is in the beginning of Libra, the moon, at it's full, is in the beginning of Aries; and as the mean motion of the moon in it's orbit is about 13° in a day, put a patch on the firſt point of Aries, and another at the diſtance of 13° from it; bring the former patch to the horizon, and then turn the globe till the other comes to it, and the motion of the index will ſhow about 17 minutes, which is the difference of the times of riſing on two ſucceſſive nights at that time. This ſmall difference ariſes from the ſmall angle which that point of the ecliptic, or moon's orbit, makes with the horizon at it's riſing If you continue the patches at every 13° till you come to Libra, you will find the difference of the times of riſing will increaſe up to that point, and there the difference will be found to be about 1*h*. 17′; for this point of the ecliptic makes the greateſt angle with the horizon at it's riſing. Hence, whenever the moon comes to the firſt point of Aries, there will be the leaſt difference of the times of it's riſing; and this happens at the time of the full moon, when the full moon happens about September 21. That point of the ecliptic which riſes at the leaſt angle with the horizon will appear to ſet at the greateſt, and therefore when there is the leaſt difference in the times of riſing, there will be the greateſt difference in the times of ſetting; and the contrary.

To find the Time of the Year when a Star riſes or ſets Coſmically *and* Achronically.

788. Rectify the globe to the latitude of the place, and bring the ſtar to the horizon on the eaſt ſide, and ſee what degree of the ecliptic cuts the horizon, and upon the horizon ſeek what day of the month that degree anſwers to, and that is the day when the ſtar riſes *coſmically*; bring the ſtar to the weſtern horizon, and the degree of the ecliptic cut by the horizon will give the day when it ſets *coſmically*. Bring the ſtar to the eaſtern horizon, and the degree of the ecliptic which cuts the weſtern horizon will give the day when the ſtar riſes *achronically*; and if you bring the ſtar to the weſtern horizon, the degree of the ecliptic cut by the eaſtern horizon ſhows the day when the ſtar ſets *achronically*.

To find the Time of the Year when a Star riſes or ſets Heliacally.

789. Having rectified the globe to the latitude of the place, bring the ſtar to the eaſtern horizon, and apply the quadrant of altitude to the weſtern ſide, ſo that it may cut the ecliptic 12° above the horizon, then will the oppoſite point of the ecliptic be 12° below the horizon, and the day correſponding to that point is the day when the ſtar riſes *heliacally*; bring the ſtar to the weſtern horizon, and apply the quadrant of altitude to the eaſtern to cut the ecliptic 12° above the horizon, and the oppoſite point will give the day when the ſtar ſets *heliacally*. This is for a ſtar of the firſt magnitude, which may be ſeen when the ſun is about 12° below the horizon; but for one of the ſecond, third, fourth, fifth, or ſixth magnitude, the ſun muſt be 13°, 14°, 15°, 16° or 17° below the horizon.

790. Since the time of SENEX, no new globes have been made of a large ſize. Meſſrs. W. and S. JONES, Mathematical and Philoſophical Inſtrument Makers in Holborn, London, are therefore now preparing a very fine ſet of new globes of 18 inches diameter, containing all the lateſt aſtronomical and

geographical diſcoveries. The places upon the terreſtrial globe are all laid down by an eminent Geographer, with their names in Engliſh; and upon the celeſtial globe there are about 6000 ſtars very accurately laid down for the year 1800, from the beſt obſervations; to which are added, many cluſters of ſtars, and nebulæ, with the figures of the conſtellations. The ſtars are marked with BAYER's and M. de la CAILLE's letters. An improved apparatus will be added for the ſolution of problems. Theſe globes will be a very valuable acquiſition to the public.

CHAP.

CHAP. XXX.

ON THE DIVISION OF TIME.

Art. 791. THE revolution of the earth about the ſun divides time into aſtronomical *Years*; the revolution of the moon about the earth divides it into aſtronomical *Months*; and the rotation of the earth about it's axis divides it into aſtronomical *Days*; theſe, which are alſo called *Natural Days*, include a common day and night. Theſe natural days are ſubdivided by clocks into *Hours*, *Minutes*, *Seconds*, *&c.* The firſt object in the regulation and diviſion of time is to keep the ſame ſeaſons to the ſame months, ſo that the middle of ſummer may always happen towards the end of June, and the middle of winter towards the end of December. But before the ſun's motion was tolerably well known, it was not eaſy to accompliſh this. Some of the ancients formed a *Lunar* year, conſiſting of 12 ſynodic lunar months, or 354 days, at the end of which they made the year begin again. But finding that this year would not agree with the ſeaſons, to correct it, they firſt added a month every three years; afterwards, three months every eighth year; and laſtly, eight months every 19 years. Theſe were called *Luni-ſolar* years, and were uſed by the Jews and Romans. The *Egyptian* year conſiſted of 365 days; they had 12 months of 30 days each, and then added five days more. The year which NUMA introduced amongſt the Romans was the luni-ſolar year, adding to the lunar year of 354 days, 22 days every two years, inſerting them as an intercalary month after February every other year, according to his life in PLUTARCH. But through the ignorance or negligence of the Prieſts, who had the care of theſe matters, the corrections, called intercalations, neceſſary for preſerving the agreement between the luni-ſolar year and the ſeaſons, were either omitted, or ſo improperly applied, as to produce great diſorders in the Roman calendar. Therefore JULIUS CÆSAR, to whom, when *Pontifex Maximus*, the care of theſe things belonged, reſolved to prevent, as far as he could, the like errors for the future. Accordingly, after having reſtored all the feſtivals to their proper ſeaſons, he, by the aſſiſtance of SOSIGENES, an Aſtronomer of Alexandria, cauſed the old luni-ſolar year of NUMA to be entirely laid aſide, and ſubſtituted, inſtead thereof, the Egyptian ſolar year of 365 days, with the correction of an additional day every four years, it having been found that the true *tropical* year,

by which the ſeaſons are governed, exceeds 365 days by ſix hours. This is called the *Julian* year. To add a day every fourth year, he cauſed the twenty-fourth day of February, which was the ſixth (ſextus) of the calends before March, to be reckoned twice. Hence this year was called *Biſſextile*; it is alſo now commonly called *Leap-Year*. In our calendar, this day is added every fourth year to the end of February. This civil year immediately came into uſe throughout all Europe.

792. But time ſhowed that this correction was not ſo perfect as it was firſt thought to have been; for it was found that the equinoxes and ſolſtices happened earlier by ſome days than they did in ſome former diſtant years; and more accurate obſervations of the ſun diſcovered that the true tropical year was not 365*d*. 6*h*. but 365*d*. 5*h*. 48′. 48″. The tropical year was therefore thought to be longer than it really was by 11′. 12″, which in about 129 years would amount to a whole day, and cauſe the equinoxes to fall ſooner by one day.

793. Pope GREGORY XIII. therefore ſet about the correction, from a deſire that the moveable feaſt of Eaſter ſhould happen as nearly as poſſible at the ſame times of the year reſpectively, with thoſe at which it had been kept for ſome years after the general Council of Nice, which was holden in the year 325. But this could not be corrected without affecting the civil year in ſuch a manner, that the vernal equinox ſhould then, and at all future times, fall on, or as near as poſſible to, March 21, as it did at that general Council, but which had then anticipated by 10 days. For this purpoſe, he cauſed 10 days to be dropped in October 1582, and by this means the vernal equinox was reſtored to March 21. And having conſulted with the Aſtronomers, he ordered that three ſucceſſive centenary years, which, according to the *Julian* account, would have been *biſſextiles*, ſhould be common years, but that every fourth centenary year ſhould be, as it otherwiſe would have been, a *biſſextile* year. By this means, the difference between the *civil* and *tropical* accounts for the ſpace of 400 years will not differ ſo much as two hours, and will not amount to a whole day in leſs than 5082 years, at the end of which time it will be neceſſary to make a correction for this day. The civil year thus corrected took place in moſt parts of Europe many years ago, but it did not take place in England till the year 1752, at which time a correction of 11 days, which, as will appear, then became neceſſary, were applied, and ſo many days were taken from September, the *third* day of that month being called the *fourteenth*. This is called by us the *New Stile*, and that in uſe before, or the *Julian* account, is called the *Old Stile*. As *leap-year* happens every fourth year, and every hundredth year was a leap-year in the *Julian* account, therefore every year which is diviſible by 4, became a leap-year. Now theſe centenary years,

years, which, in the *Gregorian* account, are not to be leap-years, are 1700, 1800, 1900, 2100, 2200, 2300, 2500, 2600, 2700, &c. therefore as the year 1700 happened between the time of the correction by GREGORY, and that made by us, the *Gregorian* account had left out one day in that year which the *Julian* had not; therefore the *Gregorian* account having, at the time it took place, left out 10 days, we were obliged to leave out 11 days to bring our account to agree with that.

794. Amongſt different nations, the *beginning* of the year varied, as well as the *length*. The *Jews* began their *eccleſiaſtical* year with the new moon of that month, whoſe full moon happened next after the vernal equinox. The *Church of Rome* begin their year on the Sunday that falls upon the ſaid full moon, or that happens next after it; or on Eaſter Sunday. The *Jews* begin their *civil* year with the new moon which has it's full moon happening next after the autumnal equinox. The *Grecians* began their year with the new moon which happened next after the ſummer ſolſtice. The *Romans*, according to PLUTARCH, began their year at March, from the time of ROMULUS to NUMA, who changed the beginning to January. ROMULUS made the year conſiſt of only ten months, as appears from the name of the laſt, *December*, or the tenth month; and that *March* was the firſt, is evident, becauſe they called the fifth from it *Quintilis*, the ſixth *Sextilis*, and the reſt in their order. Moreover, it is very reaſonable to think that *March*, dedicated to *Mars*, would ſtand firſt; and that *April*, which has it's name from *Aphrodite*, or *Venus*, would ſtand ſecond. Although ſome, as PLUTARCH obſerves, think it comes from *aperire*, TO OPEN; that being the time when the bloſſoms begin to open. The firſt month of the *Egyptian* year began on our Auguſt 29. The *Arabic* and *Turkiſh* year began on July 16. The ancient Clergy made March 25, the beginning of the year.

795. The firſt diviſion of the civil year is into civil months, of which there are twelve. Theſe cannot be of an equal length, becauſe the number of days in a year is not diviſible by 12. There are therefore in every year, *ſeven* months of 31 days each, *four* of 30 days each, and in the common years *one* of 28 days, but which contains 29 in every leap-year. Theſe are the months which are formed for civil purpoſes. But the ſpace of 28 days is alſo called a month, and it is by the diviſion of this into four equal parts, that the year is ſubdivided into *Weeks*. A week therefore conſiſts of ſeven days. Hence a common year conſiſts of 13 of theſe months, or 52 weeks, and 1 day, and a leap-year of the ſame and two days.

796. The days into which the civil year is divided are called *Natural*, and contain 24 hours. But there is a day called *Artificial*, which is the time from ſun riſe to ſun ſet. The natural day is either *aſtronomical* or *civil*. The

aſtronomical

aſtronomical day begins at noon. The *Britiſh*, *French*, *Dutch*, *Germans*, *Spaniards*, *Portugueſe*, and *Egyptians*, begin the civil day at midnight; the ancient *Greeks*, *Jews*, *Bohemians*, and *Sileſians*, began it at ſun-ſetting, as do the modern *Italians* and *Chineſe*; and the ancient *Babylonians*, *Perſians*, *Syrians*, with the modern *Greeks*, at ſun-riſing. A *natural* day is divided into 24 equal parts, called *Hours*; theſe hours are, by almoſt all nations, divided into 60 equal parts, called *Minutes*, and theſe ſubdivided into 60 equal parts, called *Seconds*, *&c.* But the *Jews*, *Chaldeans*, and *Arabians*, divided the hour into 1080 equal parts, called *Scruples*.

797. As Aſtronomers begin their computations of the motions of the heavenly bodies from ſome fixed points, ſo there are certain points of time from which hiſtorians begin to reckon. Theſe points of time are called *Epochs*, or *Æras*, and generally ariſe from ſome remarkable event. The firſt *Æra* is the *Creation of the World*. Hiſtorians differ a little in their eſtimation of this time, making it from 3950 to about 4000 years before CHRIST. The *Æra* of the *Olympiads* is the moſt famous of the profane ones, which is placed 776 years before CHRIST, and from which the *Greeks* reckoned. The *Æra* of the *Building of Rome* was 752 years before CHRIST, which the *Romans* uſed. The *Æra* of NABONASSAR was 747 years before CHRIST, from which time the *Chaldeans* and *Egyptians* reckoned their years. The *Æra* which we make uſe of is called the *Chriſtian Æra*, becauſe it began at the *Birth of* CHRIST; not indeed on the very day he was born, but ſeven days after, for the day of his nativity was on December 25, and we begin our reckoning from January 1, the next year. The *Æra* of the *Julian* year was 45 years before this, when JULIUS CÆSAR rejected the old *Roman* year, and ordered the *Julian* year to be obſerved all over the Roman empire. The *Turkiſh Æra* is the *Hegira*, or *Flight of* MAHOMET, 622, A.C. The *Perſian Æra* is called *Yeſdegird*, 631, A.C.

798. But beſides the meaſures of time by *Years*, *&c.* it was found convenient to introduce the uſe of *Cycles*, that is a circulation of time between the returns of the ſame event. The *Cycle of the Sun* is a ſpace of 28 years, in which time the days of the month return again to the ſame days of the week, and the ſun's place to the ſame degrees of the ecliptic on the ſame days, ſo as not to differ 1° in 100 years; and the leap-years return again in reſpect to the days of the week on which the days of the months fall. Theſe things ariſe from hence. If 365, the number of days in a common year, be divided by 7, there remains 1, which ſhows that the laſt day of the year is the ſame day of the week as the firſt. Now it is cuſtomary to place againſt the ſeven days of the week, the firſt ſeven letters of the alphabet *A*, *B*, *C*, *D*, *E*, *F*, *G*, placing *A* always againſt the firſt day of the year, and therefore as they were continued through the year, it is evident that the ſame letter *A* muſt ſtand againſt the

firſt

firſt and laſt days of the year. Hence, if the *firſt* of January be a *Sunday*, and *A* ſtands againſt it, then will *A* point out every Sunday in that year. But as the firſt day of the next year will be on a *Monday*, againſt which *A* is placed, *G* will ſtand againſt the firſt Sunday, and conſequently againſt every Sunday in that year. For the ſame reaſon, as the firſt day of the next year is a *Tueſday* and marked with an *A*, *F* will ſtand againſt every Sunday in the year; and ſo on. Therefore the *Sunday Letters* will come on in a retrograde order, *A*, *G*, *F*, *E*, *D*, *C*, *B*, in the ſucceſſive years; hence theſe are called *Dominical* letters. This would be the caſe if all the years were of 365 days; but in *Leap-Year* there are 366 days, and the additional day thus taken is marked with the ſame letter in order that *A* may be the laſt letter of the year, which neceſſarily throws the Sunday letter one letter back for the reſt of the year. Hence, in every leap-year there are two *dominical letters*. As therefore the regular change of the Sunday letter which would be otherwiſe completed in ſeven years, is thus interrupted every four years, the whole change will not be compleated in leſs than 7×4, or 28 years. But this will be ſometimes interrupted, becauſe every three centenary years out of four are not leap-years. The dominical letter however may be found by Table I. (given by Mr. FERGUSON in his *Aſtronomy)* for every year of the *New Stile* to the year 4000. The year of our Saviour's birth, was the ninth of this cycle; therefore to find the year of this cycle, add 9 to any year and divide the ſum by 28, and the quotient ſhows the number of cycles elapſed ſince his birth, and the remainder is the cycle for the year; if nothing remains, the cycle is 28.

799. The *Cycle of the Moon*, ſometimes called the *Metonic Cycle* from the inventor METON, is a revolution of 19 years, in which time, the conjunctions, oppoſitions, and all other aſpects of the moon, return on the ſame days of the months as they did 19 years before, but about $1\frac{1}{2}$ hour ſooner. The ancients formed this *cycle* thus. Taking any year for the cycle, they obſerved all the days on which the new moons happened through the year, and againſt each ſuch day they placed the number 1; in the ſecond year of the cycle they did the ſame, placing the number 2; and proceded in like manner through the cycle of 19 years. This being done for one cycle, the ſame numbers were fitted to the calendar to ſhow the new moons in every future cycle; and, upon account of their excellent uſe, they were written in *Gold*, and from thence called *Golden Numbers*. But the difference of about $1\frac{1}{2}$ hour in 19 years increaſes to a whole day in about 312 years, ſo that this cycle can only hold for that time. For as the new and full moons anticipate a day in that time, the Golden Numbers ought to be placed one day earlier in the calendar for the next 312 years to come. It was thought proper however to make this correction at the end of whole centuries; accordingly they put the new moon

forward

forward one day at the end of each 300 years for ſeven times ſucceſſively, which makes 2100 years; and to account for the odd 12½ years, they deferred putting the moon forward to the end of 400 years, making the period of 2500 years = 8 × 312½. The *Golden Numbers* were properly placed by the Council of Nice, A. D. 325; the anticipation, which has been neglected ever ſince, is now become almoſt five days, and therefore all the Golden Numbers ought now to be placed five days higher in the calendar for the *Old Stile*, than they were at the above-mentioned Council; or ſix days lower for the *New Stile*. But becauſe the lunar cycle of 19 years ſometimes includes five leap-years and ſometimes four, it is impoſſible to have a correct Table of all the numbers unleſs it be extended to four times 19, or 76 years, in which there are 19 leap-years without a remainder. And in this caſe it muſt be adapted to the Old Stile, becauſe in every centenary year not diviſible by 4, the regular courſe of the leap-year is interrupted in the New Stile. Table II. (given by Mr. FERGUSON) ſhows the *Golden Number* from the *Chriſtian Æra* to the year 3800.

800. The *Epact* is the moon's age in days at the beginning of the year. Let a new moon happen on January 1, then the *Epact* is nothing. Now as 12 lunations are completed in 354 days, it is plain that the epact, or moon's age would be 11 at the beginning of the ſecond year; 22 at the beginning of the third; and 33 at the beginning of the fourth. But as one lunation is never more than 29½ days, the epact muſt always be leſs than 30; therefore ſubtracting 30 from 33 there remains 3 for the epact for the fourth year. And by proceeding thus for 19 years the epacts will ſtand thus, 0, 11, 22, 3, 14, 25, 6, 17, 28, 9, 20, 1, 12, 23, 4, 15, 26, 7, 18, 0; in the nineteenth year the difference amounts to 29 days, and therefore the month which is ſubtracted muſt conſiſt only of 29 days in order that the epact may begin again, as it muſt, the new moon falling then on January 1. Theſe epacts being placed againſt the days of the months in the calendar, on which the new moons fall in each year, anſwer the ſame purpoſe as the Golden Numbers. But it is liable to be interrupted every 310 years for the ſame reaſon, the new moon having then anticipated a whole day, and therefore on the firſt year of the cycle the moon would be one day old on the firſt of January; therefore the epact would be increaſed by 1, and ſtand thus, 1, 12, 23, 4, &c. But this arrangement would be interrupted by the omiſſion of the leap-year at the end of every three centuries out of four; for theſe years being a day leſs than by the *Julian* account, the new moons would happen a day later, and conſequently make the epact one leſs. The moon's age here ſuppoſed is that of the mean new moon, and conſequently it does not agree with the true age of the moon as computed in our Ephemeris, which ſometimes

may

may differ a day; according to the rule therefore by which we from hence find Eaſter, that feſtival is not always found to agree with the time deduced from the new moon, as put down in our Almanacs. In the correction of the Britiſh Calendar, we uſe the Golden Numbers, omitting the Epacts; and have placed the Golden Numbers, not againſt the days of the new moon, but of the full moons, and only againſt the full moons in the paſchal months, March and April, in order to find Eaſter.

801. The *Indiction* is a cycle of 15 years, but has no dependance on the motion of the heavenly bodies. It was uſed by the Romans for indicating the times of certain payments, made by the ſubjects to the Republic, and was eſtabliſhed by CONSTANTINE in the year 312. Why it was confined to 15 years, or upon what occaſion it was inſtituted, is not known.

802. From the multiplication of the *Solar* cycle of 28 years into the *Lunar* of 19 and *Indiction* of 15, ariſes the *Julian* period of 7980 years, in which time they all return again in the ſame order. Now the *Dionyſian*, or vulgar æra of CHRIST's birth was at the end of the year of the Julian period 4713, and therefore the firſt year of his age was the 4714th year of the ſaid period. Therefore if to any year of CHRIST we add 4713, it gives the year of the Julian period; or ſubtract, for any time *before* CHRIST. The year of our Saviour's birth, according to the vulgar æra, was the ninth of the *Solar Cycle*, and the firſt year of the *Lunar Cycle*. Hence, to find at any time the year of the *Solar* cycle, add 9 to the given year and divide the ſum by 28, and the quotient is the number of cycles elapſed ſince his birth, and the remainder is the cycle for the given year, and if nothing remain, the cycle is 28. And to find the *Lunar* cycle, add 1 to the given year of CHRIST and divide the ſum by 19, and the quotient is the number of cycles elapſed from the time of CHRIST, and the remainder is the cycle for the given year, or the Golden Number, and if nothing remain, 19 is the cycle. The 312th year after the birth of CHRIST was the firſt of the Roman *Indiction*; ſubtract therefore 312 from the given year, and divide the difference by 15, and what remains is the Indiction for the given year, and if nothing remain, the Indiction is 15.

803. The *Cycle of Eaſter*, called alſo the *Dionyſian* period, is the product of the *Solar* and *Lunar* cycles of 28 and 19 years = 532 years. If the new moons did not anticipate upon this cycle, as in Art. 799. *Eaſter Day* would always be the Sunday next after the firſt full moon which follows March 21. But on account of that, before the alteration of the Stile, the *Eccleſiaſtical Eaſter* happened, within this laſt century, a week different from the *true Eaſter*. But this is now remedied in the *Common Prayer Book*, by making the Table, which uſed to find *Eaſter for ever*, of no longer uſe than the lunar difference will admit of.

804. The *earlieſt* Eaſter is March 22, and the *lateſt* is April 25; for *Eaſter Sunday* is always the firſt Sunday after the full moon, which happens *upon* or *next* after March 21. Within theſe limits there are 35 days, and the number belonging to each is called the *Number of Direction*. To find this number, take the *Dominical Letter* for the given year from Table I. and from Table II. take out the *Golden Number*; laſtly, with theſe enter Table III. and you get the *Number of Direction*; which added to March 21, gives the day of *Eaſter Sunday*. Thus it appears that the Golden Number for 1757 is 10, and the Sunday Letter is *B*; hence the *Number of Direction* is 20, which added to March 21, gives April 10, for *Eaſter Sunday* that year. In *Leap-Year*, there are two *Dominical Letters*, the firſt of which takes place before February 29, and the laſt, for the following part of the year.

TABLE

The Dominical Letter, New Stile, for 4000 Years after the Chriſtian Æra.

After CHRIST.				Hundreds of Years.			
Years leſs than an Hundred.				100 200 300 400 500 600 700 800 900 1000 1100 1200 1300 1400 1500 1600 1700 1800 1900 2000 2100 2200 2300 2400 2500 2600 2700 2800 2900 3000 3100 3200 3300 3400 3500 3600 3700 3800 3900 4000		
				C	E	G	B A
1	29	57	85	B	D	F	G
2	30	58	86	A	C	E	F
3	31	59	87	G	B	D	E
4	32	60	88	F E	A G	C B	D C
5	33	61	89	D	F	A	B
6	34	62	90	C	E	G	A
7	35	63	91	B	D	F	G
8	36	64	92	A G	C B	E D	F E
9	37	65	93	F	A	C	D
10	38	66	94	E	G	B	C
11	39	67	95	D	F	A	B
12	40	68	96	C B	E D	G F	A G
13	41	69	97	A	C	E	F
14	42	70	98	G	B	D	E
15	43	71	99	F	A	C	D
16	44	72		E D	G F	B A	C B
17	45	73		C	E	G	A
18	46	74		B	D	F	G
19	47	75		A	C	E	F
20	48	76		G F	B A	D C	E D
21	49	77		E	G	B	C
22	50	78		D	F	A	B
23	51	79		C	E	G	A
24	52	80		B A	D C	F E	G F
25	53	81		G	B	D	E
26	54	82		F	A	C	D
27	55	83		E	G	B	C
28	56	84		D C	F E	A G	B A

TABLE II.

SHOWING

THE GOLDEN NUMBER

(Which is the ſame both in the Old and New Stiles)

FROM THE CHRISTIAN ÆRA TO A.D. 3800.

		Years leſs than an Hundred.																		
Hundreds of Years.		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
		19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37
		38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56
		57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
		76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94
		95	96	97	98	99														
0	1900	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
100	2000	6	7	8	9	10	11	12	13	14	15	16	17	18	19	1	2	3	4	5
200	2100	11	12	13	14	15	16	17	18	19	1	2	3	4	5	6	7	8	9	10
300	2200	16	17	18	19	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
400	2300	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	1
500	2400	7	8	9	10	11	12	13	14	15	16	17	18	19	1	2	3	4	5	6
600	2500	12	13	14	15	16	17	18	19	1	2	3	4	5	6	7	8	9	10	11
700	2600	17	18	19	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
800	2700	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	1	2
900	2800	8	9	10	11	12	13	14	15	16	17	18	19	1	2	3	4	5	6	7
1000	2900	13	14	15	16	17	18	19	1	2	3	4	5	6	7	8	9	10	11	12
1100	3000	18	19	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1200	3100	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	1	2	3
1300	3200	9	10	11	12	13	14	15	16	17	18	19	1	2	3	4	5	6	7	8
1400	3300	14	15	16	17	18	19	1	2	3	4	5	6	7	8	9	10	11	12	13
1500	3400	19	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1600	3500	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	1	2	3	4
1700	3600	10	11	12	13	14	15	16	17	18	19	1	2	3	4	5	6	7	8	9
1800	3700	15	16	17	18	19	1	2	3	4	5	6	7	8	9	10	11	12	13	14

TABLE III.

SHOWING

THE NUMBER OF DIRECTION,

For finding Eaſter Sunday by the Golden Number and Dominical Letter.

Dominical Letters.	Golden Numbers.																		
	1 =	2 =	3 =	4 =	5 =	6 =	7 =	8 =	9 =	10 =	11 =	12 =	13 =	14 =	15 =	16 =	17 =	18 =	19 =
A	26	19	5	26	12	33	19	12	26	19	5	26	12	5	26	12	33	19	12
B	27	13	6	27	13	34	20	13	27	20	6	27	13	6	20	13	34	20	6
C	28	14	7	21	14	35	21	7	28	21	7	28	14	7	21	14	28	21	7
D	29	15	8	22	15	29	22	8	29	15	8	29	15	1	22	15	29	22	8
E	30	16	2	23	16	30	23	9	30	16	9	23	16	2	23	9	30	23	9
F	24	17	3	24	10	31	24	10	31	17	10	24	17	3	24	10	31	17	10
G	25	18	4	25	11	32	18	11	32	18	4	25	18	4	25	11	32	18	11

This Table is adapted to the New Stile.

BRITISH 2 AU92 MUSEUM

ADDENDA.

ADDENDA.

THE Rule in Art. 107. may be thus proved. Let l = the latitude of the place, d = the ſun's declination, h = the ſun's altitude, x = the hour angle from noon; then by Spherical Trigonometry, ſin. h — ſin. $l \times$ ſin. d — coſ. $l \times$ coſ. $d \times$ coſ. $x = 0$; and by ſuppoſing l and h to remain the ſame, and d and x to vary, we have, by taking the fluxion, $\overline{\text{ſin. } l \times \text{coſ. } d - \text{coſ. } l \times \text{ſin. } d \times \text{coſ. } x} \times \dot{d} - \text{coſ. } l \times \text{coſ. } d \times \text{ſin } x \times \dot{x} = 0$, hence $x = \overline{\frac{\text{tan. } l}{\text{ſin. } x} - \frac{\text{tan. } d}{\text{tan. } x}} \times \dot{d}$, and this being true for the fluxions, it will be nearly true for the increments. If the declination be ſouth, the ſign of the ſecond term becomes +, the latitude being north; if the latitude be ſouth, + is uſed when the declination is north, and — when ſouth. This appears from attending to the ſigns of the quantities in the different circumſtances. As this is the variation of the hour angle in the whole time between the obſervations ariſing from the variation of the declination, it's half muſt be applied to the middle interval in order to find the time of apparent noon.

In like manner, all ſpherical Problems reſpecting the cotemporary variations of quantities may be very readily ſolved, by taking the trigonometrical equation involving the given quantities, and putting it's fluxion = 0, making thoſe quantities only variable which are ſuppoſed to vary.

For example, in Art. 93. if we aſſume the ſame trigonometrical expreſſion, and make h and x vary, we have, coſ. $h \times \dot{h}$ — coſ. $l \times$ coſ. $d \times$ ſin. $x \times \dot{x} = 0$, hence $\dot{x} = \frac{\text{coſ. } h \times \dot{h}}{\text{coſ. } l \times \text{coſ. } d \times \text{ſin. } x}$; but (as the ſines of the ſides are as the ſines of the oppoſite angles) coſ h : coſ. d :: ſin. x : ſin. azim. $= \frac{\text{coſ. } d \times \text{ſin. } x}{\text{coſ. } h}$; hence $\dot{x} = \frac{\dot{h}}{\text{ſin. azim.} \times \text{coſ. } l}$.

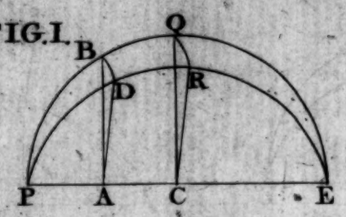
In page 416, before Art. 623. add the following directions for finding the poſition of the zenith of the place. Deſcribe a ſmall circle upon paper, repreſenting the ſun's diſc, and draw a line upon it repreſenting the path of *Venus*, and let the center of the circle be laid upon a globe on the ſun's place in the ecliptic, with the path of Venus pointing to her deſcending node; then from the horary angle and the latitude of the place, the ſituation of the place upon the globe in reſpect to the ſun and Venus will immediately appear.

CORRIGENDA.

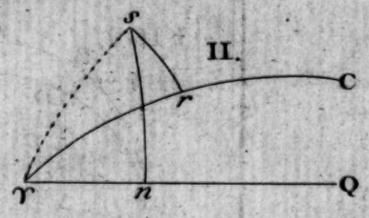
PAGE 18. line 18. for *a* read a.
21. line 20. after *visible* put a *semicolon*, and instead of *so that*, put, *draw a d e parallel to E Q and*.
32. line 5. and 8. for *a g c e* read *a g c d*.
56. last line, for *heroidical* read *spheroidical*.
57. last line but four, for 2′ read 2″.
71. line 15. for *parallax computed for* read *longitude seen from*.
112. to Art. 129. add, *the same is true at the perihelion Q*.
144. in the Table, col. 3. for 35″,5 read 34″,5. And it must be observed, that the mean longitudes here put down are for the meridian of Paris.
196. last line but one, for *y* read *g*.
197. line 10. and 12. for *y* read *g*,
240. for Dr. POUND read Mr. POUND; also Art. 418. is put twice.
249. in the margin against line 8. put FIG. 99.
252. in the margin against the last line but one, put FIG. 100.
298. last line for 25″ read 18″.
316. line 20. for *v* read *V*.
336. line 18. for 3 read 10, and for 10 read 3.
360. line 3. for *SM* read *SN*.
429. line 23. for *exibits* read *exhibits*.
432. line 12. for *impractible* read *impracticable*; and line 26. for *a n, a p*, read *b n, c p*.
476. in the Table, col. 2. for 161°. 44′. 41″,1 read 161°. 43′. 44″,1.

BRITISH MUSEUM 2 AUG 92

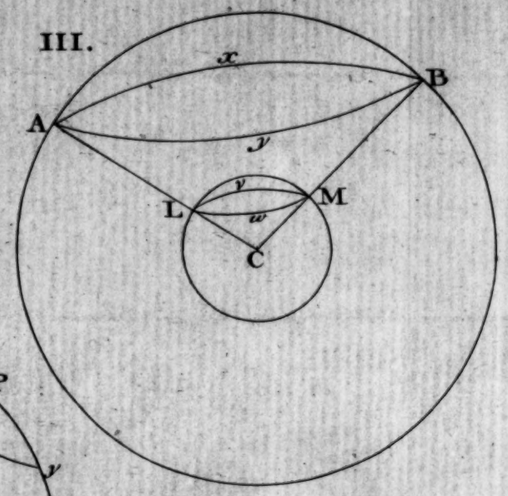
FIG. I.



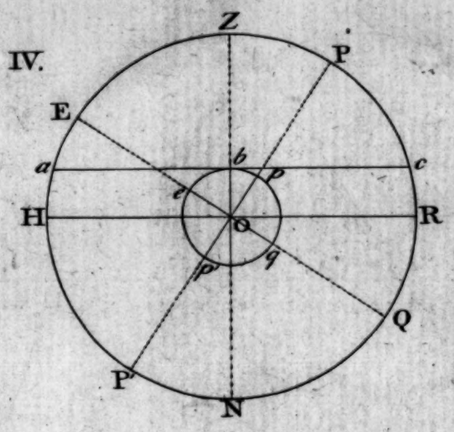
II.



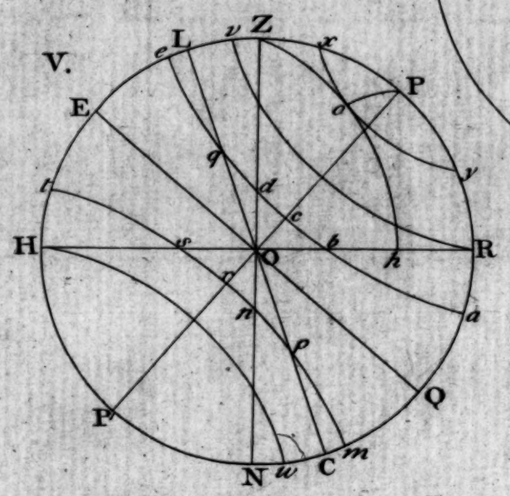
III.



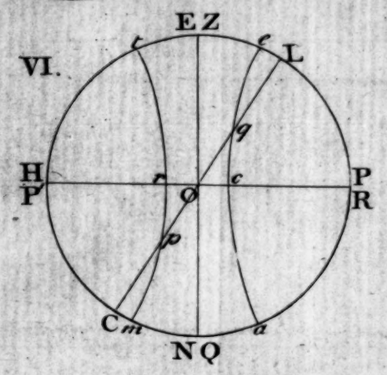
IV.



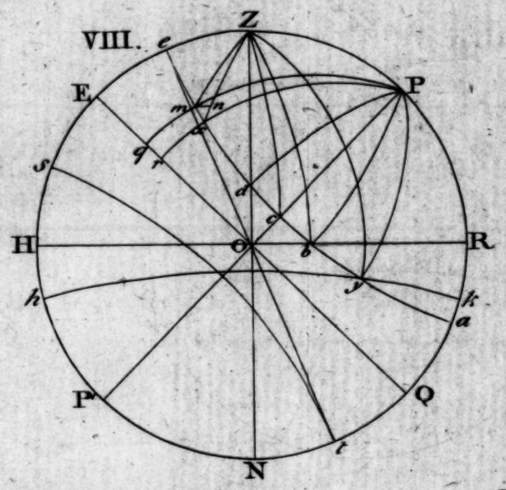
V.



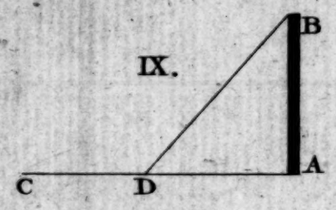
VI.



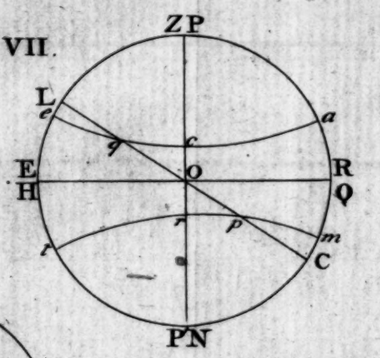
VIII.



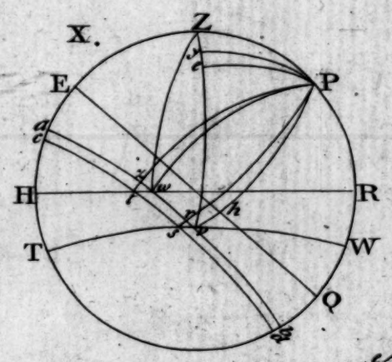
IX.



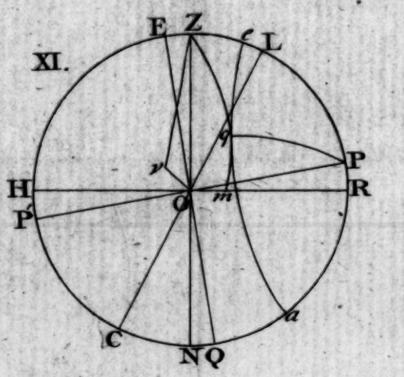
VII.



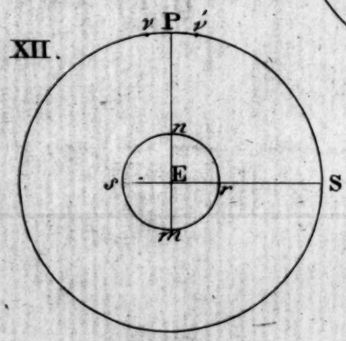
X.



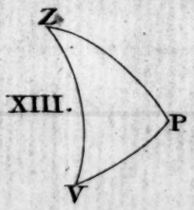
XI.



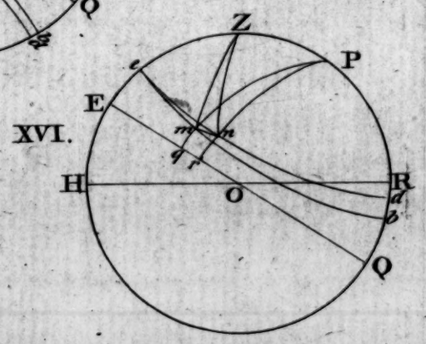
XII.



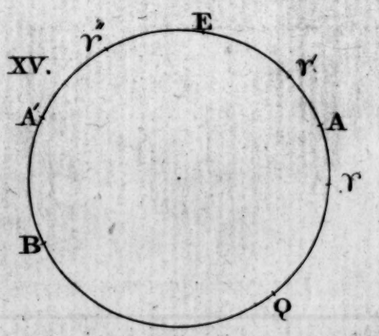
XIII.



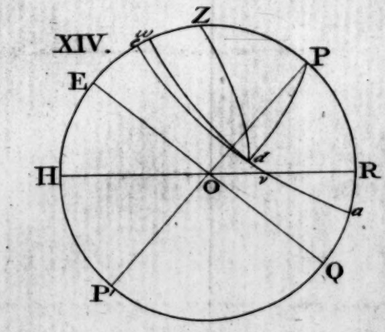
XVI.



XV.

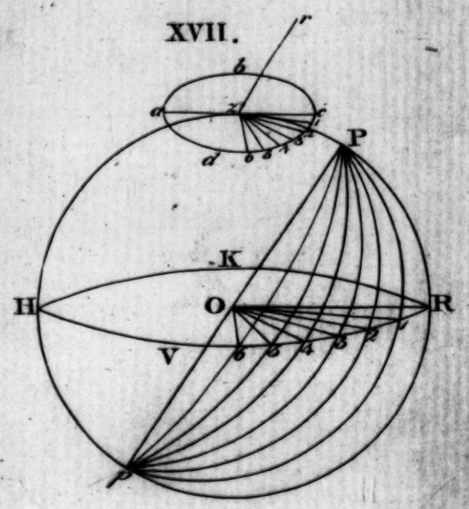


XIV.

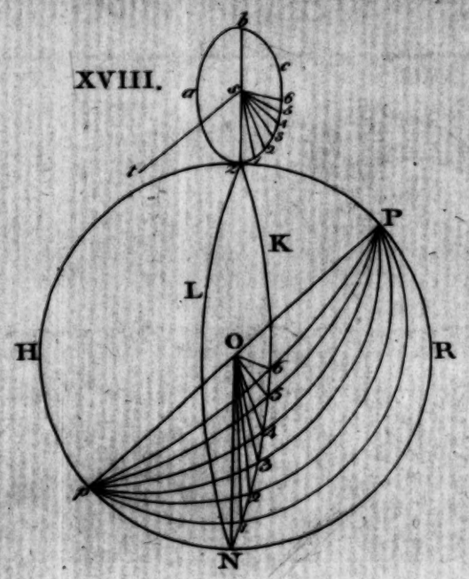


Ashby Sculp. London.

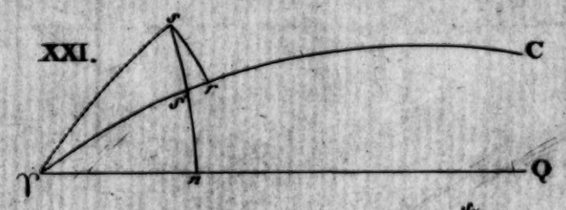
XVII.



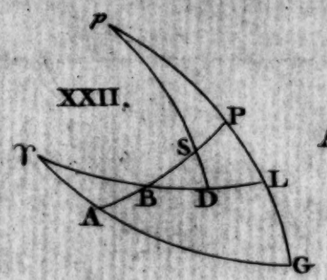
XVIII.



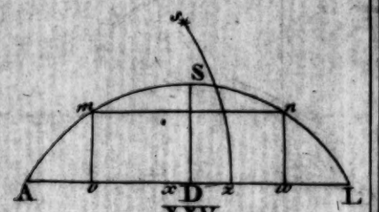
XXI.



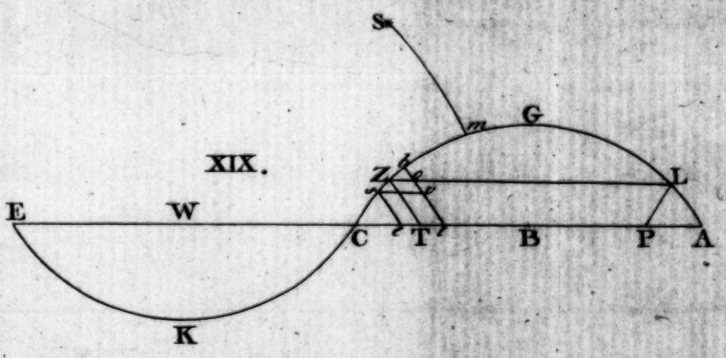
XXII.



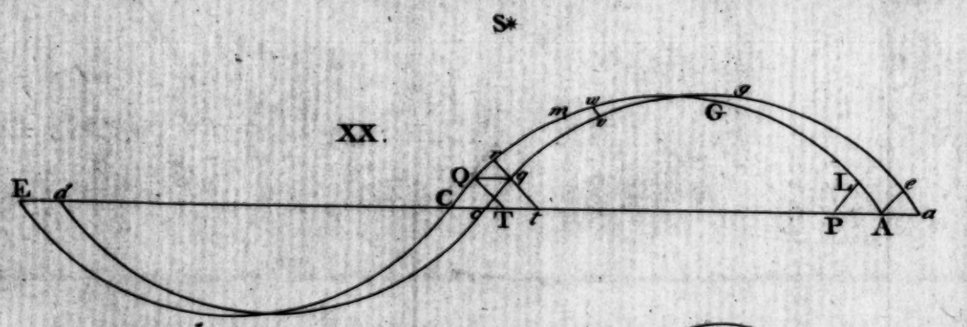
XXV.



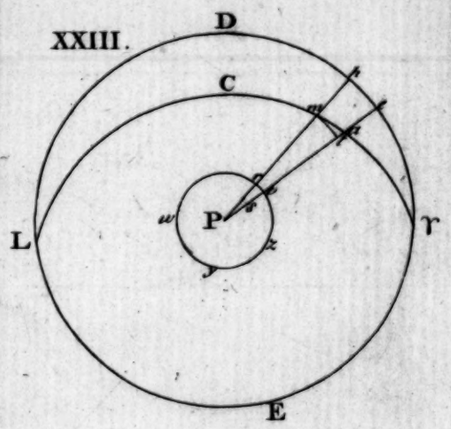
XIX.



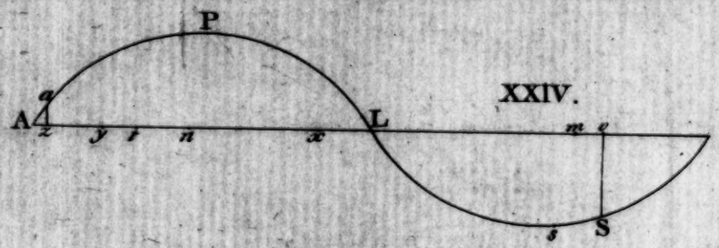
XX.



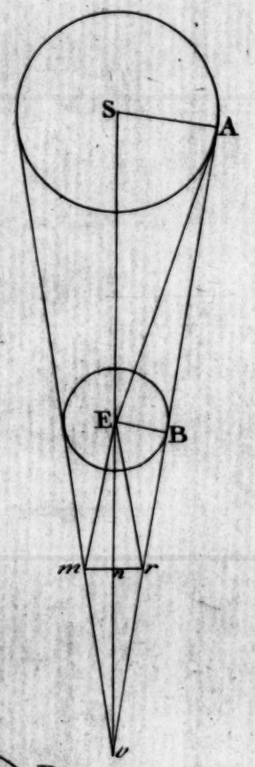
XXIII.



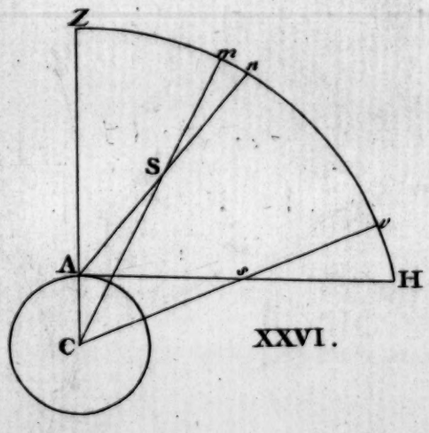
XXIV.



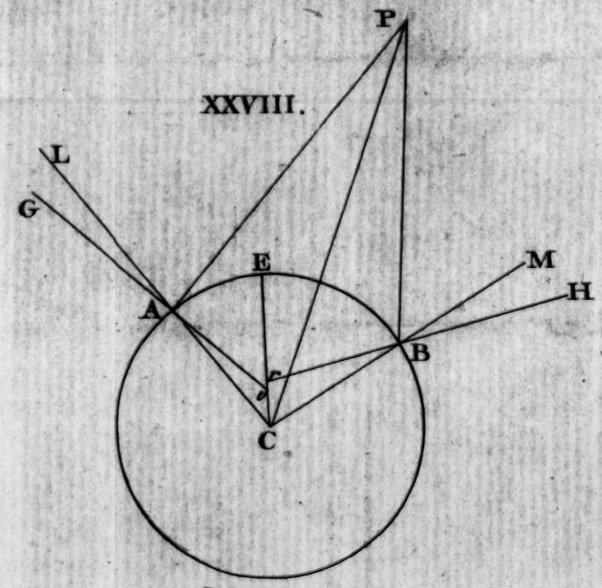
XXVII.



XXVI.



XXVIII.



XXIX.

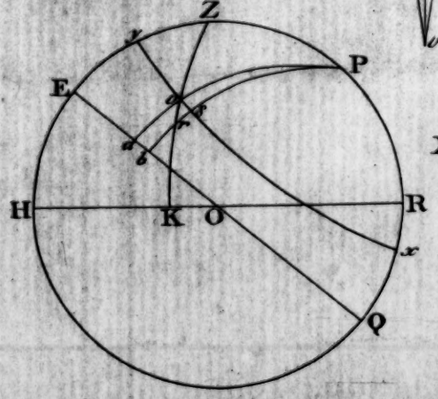
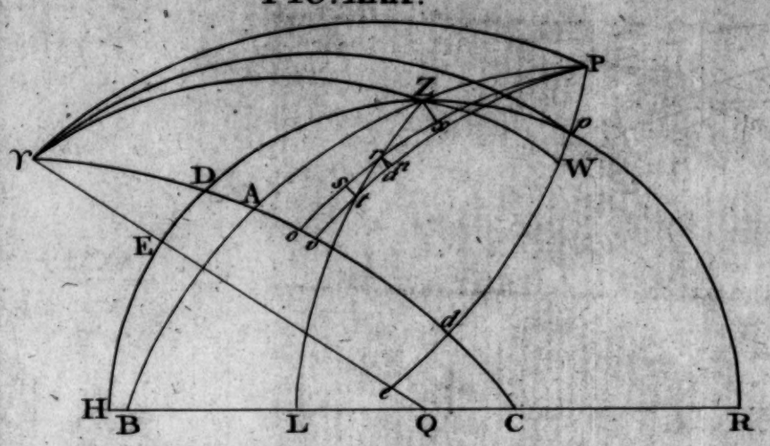
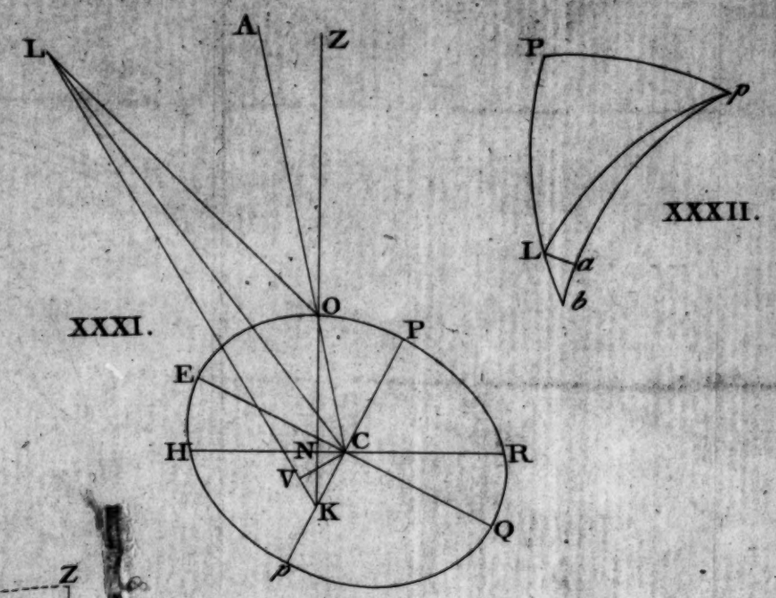


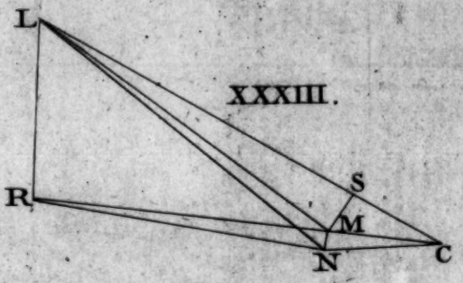
FIG. XXX.



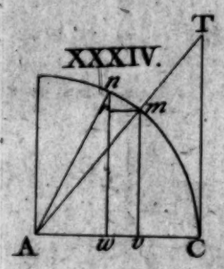


XXXI.

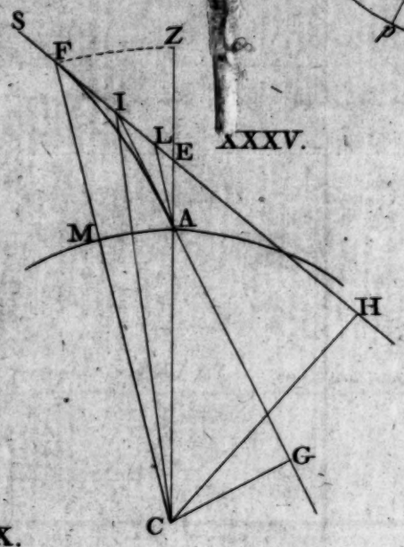
XXXII.



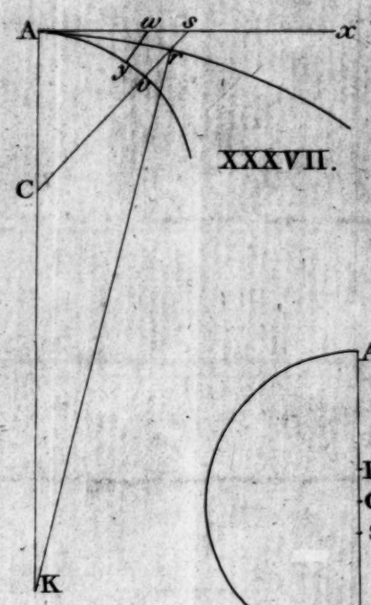
XXXIII.



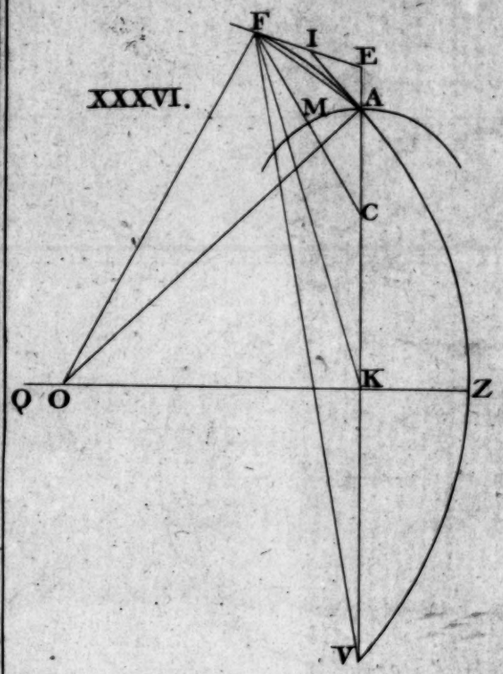
XXXIV.



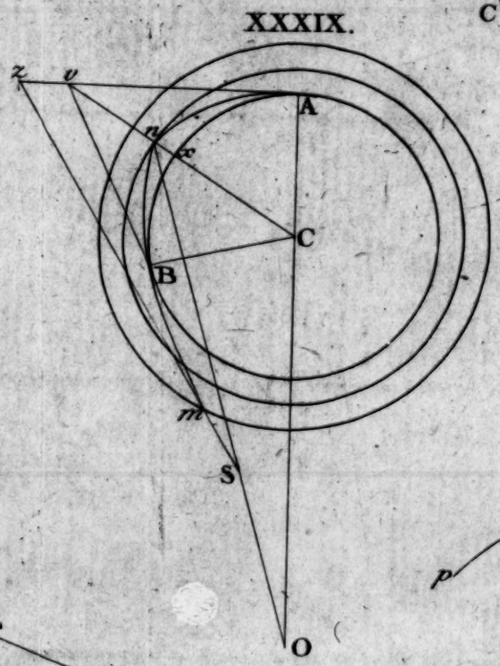
XXXV.



XXXVII.

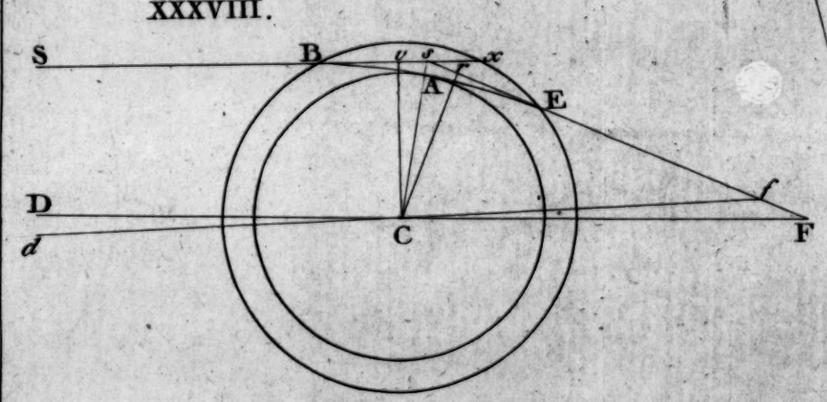


XXXVI.

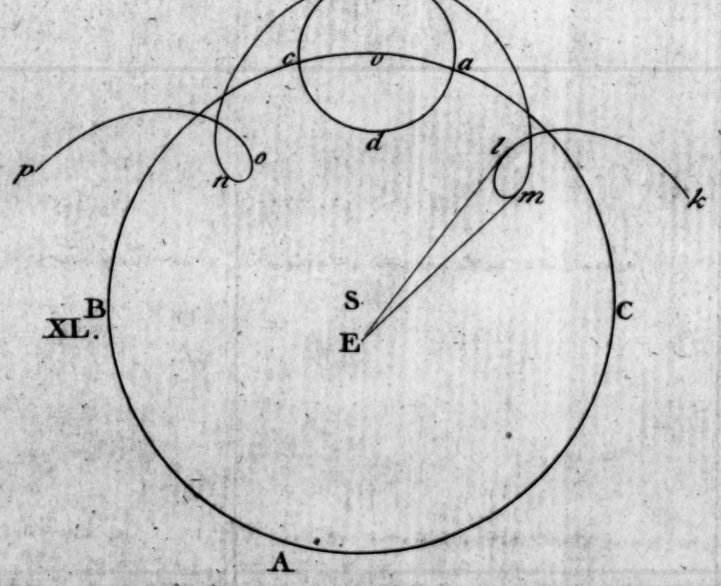


XXXIX.

XLI.

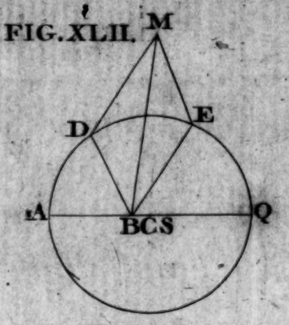


XXXVIII.

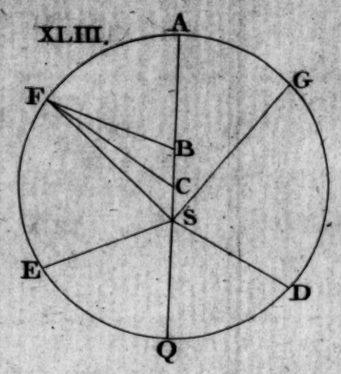


XL.

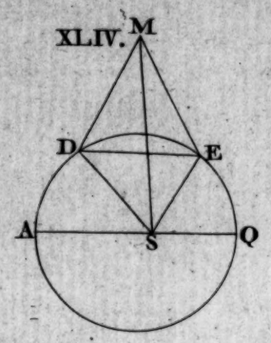
FIG. XLII.



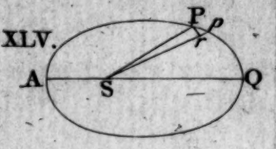
XLIII.



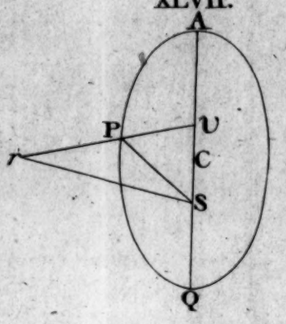
XLIV.



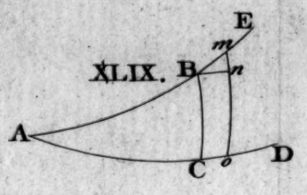
XLV.



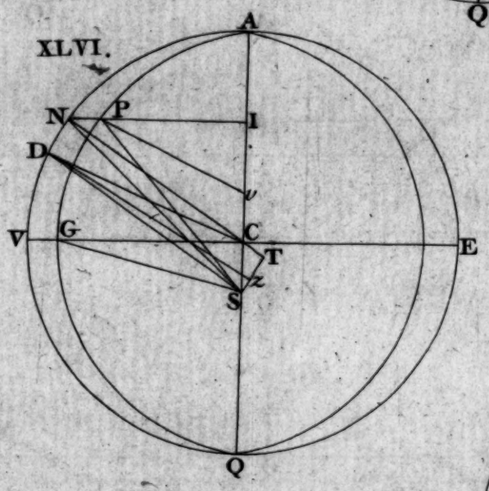
XLVII.



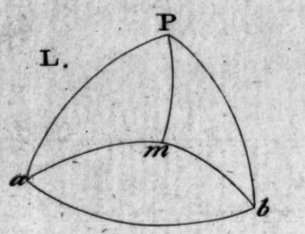
XLIX.



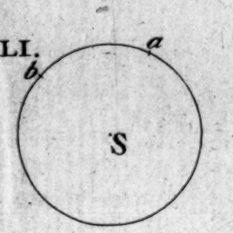
XLVI.



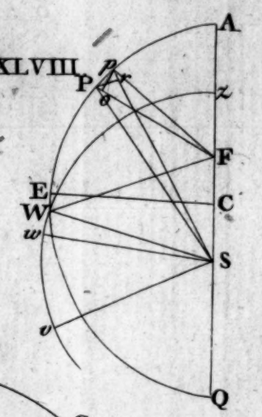
L.



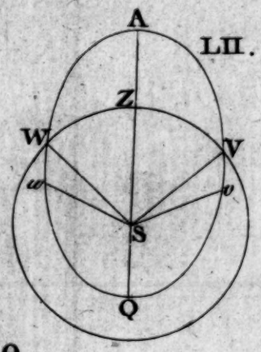
LI.



XLVIII.



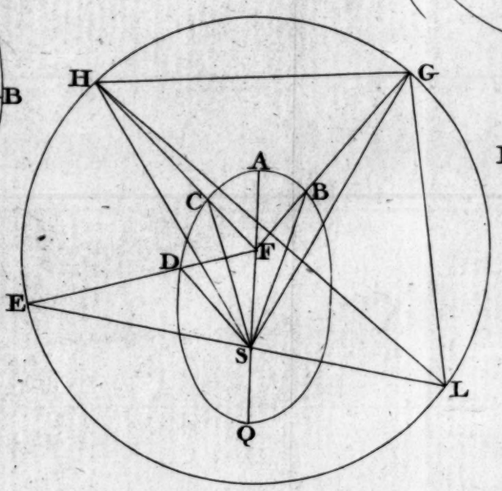
LII.



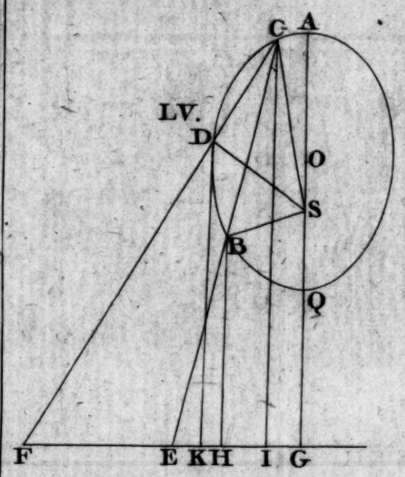
LIII.



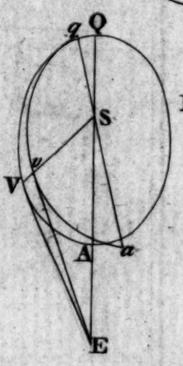
LIV.



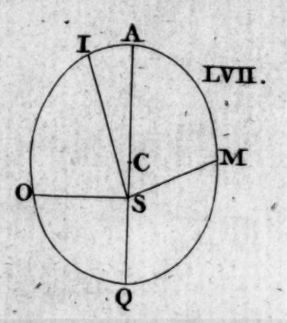
LV.



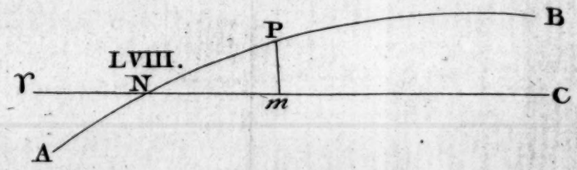
LVI.



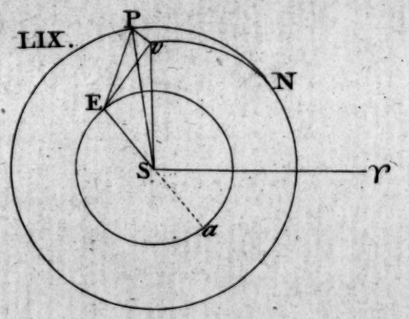
LVII.



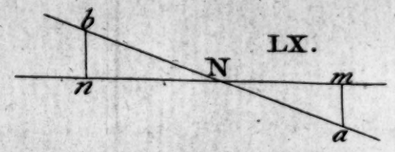
LVIII.



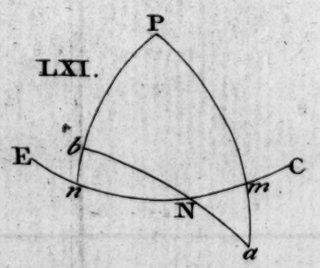
LIX.



LX.



LXI.



LXII.

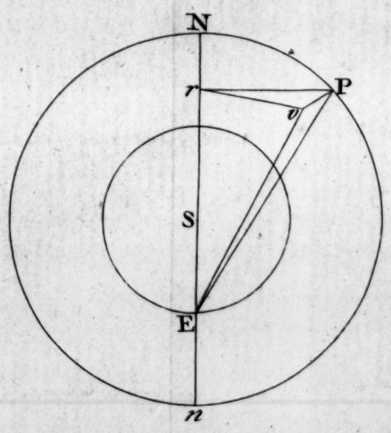
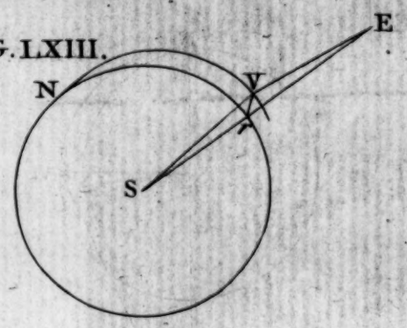
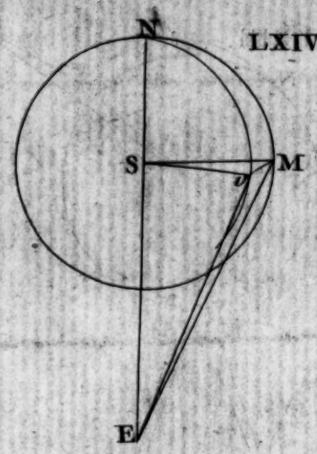


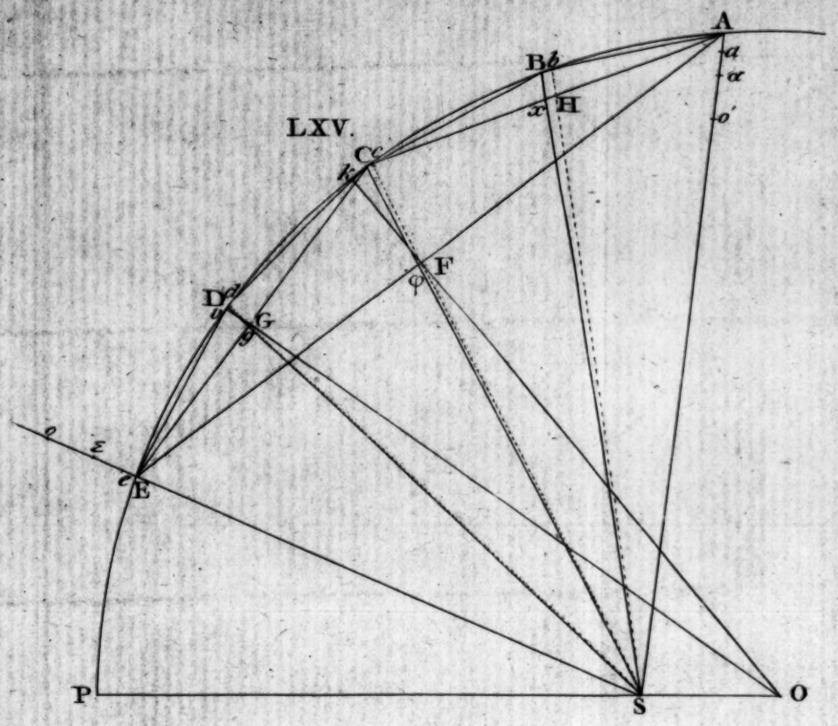
FIG. LXIII.



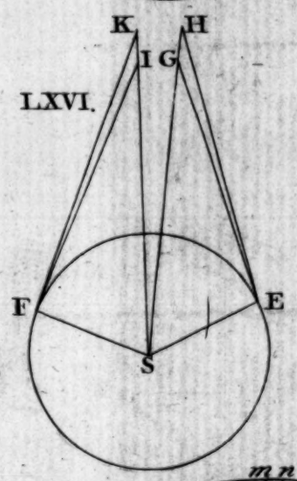
LXIV.



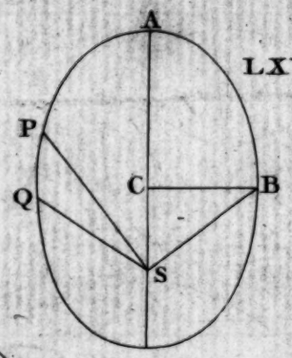
LXV.



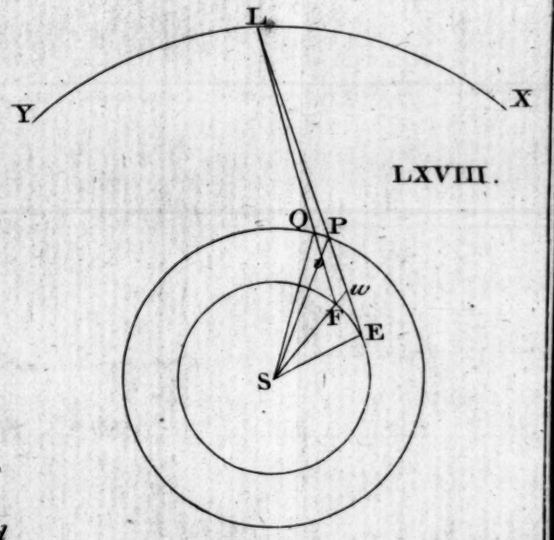
LXVI.



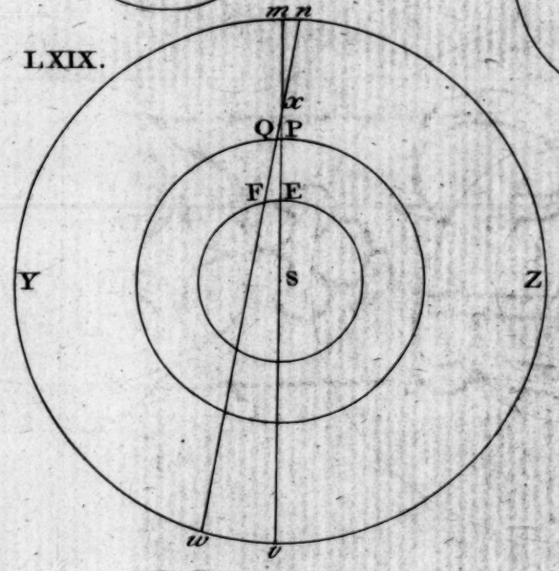
LXVII.



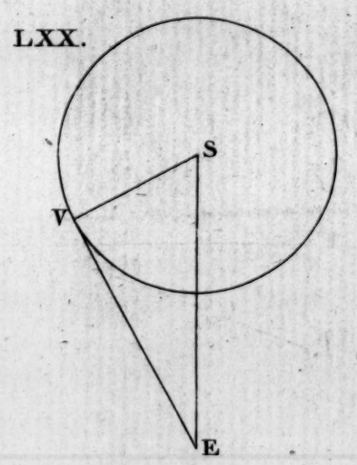
LXVIII.



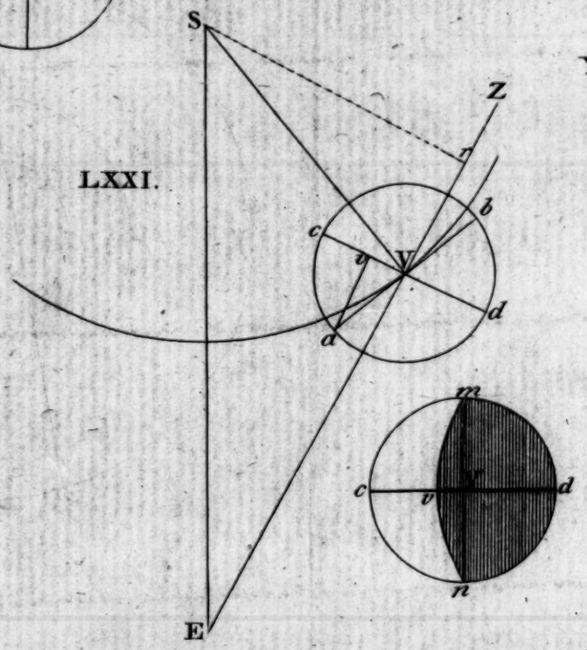
LXIX.



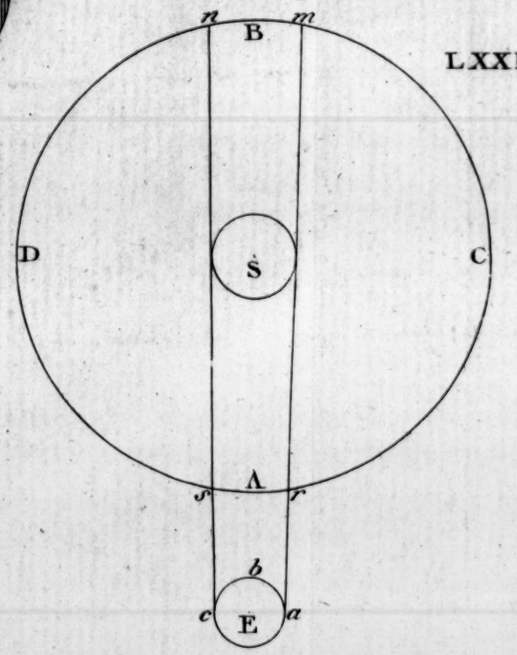
LXX.



LXXI.



LXXII.



LXXIII.

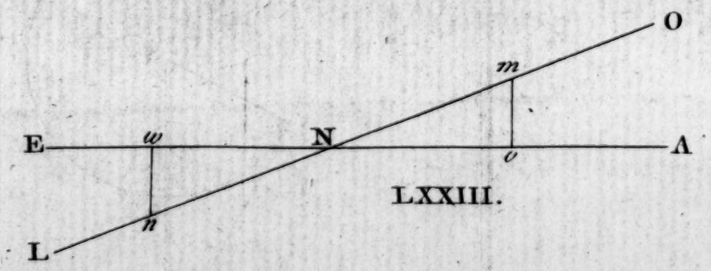
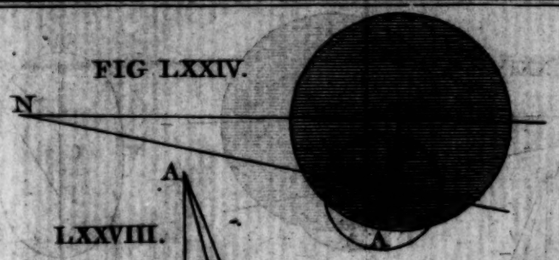


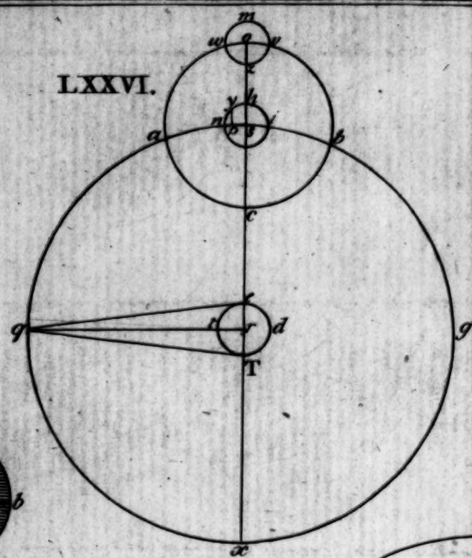
FIG LXXIV.



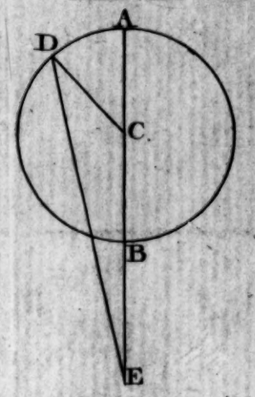
LXXV.



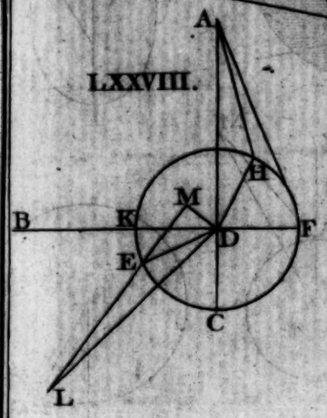
LXXVI.



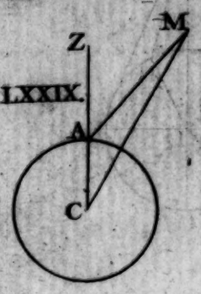
LXXVII.



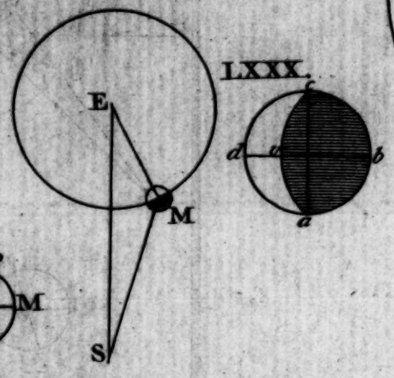
LXXVIII.



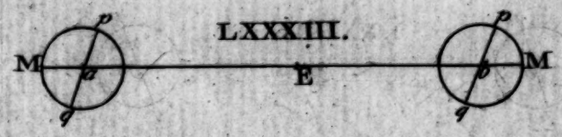
LXXIX.



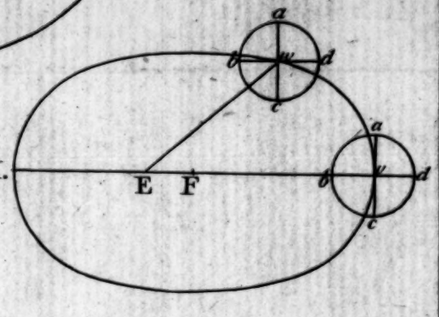
LXXX.



LXXXIII.



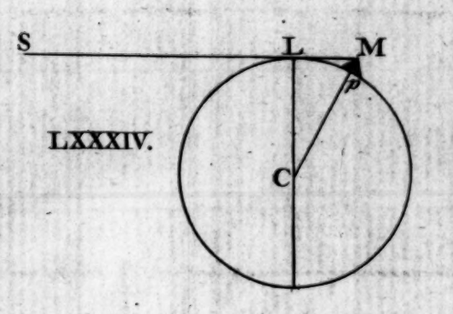
LXXXII.



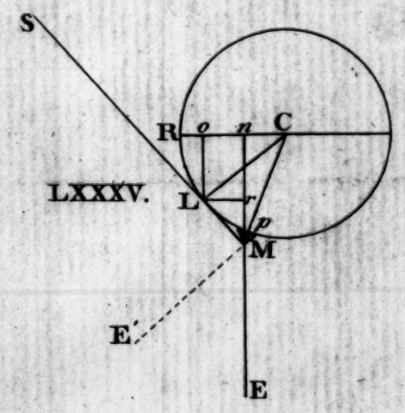
LXXXI.



LXXXIV.



LXXXV.



LXXXVI.

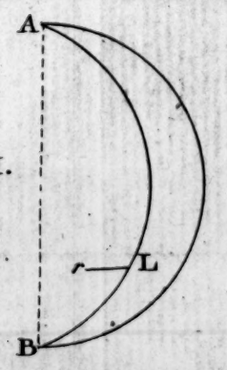
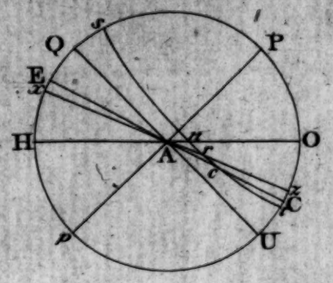
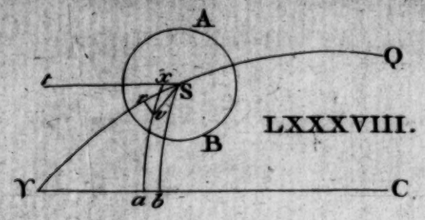


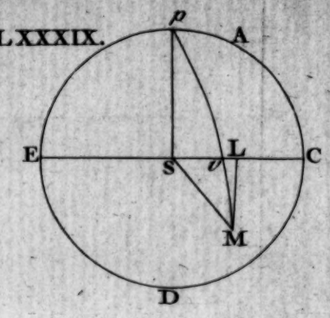
FIG. LXXXVII.



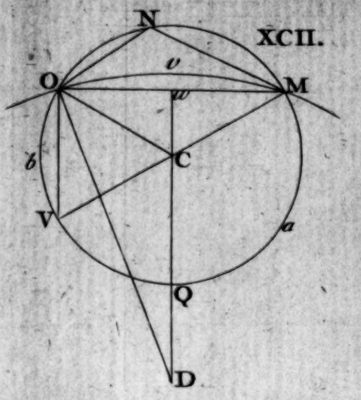


LXXXVIII.

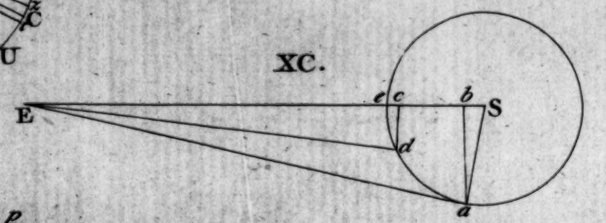
LXXXIX.



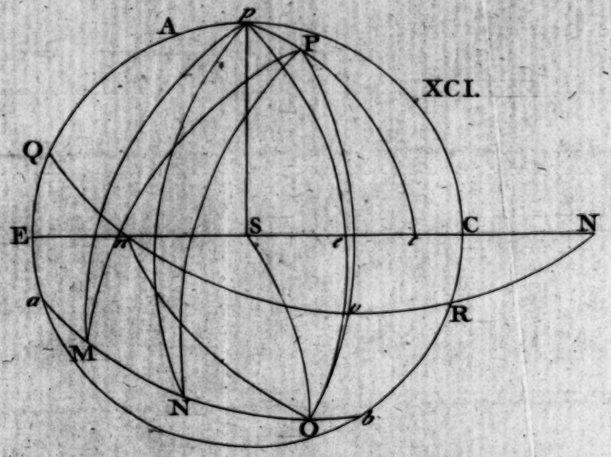
XCII.



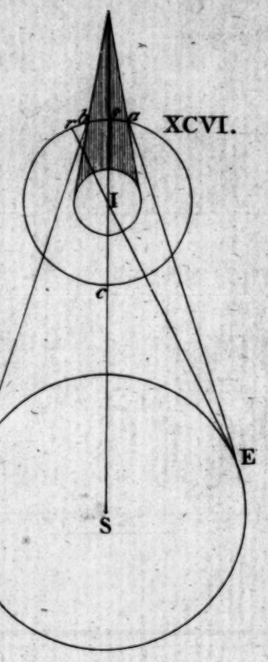
XC.



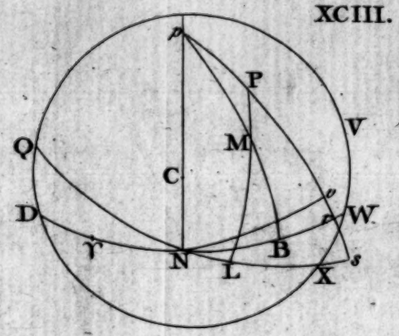
XCI.



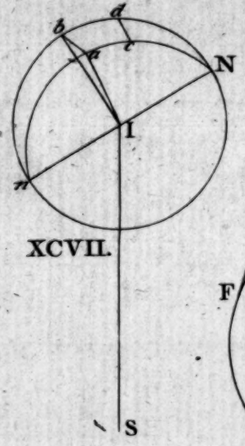
XCVI.



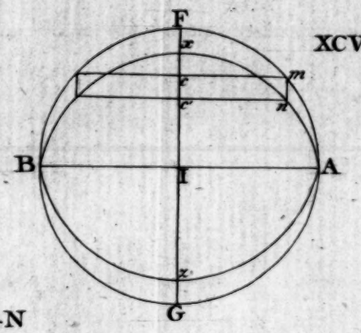
XCIII.



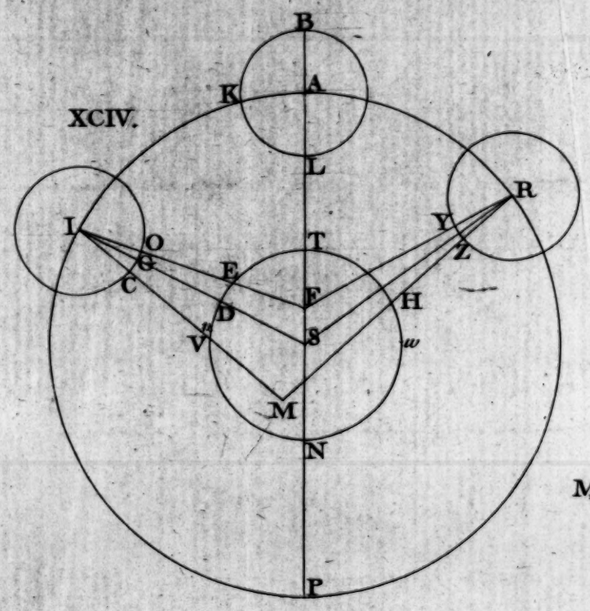
XCVII.



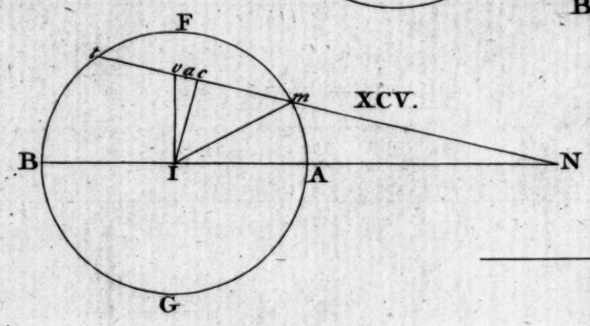
XCVIII.



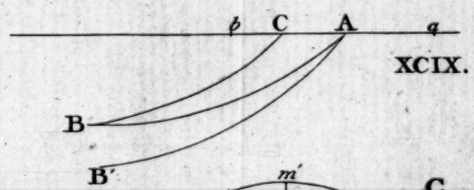
XCIV.



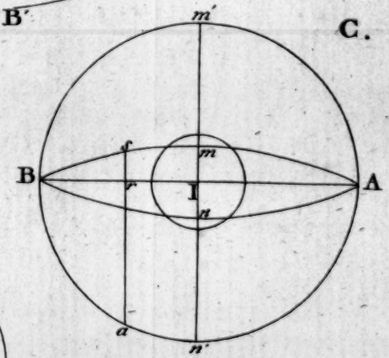
XCV.



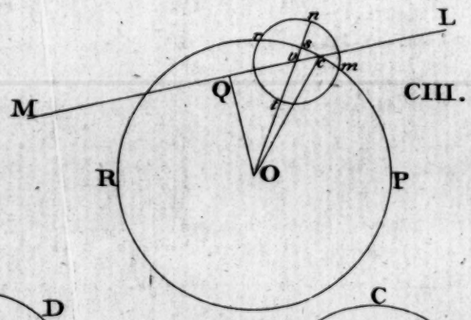
XCIX.



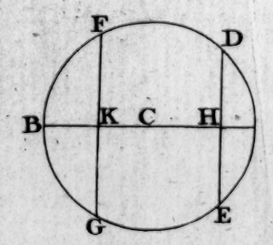
C.



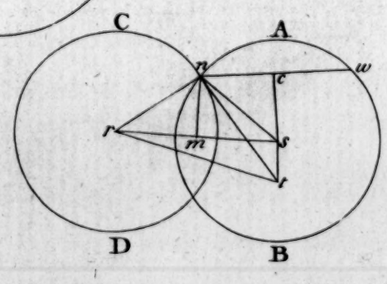
CIII.



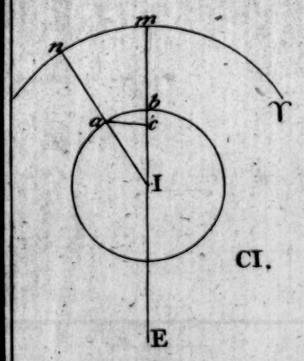
CIV.

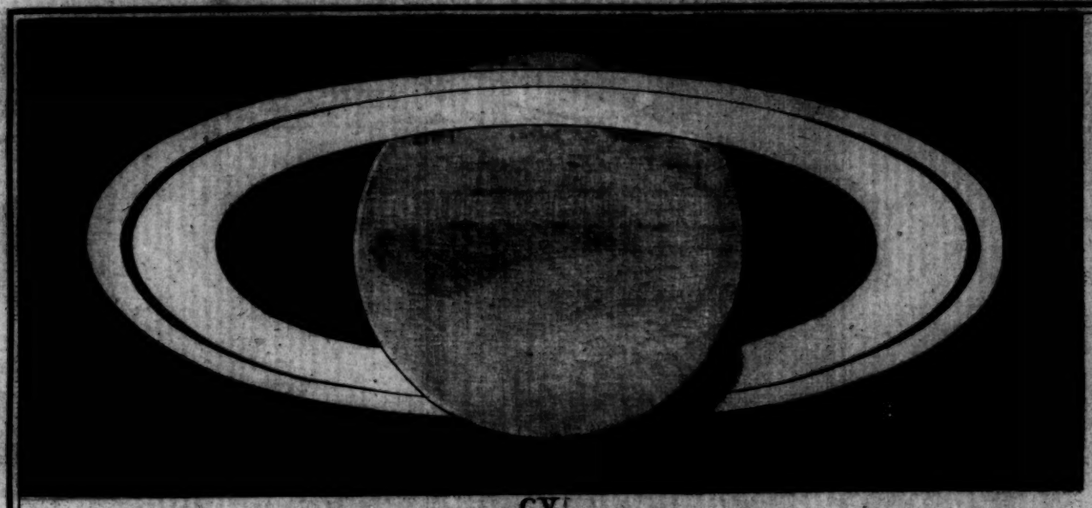


CII.

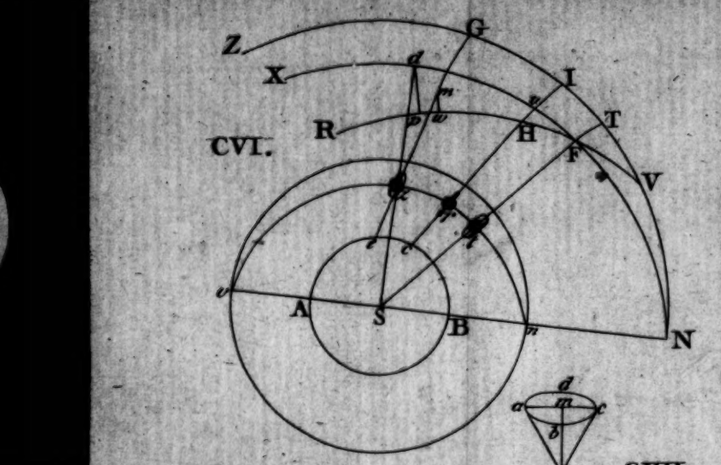


CI.

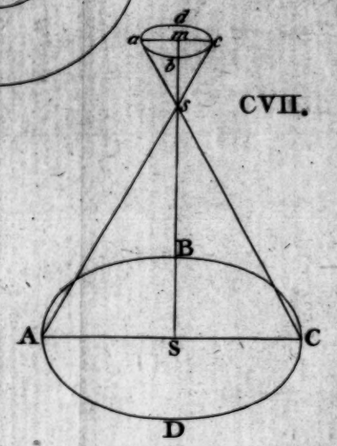




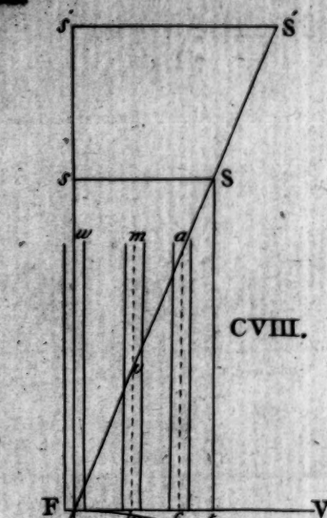
CV.



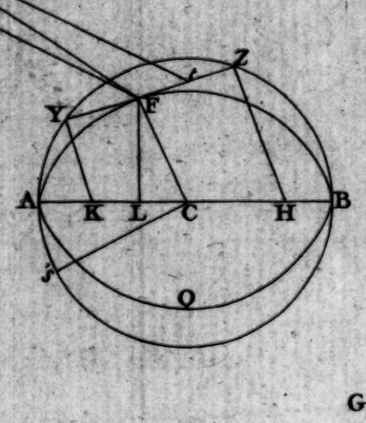
CVI.



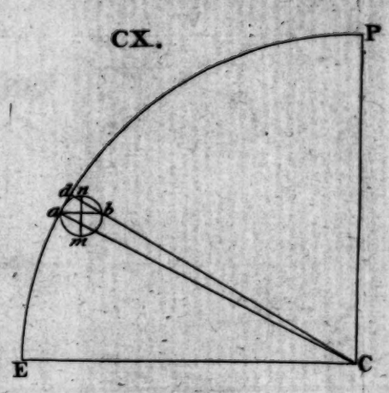
CVII.



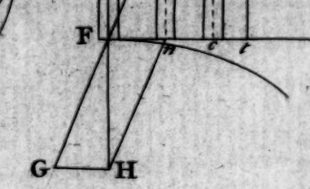
CVIII.

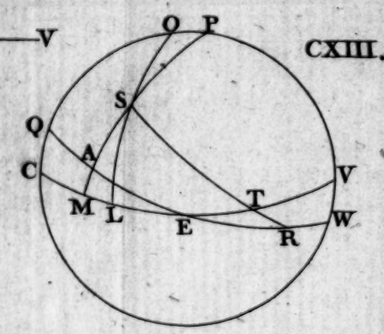


CIX.

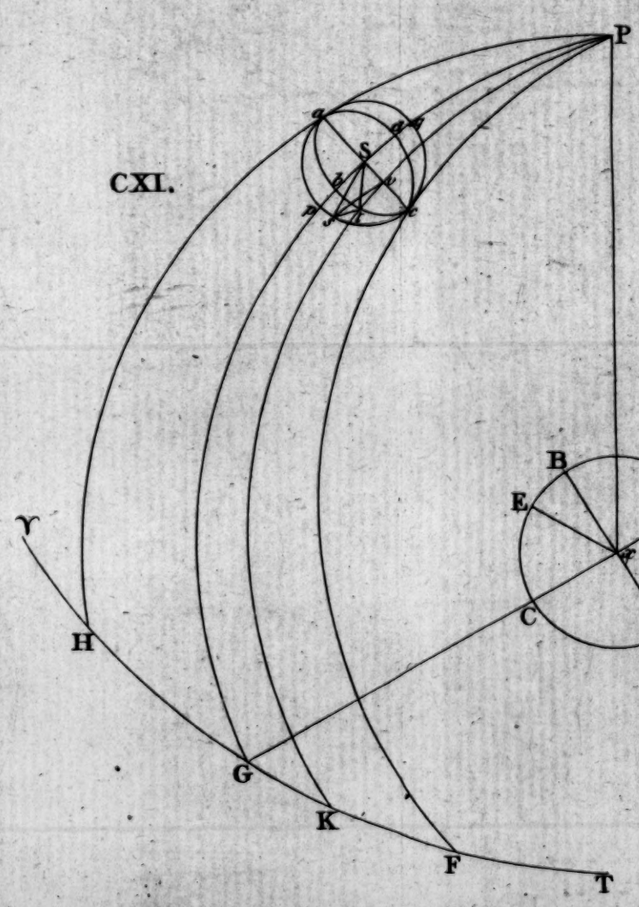


CX.

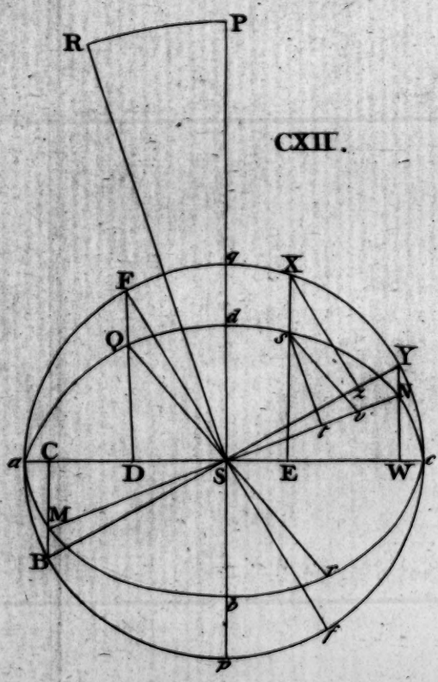




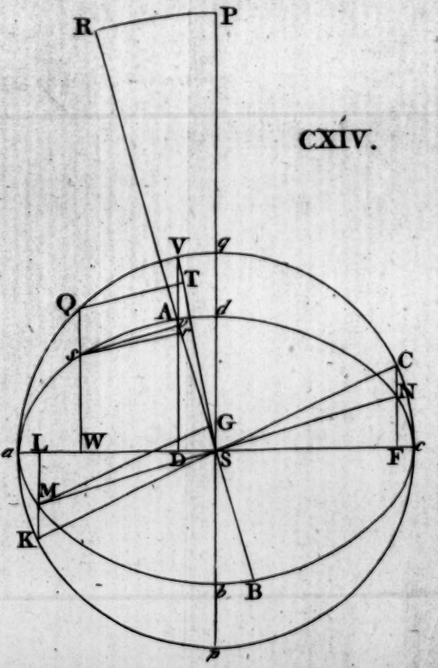
CXIII.



CXI.

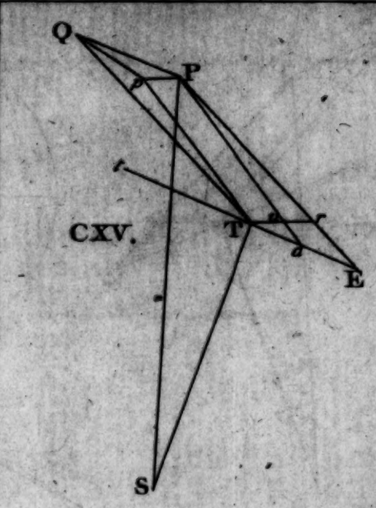


CXII.

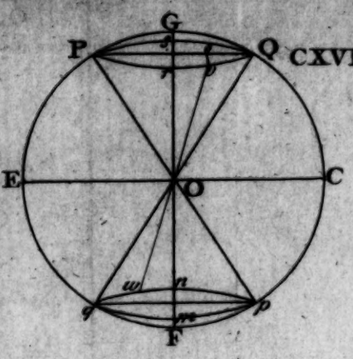


CXIV.

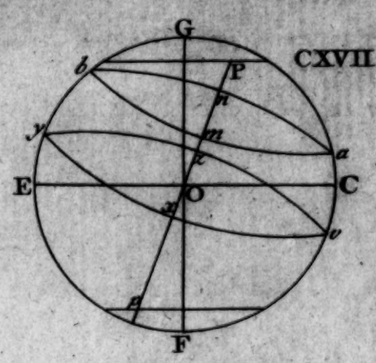
CXV.



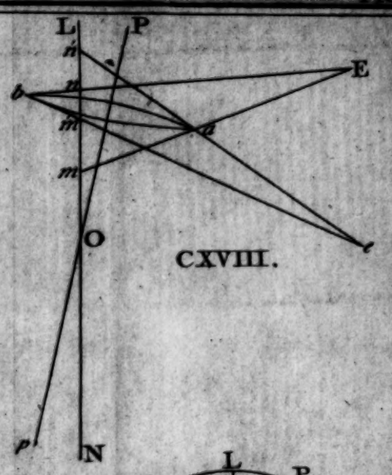
CXVI.



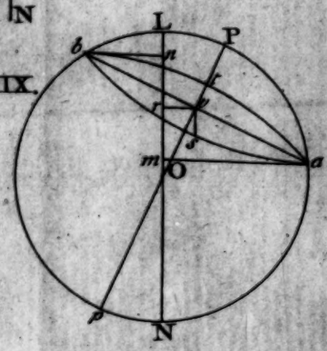
CXVII.



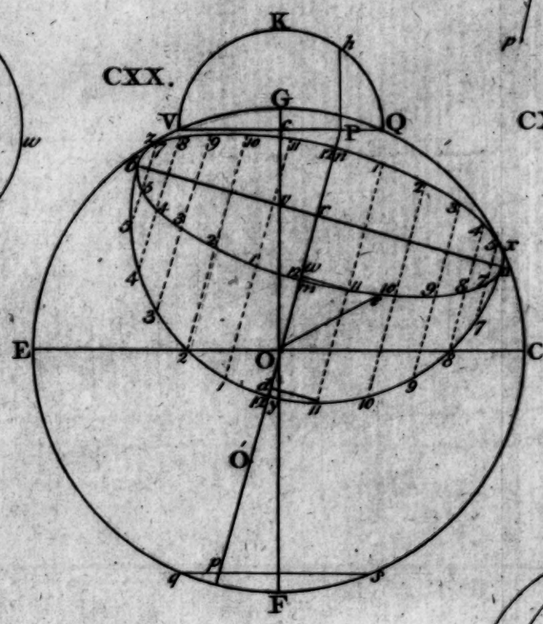
CXVIII.



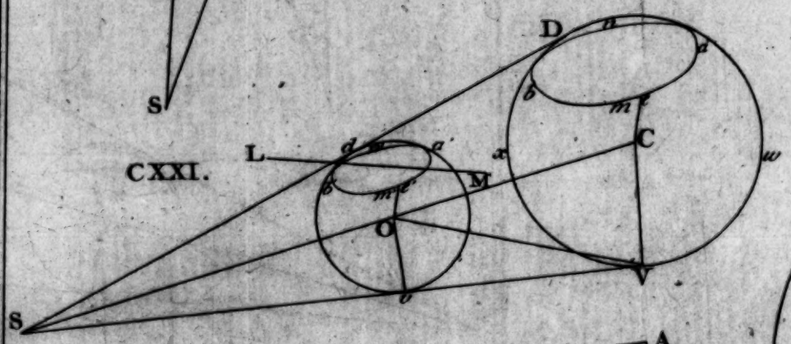
CXIX.



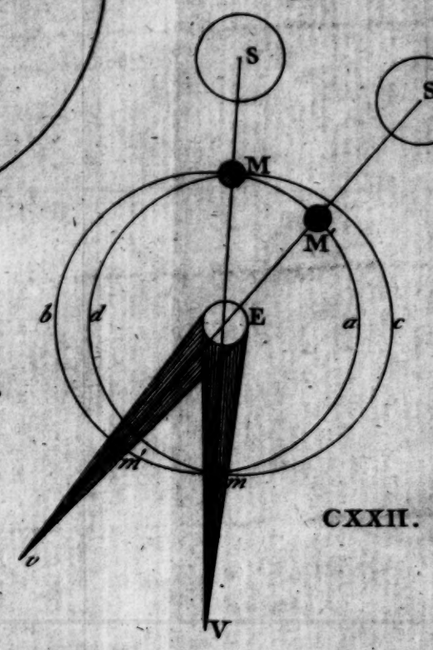
CXX.



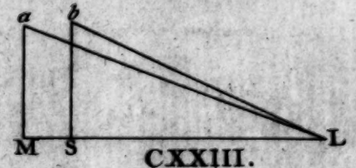
CXXI.



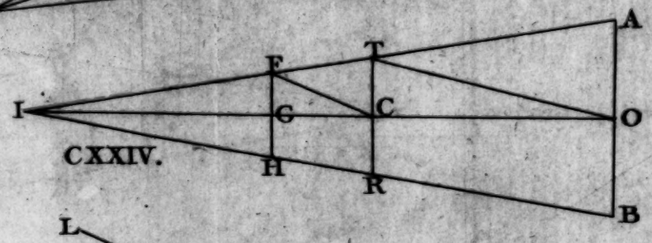
CXXII.



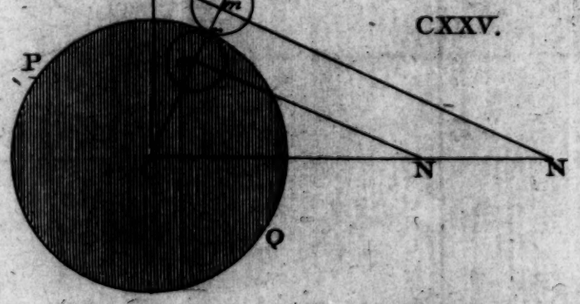
CXXIII.



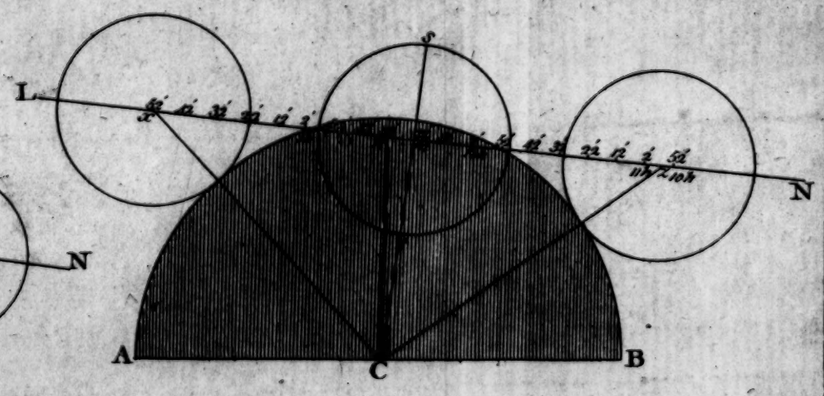
CXXIV.



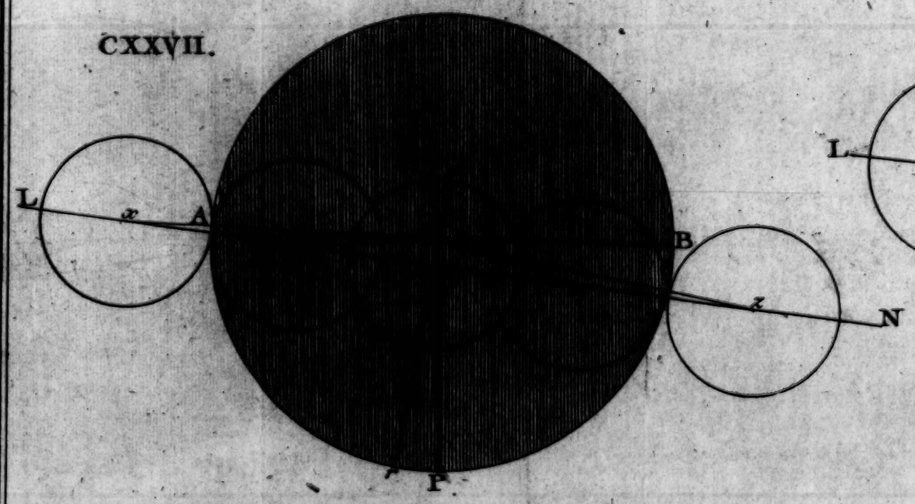
CXXV.



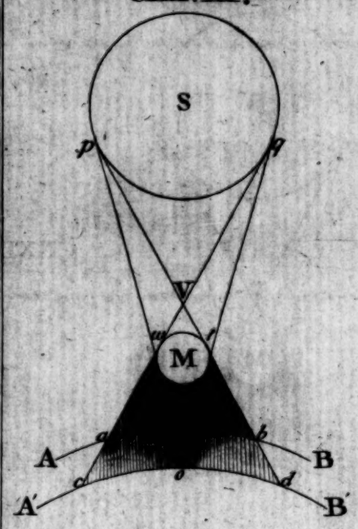
CXXVI.



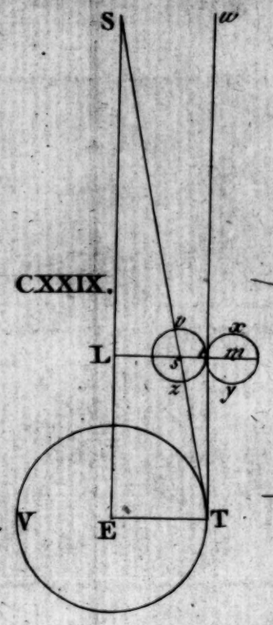
CXXVII.



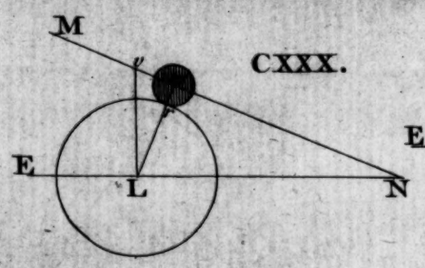
CXXVIII.



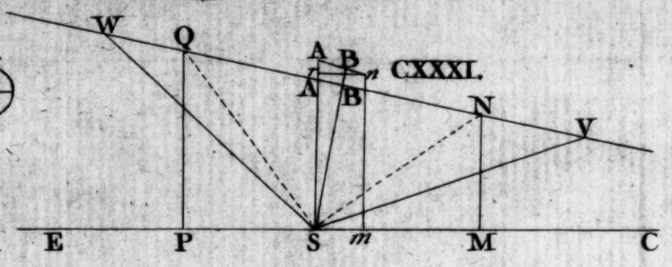
CXXIX.



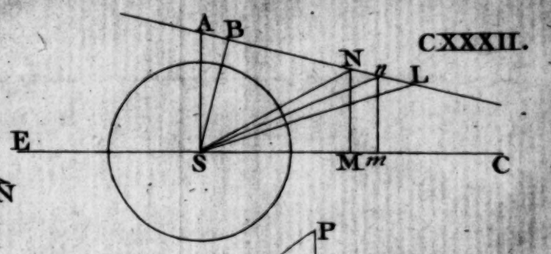
CXXX.



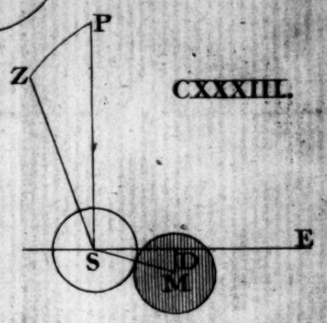
CXXXI.



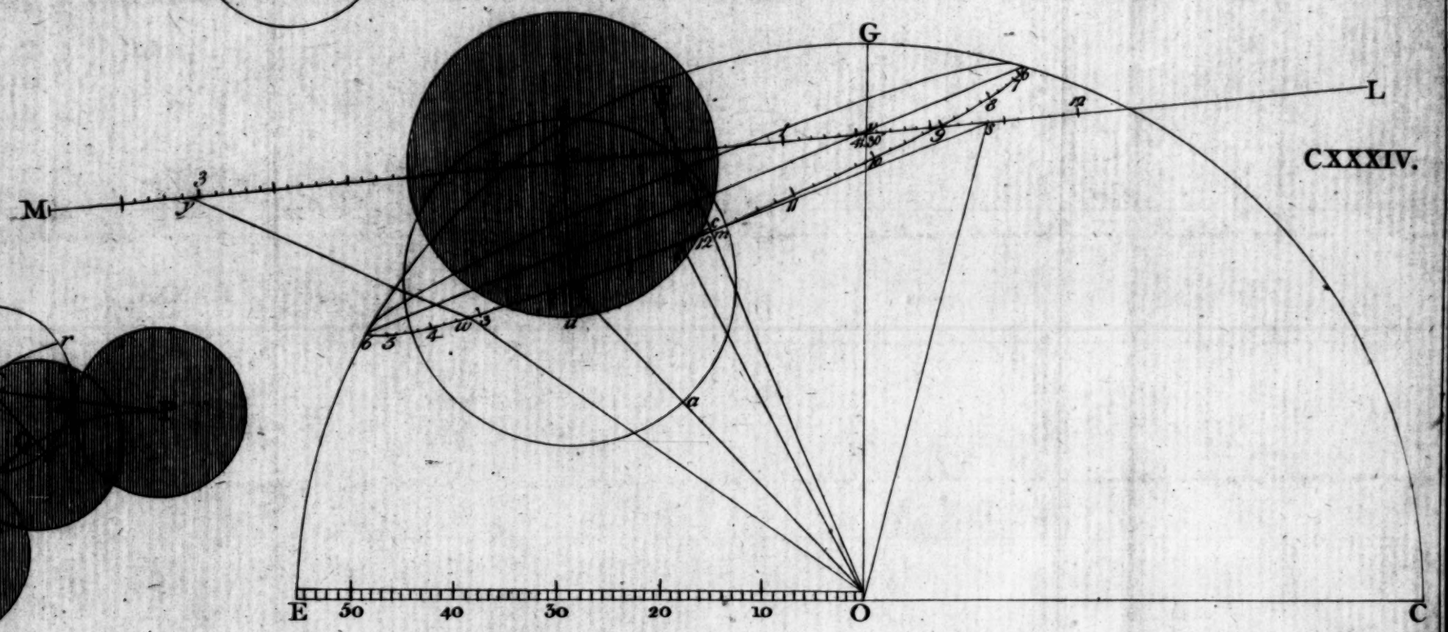
CXXXII.



CXXXIII.

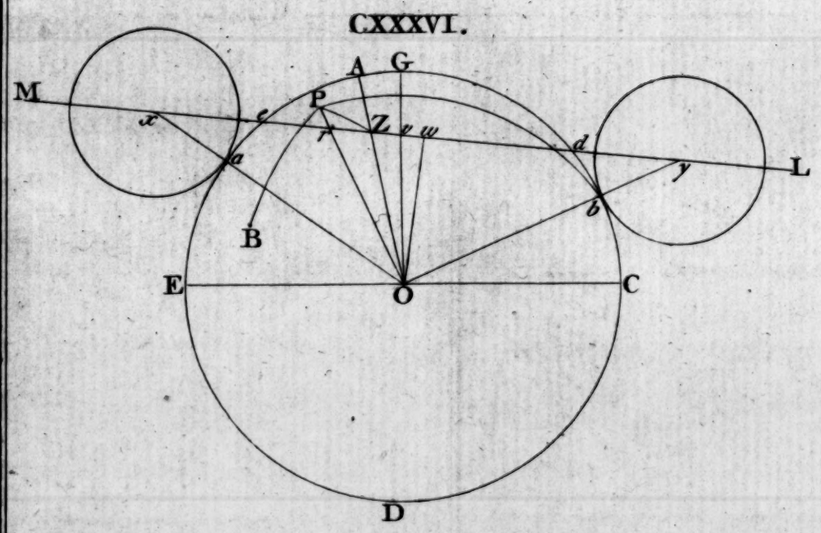


CXXXIV.

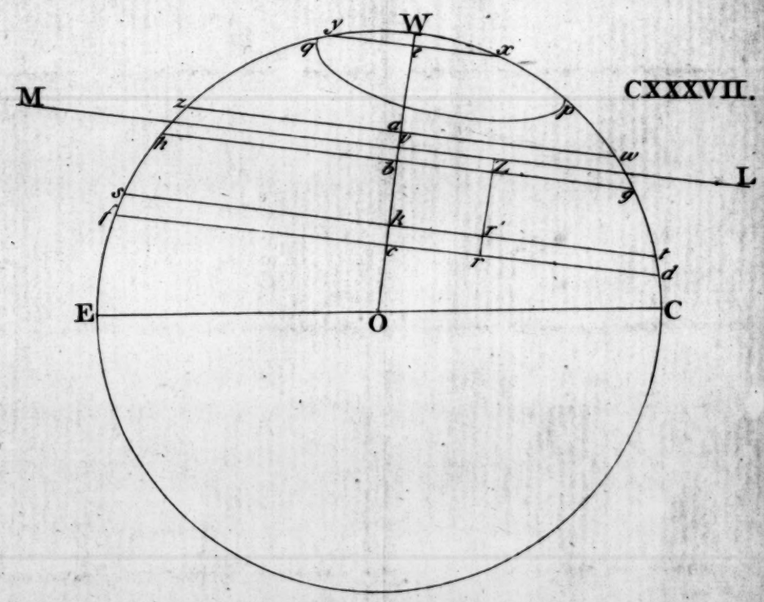


CXXXV.

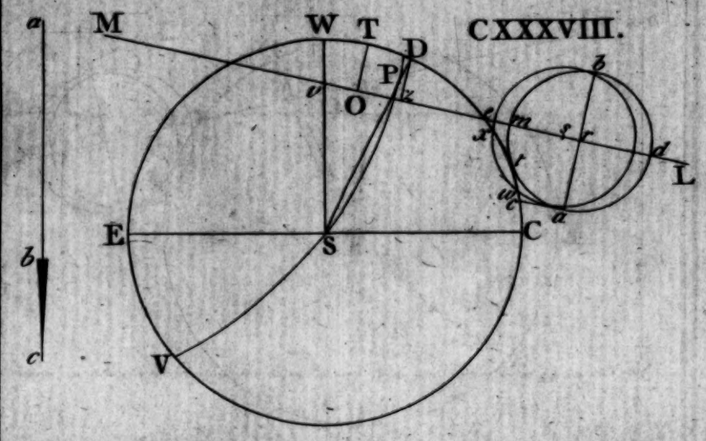
CXXXVI.



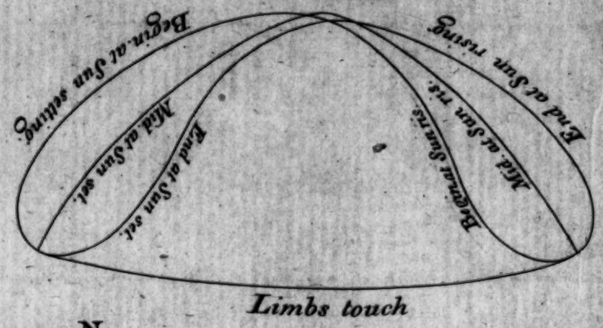
CXXXVII.



CXXXVIII.



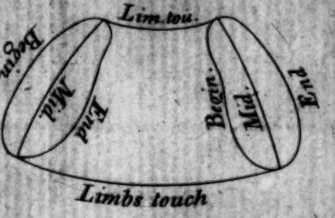
CXXXIX.



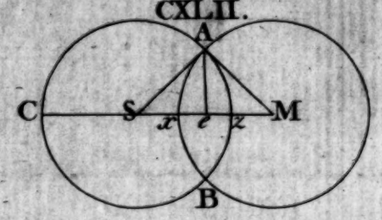
CXL.



CXLI.



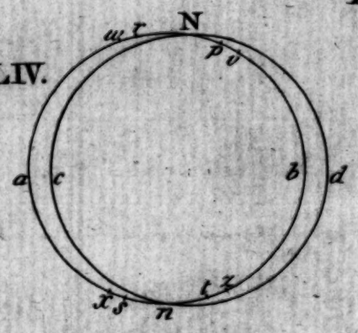
CXLII.



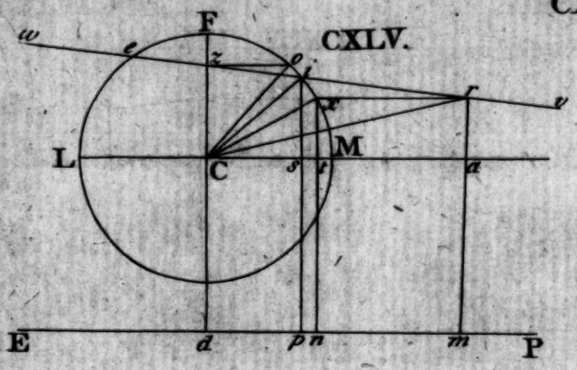
CXLIII.



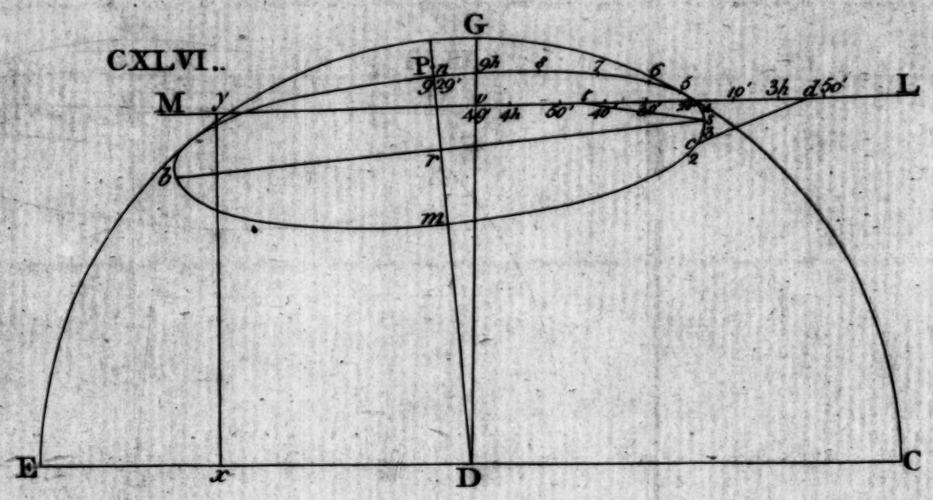
CXLIV.



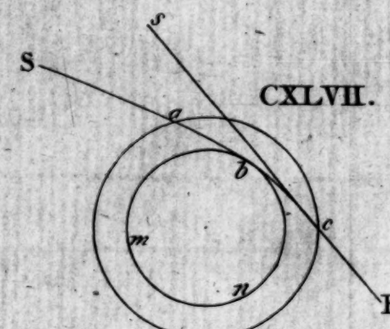
CXLV.



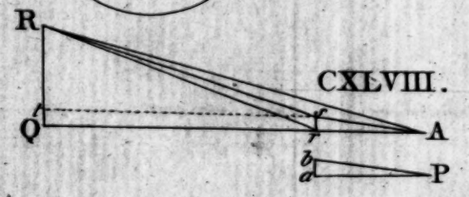
CXLVI.



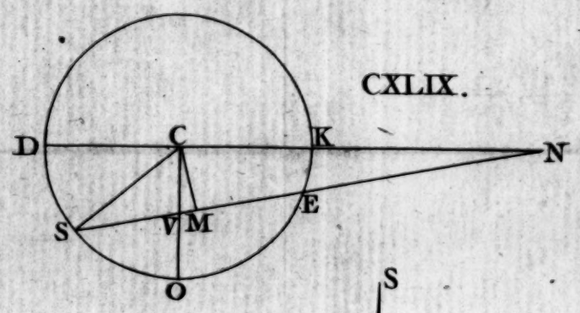
CXLVII.



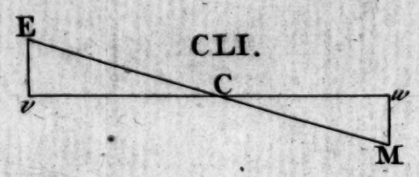
CXLVIII.



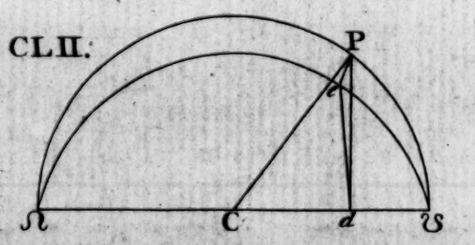
CXLIX.



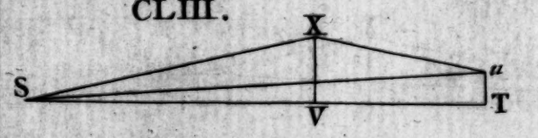
CLI.



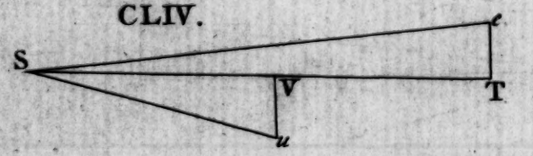
CLII.



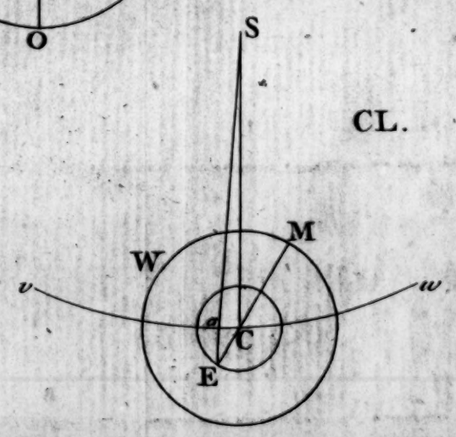
CLIII.



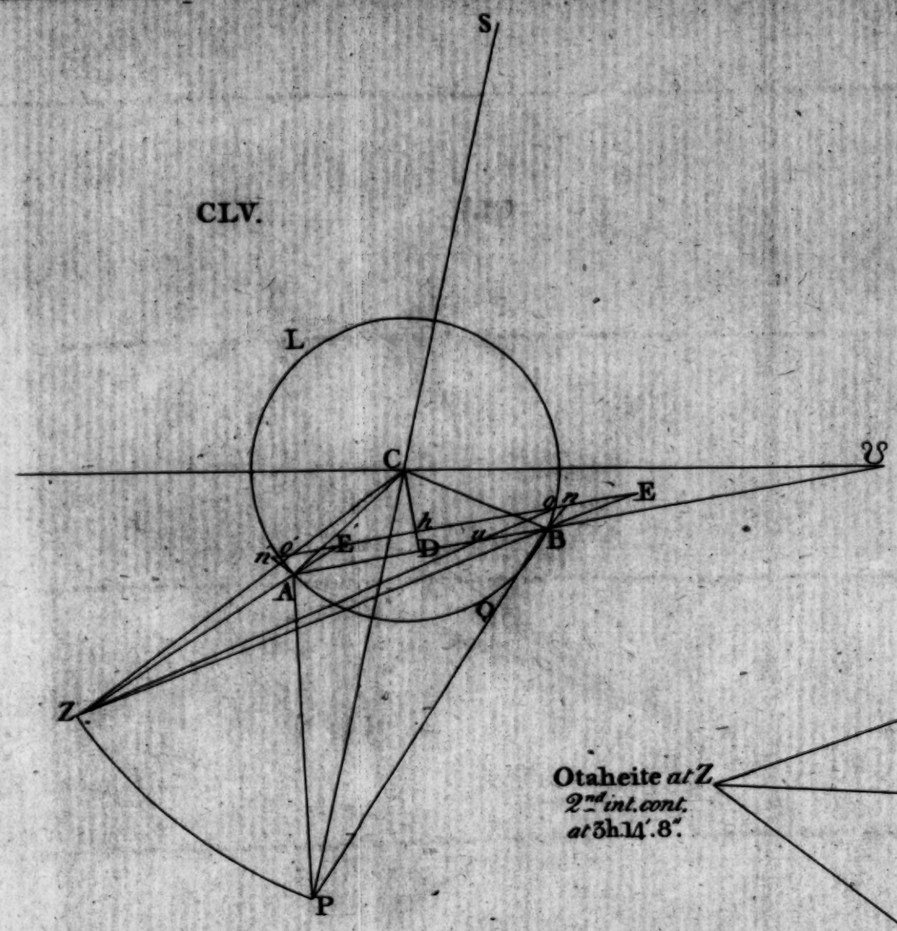
CLIV.



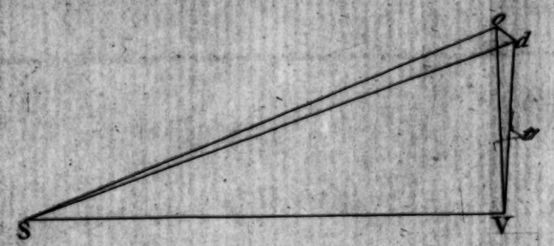
CL.



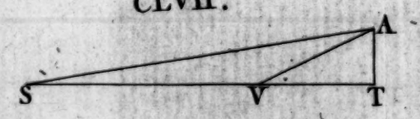
CLV.



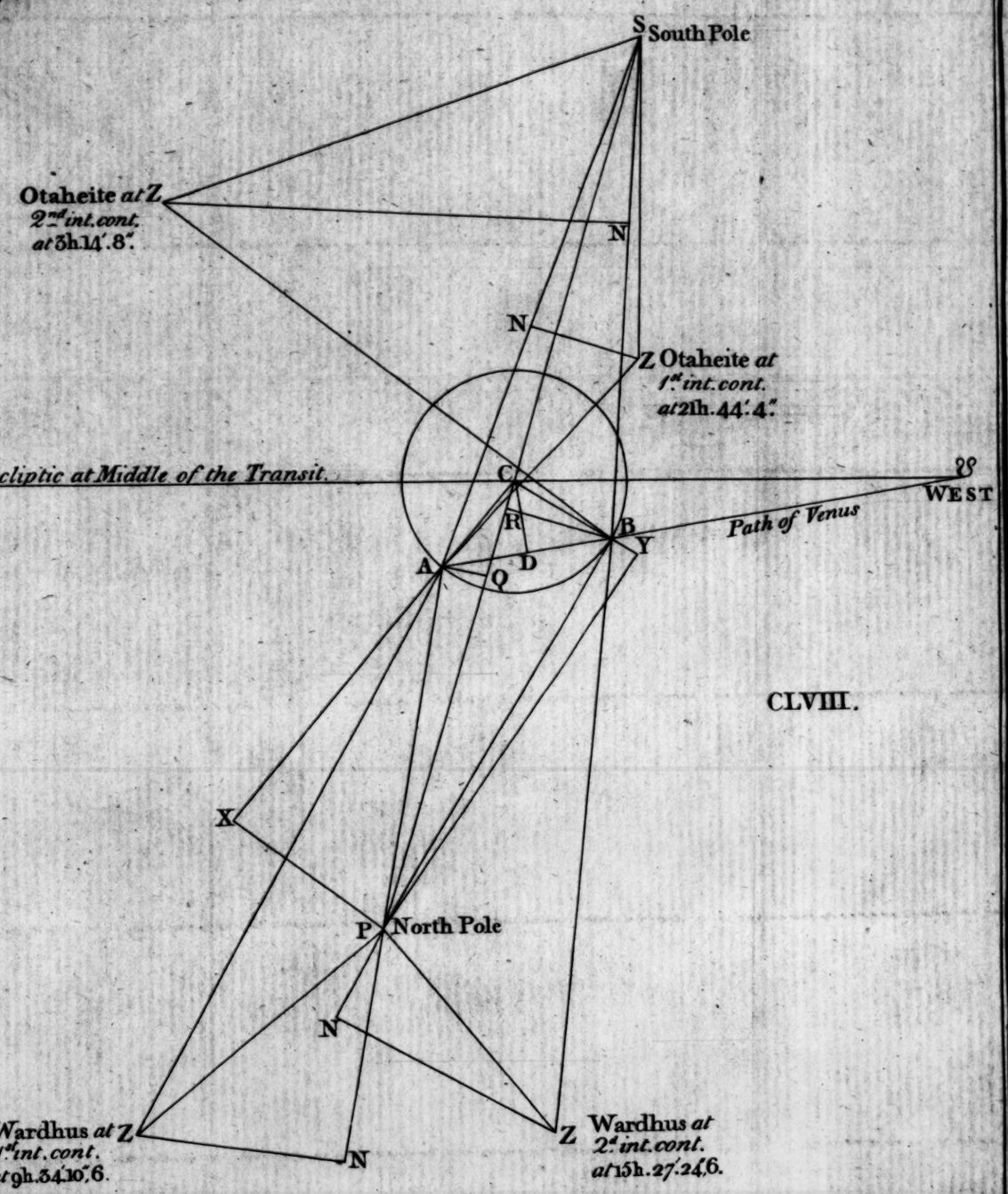
CLVI.



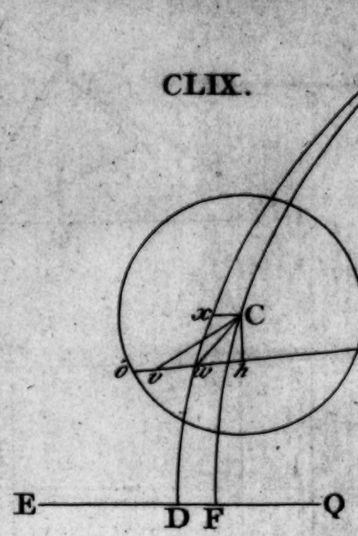
CLVII.



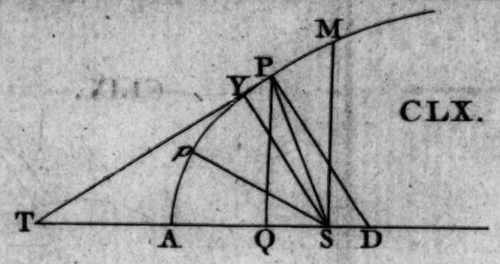
CLVIII.

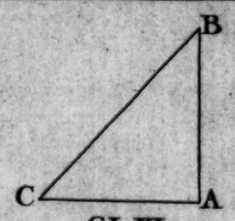


CLIX.



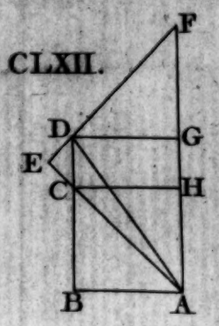
CLX.

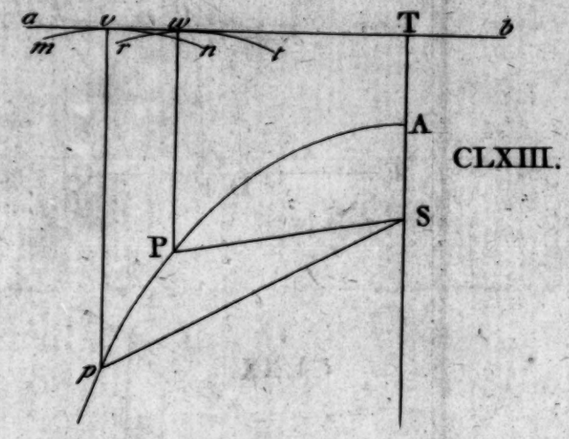




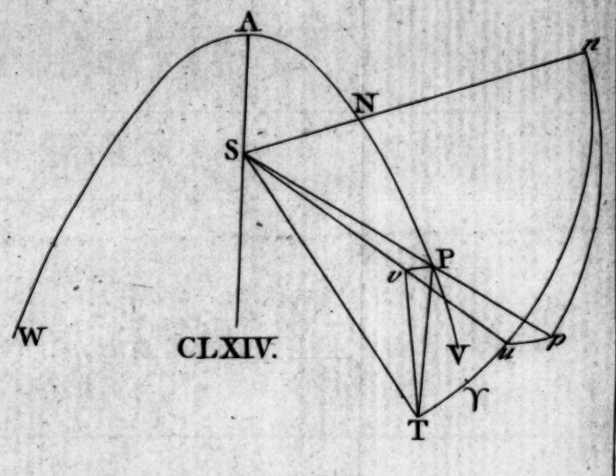
CLXI.

CLXII.



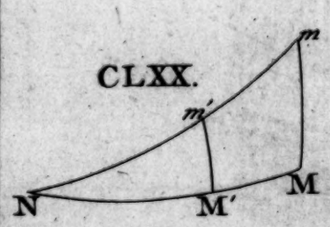


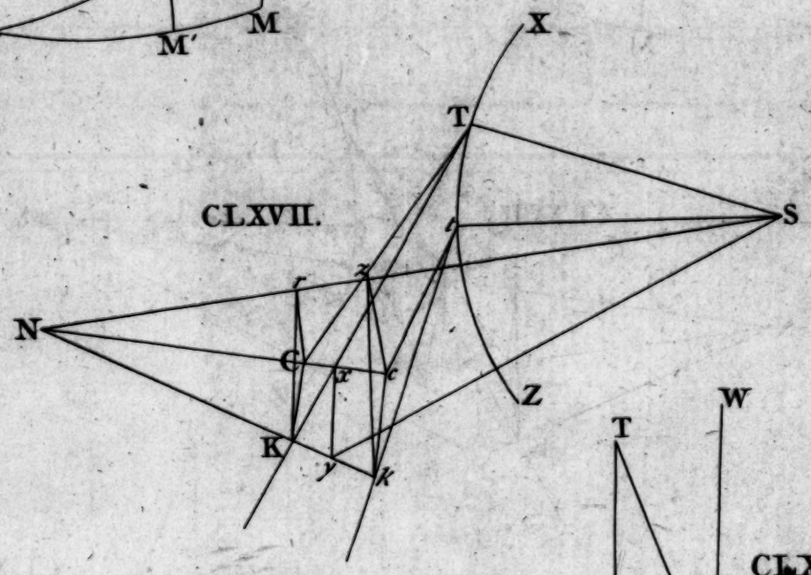
CLXIII.



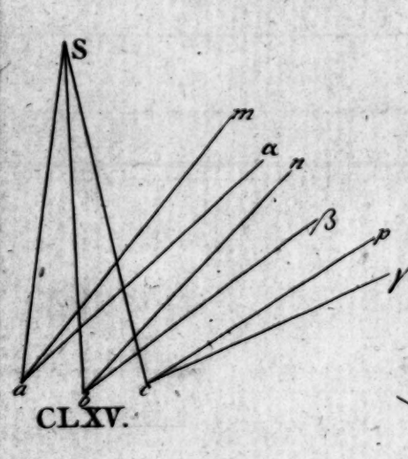
CLXIV.

CLXX.

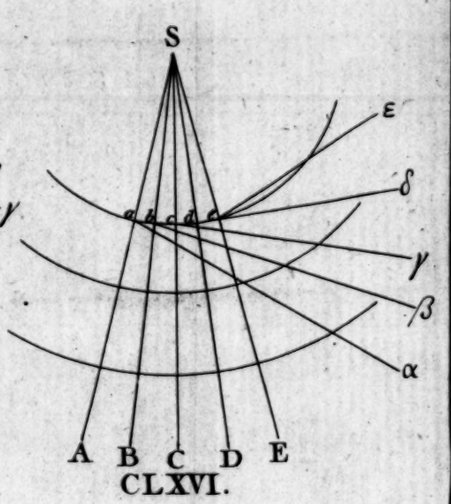




CLXVII.



CLXV.

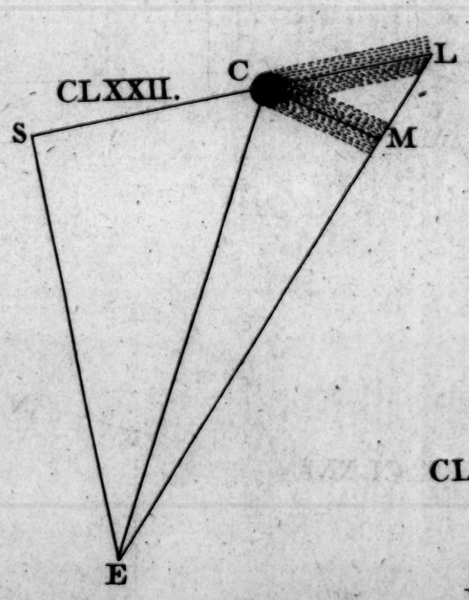


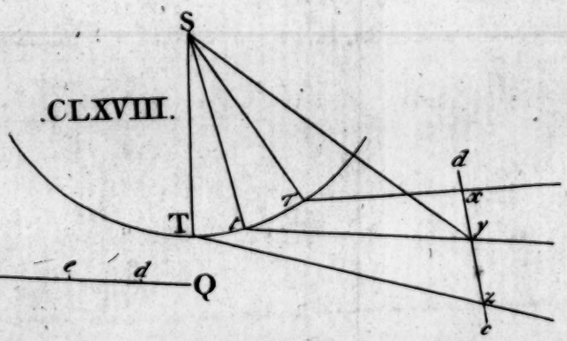
CLXVI.



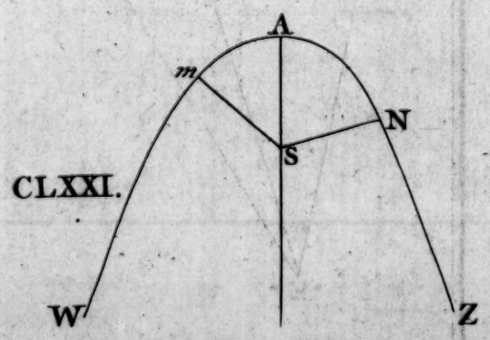
CLXXIII.

CLXXII.

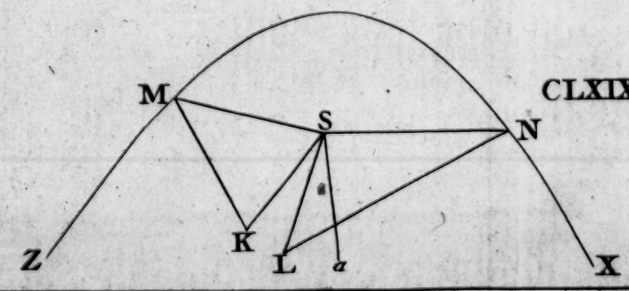




CLXVIII.

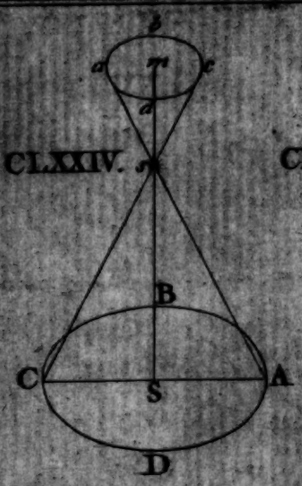


CLXXI.

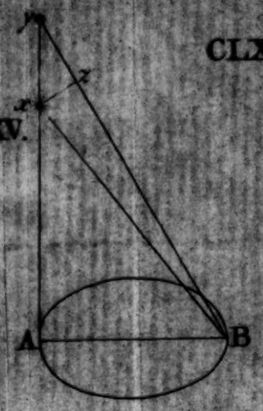


CLXIX.

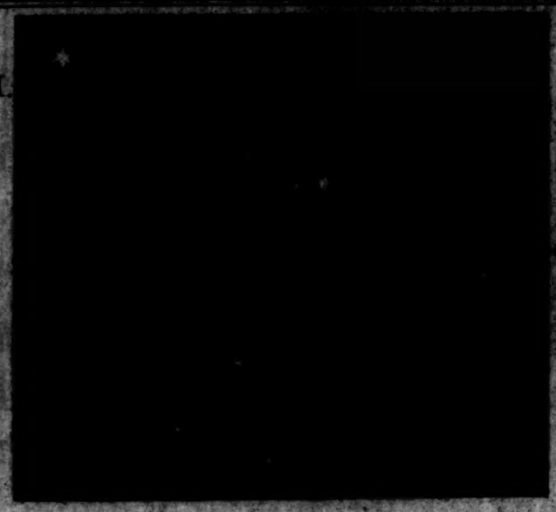
CLXXIV.



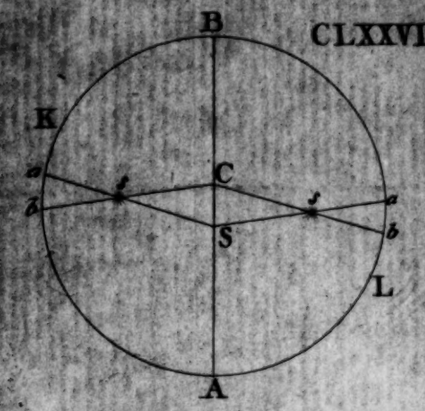
CLXXV.



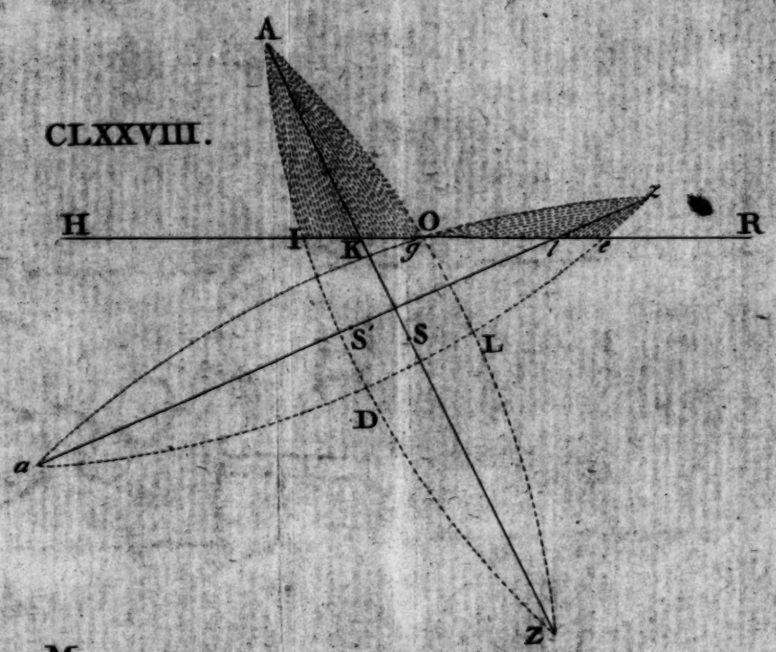
CLXXVI.



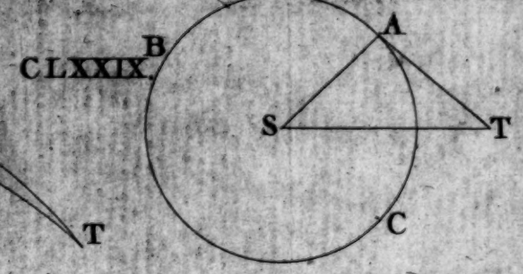
CLXXVII.



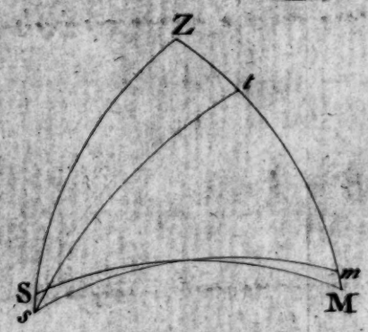
CLXXVIII.



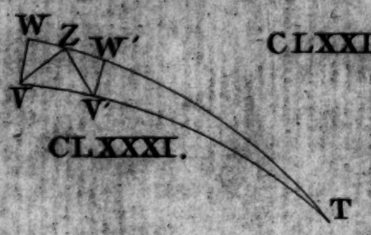
CLXXIX.



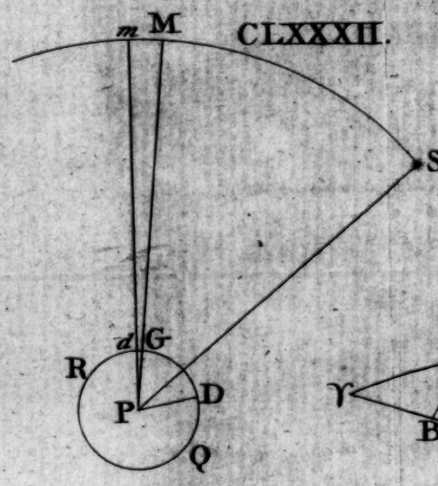
CLXXX.



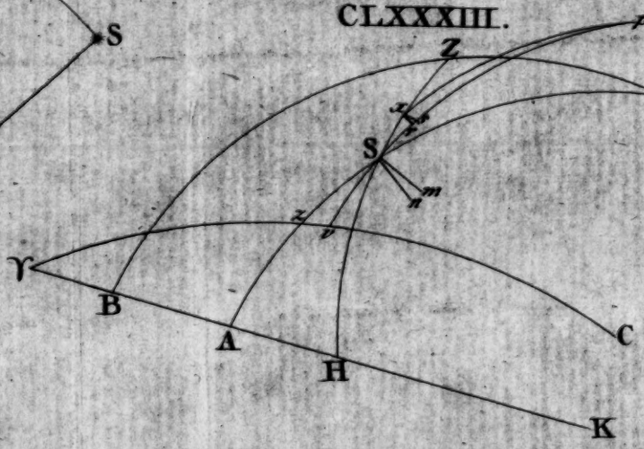
CLXXXI.



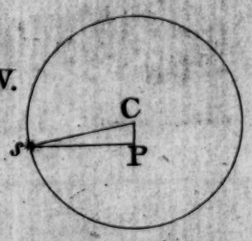
CLXXXII.



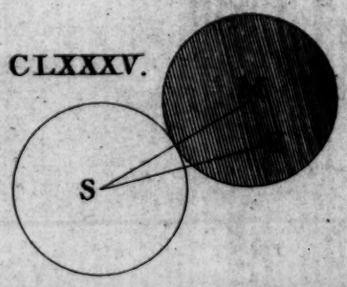
CLXXXIII.



CLXXXIV.



CLXXXV.



CLXXXVI.

